Midterm Solutions

1 Question 1 (30 points)

The inverse demand facing a farm sector is denoted by $p = 20 - x$ where x is output and p is output price. The sector's activities also generate positive externalities and the marginal externality benefits is $MEB = 10 - 0.5x$. The marginal cost of production is $MC = 2 + x$.

a: How much output will be produced and at what consumer price if the industry operates as a monopoly in the output market? (4 points)

Solution: Monopoly will achieve its optimal quantity when the marginal revenue (MR) equals the marginal cost (MC).

$$\begin{cases} p = 20 - x \Rightarrow MR = 20 - 2x \\ MC = 2 + x \end{cases}$$
$$MR = MC \Rightarrow 20 - 2x = 2 + x \Rightarrow \mathbf{X_m = 6} \text{ and } \mathbf{P_m} = 20 - X_m = 14$$

b: What will be the welfare loss in this case, relative to the socially optimal outcome? (8 points)

At the socially optimality, the marginal social benefit (MSB) equals the marginal social cost (MSC). In this case, the marginal social benefit includes the marginal private benefit (MPB) and the marginal externality benefit (MEB).

$$\begin{cases} MSB = MPB + MEB = [20 - x] + [10 - 0.5] = 30 - 1.5x \\ MSC = 2 + x \end{cases}$$
$$MSB = MSC \Rightarrow 30 - 1.5x = 2 + x \Rightarrow \mathbf{X_s} = 11.2 \text{ and } \mathbf{P_s} = 20 - 11.2 = 8.8$$

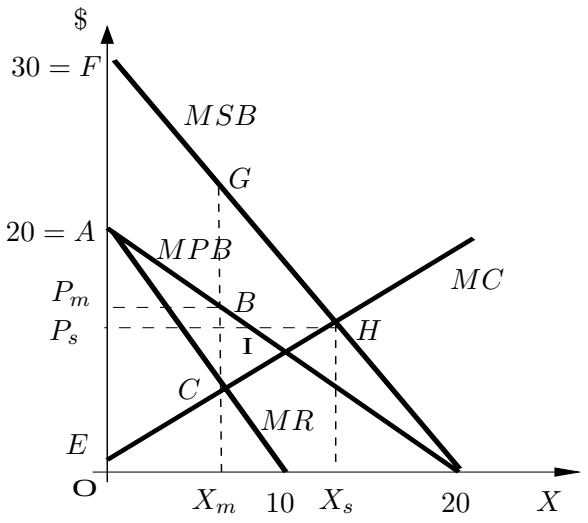

Figure 1: Welfare Comparison between Monopolistic and Socially Optimal Outcomes

As shown by Figure 1, we can identify the area for the deadweight loss in a comparison between the monopolistic and socially optimal outcomes.

- The net social welfare under the social optimality is given by

$$SW_s = \text{total benefits} - \text{total costs} = \Delta FHX_sO - \Delta EHx_sO = \Delta FHE$$

- The net social welfare under monopolistic outcome is given by

$$\begin{aligned} SW_m &= \text{consumer surplus} + \text{producer surplus} + \text{externality benefits} \\ &= \Delta ABP_m + \Delta P_mBCE + \Delta FGBA = \Delta FGCE \end{aligned}$$

Therefore, the welfare loss relative tot he socially optimal outcome is given by

$$\begin{aligned} SW_m - SW_s &= \Delta FGCE - \Delta FHE = -\Delta GHC \\ &= -\frac{1}{2}\left[MSB(X_m) - MC(X_m)\right](X_s - X_m) \\ &= -\frac{1}{2}\left[(30 - 1.5X_m) - (2 + X_m)\right](X_s - X_m) \\ &= -\frac{1}{2}\left[(30 - 1.5 \times 6) - (2 + 6)\right](11.2 - 6) \\ &= -33.8 \end{aligned}$$

c: What policies can be used to attain the social optimum? How will the welfare of various groups affected by these policies? (10 points)

There are several policies can be used to attain the social optimum such as subsidies and quota.

- **Subsidies**: The optimal subsidies are given by

  $S_{optimal} = MC(X_s) - MR(X_s) = (2 + X_s) - (20 - 2X_s) = (2 + 11.2) - (20 - 2 * 11.2) = 15.6.$

  As shown by Figure 1, Table 1 demonstrates the welfare analysis.

