

Chapter #12: Concepts of Nonrenewable Resources

Contents:

Two-Period Nonrenewable Resource Model with Extraction Costs

Two-Period Nonrenewable Resource Model with Open Access

Policies to Correct Open Access Market Failures

Two-Period Nonrenewable Resource Model with Monopoly

Two-Period Nonrenewable Resource Model with Changing Demand

Two-Period Nonrenewable Resource Model with a Backstop Technology

Sketch of An "n-Period" Model of Nonrenewable Resources

Summary of Nonrenewable Resource Model Results

Two-Period Nonrenewable Resource Model with Extraction Costs

Utility Maximization

Assume that we are concerned with a nonrenewable resource under unsatiated demand in two periods, $t=0$ and $t=1$. $B(X_t)$ is the *gross* benefit associated with using X_t amount of the resource in period t . Now, let c denote marginal extraction cost of the resource. Hence, the *Net Benefit* for period t becomes: $B(X_t) - cX_t$,

We can maximize social welfare by solving:

$$\max_{X_0, X_1} NPV[SW(X_0, X_1)] = B(X_0) - c X_0 + \frac{1}{1+r} [B(X_1) - c X_1]$$

subject to the constraint of the total available resource, S : $S_0 = X_0 + X_1$.

The Lagrangian equation for this problem is:

$$L = B(X_0) - c X_0 + \frac{1}{1+r} [B(X_1) - c X_1] + \lambda(S_0 - X_0 - X_1)$$

The F.O.C.'s are:

$$(1) \quad L_{X_0} = B_x(X_0) - c - \lambda = 0.$$

$$(2) \quad L_{X_1} = \frac{B_x(X_1) - c}{1+r} - \lambda = 0.$$

$$(3) \quad L_\lambda = S_0 - X_0 - X_1 = 0.$$

where $B_x(X_t) = MB$ of using X_t amount of the resource in period t , and $c = MC$.

Understanding the Relationships

Equation (1) states that the price of the mineral resource, P_0 [which equals $B_x(X_0)$] equals marginal mining cost, c , plus the shadow cost of the resource constraint, λ or: $P_0 = c + \lambda$. The shadow cost of the resource constraint is also called the **user cost** in dynamic problems. It is the opportunity cost of not being able to use the marginal unit of the resource in the future if you use it today. Stated in another way, the shadow price is the benefit one could gain from increasing the constraint of initial resource by one unit.

Notice from Equation (2) that higher interest rates reduce the user cost, as before. A reduction in user cost implies that one should use more of the resource today and less in the future. If one uses more of the resource today and less in the future, then the resource will be more plentiful today and more scarce in the future, so the price of the resource will be lower today and higher in the future. Hence, an increase in the interest rate shifts a larger share of consumption from the future to the present, lowering the price today, but raising the price in the future.

Impacts of Extraction Costs

It is clear from both equation (1) and Equation (2) that higher extraction cost reduces the net marginal benefit associated with using the resource in either time period, which in turn:

- **reduces resource use and raises prices in both periods.**
- **reduces the user cost**, increasing the incentive to shift some consumption from the future to today.

With very high extraction cost, the entire resource stock may not be used by the end of the last time period, in which case the shadow cost of the resource constraint, λ will equal zero. We can draw this conclusion because the cost of extraction will be greater than the benefits of resource use.

Without extraction costs, the price of an optimally-managed nonrenewable resource, P , will grow at the rate of interest, r :

$$\frac{P_1 - P_0}{P_0} = r$$

By combining Equations (1) and (2) we find that, with extraction costs, the price of an optimally-managed nonrenewable resource minus extraction costs, or royalty, will grow at the rate of interest:

$$\frac{R_1 - R_0}{R_0} = r, \text{ where } R_i = (P_i - c)$$

This implies that, with extraction costs, the price of an optimally-managed natural resource may grow at a rate less than the rate of interest.

Sample Problem

Let's clarify this point with an example. Assume a competitive resource market with linear demand for a nonrenewable resource. Recall that $B_X(X)$ represents marginal benefit and that marginal benefit is the demand curve. In this problem, marginal benefit will have a linear form:

$$B_X(X) = a - bX$$

Substituting $B_X(X)$ into the FOC's (1) and (2) from the previous section, we can solve the system of equations for the optimal values of resource to use in terms of all other variables. Equating equations (1) and (2), we have:

$$a - bX_0 - c = \lambda = \frac{1}{1+r} [a - bX_1 - c]$$

Using the relationship $S_0 - X_0 = X_1$ given by equation (3), we have:

$$a - bX_0 - c = \frac{1}{1+r} [a - b(S_0 - X_0) - c],$$

which can be used to solve for X_0^* , as:

$$\begin{aligned} X_0 &= \frac{(a - c)(1 + r) - a + b(S_0 - X_0) + c}{b(1 + r)} \\ X_0 + \frac{X_0}{1 + r} &= \frac{(a - c)(1 + r) - a + bS_0 + c}{b(1 + r)} \\ X_0^* &= \frac{(a - c)r + bS_0}{b(2 + r)} \end{aligned} \tag{4}$$

From equation (3), we know that:

$$\begin{aligned}
X_1^* &= S_0 - X_0^* \\
&= \frac{bS_0(2+r) - (a-c)r - bS_0}{b(2+r)} \\
X_1^* &= \frac{bS_0(1+r) + (a-c)r}{b(2+r)} \tag{5}
\end{aligned}$$

