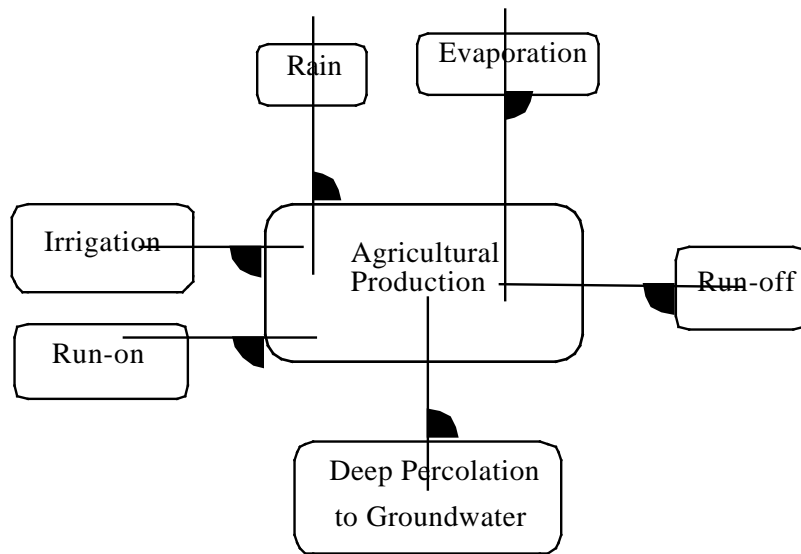# Chapter #17: Irrigation Economics

Contents: **General Overview**
An Example of Technology Choice Under Markets
Some Stylized Facts About Irrigation
Queuing Vs. Markets: A Numerical Example
Government Appropriates Water Rights

General Overview

The economics of irrigation is an important part of water economics in the U.S., because irrigation accounts for the majority of agricultural water use and agriculture uses 80% of annual water supply. The analysis of irrigation water demand requires a basic knowledge of the hydrologic cycle as it affects agricultural production.

Figure 17.1: The Hydrologic Cycle

The use of irrigation water depends on:
- Economics (prices and costs)
- Crop Selection
- Land Quality and Environmental Conditions
- Irrigation Technology

Water Management Choices depend on:
- Type of Crops
- Irrigation Technology
- Level of Water Availability

Some Stylized Facts About Irrigation

Irrigation water is measured in **"acre-feet," AF**, which is the amount of water needed to cover one acre of land to a one foot depth (before water is lost to percolation).

Irrigation Efficiency measures the percentage of water that is actually consumed by the crop.

Typical Water Use of Common Crops:

Heavy water users:	Alfalfa:	5-7 AF/year
	Rice	
Medium water users:	Fruits:	2.5-4 AF/year
	Cotton:	2.5-4 AF/year
	Vegetables:	2-3.5 AF/year
Low water users:	Wheat:	1.8-2.5 AF/year

Notice that there are large water savings moving from Rice or Alfalfa to Wheat.

Irrigation efficiencies of several irrigation technologies

Gravitational:	Furrow	.65	
	Border	.65	
Sprinkler:	Manual move	.8	
	Center pivot	.8	With field crops
Low volume:	Drip	.95	Not used with alfalfa, wheat
	LEPA	.9	Used in field crops
	Mini-sprinkler	.9	Used with trees

Water Saving Technology is initially very expensive to install. Over the lifetime of the system, the farmer gains from the technology through higher irrigation efficiency which defrays some of the cost of water. However, the correct incentives may not exist to stimulate investment in water-saving technology, since water is not sold in markets (price does not necessarily reflect MB).

Currently, the price of water is set administratively and is not the result of the maximizing behavior of economic agents. Water in

agricultural uses is also heavily subsidized, which has the following implications:
- Low water prices benefit users by providing a cheap source of water
- Low water prices creates an inefficient incentive to adopt new, water-saving technologies.

How The Choice of Irrigation Technology Affects Output
Water is applied to the surface, percolates through the soil, and is taken up by the root system.
- If the soil is dry and a bucket of water is poured on it, most of it will fail to permeate the soil, but will instead exit the land in the form of runoff.
- If the soil is first moistened, it much more readily absorbs water

This is the principle behind **drip irrigation**: it applies water much more slowly so that the crop can absorb it better. Most of the water applied in a drip irrigation system is absorbed by the plant. On the other hand, drip irrigation systems are very expensive to set up.

In contrast, a **sprinkler** distributes water much more unevenly (through space *and* time). Less of the water applied is utilized by crops, thus, greater evaporation and runoff. Yet, sprinkler systems are relatively inexpensive to set up.

Finally, the least efficient forms of irrigation are gravitational systems such as flood or furrow, which *use the concept of acre foot quite literally* by pooling water on a portion of land. Gravitational systems are less water efficient than sprinkler or drip irrigation, because of greater evaporation and surface runoff.

A simple model of irrigation technology choice
Agricultural production is a function of the effective water taken up by the crop. It does not matter how much water is applied, because what matters is the water available to the roots.