Table 1: Welfare analysis for monopoly with and without subsidies

	Without any regulation	With subsidies	Change
Consumer surplus	$ABP_m = 18$	$ARP_s = 62.72$	$P_mBRP_s = 34.72$
Producer surplus	$P_mBCE = 54$	$P_sNE - NHR + JHST = 94.56$	50.56
Externality benefit	$AFGB = 51$	$GHRA = 80.64$	$GHRB = 29.64$
government net gain	0	$-JHST = -81.12$	-81.12
social welfare	$FGCE = 123$	156.80	$GHC = 33.8$

Therefore, consumers, producers, and the environment all gain from subsidies, while government loses.
Notes:

1. Producer surplus has three elements including:
   - P_sNE is the surplus of producing units at which the marginal benefit is at least as great as the marginal cost.

- NHR is the loss due to the fact that this monopoly produces additional $x_s - x_m$ units where the marginal cost exceeds the marginal benefit if selling at p_s.
- $JHST$ is the gain from subsidies.

2. The net welfare gain from subsidies is exactly represented by the area GHC if you carefully work out the area for producer surplus.

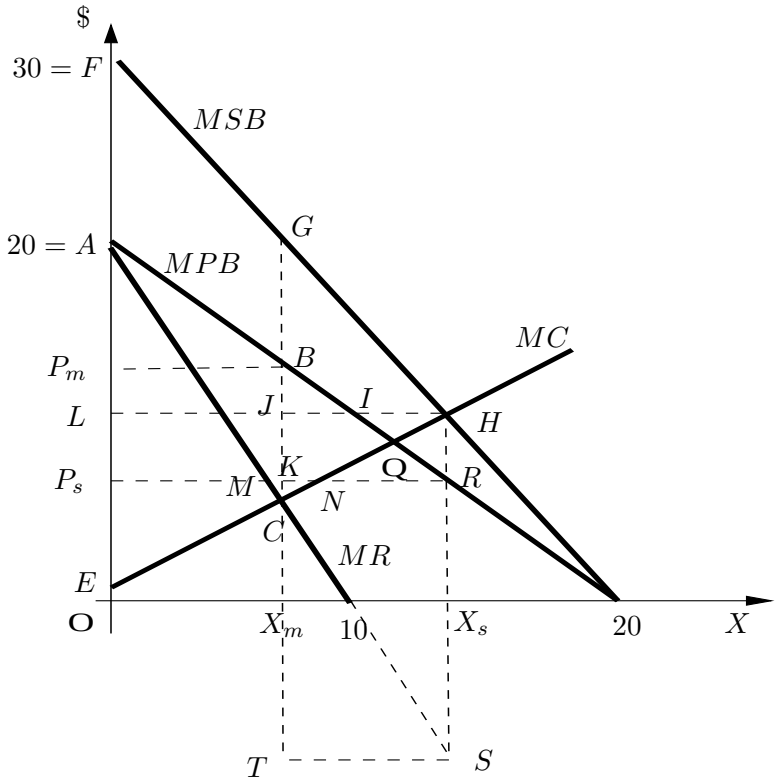

Figure 2: Welfare Comparison between Monopolistic and Socially Optimal Outcomes

- **quota**: The government sets up production quota to the monopolistic producers such that they should produce the socially optimal level (11.2). If the government has no supplementary policy, producer surplus is represented by $P_sNE - NHR = 3.52 > 0$, which implies that this monopolistic firm will still stay in business. Following the same principle, we obtain the following welfare analysis in Table 2. Therefore, consumers and the environment gain from quota, and producers lose. This direct control has no effect on governmental welfare.

d: How much will be produced under competition and at what price? (4 points)

At the competitive outcome without any regulation, the marginal private benefit (MPB) equals the marginal cost (MPC).

$$MPB = MPC \Rightarrow 20 - x = 2 + x \Rightarrow \mathbf{X_c} = 9 \text{ and } \mathbf{P_c} = 20 - 9 = 11 \tag{1}$$

Table 2: Welfare analysis for monopoly with and without quota

	Without any regulation	With quota	Change
Consumer surplus	$ABP_m = 18$	$ARP_s = 62.72$	$P_mBRP_s = 34.72$
Producer surplus	$P_mBCE = 54$	$P_sNE - NHR = 13.44$	-50.48
Externality benefit	$AFGB = 51$	$GHRA = 80.64$	$GHRB = 29.64$
government net gain	0	0	0
social welfare	$FGCE = 123$	156.80	$GHC = 33.8$

e. What policies are needed to move from the competitive outcome to the optimal outcome? Who will be the gainers and losers? (4 points)

Solution: Both subsidies and quota can move the competitive to the socially optimal outcome. If subsidies are chosen, consumers, producers, and the environment are gainers, and government incurs cost of subsidies. If quota is chosen, consumers and the environment gain but producers lose.