And, we can also solve for λ^* using either equation (1) or (2). Using equation (1):

$$\begin{aligned}
\lambda^* &= a - bX_0^* - c \\
&= \frac{(a-c)(2+r) - (a-c)r + bS_0}{(2+r)} \\
\lambda^* &= \frac{2(a-c) + bS_0}{(2+r)} \tag{6}
\end{aligned}$$

Recall that, for a competitive market, Price = Marginal Benefit at the market clearing quantity level. Hence, substituting the values for X_0^* and X_1^* into the marginal benefit functions, we can calculate the (nominal) price in each time period:

$$\begin{aligned}
P_0^* &= a - b(X_0^*) \\
&= \frac{a(2+r) - (a-c)r - bS_0}{2+r} \\
P_0^* &= \frac{2a + rc - bS_0}{2+r} \tag{7}
\end{aligned}$$

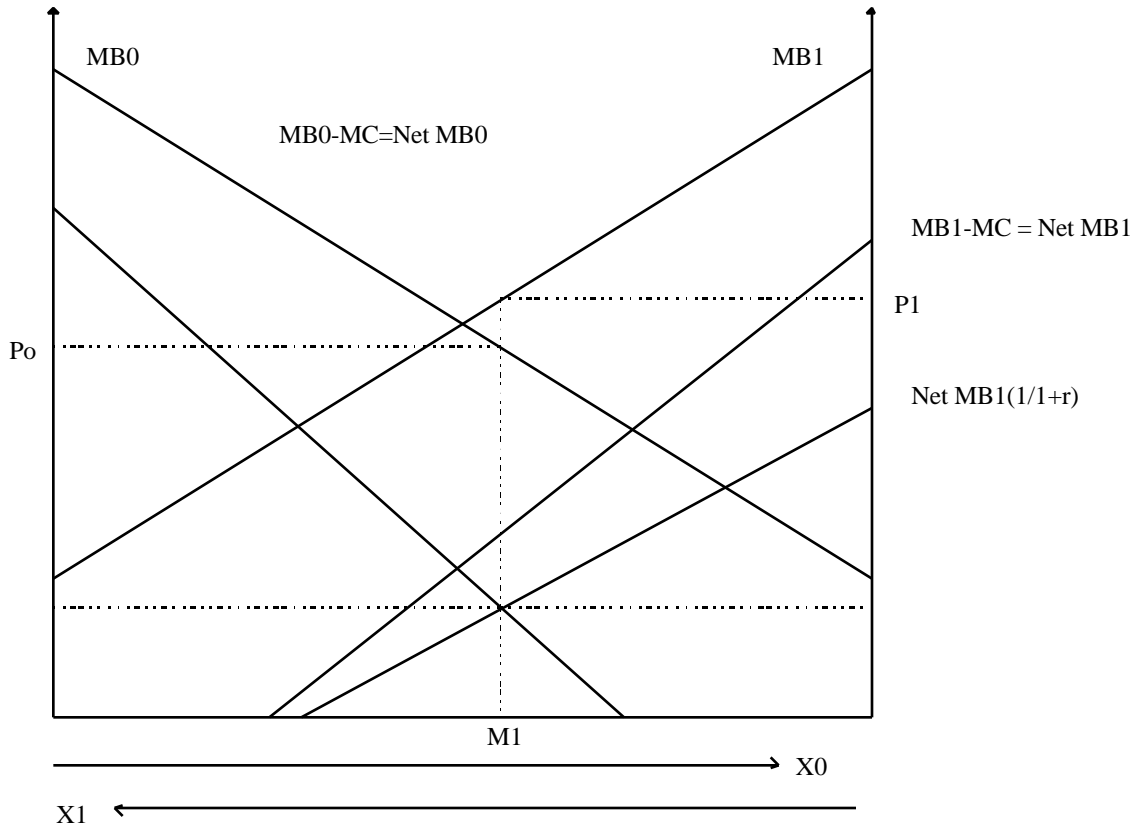
$$\begin{aligned}
P_1^* &= a - b(X_1^*) \\
&= \frac{a(2+r) + (a-c)r - bS_0(1+r)}{2+r} \\
P_1^* &= \frac{(1+r)[2a - bS_0] - rc}{2+r} \tag{8}
\end{aligned}$$

Using (7) and (8), we can show that the initial price, P_0 , is smaller when the initial resource stock is larger or when the discount rate is larger. Note that this is implied by the decreasing value of price in period zero with an increase in initial stock or a decrease in the interest rate:

$$\frac{dP_0^*}{dS_0} = \frac{-b}{2+r} < 0, \text{ and}$$

$$\begin{aligned}\frac{dP_0^*}{dr} &= \frac{c}{2+r} - \frac{[2a+rc-bS_0]}{(2+r)^2} \\ &= \frac{-[2(a-c)-bS_0]}{(2+r)^2} < 0\end{aligned}$$

Figure 12.1: Effects of Extraction Costs



Other Considerations

The extraction cost model can be modified to accommodate other specific applications. For example, in the case of a non-replenishable ground water aquifer, where you have additional costs associated with treatment and shipment, the price of water would reflect the sum of the various other marginal costs:

$$\begin{aligned} \text{water price} = & \text{user cost} \\ & + \text{marginal extraction (pumping) cost,} \\ & + \text{marginal shipment (conveyance) cost,} \\ & + \text{marginal treatment cost,} \end{aligned}$$

Hence, our model would predict that the existence of these costs would cause the price of water to grow at less than the rate of interest, since royalties will increase at the rate of interest, where $Royalty = Price - Pumping - Conveyance - Treatment$, (per unit).