Effective Irrigation Water is the quantity of water actually taken up by the crop. Effective water is the product of two components:
- **applied irrigation water** (the quantity of water applied), and
- **irrigation efficiency** (the *fraction* of applied water taken up by the crop):

Let the per-acre production function for an agricultural product be:

$$y = f(e)$$

where:

    y = agricultural output per acre
    e = effective irrigation water per acre

and where we assume $f_e > 0$ and $f_{ee} < 0$, i.e., water has a positive marginal product that increases at a decreasing rate.

    Effective water equation: $e_i = a*h(i,q,c)$, where:

      a = **applied water** per acre
      $h(i, q, c)$ = **irrigation efficiency**
      i = irrigation technology, where we assume two possible irrigation technologies, labeled with an index variable i:

          Traditional technology: i=1
          Modern technology: i=2

      (dh/di) > 0:   an increase in irrigation technology increases irrigation efficiency (higher i results in higher irrigation efficiency).
      q = quality (of land or water). Land quality has many dimensions, such as water-holding capacity, soil quality and topographical conditions such as slope.

        $h_q > 0$, thus, an increase in land or water quality increases irrigation efficiency.
      c = climate variables (temperature, humidity, etc.)

        h_c is ambiguous, thus, an increase in a climate variable may increase or decrease irrigation efficiency, depending on the particular climate variable considered. (high wind and sun energy, for example, tend to decrease irrigation efficiency, due to greater evaporation rates).

    The farmer's per-acre profit-maximization problem can be expressed as the following **discrete/continuous choice problem**:

$$\max_{ia} \quad = Pf(e) - (w + z_i)a - k_i$$

$$\max_{ia} \quad = Pf(ah(i,q,c)) - (w + z_i)a - k_i$$

where: = profit
          P = output price
          z_i = cost of water pumping and pressurization

k_i = fixed cost of technology i

w = price of water

Assumptions:
(a) $h(i=1) > h(i=0)$ Irrigation efficiency is higher with modern technologies.
(b) $k_1 > k_0$ Modern technology requires higher fixed cost.
(c) $z_1 > z_0$ Pumping and pressurization cost may be higher with modern technology.

For a given *i,* optimal water use is determined by the F.O.C.:

$$\frac{d}{da} = Pf_e e_a - \left(w + z_i\right) = Pf_e h(i,q,c) - \left(w + z_i\right) = 0$$

where f_e and e_a are the appropriate partial derivatives (use chain rule). Rearranging:

$$\frac{w + z_i}{h(i,q,c)} = Pf_e$$

which says that at the optimum the MC of *effective* water is equal to the MVP of *effective* water.

The Technology Adoption Decision

The decision to adopt modern irrigation technology depends on a number of parameters in the model, for example, land quality. At some level of land quality, all else being equal, switching technologies will maximize profits. This level of land quality is called the **switch point**. (Similarly, other parameters in the model have other, analogous switch points.)
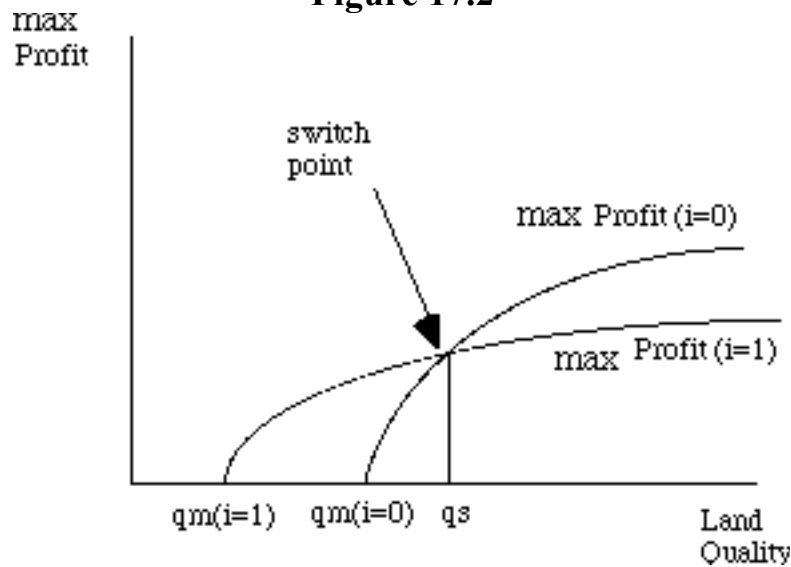
Figure 17.2

Figure 17.2 shows that it is not profitable to farm land of quality $<$ $q_m(i{=}1)$ regardless of the type of irrigation technology. On the other hand, with **high-quality land**, either technology is profitable, although the **traditional technology is more profitable**. This is because on high quality land, the increase in yield with the modern technology is not worth the fixed cost of installing it. Where the modern technology makes a difference is on land of moderate quality, i.e., the land between $q_m(i{=}1)$ and q_s. The modern technology increases profits on land between $q_m(i{=}0)$ and q_s, and makes profitable land that was previously not worth farming, i.e., the land between $q_m(i{=}1)$ and $q_m(i{=}0)$.

The switch point is based on a comparison of Profit($i{=}0$) with Profit($i{=}1$).

