Chapter #15: Forestry Economics

General Overview

The economics of forest resources are very similar to the dynamic management of a fishery:
- Both forests and fisheries are renewable resource systems
- The economic principles that determine optimal management are very much the same

The major difference between the economics of a forest and a fishery resource are related to biological principles.

The central question of commercial or social forest economics is "When should we cut a stand of trees?"
- We will assume that the land has no available alternative use
    - If we had an alternative use, it would:
        1) introduce opportunity cost in the model

Differences Between Issues of Forestry and Fisheries

The forest problem is a problem of divestment, which means the solution calculates the optimal time to consolidate and sell the entire stock and begin the next rotation. The analogy of a forest rotation is that of a conventional crop which does not get harvested every season. However, the growth cycle of a forest resource (a period of centuries instead of months) is so long that resource owners get really impatient and discounting/dynamic analysis is important.

How Is The Forestry Problem Different From a Fishery?
1) Forestry solutions determine "when" rather than "how much"
2) Growth occurs over long time periods and can be measured
3) The forestry problem solves for the optimal time to harvest entire stock and the solution gives the optimal length of each rotation of stock

4) Property rights are secure (no open access problems)

In the forestry problem, the critical element is that the growth function is a function of time, not a function of stock.

Figure 15.1: The Forestry Growth Function
Q(t) = Volume of Timber (i.e., Board Feet or Cubic Feet)
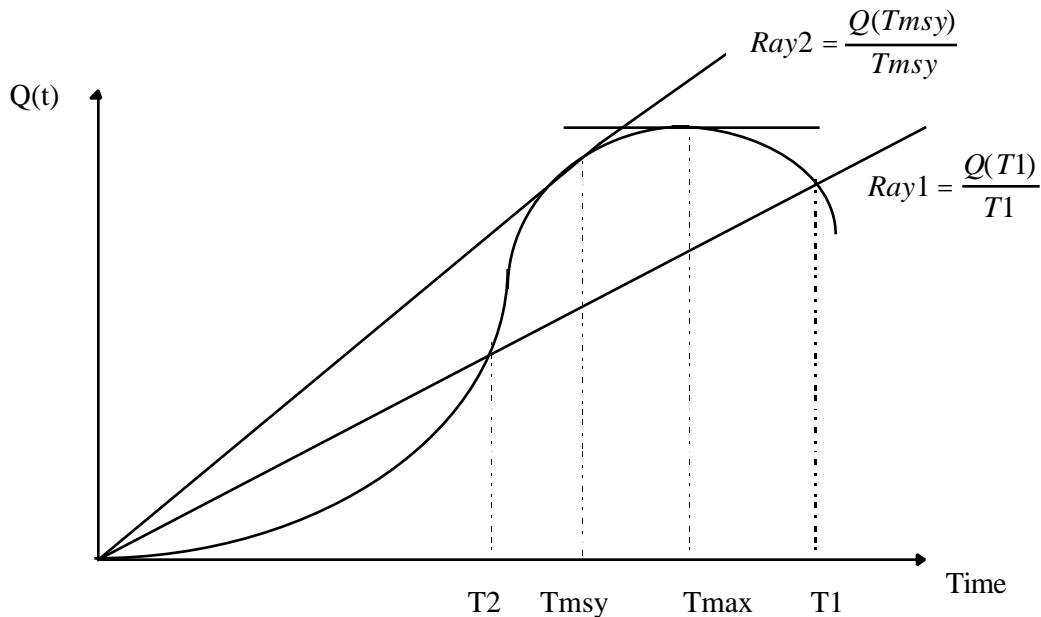
The growth function of a typical stand of trees looks like this. At first, the volume increases at an increasing rate for very young trees. Then growth of volume slows and increases at a decreasing rate. Finally, when the trees are very old, they begin to have negative growth as they rot, decay, and become subject to disease and pests.

The volume of a stand of trees is maximized at time T_{max}, with a volume $Q(T_{max})$.   Yet, this is not the volume associated with the Maximum Sustainable Yield
- MSY occurs where the growth rate equals the Average Growth *per rotation*. Recall that our goal is to re-plant new trees.
- The average growth rate of a stand, at any time, t, is:

$$A.G. = \frac{Q(t)}{t}$$

which can be shown by a ray through the origin.
- Ray 1 shows that the average growth can be achieved by either cutting at time T_1 or at time T_2, but neither time gives the Maximum Average Growth. What we want is the highest average growth over all harvests

Finding Tmsy

The MSY occurs at a rotation length that maximizes the average annual growth of the stands through time.
- Max. $\{ Q(t)/t \}$ implies the FOC:

$$\frac{d\,\dfrac{Q(t)}{t}}{dt} = \frac{Q'(t)t - Q(t)}{t^2} = 0$$

where we have used the quotient rule of calculus. Rearranging terms, we get:

$$\frac{Q(t)}{t} = Q'(t).$$

Thus, in order to harvest the MSY, we should cut the stand of trees when marginal growth equals average growth of the stand. Ray 2 shows where this condition is met, where the average growth is tangent to the growth function. In biology, this concept is known as Maximizing the Mean Annual Increment.

