

Irreversibility and Option Value

dichotomous (or discrete) choices: when a certain parameter must take on either a value of zero or one.

For example: **Building a Dam** is a dichotomous choice, since you either build a dam or you don't. *It is not possible to build half a dam.*

Irreversibility: An irreversible action is one in which the effects of the action cannot be reversed, either absolutely or because the costs of doing so would be extremely high.

Option Value: The value of retaining the option to make an irreversible decision until a future period in time. In a resource development context, the value of keeping a resource in a state of preservation as an option to develop at a later point in time. *The increase in expected net benefits as a result of making a decision after the uncertainty is resolved.*

The Benefit-Cost Method of Project Evaluation:

The Benefits and Costs of a project are evaluated by the *expected present value*:

- $PV(B) = \int_0^T E[B(t)]e^{-rt} dt$ in continuous time
- $= \sum_{i=0}^T E[B_i(X)] \frac{1}{1+r}^i$ for some project X in discrete time
- $PV(C) = \sum_{i=0}^T E[C_i(X)] \frac{1}{1+r}^i$ for the project X

We Can Compare Projects Based on the Benefit-Cost Ratio:

- If $\frac{PV(B1)}{PV(C1)} > \frac{PV(B2)}{PV(C2)} > 1$, then Build Project 1.

Problems with the Cost-Benefit criteria:

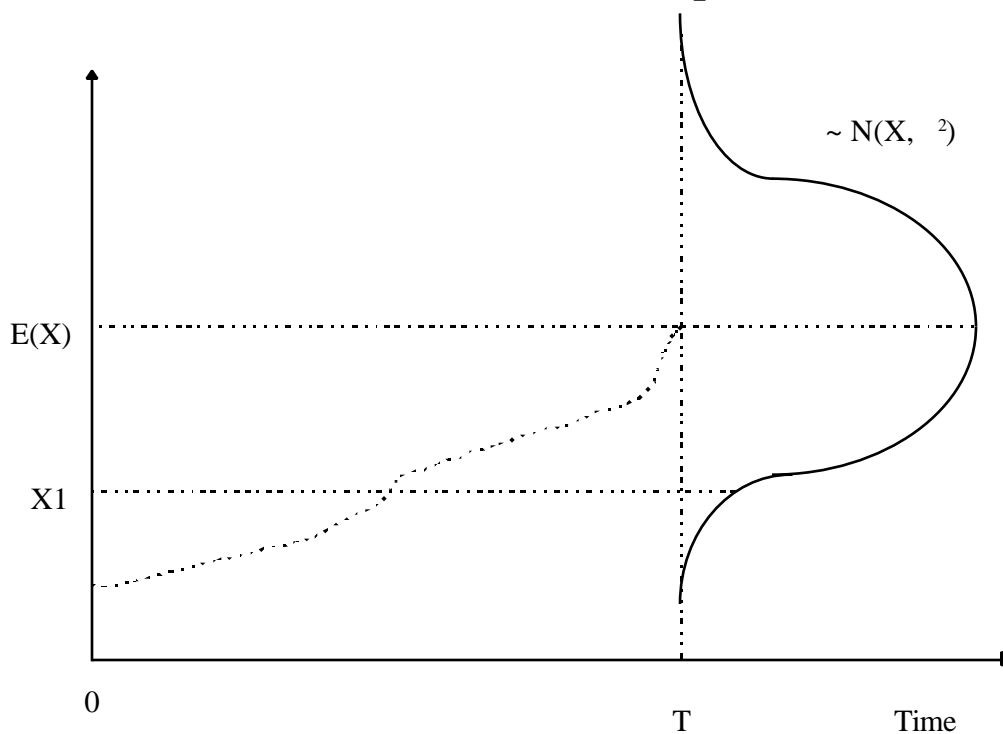
- hard to identify Benefits and Costs before building the project
 - especially if nonmarket values exist for the project
- The choice of the discount rate is also a big issue
 - often the time horizon of Benefits and Costs differ, such as when costs are paid up front and benefits accrue over time.
 - high discount rates tend to make projects look unattractive

There are three important issues Benefit-Cost analysis ignores:

- ability to delay an investment project until a later point in time
- The potential irreversibility of the investment
- The effect of uncertainty over future benefits

The existence of these 3 factors can introduce a value to delaying the investment until uncertainty has been sufficiently resolved: The value is called an **Option Value**.

The Value of A Stock Option



option value of a natural resource is like a **Call Option** in the stockmarket.