Crop Selection
The selection of agricultural crops may be viewed as another, parallel type of "technology adoption". Crop selection depends on the levels of economic parameters such as land quality and water prices. If the price of water increases, farmers are more likely to increase their acreage water-saving crops such as wheat, sorghum, vegetables, and nuts. This process is called a **switch in "biological technology."**

In general, farmers *simultaneously* choose crop type, irrigation technology and applied water use levels to maximize profits, based on the

6

levels of economic parameters such as land quality. Modern technologies are more likely to be adopted in regions with:
- Moderate to low quality land
- High value crops (fruits, nuts and vegetables)
- Low quality water
- High price water.

Some of the Effects of Technology Adoption

Recall that profit maximization requires:

$$\frac{w + z_i}{h(i)} = Pf_e$$

Now assume for simplicity that $z_0 = z_1$, (that is, pumping costs are the same) and recall from that $f_e > 0$. This implies that:

$$h(i=0) < h(i=1) \implies f_{e(i=0)} > f_{e(i=1)} \implies e_0 < e_1$$

Thus, modern technology increases the optimal level of *effective* water use. But note that a higher level of *effective* water use does not imply a higher level of *applied* water use. This is because the ratio (a/e) is smaller with modern technology, so that greater effective water can be utilized with lower applied water.

In most cases, modern technology reduce the optimal level of *applied* water use, and is therefore water-saving.

If $e_0 < e_1$, then $q_0 < q_1$. Thus, modern technology increases crop output.

If	(a) Land quality is high
and	(b) Water quality is high
and	(c) Weather is mild,

then *h*(i=1) and *h*(i=0) are not very different and the adoption of modern irrigation technology does not change the optimal levels of crop output or applied water use by very much.

If	Either	(a) land quality is low
	or	(b) water quality is low
	or	(c) weather is hot,

then adoption of modern irrigation technology may affect optimal crop

output and applied water use significantly. When land quality is low and temperature is high, the effect of adopting new technology depends on water price:

Water Price	*Increase* In Crop Output	*Decrease* In Applied Water Use
Low ($15/AF)	Minimal (0-5%)	High (30-40%)
Med ($15-$80/AF)	Medium (5-15%)	Medium (15-20%)
High ($80+/AF)	High (25-50%)	Low or negative (<5%)

An Example of Technology Choice Under Markets

Say an individual farmer is growing a crop, Y

- Crop Production Function: $Y = 30e - 0.2e^2$
- The price of y is: $P = \$80/ton$
- The price of water is: $V = \$400 / A\text{-}F$
-   $= PY - Va - F$, where a = the input 'applied water', and F = fixed costs

The farmer is trying to decide between two technologies:
- Sprinkler Irrigation is 50% efficient and costs $10,000 to install
- Drip Irrigation is 75% efficient and costs $20,000 to install

We first calculate profits under each system, then compare.

Under Sprinkler Irrigation

$$\underset{a}{Max.}\{\pi^S = \$80(30e - 0.2e^2) - 400a - \$10,000\}$$

$$subject\ to:\ \ e = 05a$$

Substituting in the constraint relating effective water to applied water:

$$\underset{a}{Max.}\{\pi^S = 800a - 4a^2 - \$10,000\}$$

FOC: $\dfrac{d\pi^S}{da} = 800 - 8a = 0$

which yields: a* = 100 A-F.

Substituting the value a* into the profit expression we get:

$$\pi^S = \$800(100) - 4(100)^2 - 10{,}000 = \$30{,}000$$

Under Drip Irrigation

$$\underset{a}{Max.}\left\{\pi^D = \$80(30e - 0.2e^2) - 400a - \$20{,}000\right\}$$

$$subject\ to:\ \ e = 0.75a$$

Substituting in the constraint relating effective water to applied water:

$$\underset{a}{Max.}\left\{\pi^D = 1{,}400a - 9a^2 - \$20{,}000\right\}$$

FOC: $\qquad \dfrac{d\pi^D}{da} = 1{,}400 - 18a = 0$

which yields: a* = 77.78 A-F.

Substituting the value a* into the profit expression we get:

$$\pi^D = \$800(77.78) - 4(77.78)^2 - 20{,}000 = \$34{,}444$$

Since $\pi^D > \pi^S$, the farmer is better off investing in a drip irrigation system. Notice:
- Drip Irrigation Uses Less Applied Water: (77.78 < 100)
- Drip Irrigation Uses More Effective Water: 0.75(77.78) = 58.34 > 50 = 0.5(100)
- Output Per Acre is Higher Using Drip Irrigation: $Y^D > Y^S$

Comparing Irrigation Under Water Markets and Queuing Systems

Suppose land quality is given as q and that we have two technologies denoted by $i = \{0, 1\}$, and

h_i = irrigation efficiency,
a_i = applied water per acre, and
e_i = effective water per acre.
L = Total water available for all acreage in a watershed
A = Total acreage of productive agricultural land in the region

The production function is:

$$y = f(e),\ where \qquad e = ah_i$$
$$e\ =\ effective\ water\ per\ acre$$
$$a\ =\ applied\ water\ per\ acre$$
$$y_m = f(e_m)\ =\ maximum\ output\ per\ acre$$

9

$e = e_m$, effective water associated with maximum yield per acre $f'(e_m) = 0$.