Two-Period Nonrenewable Resource Model with Open Access

We have found the socially optimal allocation of extraction of a nonrenewable resource stock over two periods, both with and without extraction costs, but *will a competitive market necessarily achieve this socially optimal allocation?*

If the resource industry is competitive and there is **open access** to the resource, the answer is no. In a competitive industry under open access, firms race to extract the resource. Such a case leads firms to *operate as if the user cost of mining the resource is zero*. They realize that the tradeoff that they are making is not between how much to extract in the initial period and how much to extract in later periods, but rather how much they extract in the initial period and how much *other* firms extract in the initial period.

Open access resources yield a “tragedy of the commons mentality”, in which firms think that if they do not extract a profitable unit today, someone else will beat them to it. Thus, under open access, a firm will not compare marginal benefit today to MB tomorrow, because there is no assurance that anything will be resource left over for tomorrow. Instead, they will extract a marginal unit until $MB_0 = MC$, as if they were operating in a static model with only a single period.

To mathematically represent the competitive, open access situation, firms enter the industry until price falls to minimum average mining cost. That is, until static profit is driven to zero, where:

$$\pi = PX - C(X) = 0 \quad P = \frac{C(X)}{X} = AC(X)$$

In our example where we have constant marginal mining costs, average mining cost equals marginal mining cost ($c = AC$), so that minimum average mining cost is just equal to c . Thus, the total amount extracted by the industry in the initial period, X_0 , will be determined by:

$$(10) \quad P_0 = B_X(X_0) = c.$$

Let's assume for our analysis the following linear market demand, $B_X(X) = a - bX$. The industry *would like to* extract the following open-access amount:

$$a - bX = c \quad X_{OA} = \frac{a - c}{b} \quad (11)$$

- **If demand is satiated in the initial period** ($X_0^* \leq S_0$), then X_0^* is in fact extracted in the initial period, and price falls to c ($P_0 = c$). The remaining stock ($S_1 = S_0 - X_0^*$) is simply ignored.
- **If demand is nonsatiated in the initial period** ($X_0^* > S_0$), then we must set $X_0^* = S_0$, because it is impossible to extract more than the total initial stock. Price falls to $P = a - bS_0$

In either case, if there are n firms in the industry, each firm would extract X_0^*/n and earn zero economic profits. A comparison of the optimal and open access level of extraction shows that when there is competitive, open access to a nonrenewable resource, there will be inefficient over-extraction of the resource in the initial period.

Policies to Correct Open Access Market Failures

Figure 12.2 can also be used to identify the decreased net benefits in the two periods if open access exists as opposed to established ownership. By measuring total areas under MB1 and MB, under both circumstances, we see that open access leads to lower benefits.

We can suggest three relatively effective measures for the government to correct open access market failures. These policies include (1) an output tax on the resource, (2) limitations on total mining by issuing permits, and (3) establishment of property rights to the resource for producers. While the last option appears to be the most efficient, issues of distribution of rights make the policy rather controversial.

Output Tax

An optimal output tax, t^* , would be set to equal the calculated user cost, . In order to reduce mining of the resource, this additional cost to producers will make it less profitable to produce beyond the social optimum.

- The optimal tax can be calculated for the two-period case, where $P(X) = a - bX$, and $C(X) = cX$:

$$t^* = \frac{2(a - c) + bS_0}{(2 + r)}$$

So that the outcome of the competitive, static maximization problem in period 0 is

$$Max_{X_0} \pi = P_0X_0 - cX_0 - \frac{[2(a - c) - bS_0]X_0}{(2 + r)}$$

with FOC:

$$\pi_{X_0} = (a - bX_0) - c - \frac{[2(a - c) - bS_0]}{(2 + r)} = 0$$

where the substitution can be made into the FOC for $P = a - bX_0$. The FOC can be solved for X_0^* :

$$X_0^* = \frac{(a - c)(2 + r) + 2(a - c) + bS_0}{b(2 + r)}$$

$$= \frac{(a - c)r + bS_0}{b(2 + r)}$$

as we calculated before for the social optimum. Remember that you can plug in the value for optimal initial resource use and repeat the calculations to solve for optimal tax under open access.

Issuing Permits

The government can begin determining the amount to be mined each period by asking competitive producers to bid for mining rights. As we would suspect, per unit price for the right to mine will be . Yet another way to reach the correct level of mining would be to calculate optimal levels to be harvested each period, and issue free tradable permits to producers.

Property Rights

Establishment of property rights for competitive producers. As mentioned earlier, there are more complicated distribution issues to deal with in this type of policy. However, the greatest advantage to establishing ownership is that enforcement of the policy in future periods is less tedious task. Additionally, unless extraction costs depend on stock levels, an extra tax may be necessary.

