

# Chapter #11: Natural Resources and Dynamic Systems

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## **General Overview**

**Resource Economics** addresses the allocation of natural resources over time. It does so using models of dynamic systems. A **model** is collection of variables and parameters and the equations that show how the variables and parameters are related to one another. A **system of equations** is a model containing two or more interrelated equations.

A **dynamic system** is a system that contains **time** as one of the variables. A dynamic system is a model that attempts to capture the important changes in, and changing interrelationships among, variables and parameters over time. We will use simple models of dynamic systems to study the inter-relationships among markets, natural resources, and the environment over time. The reason why it is important to do this is that there are some important types of market failure that only show up over time in a dynamic system.

But before we jump into modeling dynamic systems, let's review what we have done so far in this course from a modeling perspective. So far, we have conducted our analyses using **static models**. A model that addresses how variables interrelate at a single point in time is called a **static model**. In economics, we often use static models to examine a special point in time, namely the point in time during which an economic system is in equilibrium. These models are often called **equilibrium models**. The basic model of competitive supply and demand is an equilibrium model, because it predicts market price and market quantity when the market is in equilibrium.

We have used static models to determine the market equilibrium of an economic system, to determine the socially optimal equilibrium of the economic system and then to compare the two equilibria. When the two equilibria did not coincide, we explored some of the policies that could be

used to move the equilibrium from the inefficient market equilibrium to the socially-optimal equilibrium. As we have seen, static models can be quite useful in determining equilibria, comparing equilibria, and suggesting ways to achieve a more efficient equilibrium.

However, some types of market failure occur "between" equilibria, i.e., "on the way" from one equilibrium to the next. Resource economics uses models of dynamic systems to develop policies that can be used to correct such market failures, market failures that manifest themselves over time. Often, these market failures exist because private markets extract or harvest natural resources either too quickly or too slowly relative to the socially optimal rate of use.

### **Key Terms and Components of Dynamic Systems**

***Variable:*** Recall that a variable is an item of interest that may take on different values.

***Parameter:*** Recall that a parameter is a quantity that is constant at the location of interest, or over the time interval of interest. Examples include the interest rate, supply and demand elasticities in econometric equations.

***System:*** A collection of variables and parameters linked together through two or more equations.

***Static Systems:*** Systems that do not contain time as a parameter; such systems do not evolve over time, rather, they usually represent equilibrium situations.

***Dynamic Systems:*** Systems that contain *time* as a parameter; such systems "evolve" over time.

***State Variable:*** A state variable describes the status, or "state of being," of one of the variables in the system. There may be one or more state variables in a system.

***Initial Conditions:*** Values that the state variables take on at the beginning of the time period of interest.

***Control Variable:*** A control variable is a variable that is under the control of some individual or group. There may be one or more control variables in a system. Conceptually, individuals (or groups) manipulate control variables in an attempt to meet objectives (in the case of profit maximization, one control variable is quantity produced).

***Random variables (noise variables):*** Uncontrolled variables (for example, weather) which can assume several values with certain probabilities.

***Constraints:*** Equations (or inequalities) which limit the values that state variables or control variables can take on.

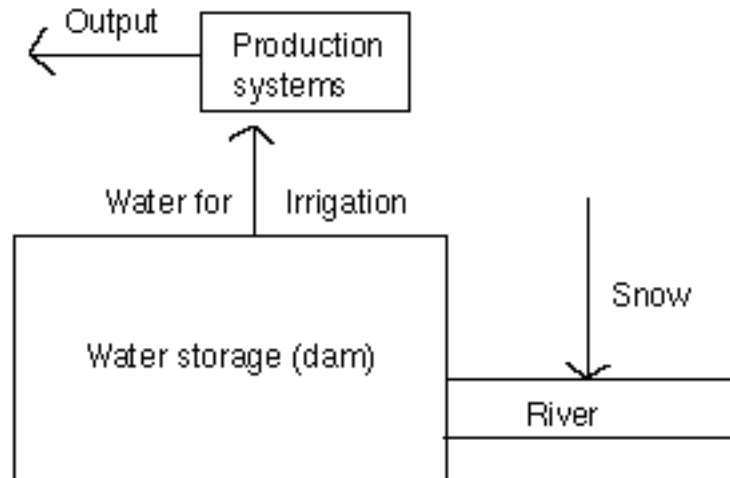
***Equation of Motion:*** An equation in a dynamic system that describes the relationship between *time* and the variables in the system. An equation of motion can be thought of as a *constraint* on the system. It constrains the variables in the system to interact with time in a particular way. The equation of motion describes how a variable changes over time.