Suppose you have the option to buy a given amount of stock at some time T in the future at a price of $X_T = E(X)$; that is, to buy the stock at the price that you and everyone else thinks the stock will be worth at time T . The distribution of prices might be known to be distributed normally, with an average price = $E(X)$.

- If you get to time period T and the stock value is below $E(X)$, such as at point X_1 , then you will not invest.
- if you get to time period T and the stock value is above $E(X)$ you will buy it at a price below market value. You gain the difference between the stock value and the price you paid for it.
- **Stock options are worth a positive amount of money even though their expected value is zero, because they eliminate downside risk!**

Option Values in an Environmental Context

In order for an option value to exist in an environmental context:

- At least a portion of the investment must be irreversible
- Uncertainty over future events must exist
- We must be able to delay investment as a viable policy choice

Option Values occur when we wait before investing in irreversible projects because the decision to wait on investment eliminates downside risk!

Investment Under Uncertainty Differs From C-B Analysis:

Gov't can invest in a project with initial cost of \$I in period $t=0$.

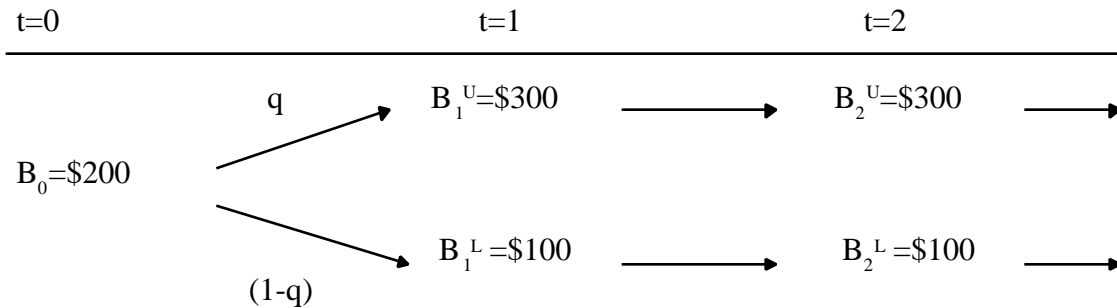
- The project will give expected benefits $E(B_t)$ and is uncertain
 - with Probability q benefits will increase by \$a in period $t=1$
 - with Probability $(1-q)$ benefits will decrease by \$a in period $t=1$
 - After period 1, the benefits will remain at that level forever

$t=0$	$t=1$	$t=2$
Pay \$I, receive B_0	$E(B_1) = q(B_0+a) + (1-q)(B_0-a)$ $= B_0 - a + 2qa$	$E(B_2) = E(B_1)$ $= B_0 - a + 2qa$

Investment Under Uncertainty (cont.)

assume: $B_0 = \$200$; $a = \$100$; $I = \$1,600$

then, we have the benefit matrix:



Say we also know that: $q = 0.5$; $r = 0.1$

Using the Cost-Benefit Criteria,

$$NPV = -I + \sum_{t=0}^{\infty} \frac{E(B_t)}{(1.1)^t} = -\$1,600 + \sum_{t=0}^{\infty} \frac{\$200}{(1.1)^t} = -\$1,600 + \$200 \sum_{t=0}^{\infty} \frac{1}{1.1^t}$$

apply the rule for the sum of an infinite series, to get:

$$\begin{aligned} &= -\$1,600 + \$200 \frac{1}{1 - \frac{1}{1.1}} \\ &= -\$1,600 + \$200(1.10) \end{aligned}$$

which implies that **NPV = \$600**.

It appears that, since the NPV of the project is positive the investment should be made.

However, we also know that, when we get to period $t=1$, benefits will either be \$100 or they will be \$300 indefinitely.

Investment under Uncertainty (cont.)

The Cost-Benefit figure of \$600 ignores the opportunity cost of investing now instead of waiting and maintaining the option to not invest should the future benefits fall.

lets recalculate the NPV assuming we wait until $t=1$.

If the benefit decreases in period 1, then we have:

$$\begin{aligned} \text{NPV} &= (1 - q) \frac{-\$1,600}{1.1} + \frac{100}{1.1} \\ &= 0.5 \frac{-1,600}{1.1} + \frac{1,100}{1.1} = \frac{-250}{1.1} < 0, \end{aligned}$$

gov't should not invest in $t=1$ when the value decreases.

If the benefit increases in period 1, then we have:

$$\begin{aligned} \text{NPV} &= q \frac{-\$1,600}{1.1} + \frac{300}{1.1} \\ &= 0.5 \frac{-1,600}{1.1} + \frac{3,300}{1.1} = \frac{\$850}{1.1} = \$773, \end{aligned}$$

gov't should invest in $t=1$ when the value increases.