Two-Period Nonrenewable Resource Model with Monopoly (Optional)

The monopoly resource owner has a different objective function than society, since the monopolist seeks to take advantage of demand conditions in order to make greater levels of profit. The objective function for the monopoly owner is:

$$\max_{X_0, X_1} \text{NPV}(\pi) = \left[P_0(X_0) X_0 - C(X_0) \right] + \frac{\left[P_1(X_1) X_1 - C(X_1) \right]}{1 + r}$$

subject to: $S_0 = X_0 + X_1$

Recall that $B_X(X_t) = P_t$. Making this substitution:

$$\begin{aligned} \max_{X_0, X_1} \text{NPV}(\pi) &= \left[B_x(X_0) X_0 - C(X_0) \right] + \frac{\left[B_x(X_1) X_1 - C(X_1) \right]}{1+r} \\ \text{subject to: } S_0 &= X_0 + X_1 \end{aligned}$$

Introducing a Lagrangian multiplier, λ , the monopoly's problem becomes:

$$\max_{X_0, X_1, \lambda} = \left[B_x(X_0) X_0 - C(X_0) \right] + \frac{\left[B_x(X_1) X_1 - C(X_1) \right]}{1+r} + \lambda (S_0 - X_0 - X_1)$$

F.O.C's: (1) $dL/dX_0 = B_x(X_0) + B_{xx}(X_0)X_0 - MC(X_0) - \lambda = 0$

(2) $dL/dX_1 = [B_x(X_1) + B_{xx}(X_1)X_1 - MC(X_1)] / [1+r] - \lambda = 0$

(3) $dL/d\lambda = S_0 - X_0 - X_1 = 0$.

Note that marginal revenue $MR(X_t) = B_x(X_t) + X_t B_{xx}(X_t)$.

Hence, we find that $MR(X_t) = MC(X_t) + (1+r)^t$

where all other terms besides the MR function are the same. The only change to the monopoly case is to equate $MR = MC + (1+r)^t$. We simply replace MB with MR to get the expressions that determine monopoly behavior.

Let's consider an example. To focus on the effects of monopoly, we assume simple linear demand, $B_x(X) = a - bX$, and zero extraction costs. A monopoly seeks to maximize the NPV of profits:

$$\begin{aligned} \max_{X_0, X_1} \text{NPV}(\pi) &= \left[(a - b X_0) X_0 \right] + \frac{1}{1+r} \left[(a - b X_1) X_1 \right] \\ \text{subject to : } S_0 &= X_0 + X_1 \end{aligned}$$

Introducing a Lagrangian multiplier, λ , the monopoly's problem becomes:

$$\max_{X_0, X_1, \lambda} L = \left[(a - b X_0) X_0 \right] + \frac{1}{1+r} \left[(a - b X_1) X_1 \right] + \lambda (S_0 - X_0 - X_1)$$

F.O.C's:

- (1) $a - 2bX_0 - \lambda = 0$
- (2) $(a - 2bX_1)/(1 + r) - \lambda = 0$
- (3) $S_0 - X_0 - X_1 = 0$.

Solving the system of F.O.C.'s, we find that:

$$X_0^M = \frac{2bS_0 + ra}{2b(2+r)} \quad X_1^M = \frac{2bS_0(1+r) + ra}{2b(2+r)}$$

Recall the social welfare maximizing outcome:

$$X_0^* = \frac{(a - c)r + bS_0}{b(2+r)}, \quad X_1^* = \frac{bS_0(1+r) + (a - c)r}{b(2+r)}$$

Comparing the monopoly outcome with the social welfare maximizing outcome in which $c = 0$, we find that monopoly leads to underextraction (and thus higher prices) in the initial period and overextraction (and thus lower prices) in the later period.

Discussion of Variables

In a static model, we usually set the FOC equal to zero, while in a dynamic model, we now set the static FOC = user cost.

Under perfect competition, zero interest rate gives $MB - MC = 0$. While $MB_i - MC_i = (1+r)^i$ holds in a dynamic model. Now $(MB-MC)$ increases at the rate of interest over time. For a monopoly situation, a static system leads to $MR - MC = 0$. Including interest rate over time changes the solution to $MR_i - MC_i = (1+r)^i$ in a dynamic model. This way, $(MR-MC)$ increases at the rate of interest over time.

So, now let us say we also have externalities, i.e., the case of the polluting mine

- When we also have pollution, the social optimal rate of extraction would be found where $MB_i - MC_i - MEC_i = (1+r)^i$; that is (MB-MS) increases at rate = r.

When $MC(X_t) = 0$, we find that marginal revenues increase at a rate equal to the interest rate. Thus, a monopoly extracts less in earlier time periods and more in later time periods. As a result, prices are initially higher under monopoly but grow at a slower rate over time. In the second period, the monopoly provides a greater amount of the resource in order to deplete all of the final stock and meet the constraint. *Hence, the second period price is actually lower under a monopoly industry structure than under a competitive structure.*

Two-Period Nonrenewable Resource Model with Changing Demand

Assume $B(X_t) = [(1+n)^t][a^*(X_t)^{0.5}]$, $B_x(X_t) = [(1+n)^t][0.5*a^*(X_t)^{-0.5}]$ where **n is the "growth rate of demand"**, perhaps representing increased population. Assume extraction costs are zero.

Now the socially optimal allocation of resource extraction over two periods is found by solving:

$$\max_{X_0, X_1} SW(X_0, X_1) = a(X_0)^{0.5} + \frac{1}{1+r} (1+n) a(X_1)^{0.5}$$

subject to: $X_1 = S_0 - X_0$.