2 Question 2 (36 points)

A community has n members. The marginal benefit of each person from clean air is $MB_x = 10 - 0.4x$.

a: Is it a public good? Why? (4 points)

Solution: Clean air is a public good. An individual cannot prevent others from using clean air (**non-excludibility**), and the use of clean air by one person will not reduce the benefit to others (**non-rivalry**).

b: What is the socially optimal quality for a community of $n = 5$ individuals if the marginal cost of improving x is $MC = 10 + 2x$ (2 points)? What is the net social welfare per member of the community if they share the cost of clean up equally? (4 points)

Solution: To get the aggregate marginal benefit of public goods, we need vertically sum all individual marginal benefits together.

$$\begin{cases} MB_5 = 5MB_x = 5[10 - 0.4x] = 50 - 2x \\ MC = 10 + 2x \end{cases} \Rightarrow MB_5 = MC \Rightarrow 50 - 2x = 10 + 2x \Rightarrow \mathbf{x_5} = 10 \qquad (2)$$

Thus, the net welfare for each member of the community is given by

$$\underbrace{\int_0^{10} [10 - 0.4x]dx}_{individual\ benefit} - \underbrace{\frac{\int_0^{10} [10 + 2x]dx}{5}}_{shared\ cost\ of\ an\ invidual} = 40 \qquad (3)$$

c: What is the socially optimal quality for $n = 10$ and $n = \infty$? (6 points)

Solution:

$$n = 10 \Rightarrow \begin{cases} MB_{10} = 10[10 - 0.4x] = 100 - 4x \\ MC = 10 + 2x \end{cases}$$
$$\Rightarrow MB_{10} = MC \Rightarrow 100 - 4x = 10 + 2x \Rightarrow \mathbf{x_{10} = 15}$$

$$n = \infty \Rightarrow \begin{cases} MB_{10} = n[10 - 0.4x] = 10n - 0.4nx \\ MC = 10 + 2x \end{cases}$$
$$MB_{n=\infty} = MC \Rightarrow 10n - 0.4nx = 10 + 2x \Rightarrow \mathbf{x_{n=\infty}} = \frac{\mathbf{10n - 10}}{\mathbf{0.4n + 2}} = 25.$$

d: Suppose that the community has two groups of citizens. Group 1 of 5 individuals whose marginal benefits is $MB_x^1 = 6 - 0.4x$ and group 2 with 5 individuals and marginal benefit is $MB_x^2 = 10 - 0.4x$. What is the socially optimal quality in this case? (8 points)

Solution: The socially optimal outcome is achieved when the marginal social benefit equals the marginal social cost.

$$\begin{cases} MSB = \begin{cases} n_1 MB_x^1 + n_2 MB_x^2 = 5[6 - 0.4x] + 5[10 - 0.4x] = 80 - 4x & \text{if } 0 \le x \le 15 \\ n_2 MB_x^2 = 50 - 2x & \text{if } 15 \le x \le 25 \end{cases} \\ MSC = 10 + 2x \end{cases}$$

$$MSB = MSC \Rightarrow \begin{cases} 80 - 4x = 10 + 2x \Rightarrow x_s = 70/6 \\ 50 - 2x = 10 + 2x \Rightarrow x_s = 10 < 15 \Rightarrow \text{invalid} \end{cases} \Rightarrow x_s = 70/6$$

e: Suppose that in the case of part d the government cannot raise taxes to pay for improving quality. Suppose that the member of group 2 form a cooperative and share the cost of improving air quality. What will be the air quality in this case? What will be welfare loss to member of each group relative to the socially optimal outcome? (12 points)

Solution: In this case, members of group 2 form a cooperative and share the cost of improving air quality. Members in the group 1 can free ride because clean air is a public good.
Group 2 will choose the optimal air quality at which their aggregate marginal benefit equals the marginal cost:

$$MB^2 = MC \Rightarrow n_2 MB_x^2 = 5[10 - 0.4x] = 10 + 2x \Rightarrow x_e = 10$$