***Solution of a System:*** The solution of a static system is a set of *values* for the state variables, where the values are expressed in terms of the system parameters, such that all equations in the system are satisfied. The solution of a dynamic system is a set of *equations*, where the equations are in terms of the system parameters, *including time*, such that all of the original equations in the system are satisfied. Thus, a dynamic system may have many solutions, depending on the specific initial conditions of the resource.

***Objective Function:*** An objective function is an equation that measures how well the system is attaining some goal or objective, usually a maximization or minimization objective. The objective function is expressed in terms of the *state variables*, *control variables*, and *parameters* of the system. For example, a policy objective might be to maximize the NPV of expected benefits generated by a dynamic resource system.

## Example of a Dynamic System

**Figure 11.1: Irrigation water system**



### ***State variables:***

- ( $S_t$ ): Denotes the level of a stock at time  $t$  (e.g., the quantity of water stored in the reservoir behind the dam at time  $t$ ).
- ( $U_t$ ): Uncontrolled inputs; inputs affecting the system but outside of our control, (e.g., rain, snow). Uncontrolled inputs may be random and vary over time.
- ( $Y_t$ ): Outputs; outcome of systems at time  $t$ ; (e.g., crops produced through a production system using irrigation water).

### ***Control variables:***

- ( $X_t$ ): Inputs whose magnitudes we can choose in our attempt to reach our objectives (e.g., the amount of water used for irrigation).

### ***Parameters:***

- ( $P$ ): Items of interest that can be taken as constant with respect to the problem at hand. Example: coefficients that measure the performance of some part of the system (e.g., the production elasticity of irrigated water, the type of production system  $S$ , that farmers use).

### ***Equation of motion:***

Next period water stock; this period water stock + rainfall - irrigation water:

$$S_{t+1} = S_t + U_t - X_t$$

This equation would be a *constraint* on irrigation system operations.

**Objective Function:**

$$\max_{X_t} NPV = \sum_{t=0}^T \frac{B_t[Y_t(P)] - C_t(X_t)}{(1+r)^t}$$

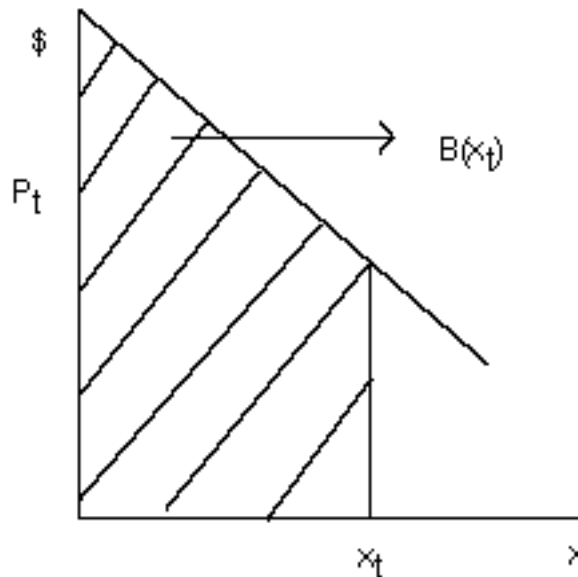
### Dynamic Models of Nonrenewable Resources

**Nonrenewable resources** are resources that have a finite stock and that do not grow naturally. For example, oil and minerals are nonrenewable resources, but trees and fish are renewable resources (because they grow/reproduce relatively quickly).