Under a queuing system

Water trading is disallowed under a queuing system. There is no incentive to adopt modern technology, since there is no water price. Water is simply diverted, as needed, according to the queue.

- Senior rights owners use water until the VMP of water $= 0$, which is the level that will *maximize yields*. Applied water use is $a_m = \dfrac{e_m}{h_0}$ per acre, the amount of applied water associated with the maximum effective water absorbed by the crop.
- Junior rights owners downstream use whatever water is left.

Under a queuing system of water rights:
- water price $= 0$
- per acre fee for water use $= \mu$

Total acres under a water rights system: $\dfrac{A}{a_m} = \dfrac{A h_0}{e_m} < L$

Total output $= \dfrac{A h_0 y_m}{e_m}$

Output price $= P$

Producer surplus $= P\dfrac{A h_0 y_m}{e_m} - u\dfrac{A h_0}{e_m}$.

Is the queuing system efficient?

Junior rights owners do not get enough water if scarcity exists.
- A unit of water would provide positive MVP on junior owners land
- The last unit of water on a senior owners land provides MVP $=0$.
  Therefore, the queuing system is Inefficient. **MVP$_J \neq$ MVP$_S$**

The queuing system leads to under-utilization and over-irrigation of land. A market system may offer a better solution, depending on transaction costs.

The switch to a market system involves costs of t dollars per acre annually in transaction costs. Costs include improved piping and improved monitoring.

Under a Market System

When all land quality is the same, the efficient solution involves applying water uniformly across all land to equate the MVP. Thus, under a market system, all land is utilized and each owner faces the choice of technology i.

• water per acre = $\dfrac{A}{L}$

• yield per acre = $y_i = f\left(h_i \dfrac{A}{L}\right)$

• price of water = VMP *of applied water* = $Pf_e h_i$

• k_i = the fixed cost of implementing the technology $(k_1 > k_0)$

The producers' annual profits per acre are:

$$\pi_i = Py_i - \frac{A}{L}Pf_e h_i - k_i - \mu - t$$

so that:

$$\pi_1 = Py_1 - \frac{A}{L}Pf_e h_1 - k_1 - \mu - t$$

$$\pi_0 = Py_0 - \frac{A}{L}Pf_e h_0 - k_0 - \mu - t$$

$$(\pi_1 - \pi_0) = P(y_1 - y_0) - \frac{A}{L}Pf_e(h_1 - h_0) - (k_1 - k_0)$$

Technology 1 is selected if:

$$(\pi_1 - \pi_0) > 0$$

Both technologies require the same water per acre, because water is evenly distributed across all acres as a result of equating the MVP. When each farmer is a small unit, the farmer does not believe that her choice of technology will affect the market price of water. When the market price of water is taken as a constant in the problem, the choice of technology can be expressed as:

Select technology 1 when: $P(y_1 - y_0) > k_1 - k_0$

Both technologies result in the same water use per acre, but the modern technology increases the yield by raising the amount of effective water received by the crop.

If the market value of the increase in yield is greater than the extra capital costs involved with investing in the new technology, the farmer should invest.

Comparing Market and Queuing Outcomes

Assume that, under market conditions, technology i is optimal and adopted by all farmers. Under a market system all arable land is utilized. The transition to market will increase irrigated land from

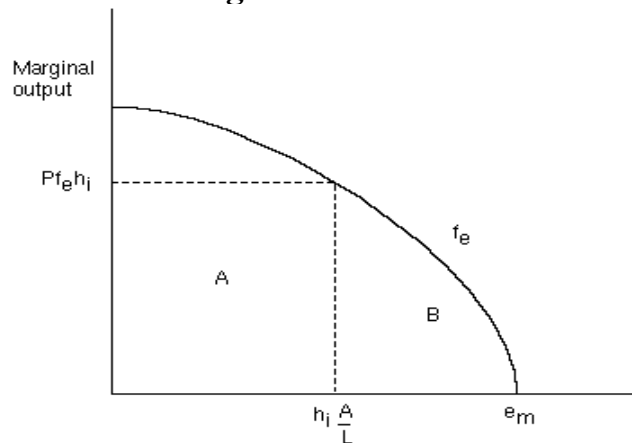$$\frac{Ah_o}{e_m} \quad to \quad L$$

Output will increase by:

$$Y = Lf\left[h_i \frac{A}{L} \right] - \frac{Ah_o}{e_m} f\left(e_m\right) > 0$$

as water is shifted from low MP land to the high MP lands of junior rights holders.

• Output per acre of senior rights owners will decrease, and
• water per acre of all users will decrease.

The reduction in output per acre is from $f(e_m)$ to $f(A/L\, e_i)$. The reduction in water use per acre is from $e_m\, h_0$ to A/L.