If we wait to invest, the NPV *today* of the investment is \$773, while if we actually *make* the investment today according to Benefit-Cost estimations, the NPV is only \$600.

- In this case, **the Option Value is \$773 - \$600 = \$173!**

Value of the Parameters is Really Important:

- Higher Investment Cost, \$I, increases the Option Value
- Higher Initial Benefit, B_0 , decreases the Option Value
- Greater Uncertainty Increases the Option Value
- Larger Probability of Decreased Future Value Increase the Option Value

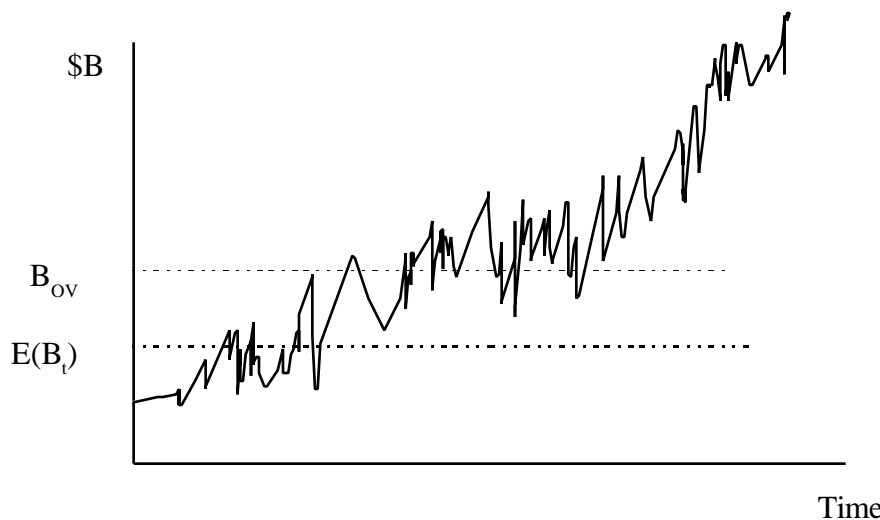
More Complicated Stochastic Processes

A **stochastic process** is a time related series of events which is subject to random variation. A stochastic process combines dynamics with uncertainty, so that the current state of the system determines only the probability distribution of future states, not the actual value.

Typically, a stochastic diffusion process is used, in which the effect of past disturbances are assumed to effect future outcomes according to some given rules of stochastic calculus.

- One of the most common stochastic diffusion processes used to model uncertain events is **Brownian Motion** (continuous time random walk process)

$$P_t = P_{t-1} + \epsilon_t.$$



The idea is that Investment should be delayed until the project is **sufficiently “Deep in the Money”**, so that the probability of the NPV dropping below zero is sufficiently small.

- The literature on Environmental Option Values calculates such Thresholds, called **hurdle rates**, shown as point B_{OV}, in which point a projects becomes sufficiently “Deep in the Money”

The Case of Uncertain Preservation Benefits

the choice of whether or not to develop an area that is currently being preserved is an irreversible decision.

$I_0 = I =$ Cost of Developing an Acre of Rainforest

$$I_1 = \begin{array}{ll} (I + a) & \text{with probability } q \\ (I - a) & \text{with probability } (1 - q) \end{array}$$

$a =$ the value of new medicine discovered in an acre of Rainforest

$q =$ the prob. that new medicine is discovered in a given acre

$r =$ the discount rate

$D =$ annual profit from a developed acre of Rainforest

- If we invest in development today:

$$NPV_0 = -I + \sum_{t=0}^{\infty} \frac{D}{(1+r)^t} \quad \text{or,} \quad NPV_0 = \frac{D(1+r) - rI}{r}$$

The NPV is positive, but once again it ignores an opportunity cost.

recalculate the NPV assuming that we wait until period $t=1$

$$NPV_{OV} = (1-q) \frac{-(I-a)}{(1+r)} + \sum_{t=1}^{\infty} \frac{D}{(1+r)^t}$$

$$\text{or,} \quad NPV_{OV} = \frac{1-q}{r(1+r)} [(1+r)D - r(I-a)]$$

Uncertain Preservation Benefits (cont.)