From the F.O.C.'s, we find that: $\frac{X_0}{X_1} = \frac{1+r}{1+n}$

$$X_0 = S_0 \frac{(1+r)^2}{(1+n)^2 + (1+r)^2}$$

$$X_1 = S_0 \frac{(1+n)^2}{(1+n)^2 + (1+r)^2}$$

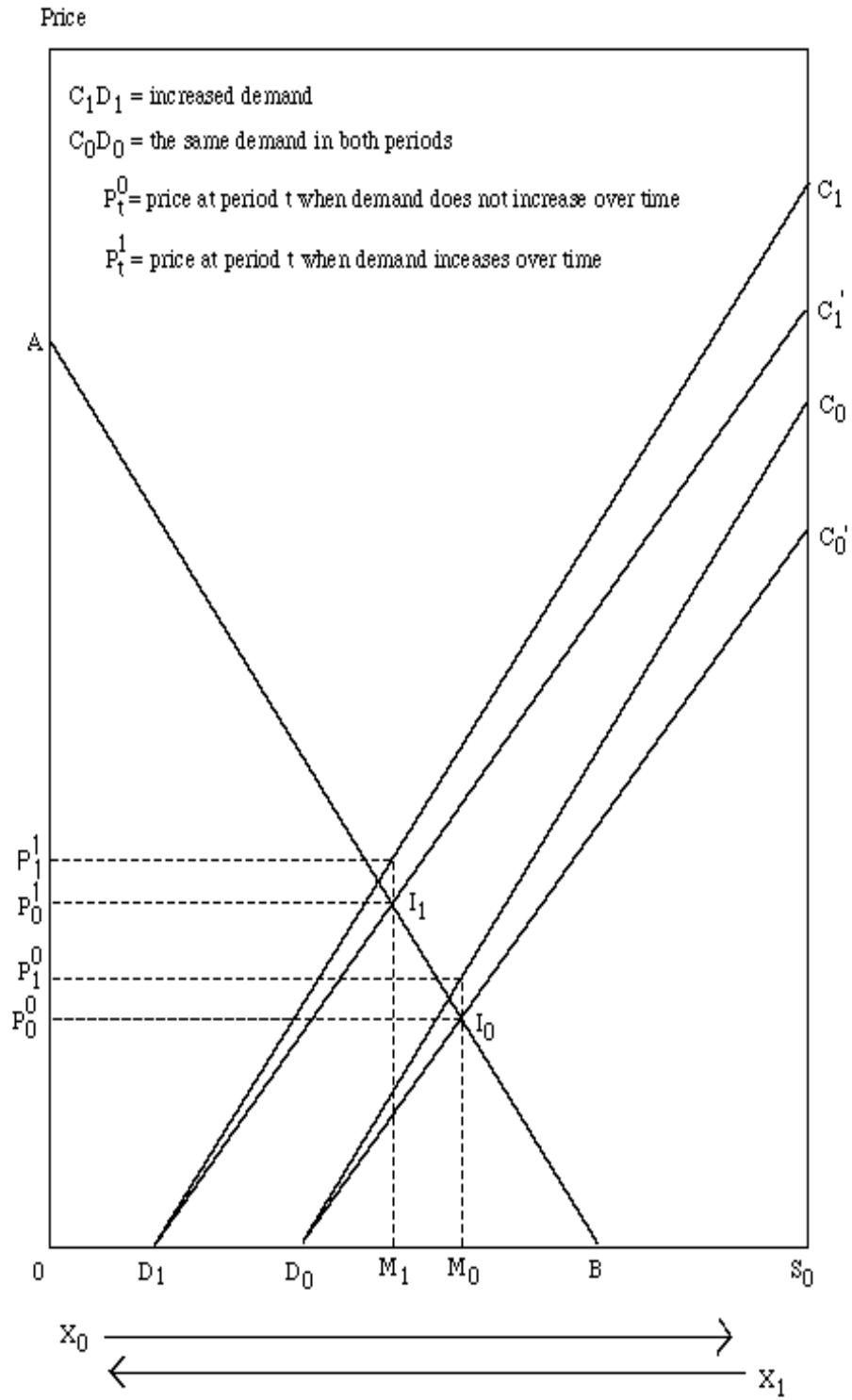
and

$$P_0 = \frac{a}{2(1+r)} \sqrt{\frac{(1+r)^2 + (1+n)^2}{S}} \quad \text{and} \quad P_1 = \frac{a}{2} \sqrt{\frac{(1+r)^2 + (1+n)^2}{S}}.$$

Thus, compared with the constant-demand case {in which $B(X) = aX^{1/2}$ }, an increase in n will reduce X_0 and increase X_1 . In addition, an increase in n will cause both P_0 and P_1 to "jump" up, but (assuming zero extraction costs) the *rate* of price increase over time will remain:

$$\frac{P_1 - P_0}{P_0} = r.$$

Figure 12.3: Effects of Changing Demand



Optimal management of nonrenewable resources in case with growing demand

Two-Period Nonrenewable Resource Model with a Backstop Technology

Assume a new technology will make an alternative resource available in the future period ($t = 1$). Let Z represent the output level of the alternative resource, which is a perfect substitute for X . Assume the marginal cost of the alternative resource is a constant, m .

An example of a backstop technology might be one in which the nonrenewable resource is fossil fuel and the backstop technology is solar power. In this case, the marginal cost of solar power as a fuel source may be relatively high, yet a switch to solar energy, or some other alternative power source, must inevitably occur since the supply of fossil fuel is finite.