Under this case, members in the first group can free ride, i.e, they benefit from the improvement of air quality without any pay. Therefore, the individual benefit of members in two groups (w_e^1, and w_e^2) are given by:

$$w_e^1 = \int_0^{x_e} MB_x^1 dx = \int_0^{x_e} [6 - 0.4x]dx = \int_0^{10} [6 - 0.4x]dx = 40$$
$$w_e^2 = \int_0^{x_e} MB_x^2 dx = \int_0^{x_e} [10 - 0.4x]dx - \frac{\int_0^{x_e}(10 + 2x)dx}{5} = \int_0^{10} [10 - 0.4x]dx - \frac{\int_0^{10}[10 + 3x]dx}{5} = 40$$

At the socially optimal outcome (question 2.d), the optimal air quality is $x_s = 70/6$, and the welfare to each member of each group (w_s^1 and w_s^2) are given as follows:

$$\begin{aligned} w_s^1 &= \int_0^{x_s} [6 - 0.4x]dx = \int_0^{70/6} [6 - 0.4x]dx = 385/9 \\ w_s^2 &= \int_0^{x_s} [10 - 0.4x]dx = \int_0^{70/6} [10 - 0.4x]dx = 735/9 \end{aligned}$$

Thus, the welfare changes to the member of each group relative to the socially optimal outcome are given by

$$\begin{aligned} \Delta w^1 = w_e^1 - w_s^1 = 40 - 140/9 = -25/9 \\ \Delta w^2 = w_e^2 - w_s^2 = 40 - 560/9 = -375/9 \end{aligned}$$

3 Question 3: Answer any three of the following four questions. (10 point for each: *Every one got extra 4 points!*)

1. How will political economy considerations affect policy choices when producers are homogeneous and there is full certainty? Explain succinctly.
   Solution: When producers are homogeneous and there is no uncertainty, government can apply various instruments to internalize extenality. These instruments can be either price control or direct quantity control, and all these instruments can be designed to full reduce externality. Therefore, the key question to policy makers is how to balance all interest groups. For example, generally speaking, polluting firms prefer subsidies rather than tax; government would prefer collecting tax; and consumers prefer instruments lowering price.

2. What will determine the choice between taxation and direct control if the policy maker is uncertain about the marginal benefits of the pollution? Explain.
   Solution: If the policy maker is uncertain about the marginal benefits of pollution, the demand elasticity plays an important role in determining the choice between taxation and direct control (standard):

   - When the marginal benefit is elastic, direct control is better than taxation.
   - When the marginal benefit is inelastic, taxation works better.

   Note: This is essentially the outcome of Weitzman Model.

3. How can education be used to reduce pollution?
   Solution: The following mechanisms related to the deduction can reduce pollution:

   - **Formal Regulations**: A better educated societies have better regulatory system in terms of monitoring, detection, and enforcement.
   - **Informal mechanisms**: A better educated citizens have a lower tolerance of the adverse effects of pollution and, thus they are more likely involved in pollution abatement.
   - **Property Rights:** A well-designed and well-protected property rights play an important role in pollution abatement. A well-educated societies are more likely to have such a legal system.
   - High educated society is good for clean technology development, adoption, and improvement.

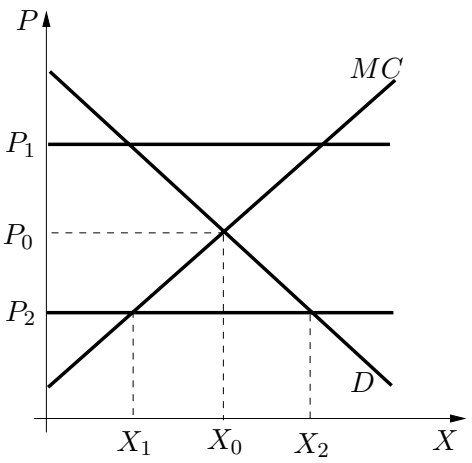

Figure 3: Aquaculture and the Environment

4. How will development of aquaculture affect the environment?
   Solution: Aquaculture is a technique to augment or provide alternative sources of fish by feeding and breeding fish and seafood.

   As shown in Figure 3, if there is no aquaculture, X_0 units of wild fish are captured and sold at price P_0. If the price of fish produced by aquaculture is P_1, it sets an upper bound on the cost of fish catching, and aquaculture will not affect the amount of wild fish captured and consumed. If the price of fish produced by aquaculture is P_2, the amount of fish consumed is X_2, which includes the amount of wild fish captured X_1 and the amount of fish produced by aquaculture $X_2 - X_1$. In this case, aquaculture reduces wild fishing from X_0 to X_1 and, thus it is a solution to overfishing.