**Key Issues:**

- Determining optimal resource allocation and pricing.
- Sources of market failure and policies to correct market failure.

**Figure 11.2: A Two-Period Model of Nonrenewable Resources**



t = time (the initial period is denoted by t = 0; the future period is denoted by t = 1)

r = interest rate

S<sub>0</sub> = initial stock of nonrenewable resource

$X_t$  = control variable, the amount of the resource consumed in period  $t$   
 $B(X_t)$  = benefit of consuming  $X_t$

The extractable resource problem is an exercise in the economics of scarcity:

- **Scarcity:** Imposes an opportunity cost on using resources today. In a natural resource system, we refer to dynamic opportunity cost as a *user cost*.
- **User Cost:** The Present Value of foregone opportunity (e.g., if you use a unit of a natural resource today, you forego the opportunity to use it tomorrow)

The User Cost thus decreases as  $r$  increases:

- The higher the interest rate, the less valuable tomorrow's benefits and the smaller the opportunity cost of using more of the resource today.
- at  $r = \text{infinity}$ , resources left for tomorrow are worth nothing and user cost = 0.
- Similarly, when there is enough of the resource to go around, so that scarcity is not an issue, the user cost = 0 (demand is said to be satiated when resource scarcity is not an issue). In this case, the dynamic model yields the same outcome as two separate static models for period 0 and 1.

The use of discounting is important in determining the optimal extraction rate of a nonrenewable resource, because the revenue a resource owner receives in period 1 is not worth as much as the revenue received in period 0. Instead, the NPV of benefits in period 1 in terms of the current

period 0, are:  $NPV = \frac{1}{1+r} B(X_1)$

### Dynamic Efficiency: The Two Period Case

An allocation of resources is said to be dynamically efficient when it maximizes the NPV of benefits. This is really the same idea as in the static Lagrange problem:

$$\text{Max. } L = B(X) - C(X),$$

except that we now have the new twist:

- $B(X)$  is now a *stream of benefits* through time,

$$\text{e.g., } B(X) = B_0 + \frac{1}{1+r} B_1 + \frac{1}{1+r}^2 B_2 + \dots + \frac{1}{1+r}^N B_N$$

- $C(X)$  is now a *stream of costs* through time

For now, we **assume zero costs** are associated with consuming the resource. The objective function is:

$$\text{Max}_{X_0, X_1} \text{NPV} = B(X_0) + \frac{1}{1+r} B(X_1).$$

**Equation of motion (constraint):**

$$S_0 = X_0 + X_1. \text{ (all resource stock is used up).}$$

Note: by assuming  $X_0 + X_1$  *exactly* equals  $S_0$ , we are implicitly assuming **unsatiated demand**, i.e., that demand would exist for additional  $S$ ,  $S > S_0$ , if it were available. We will consider the case of **satiated demand** later.

Let's consider the energy of the sun to be a non-renewable resource. At some point in time, the sun explodes. The economic perspective is that it is a waste to have not used all of the energy of the sun before it exploded. For our purposes, we will assume the sun explodes at the end of two periods.

To find the optimal  $X_0$  and  $X_1$  (quantity use of sun's energy in period 0 and 1), we need to combine the objective function with any relevant constraints to form the optimization problem:

$$\text{Max}_{X_0, X_1} \text{NPV} = B(X_0) + \frac{1}{1+r} B(X_1)$$

$$\text{subject to: } S_0 = X_0 + X_1$$

The Lagrangian expression for this problem is (where  $\lambda$  is the Lagrange multiplier):

$$L = B(X_0) + \frac{1}{1+r} B(X_1) + \lambda (S_0 - X_1 - X_0).$$

To maximize the Lagrangian expression we find the FOC's:

$$(1) \quad \frac{dL}{dX_0} = B_x(X_0) - \lambda = 0$$

$$(2) \quad \frac{dL}{dX_1} = B_x(X_1) \frac{1}{1+r} - \lambda = 0$$

$$(3) \quad \frac{dL}{d} = S_0 - X_1 - X_0 = 0$$

This is a system of three equations (1), (2), and (3) in three unknowns  $X_0$ ,  $X_1$  and  $\lambda$ . The system can be solved for  $X_0$ ,  $X_1$  and  $\lambda$  in terms of the parameters of the system. An often useful step in this process is to set FOC (1) = FOC (2) and eliminate  $\lambda$  to obtain:

$$(4) \quad B_x(X_0) = \frac{1}{1+r} B_x(X_1)$$

- Then use (3) and (4) to solve for  $X_0$  and  $X_1$ , and, finally,
- Substitute  $X_0$  back into (1) to find  $\lambda$ .

We can find  $P_0$  and  $P_1$  by recalling that:

$$(5) \quad B_x(X_t) = MB \text{ of } X \text{ at time } t = P_t$$

Rearranging (4), we get:  $(1+r) B_x(X_0) = B_x(X_1)$

Substituting  $P_0$  for  $B_x(X_0)$  and  $P_1$  for  $B_x(X_1)$ , we find:

$$(1+r) P_0 = P_1, \text{ or}$$

$$\frac{P_1 - P_0}{P_0} = r$$

**Thus, we find that when dynamic efficiency is met** (i.e., the optimal quantities are being consumed in each period), **the price increases at the rate of interest.** In addition, from (1) and (5) we see that the shadow price of  $S_0$ ,  $\lambda$ , is equal to  $P_0$ . From (2) and (5) we see that the shadow value is also equal to the *present value* of  $P_1$ . In other words,  $\lambda = P_0 = P_1/(1+r)$ . Thus, the solution to the nonrenewable resource problem equates the NPV of benefits across all time periods in the horizon, which must be the case:



- If  $P_0 > P_1/(1+r)$ , the owner should extract more today; invest the money at  $r$ .
- If  $P_0 < P_1/(1+r)$ , the owner should leave more in the ground to extract tomorrow. The rate of return of holding resource stock in the ground is  $IRR > r$ .
- Therefore, in equilibrium, it must be the case that  $P_0 = P_1/(1+r)$ ; produce today until  $MB_0 = PV(MB_1)$

Remember, the intuition for  $\lambda$  is that  $\lambda$  is the user cost of the resource. The solution to the dynamic problem equates the user cost of extracting the resource across all time periods.

We'll try the following numerical example:

Suppose  $B(X) = a\sqrt{X}$

then  $B_x(X) = \frac{a}{2\sqrt{X}}$ .

noting that  $X_1 = S_0 - X_0$  from (3),

$X_0$  can be found by using  $B_x(X)$  with eqn's (3) and (4) :

$$\frac{a}{2\sqrt{X_0}} = \frac{a}{2(1+r)\sqrt{S_0 - X_0}} \quad \frac{S_0 - X_0}{X_0} = \frac{1}{(1+r)^2}$$

$$(6) \quad X_0 = S_0 \frac{(1+r)^2}{1+(1+r)^2}$$

Substitute  $X_0$  back into eqn (3) to find  $X_1$  :

$$(7) \quad X_1 = \frac{S_0}{1+(1+r)^2}$$

Substitute  $X_0$  back into  $B_x(X_0)$  to find :

$$(8) \quad P_0 = \frac{a}{2} \sqrt{\frac{1+(1+r)^2}{S_0(1+r)^2}}$$

From (6) and (7) we see that if  $S_0$  increases, then both  $X_0$  and  $X_1$  increase. If  $r$  increases, then  $X_0$  increases and  $X_1$  decreases. From (8) we see that if  $r$  increases, then  $P_0$  decreases.