In the history of energy many backstop technologies have been introduced. The basic modification of our earlier results is that we will find when resource owners know a new technology will soon be introduced, the extraction rate will accelerate, so that more of the resource is consumed in earlier periods. This is because having a backstop technology is like having a larger stock of the resource. As S_0 increases, prices decrease in both periods.

The social optimization problem with a backstop technology is:

$$\max_{X_0, X_1} SW(X_0, X_1) = [B(X_0) - C(X_0)] + \frac{1}{1+r} [B(X_1 + Z_1) - C(X_1) - m Z_1]$$

subject to: $S_0 = X_1 + X_0$. (assume there is no constraint on the availability of Z)

The Lagrangian problem becomes:

$$\max_{X_1, X_0, Z, \lambda} L = B(X_0) - C(X_0) + \frac{1}{1+r} [B(X_1 + Z_1) - C(X_1) - m Z_1] + \lambda [S_0 - X_0 - X_1]$$

The F.O.C.'s are:

$$(1) \quad \frac{\partial L}{\partial X_0} = B_X(X_0) - C_X(X_0) - \lambda = 0.$$

$$(2) \quad \frac{\partial L}{\partial X_1} = \frac{1}{1+r} [B_X(X_1 + Z_1) - C_X(X_1)] - \lambda = 0.$$

$$(3) \quad \frac{\partial L}{\partial Z} = \frac{1}{1+r} [B_Z(X_1 + Z_1) - m] = 0.$$

$$(4) \quad \frac{\partial L}{\partial \lambda} = S_0 - X_0 - X_1 = 0$$

F.O.C. (1) states that, at the optimum:

$$(5) \quad P_0 = B_X(X_0) = C_X(X_0) +$$

Output price

= Marginal benefit

= Marg. Extraction Cost + User Cost of Consumption in Period 0.

F.O.C. (2) states that, at the optimum:

$$(6) \quad P_1 = B_X(X_1 + Z) = C_X(X_1) + (1 + r)$$

Output price

= Marginal benefit

= Marg. Extraction Cost + User Cost of Consumption in Period 1.

F.O.C. (3) reduces to:

$$(7) \quad P_1 = m$$

Thus, the price of X in period 1 equals the price of Z in period 1. From equations (5) and (6), we find:

$$P_1 - C_X(X_1) = (1 + r) (P_0 - C_X(X_0)).$$

$$(8) \quad P_0 = C_X(X_0) + \frac{1}{1+r} [m - C_X(X_1)] \text{ and}$$

$$(9) \quad \text{User Cost} = P_0 - C_X(X_0).$$

From these relationships we can confirm the following points.

For $C_X = 0$:

- If $X_1 > 0$, $P_0 = \frac{m}{1+r}$. (i.e., the price in the first period is smaller than the cost of the backstop technology).
- As m becomes smaller, X_0 increases, X_1 declines, and Z increases.
- If m is sufficiently low (below the extraction cost of X) the resource, X , will be used only at the first period, with $P_0 > m$. In this case some of the exhaustible resource may be left unused.

Numerical Example

$$B(X) = a - \frac{b}{2}X^2 \quad C(X) = cX$$

$$B_X(X) = a - bX \quad C_X = c$$

From (6), $P_1 = m$.

$$\text{From (7), } P_0 = c + \frac{1}{1+r} [m - c] = a - bX_0 \quad X_0 = \frac{1}{b} \left[a - \frac{m}{1+r} - \frac{rc}{1+r} \right] < S$$

$$X_1 = S - X_0; \quad Z = \frac{a - bX_1 - m}{b},$$

Given $a = 20$, $b = .5$, $c = 0$, $S = 50$, $r = .2$:

$$\text{If } m = 6, \quad X_0 = 2 \left[20 - \frac{6}{1.2} \right] = 30 \quad P_0 = 5 \quad X_1 + Z = 28 \quad X_1 = 20 \quad Z = 8.$$

$$\text{If } m = 7.2, \quad X_0 = 2 \left[20 - \frac{7.2}{1.2} \right] = 28 \quad P_0 = 6 \quad X_1 + Z = 25.6 \quad X_1 = 22 \quad Z = 3.6.$$

Higher m reduces X_0 and Z increases prices and X_1 . For example, if $m = 6$ and $c = 3$,

$$X_0 = 2 \cdot 20 - \frac{6}{1.2} - \frac{.2}{1.2} \cdot 3 = 29 \quad P_0 = 5.5 \quad = 2.5 \quad X_1 + Z = 28 \quad X_1 = 21 \quad Z = 7.$$

The Effect of Uncertainty of Backstop Technology

It is important to consider the reliability of the backstop technology anticipated within the model we are working with. The more certain that a backstop technology will be made available, the more resource owners mine in the present with current technology. The less certain the backstop technology, the less it constrains the behavior of the resource owner so that less is extracted in the current period and the solution approaches that with no backstop technology on the horizon.

Sketch of An "n-Period" Model of Nonrenewable Resources (Optional)

Initial Assumptions

- (1) Assume zero costs.
- (2) Assume T time periods.
- (3) Assume competitive market for nonrenewable resource.

Objective function

$$\max_{X_0, X_1, \dots, X_T} \text{NPV}(X_0, X_1, \dots, X_T) = B(X_0) + \frac{B(X_1)}{1+r} + \frac{B(X_2)}{(1+r)^2} + \dots + \frac{B(X_T)}{(1+r)^T}.$$

Equation of motion constraints: $S_{t+1} - S_t = X_t$, $t = 0, T-1$.