Figure 17.3

In the transition to the market, $e_m - h_i \dfrac{A}{L} \quad h_0 \dfrac{A}{e_m}$ units of water which were used to produce the output associated with area B in the figure are allocated to irrigate new lands. Overall, output is increasing because the water that was used under queuing to produce output associated with region B of the figure is used under markets to produce output in region A

12

on new land that is brought into production. Obviously, the marginal productivity of this water increases.

If the senior rights owners have to buy water under markets, they are losing from the transition. Under water markets, they now have lower yields, they now have to pay for water, and they also must pay to adopt the new technology, since doing so is now optimal. Their loss per acre is:

$$Pf(e_m) - Pf\left(h_i \frac{A}{L}\right) + Pf_e h_i \frac{A}{L} + k_i + u.$$

But if the senior rights are given the property rights to the water, they may win. They still have lower output than under queuing, but the gain from selling excess water may overcome this output loss. Their income per acre will be:

$$Pf\left(h_i \frac{A}{L}\right) + \left(e_m - h_i \frac{A}{L}\right) Pf_e h_i \frac{A}{L} - t - k_i$$

If the transaction costs are high, there is no incentive to switch to a system of water markets. Namely, if:

$$t > \frac{P\left(Y + k_i L - k_0 A / (h_0 e_m)\right)}{L}$$

When transaction costs per acre exceed the per acre change in output plus the cost of adopting the optimal modern technology less the cost savings of senior owners not adopting the conventional technology, water markets may be inefficient.

Because markets for final products have negatively sloped demand, the transition from queuing to markets will also reduce the market price of agricultural commodities. Senior rights owners may thus lose, even if they sell water because of the price decline of their output. Producers as a whole may actually lose, but consumer surplus will increase.

Numerical Example of Market vs. Queuing (based on a study by Zilberman)
In this example we have:
>   4 technologies;
>   2 demand elasticities for output; and
>   2 transaction cost levels:

	900	900	1050	1050
Land Base (10^3 acres)	900	900	1050	1050
Demand Elasticity	1	50	1	50
Queuing Outcomes				
Output (10^6 lbs.)	936	936	936	936
Output Price ($/lb.)	0.75	0.75	0.75	0.75
Irrigated Land (10^3 acres)	720	720	720	720
Producer Profits (10^6 dollars)	342	342	342	342
Market Outcomes Adjustment Costs = $5/acre				
Output (10^6 lbs.)	1159	1161	1161	1344
Output Price ($/lb.)	0.572	0.746	0.57	0.744
Water Price ($/AF)	62.0	73.75	63.7	118.4
Irrigated Land (10^3 acres)	900	900	902	1050
Technologies	2	2, 3	2	3
Senior Rights Net Profits (10^6 dollars)	5.3	139	0	43.4
Senior Rights Gross Profits (10^6 dollars)	191	361	191.1	398.5
Percent Gain in Social Welfare	5.4%	16.3%	5.4%	23.8%
Market Outcomes Adjustment Costs = $50/acre				
Output (10^6 lbs)	1150	1160	1150	1342
Output Price ($/lb.)	0.579	0.746	0.579	0.744
Water Price ($/AF)	53.5	81.0	53.5	118.4
Irrigated Land (10^3 acres)	890	900	890	1050
Technologies	2	2, 3	2	3
Senior Rights Net Profits (10^6 dollars)	0	107	0	10.9
Senior Rights Gross Profits (10^6 dollars)	160.7	328.2	160.7	366.1
Percent Gain in Social Welfare	- 0.5%	4.6%	- 0.5%	10.3%

The example shows:

   (1) Under markets, acreage and output grow and output prices
       go down.

(2) If demand elasticity is low, the decline in output price may
    be substantial and producers may lose. The percent gain in
    social welfare is highest in elastic demand markets
(3) If transaction costs are high, the introduction of markets
    may not be worth while.

Figure 17.4

P = 0.90
u = 0

SW(M)

SW(Q)

Social Welfare
(million dollars)