- Suppose  $r = 0.1$ ,  $S_0 = 100$  and  $a = 10$ , then:

$$X_0 = 54.75, X_1 = 45.25, P_0 = 0.68 \text{ and } P_1 = 0.74$$

- If  $r$  increases to  $r = 0.5$ , then:

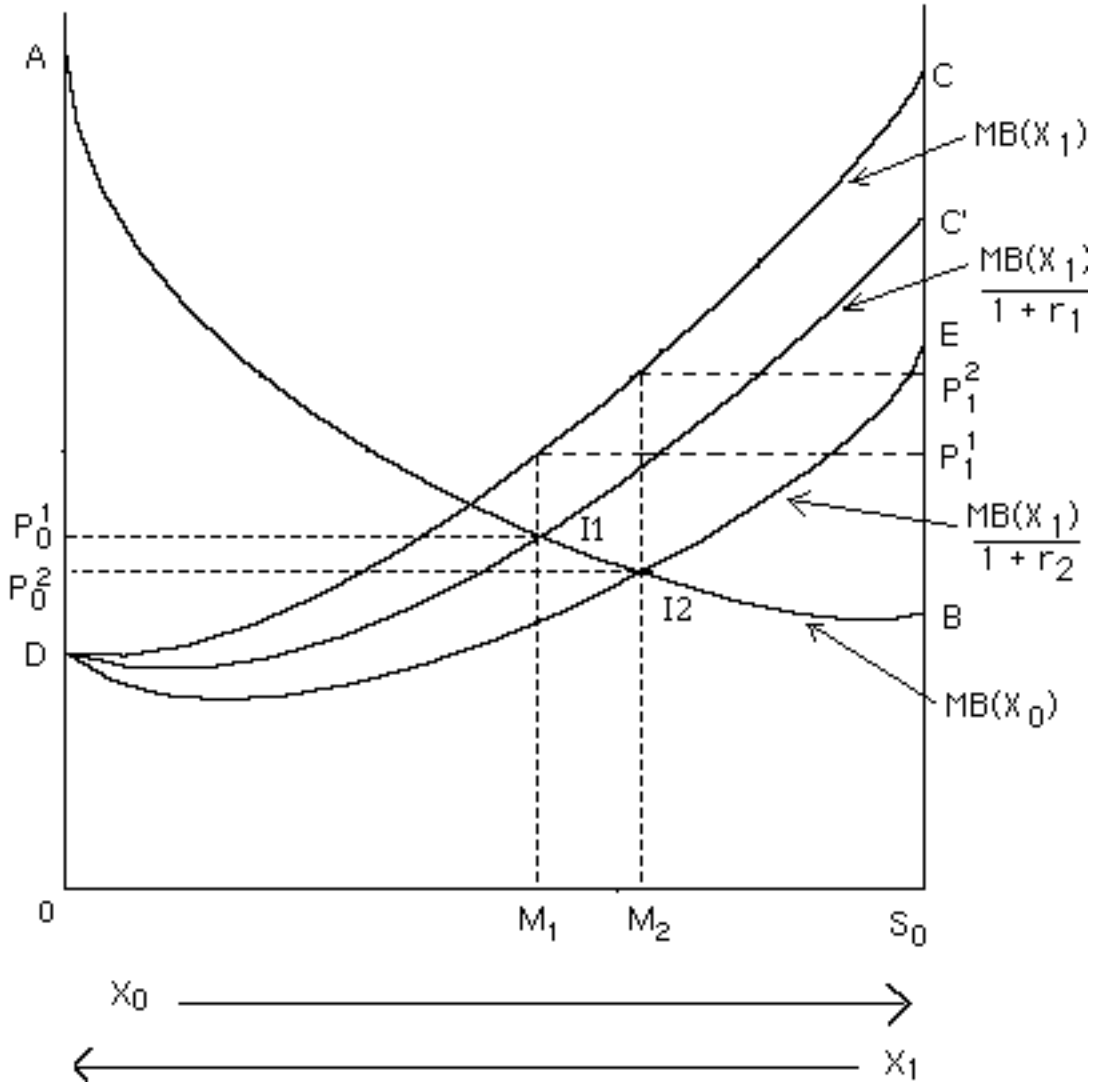
$$X_0 = 69.3, X_1 = 31.7, P_0 = 0.6 \text{ and } P_1 = 0.9$$