We can combine equation of motion constraints into a single constraint:

$$X_1 + X_2 + \dots + X_T = S_0.$$

Lagrangian problem becomes:

$$\max_{X_0, X_1, X_2, \dots, X_T, \lambda} L = B(X_0) + \frac{1}{1+r} B(X_1) + \frac{1}{(1+r)^2} B(X_2) + \dots + \frac{1}{(1+r)^T} B(X_T) + \lambda (S_0 - X_0 - X_1 - \dots - X_{T-1})$$

or,

$$= \text{Max.}_{X_i} \sum_{i=0}^T \frac{1}{1+r} B(X_i) + \lambda \left(S_0 - \sum_{i=0}^T X_i \right)$$

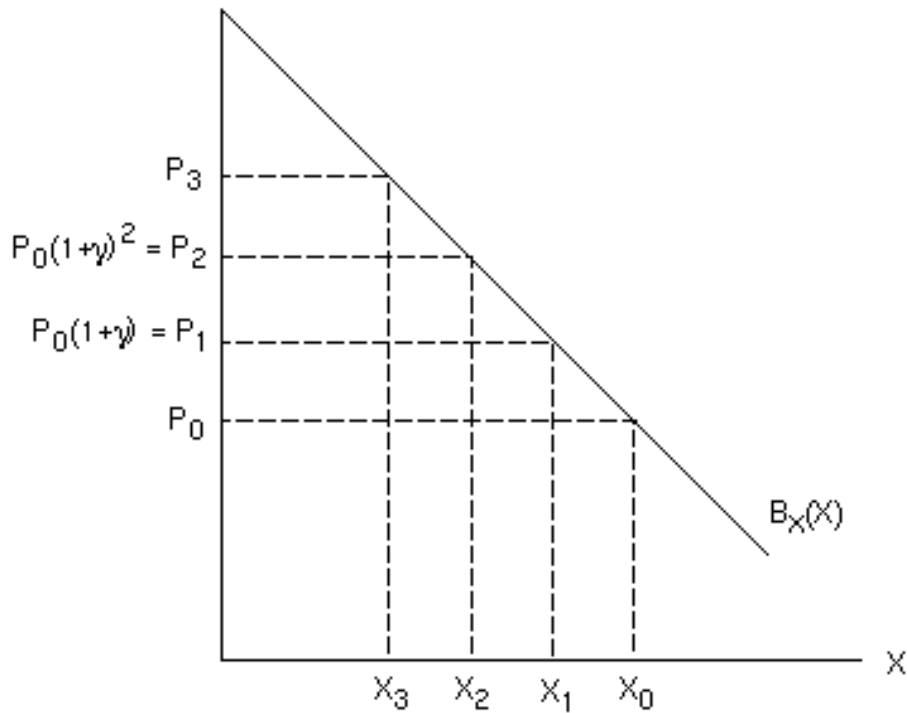
We can find the FOC's for the preceding Lagrangian problem in the customary manner.

- After finding the FOC's, we can rearrange them to derive the following optimal decision rules:

$$\begin{aligned}
 B_x(x_0) &= \\
 B_x(x_1) &= (1 + r) \\
 B_x(x_2) &= (1 + r)^2 \\
 &\cdot \\
 &\cdot \\
 &\cdot \\
 B_x(x_t) &= (1 + r)^t
 \end{aligned}$$

where $X_0 + X_1 + X_2 + \dots + X_T = S_0$ by the FOC of the constraint.

Figure 12.4:
Price Rises at the Rate of Interest and Extraction Decreases Over Time



The optimal decision rules are expressed in terms of marginal benefits $B_x(X_t)$. Recall that marginal benefit is equal to price for a

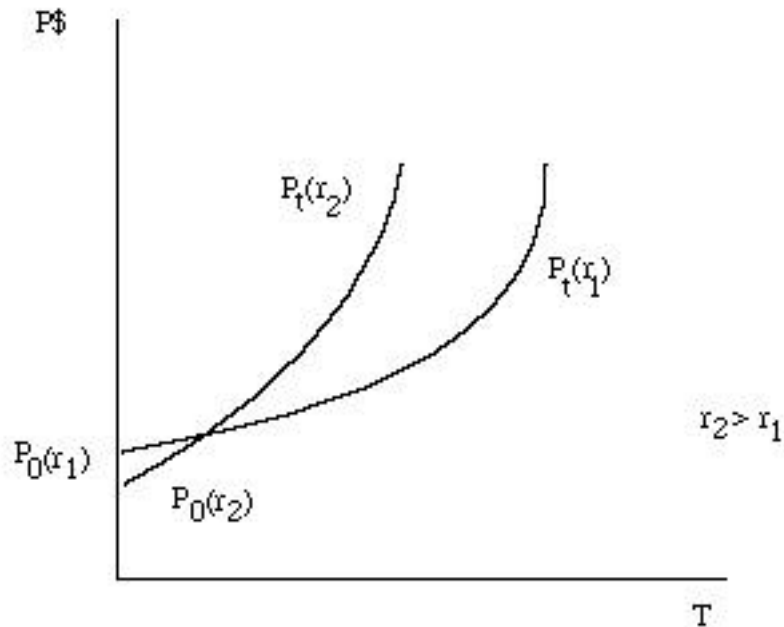
competitive industry. Hence, substituting price for $Bx(X_t)$ in the optimal decision rules derived on the previous page, we find that:

$$P_t = P_0(1 + r)^t,$$

or, $\frac{P_t - P_{t-1}}{P_{t-1}} = r$, t , i.e., the price rises at the rate of interest