Available Water (million AF)
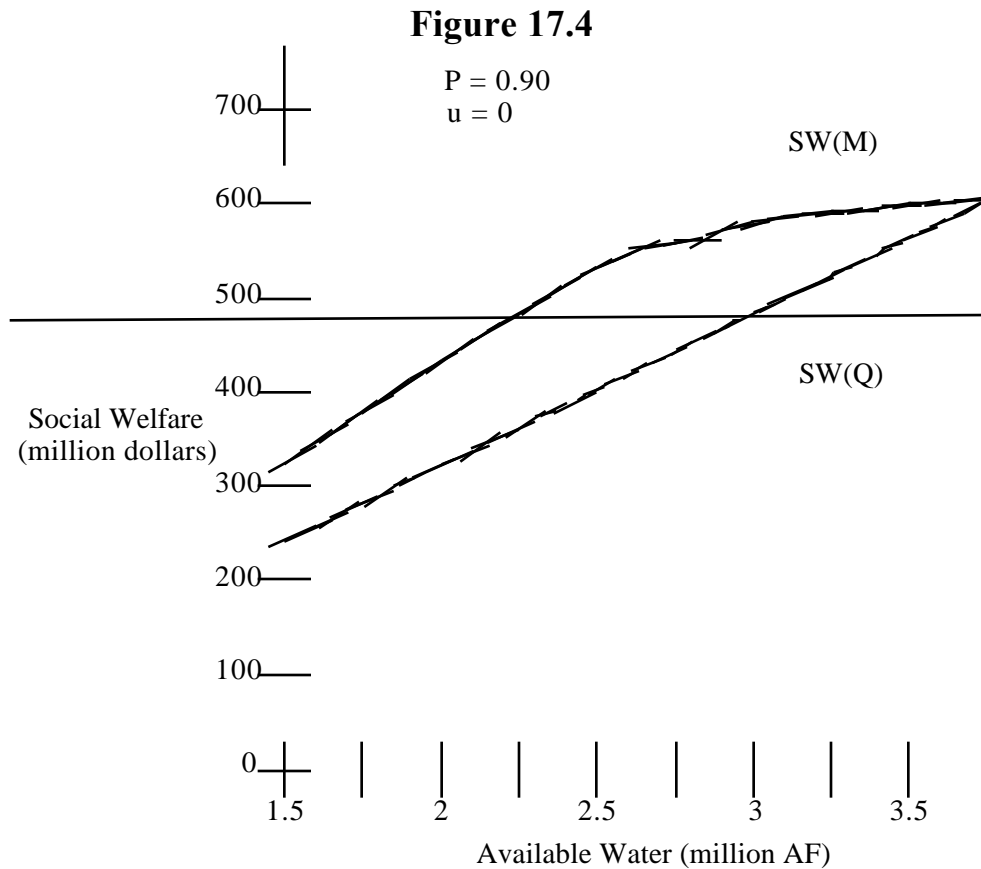
P = Output Price
u = Transaction Costs

Figure 17.4 demonstrates gains in social welfare from transition to
markets as a function of overall water. It assumes zero transaction costs.
When 2.1 million AF are available, social surplus increased by 130 million
dollars (30%). There is no gain in welfare when total water exceeds 3.8
million AF, because, at this point water scarcity is not a problem.
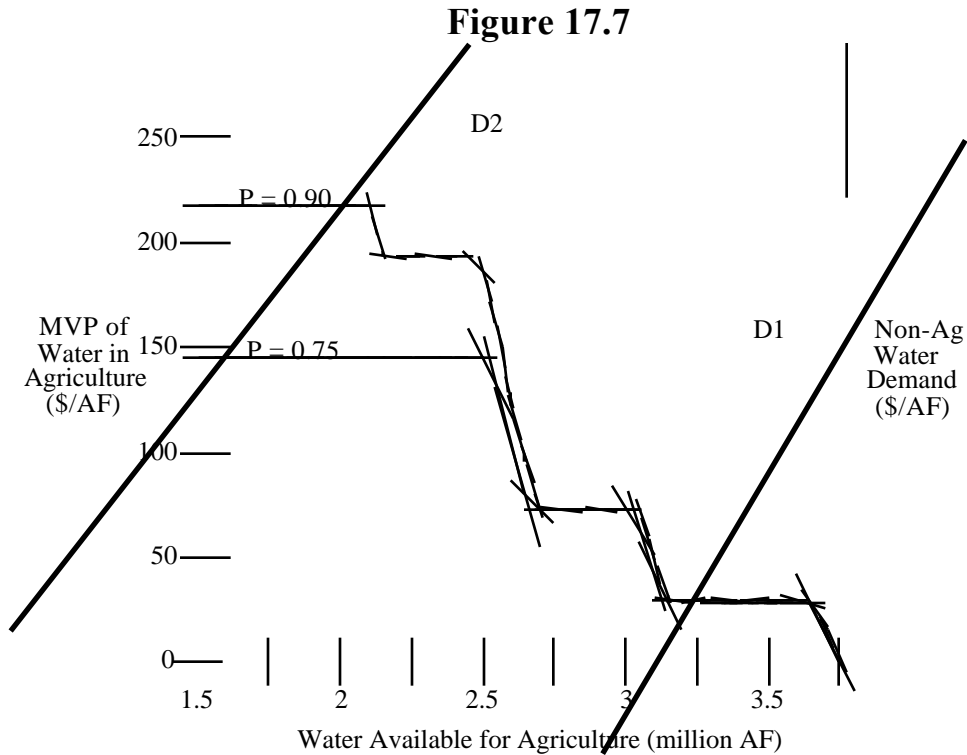
Figure 17.5
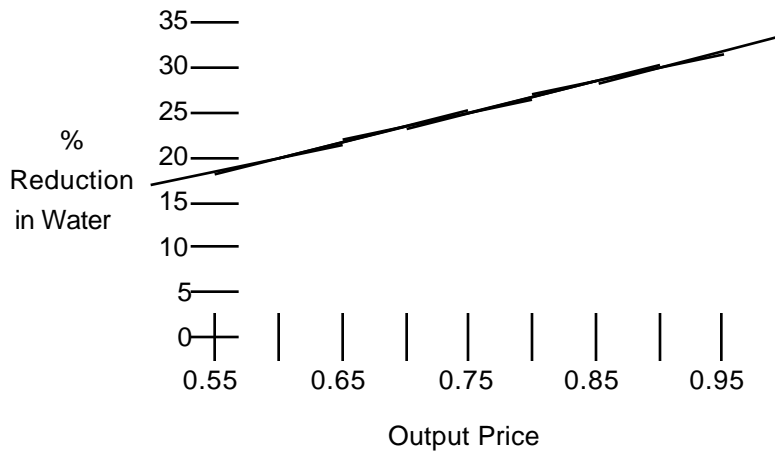
Figure 17.6
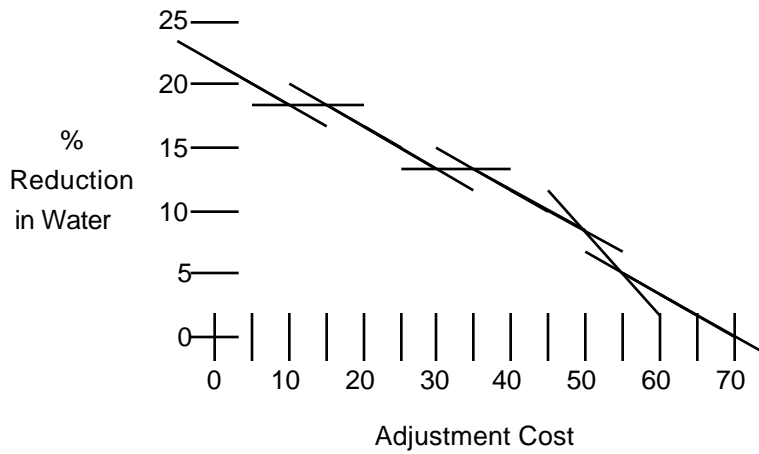
The Figures hold Social Welfare Constant