- If  $r$  increases to  $r = 1$ , then:

$$X_0 = 80, X_1 = 20, P_0 = 0.56 \text{ and } P_1 = 1.12$$

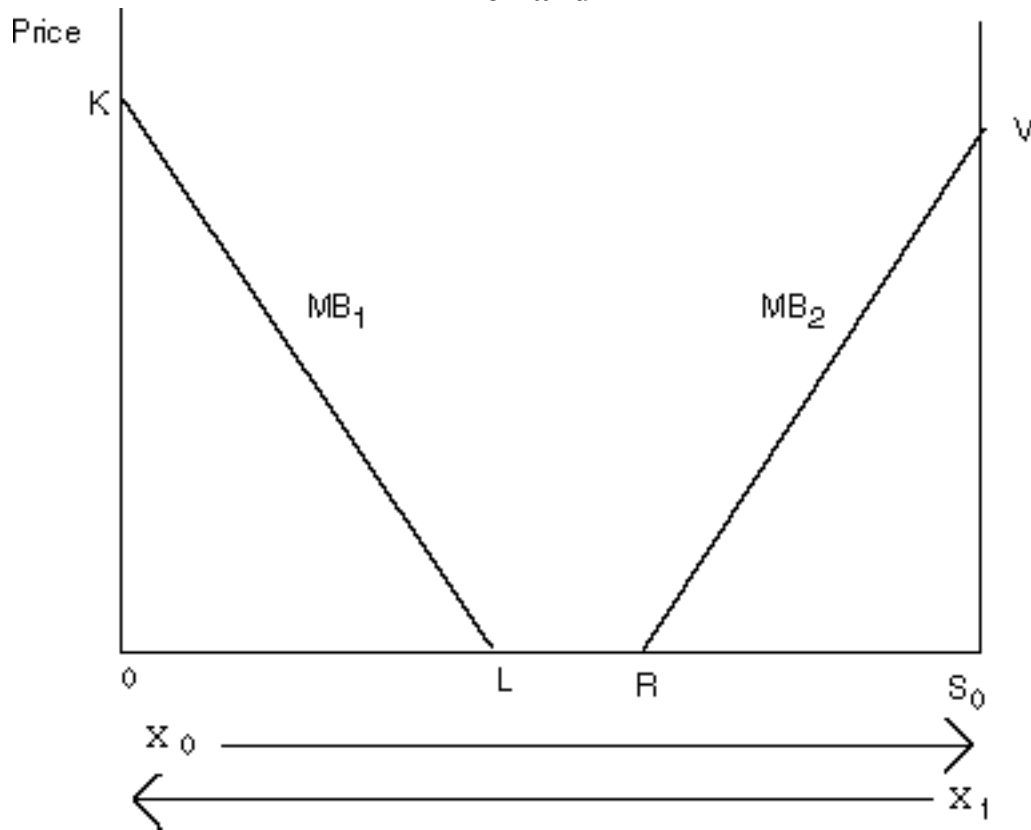
**Figure 11.3: Two-Period Non-renewable Resource Model with Unsatiated Demand**

Price



- For P's: superscript = discount rate; subscripts = time period.
- For I's, M's, subscripts = discount rate.
- $r_2 > r_1$  and  $I_1 < I_2$ .
- A lower discount rate implies:
  - $P_0^1 > P_0^2$  Higher price in the initial period.
  - $P_1^1 < P_1^2$  Lower price in the second period.
  - $M_1 < M_2$  Less resource is used in the initial period.

**Figure 7.4 - Two-Period Non-renewable Resource Model with Satiated Demand**



When  $S_0$  is so large that  $B_{x_0}$  and  $\frac{1}{1+r} B_{x_1}$  do not intersect at positive  $P$ , then:

- $X_0$  is solved for by setting  $B_X(X_0) = 0$
- $X_1$  is solved for by setting  $B_X(X_1) = 0$

This solution is identical to the solution of two individual static maximization problems, performed separately, in period 0 and period 1. Note that there is no user cost here, because the MB curves fail to intersect. That is, there is no scarcity in a nonrenewable resource model with satiated demand.