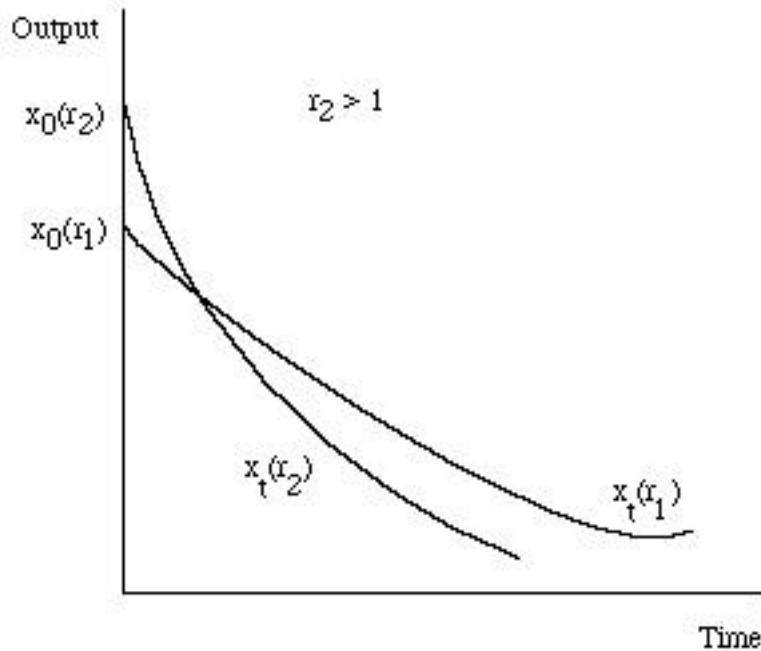
Note that as price, P_t , rises over time, the amount of the resource that is extracted in each period, X_t , will decline over time accordingly.

Figure 12.5:
Higher interest rates lead to faster price increases but lower initial prices.



When r is larger, more is extracted in earlier time periods and less is extracted in later time periods. As a result, prices rise faster over time, but the *initial price is lower* because the *initial level of extraction is larger*. A larger extraction in the initial period drives down the market price for the resource in the initial period.

Figure 12.6:
Higher interest rates lead to "faster exploitation" of resource stock.



Summary of Nonrenewable Resource Model Results

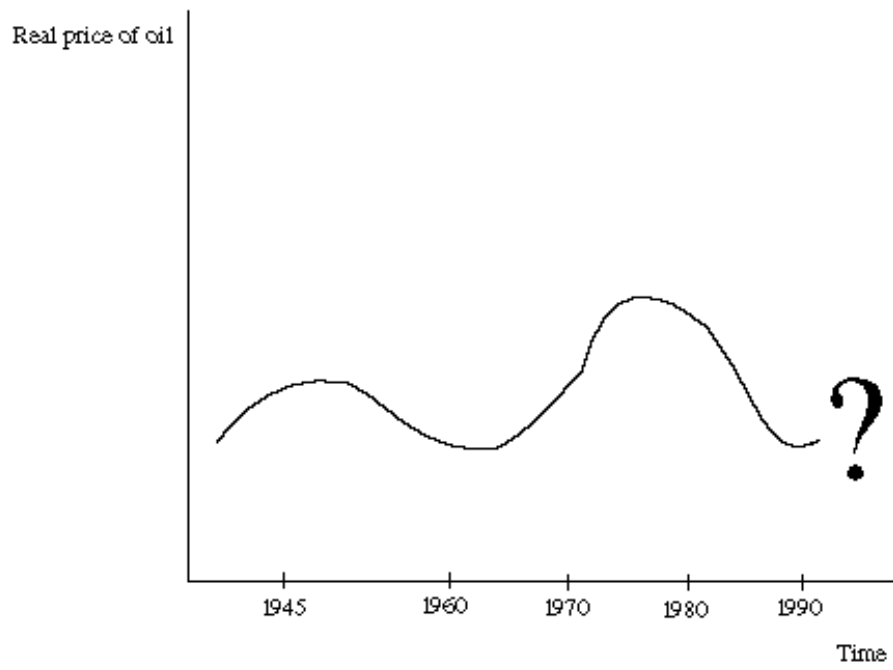
Present price of exhaustible resource (P_0):

- Declines with r .
- Declines with extraction cost.
- Increases as demand increases.
- Decreases as new stocks are discovered.
- Declines as new extraction technologies are developed.
- Declines as backstop technologies are developed.
- Increases as industry gets more monopolistic.
- Declines as alternative products get cheaper.

Example: The Case of Oil

Oil is an exhaustible resource, and its price dynamics are consistent with theory. During 1940-1960, prices went down as new fields were discovered. In 1973, price went up due to the formation of the OPEC oil cartel, which has many characteristics in common with monopoly ownership. Reduction in demand, new oil discoveries, and quantity dumping by several OPEC members led to further price declines in the 1980s.

Figure 12.7: The Price of Oil Over Time



Price went up because of cartel, increase of demand, and supply reduction.

Price went down because of reduction in demand, new discoveries, and alternative energy