Figures 17.5 and 17.6 show the amounts of reduction in agricultural water that will keep welfare unchanged if industry moves from queuing to markets - as a function of output price and transaction cost. Outcomes in the region below the lines are welfare improving.

We can think of reductions in water as a result of increasing scarcity.

Figure 17.7

The demand functions for water can be derived according to the price of output and the available technology. The demand function has several steps and each is associated with a different technology.

Figure 17.7 shows that when urban demand is small (D1), gains from markets are not spectacular, and traditional technology is optimal since the MVP of water in alternate uses is small. When water demand is high (D2), however, introduction of markets will lead to adoption of modern technologies and will reduce agricultural water use substantially.

Today we have a transition from queuing to markets in other environmental amenities, including air quality.

Polluters have assumed they have pollution rights which were established "de facto" chronologically. Now, as air quality becomes scarce, they need pollution permits, and there is trading. Introduction of pollution trading will increase efficiency - if welfare gains from markets are sufficient to offset transaction costs.

In general, there will be less objection to markets among users of scarce resources if existing users of the resource (senior rights holders or polluters) are given the right to sell permits.

Figure 17.8: Comparison of Water Rights (Queuing) and Water Markets[*]

Criterion	Queuing	Market
Allocation mechanism	Uses are assigned by seniority. Trading is disallowed. No limit on consumption use	Water is traded at going price.
Water pricing	Fixed fee per acre (u). Zero price per acre feet.	Fixed fee per acre (μ) determined by supply and demand ($W = pf'h_i$).
Water choice problem	$\max_{a,i} Pf(a_i h_i) - u - c_i$	$\max_{a,i} Pf(h_i a_i) - Wa_i - c_i - t - \mu$
Transaction costs	0	t per acre
Technology	Traditional, $i = 0$	Modern may be adopted, $i > 0$, if $c_i - c_0 < P(y_i - y_0) - W(a_i - a_0)$
Water per acre	Maximizing yield per acre, $a = e_m$.	Aggregate water/ag. land, $a = A/L$
Output per acre	$y = y_m$	$y = f(A/L \; h_i)$ Smaller than under queuing
Acreage	Not fully utilized, $L_Q = Ah_0/e_m$	Fully utilized, $L_M = L$

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[*]Under the limiting assumptions of the model presented in class. Namely, there is one land quality. Its irrigation efficiency under traditional technology is h_0 and under the modern one is h_1. Under the modern technology, there is a unique optimal solution. Aggregate land L and water A are given.

Aggregate output	Smaller than under market, $$Y_Q = \frac{Ah_0}{e_m}\,y_m$$	$$Y_M = Lf\left(A/L\ h_i\right)$$
Output price	$D(Y_Q)$ Greater than market if demand is negatively sloped.	$D(Y_M)$
Social welfare	Lower if transaction costs are not high.	Higher if transaction costs are sufficiently low.* $$t > \frac{P\ Y + c_i L - c_0\left(Ah_0/e_m\right)}{L}$$
Income of senior rights owners under transferable rights.	$$\frac{Ah_0}{e_m}\left[Pf\left(e_m\right) - \mu\right]$$	$$\frac{Ah_0}{e_m}\left[Py_i - t - c_i\right.$$ $$\left. -u + W\,\frac{e_m}{h_0} - a_i \right]\;†$$ Incomes under market are higher if demand is not very inelastic, and transaction costs are sufficiently low.
Income of senior rights owners when they do not retain water rights.	$$\frac{Ah_0}{e_m}\left[Pf\left(e_m\right) - \mu\right]$$	$$\frac{Ah_0}{e_m}\left[Py_i - Wa_i - t - c_i - u\right]$$ † lose from transition to market

a_i = applied water, e = effective water, c_i = per acre cost of technology i, and t = transaction cost.