Figure 15.2: Harvest Pattern Over Time
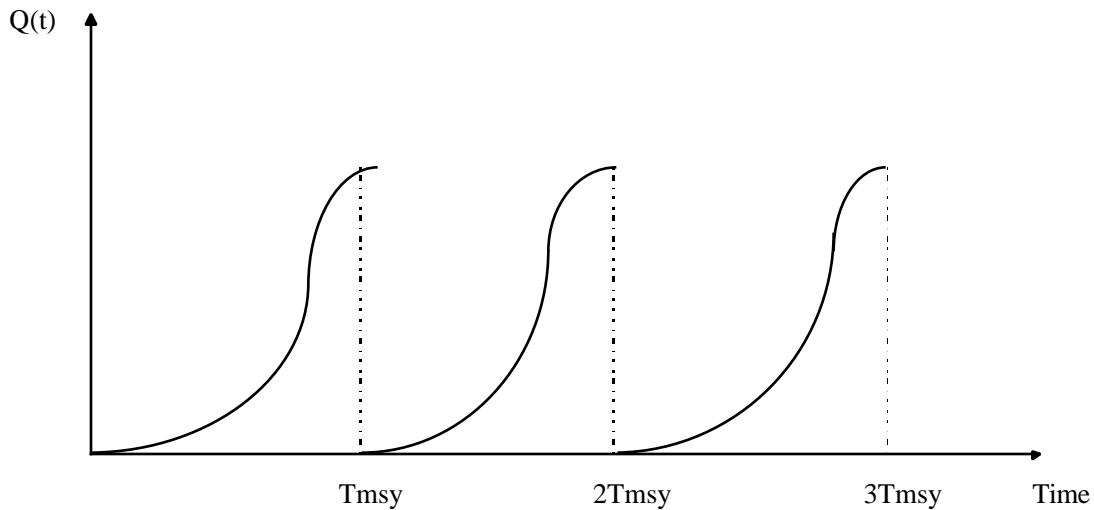
Graphically, the harvest pattern over time is shown above. As in the case of the fishery when moving from two time periods to T periods, the optimal dynamic solution is to replicate a single optimal decision many times.

The Economic Decision to Harvest a Stand

Since the value of the stand grows over time like a conventional asset, such as a stock or money in an interest bearing bank account, the optimal solution will occur where the value of the forest asset is in equilibrium with other assets in the economy. That is, we want to incorporate into the analysis the rate of time preference of the forester, (i.e., the discount rate).

The Forestry Optimization Problem

Because the growth rate is a function of time, it is necessary to formulate the optimization problem in continuous time:

- In discrete time, we find that $P_t = \dfrac{1}{1+r}^{\,t}$

- The continuous time analog to this is found as $\partial t \quad 0$:

$$as\ \partial t \quad 0, \quad \left(\frac{1}{1+r}\right)^{t} \quad e^{-rt}$$

In the economic optimization problem, we want to maximize the present value of the forest stand with respect to the time period of harvest. That is, we want to find the point in time where the NPV is maximized.

The Optimal Single Rotation
We will first look at the problem as a single rotation. In the single rotation, there is no opportunity cost incurred by failing to plant the next stand of trees at the optimal time.

Thus, the problem is really that of determining the optimal time to harvest a crop. Suppose a crop is planted at time t=0 and grows in value to PQ(t) at time t. The goal is to find the harvest time that will maximize the NPV of a single rotation.

Let P = constant price per pound of the crop.
There are no harvesting costs, so that PQ(t) = Total Revenue

The Objective Function is:
$$\underset{t}{Max.}\left\{\pi = e^{-rt}PQ(t)\right\}$$

with the FOC:
$$\frac{d\pi}{dt} = PQ'(t)e^{-rt} + PQ(t)e^{-rt}(-r) = 0$$

which can be written as:
$$PQ'(t) = rPQ(t)$$

or, MB of waiting (value of new growth) = MC of waiting (lost interest on Total Revenue)

If the forest manager delays the harvest, she will not earn interest on revenues PQ(t). If the forest manager delays the harvest, she will gain the value of new growth Q'(t). This is the trade off to consider in forestry economics. We can rearrange the optimality condition to get:
$$\frac{Q'(t)}{Q(t)} = r$$

which states that the percent rate of growth in volume should equal the discount rate. Profit maximization therefore dictates that the stand should be harvested when the percentage rate of growth of crop value equals the value of alternative investments.

• If $\dfrac{Q'(t)}{Q(t)} > r$, then the crop is increasing in value quicker than

market investments and the farmer should delay the harvest decision.

• If $\dfrac{Q'(t)}{Q(t)} < r$, then market investments are increasing in value

quicker than the growth in value of the crop (harvesting should have already occurred)

The Case of An Infinite Forest Rotation

The relevant case for most foresters is that of continuous stand rotation over time.

When the forester plans to replant a new forest stand immediately after cutting the old one, there is now an opportunity cost that must be considered: the opportunity cost of future rotations.

Before we begin, we need to review a calculus identity that we will use:

• The sum of an infinite series is $For\ |X| < 1,$

$$\sum_{i=0} X^i = \left(1 + X + X^2 + X^3 + ...\right) = \frac{1}{1 - X}$$

• In our problem of infinite rotation we will use this as follows:

$$\sum_{i=0} e^{-irT} = \left(1 + e^{-rT} + e^{-2rT} + e^{-3rT} + ...\right) = \frac{1}{1 - e^{-rT}}$$

where T is the length of each rotation.

The infinite rotation problem is commonly called the **Faustmann Rotation** after the German Forester from the early 1900's. Assume a constant net price (or profit) per cubic foot of timber. That is, if harvesting or replanting costs exist, then what we hold constant is:

Net Price = Price - per unit harvesting and replanting costs

Let:

P = the constant price per cubic foot of timber
Q = the volume of timber (in cubic feet)

We can now write forester profit as:

$$\pi = PQ(T)e^{-rT} + PQ(T)e^{-2rT} + PQ(T)e^{-3rT} + \ldots$$

$$= PQ(T)\left[e^{-rT} + e^{-2rT} + e^{-3rT} + \ldots\right]$$

$$= PQ(T)e^{-rT}\left[1 + e^{-rT} + e^{-2rT} + \ldots\right]$$

$$= PQ(T)\frac{e^{-rT}}{1 - e^{-rT}}$$

$$= \frac{PQ(T)}{e^{rT}\left(1 - e^{-rT}\right)}$$

$$= \frac{PQ(T)}{e^{rT} - 1}$$

The Optimization Problem is: $\underset{T}{Max.}\ \pi = \dfrac{PQ(T)}{e^{rT} - 1}$

with the FOC:

$$\frac{d\pi}{dT} = \frac{PQ'(T)}{e^{rT} - 1} + \frac{PQ(T)(-1)(r)(e^{rT})}{\left(e^{rT} - 1\right)^2} = 0$$

which can be re-arranged to yield:

$$PQ'(T) = \frac{PQ(T)re^{rT}}{e^{rT} - 1} \qquad = \frac{rPQ(T)}{1 - e^{rT}}$$

We can now cross-multiply and write the optimality condition as:

$$PQ'(T) \quad = \quad rPQ(T) \quad + \quad PQ'(T)e^{-rT}$$

MR of delaying $=$ MC of waiting $+$ MC of delaying future income stream.

The first two terms are identical to those in the single stand, or cropping decision. The last term represents an additional opportunity cost of delaying the harvest. As delaying the current harvest also delays income

received from future harvests. Therefore, the optimal rotation time, T, requires the forester to equate the marginal value of waiting to the marginal cost of delaying the harvest of current *and* future stands.

In general, T* < T* (single rotation) < Tmsy. It is also important to analyze the effect of different parameters of the harvesting decision.

An Increase in the Price of Timber
An increase in P will tend to shorten the rotation length, because higher timber prices increase the profitability of each harvest. Cutting trees earlier moves the profit of future harvests closer to the present.

An Increase in the Interest Rate
An increase in r will tend to shorten the optimal rotation length, because the forest owner is now relatively more impatient. The owner is now more eager to move profit up into the present.

An Increase in Harvesting Costs
- Recall how we absorbed harvesting costs into the Net Price.
- Thus, an increase in c is analogous to a decrease in Price
- An increase in c will tend to increase the rotation length, because cutting trees has now become less profitable
    - the owner wishes to delay paying future harvesting costs