†Assuming that the modern technology is adopted.

Queuing Vs. Markets: A Numerical Example

Say the farm production function is: $y = 8e - 2e^2$

There are two technologies available**:**
- The traditional technology has a 50% water efficiency: $h_0 = 0.5$
- The modern technology has a 60% water efficiency: $h_1 = 0.6$

Suppose for two technologies: $P = 125$, $\mu = 100$, $t = 50$.

Total water stock is, $A = 6000$ A-F,

Total land stock, $L = 2000$.

Queuing Outcome
- Technology 0 is chosen.

- Senior water rights holders maximize yield:
$$Max_{e}\{y = 8e - 2e^2\}$$

   which has the FOC: $\dfrac{fy}{fe} = 8 - 4e = 0$

   implying that: $e_m = 2$, $y_m = 8$,

- Water applied per acre is: $a_m = \dfrac{e_m}{h_0} = \dfrac{2}{0.5} = 4.$

- Acreage utilized simply depends on how far down the river the water flows:
$$AQ = \frac{6,000}{4} = 1500.$$

- Aggregate output: $Y_Q = y_m(A_Q) = 8(1,500) = 12,000.$

- Income per acre: $\_Q = 8(125) - 100 = \$900.$

- Total Farm Income $= (1,500 \text{ acres})(\$900/\text{acre}) = \$1,375,000.$

Market Outcome

First consider the case when only the traditional technology is available:

- Declining MVP of water and homogeneous land quality implies it is optimal to distribute water evenly across all land

$$\text{applied water per acre:} \quad a = \frac{6000}{2000} = 3$$

- Effective water: $e* = 3(0.5) = 1.5$

- Output per acre: $y* = 8(1.5) - 2(1.5)^2 = 7.5$

- Aggregate Output: $Y_M = 2000 \cdot 7.5 = 15000$.

(Yield per acre is smaller than under queuing but total output is greater.)

- Total income $= \underset{py \qquad\quad \mu \quad\; t}{2000[7.5 \cdot 125 - 100 - 50] = 1575000}$

- Water price = VMP = $p\frac{\partial f}{\partial e}h_0 = 125(8 - 4 \cdot 1.5)\cdot.5 = 125$.

Say senior rights owners are given the right to sell surplus water:

- Senior rights owner owns 4 A-F of water foot per acre, uses 3 A-F under a system of water markets and sells the remaining A-F for $125:

$$\pi* = \$125(7.5) - 100 - 50 + 1(\$125) = \$912.50$$

The senior rights owner makes greater profit than under a queuing system. Junior rights owners' profit per acre is:

$$\pi* = \$125(7.5) - 100 - 50 - 3(\$125) = \$537.50$$

The junior rights owner earns zero profit under a queuing system.

Now consider the case when the traditional technology *is* available. If modern technology is available, it will be adopted because of extra output it generates, provided the fixed costs of adopting it are sufficiently small.

- Effective water: $a = \dfrac{e}{h_1} \qquad 3 = \dfrac{e*}{0.6} \qquad e* = 1.8$

- Output per acre: $y* = 8(1.8) - 2(1.8)^2 = 7.92$

- Aggregate output under market: $Y* = (2{,}000 \text{ acres})(7.92/\text{acre}) = 15{,}840$

- Total income $= \underset{py}{1500(7.92 \underset{\mu}{\,125} \underset{t}{-100} - 50 \underset{c}{-30})} = 1620000$

- Water price $= p\dfrac{\partial f}{\partial e}\,h_1 = 125[8 - 4(1.8)]\,.6 = 125\;.8\;.6 = 60.$

Assuming, as before, that senior rights owners are allowed to sell their surplus water:
  - Senior rights owners' income $=$
  $$(792)125 - 100 - 50 - 30 + 60 = \$870$$

  - Junior rights owners' income $= 7.92\;125 - 100 - 50 - 30 - 180 = 630.$

Implications

1) Establishing water markets is welfare improving.

2) The adoption of modern technology is in the best interest of society. Total farm income is higher when farmers adopt new technology.

3) Adopting modern technology is in the interest of junior rights holders

4) Adopting modern technology is not in the interest of senior rights holders. Senior rights owners profit from a market system, but only when modern technology is not adopted.

Government Appropriates Water Rights

If the government is given the water rights, then both senior and junior rights holders must purchase water. The water price is still $P = VMP$ in either case.

Profit per acre is now the same for the both senior and junior rights holders, which is what we already calculated for junior rights holders above. Therefore, it is now in the best interest of senior rights holders to invest in the new technology. Although the allocation of property rights achieves the same result in terms of allocative efficiency: $P = MVP =$ marginal profit per acre (an application of the Coase Theorem). Different incentives are created for technology investment.