We can now calculate the value of the option to delay development:

- **Option Value** = $NPV_{OV} - NPV_0$

$$\begin{aligned}
 &= \frac{1-q}{r(1+r)} [(1+r)D - r(I-a)] - \frac{D(1+r) - rI}{r} \\
 &= \frac{(1-q)[(1+r)D - rI + ra] - (1+r)^2 D + r(1+r)I}{r(1+r)} \\
 &= \frac{rI(r+q) - (1+r)D(r+q) + ra(1-q)}{r(1+r)}
 \end{aligned}$$

or, **Option Value** = $\frac{(r+q)[rI - (1+r)D] + ra(1-q)}{r(1+r)}$

This expression gives the value of retaining a natural resource in a state of preservation for a single period before deciding whether or not to develop it.

The Effect of Alternate Parameters on Option Value:

The effect of a larger value for new medicine:

$$\bullet \frac{dOV}{da} = \frac{1 - q}{1 + r} > 0$$

As the opportunity cost of development (foregone medicine) increases, the Option Value of delaying development increases.

The Effect of Larger Development Benefits:

$$\bullet \frac{dOV}{dD} = \frac{-(r + q)}{r} < 0$$

As the benefits from development increase, the option value of maintaining acreage in preservation decreases.

The Effect of Larger Development Costs:

$$\bullet \frac{dOV}{dI} = \frac{r + q}{1 + r} > 0$$

As the cost of development increases, the option value of maintaining acreage in preservation increases.

Effect of a Higher Probability of Discovering New Medicine:

$$\bullet \frac{dOV}{dq} = \frac{rI - (1 + r)D - ra}{r(1 + r)} = \frac{I - a}{1 + r} - \frac{D}{r} < 0 \quad (\text{from } NPV_{OV} > 0)$$

as the probability of finding new medicine increases, the Option Value of maintaining acreage in preservation decreases.

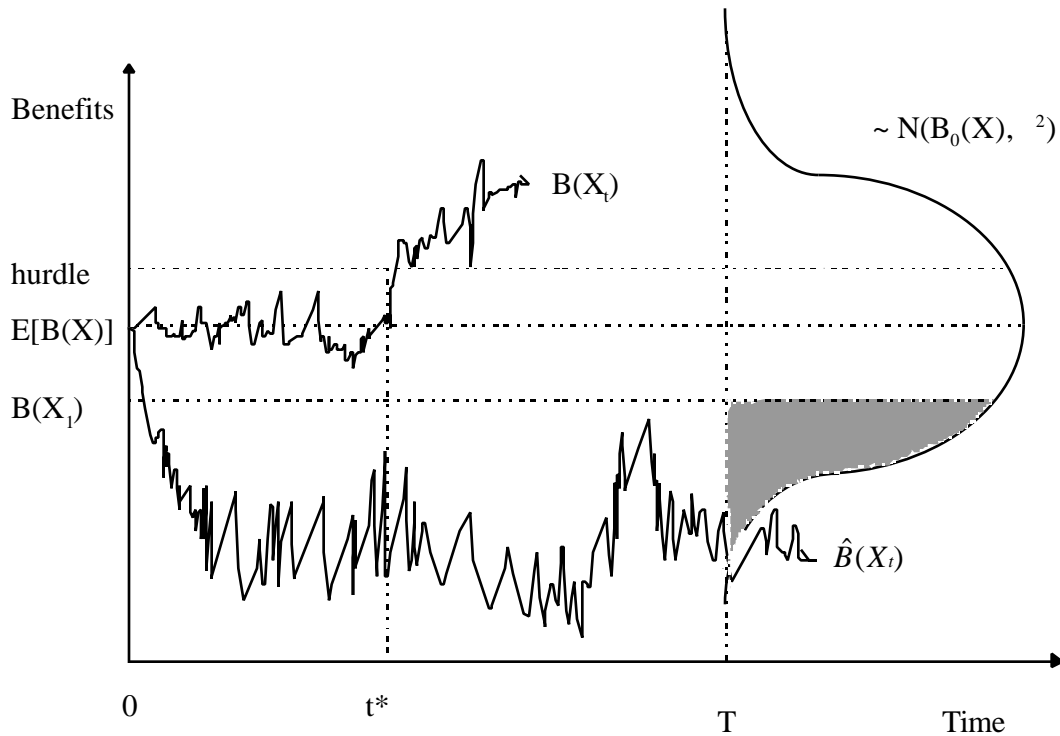
The Effect of a Higher Discount Rate:

$$\bullet \frac{dOV}{dr} = \frac{r^2(1 - q)(I - a) + (1 + r)^2 Dq}{[r(1 + r)]^2} > 0, \quad (\text{quotient rule})$$

as rate of time preference increases, the Option Value of maintaining acreage in preservation increases.

Option Values In a More General Context

Consider a development project that has the following distribution of benefits at some time T.



Time period T is some hypothetical point far off into the future at which time the net benefit of development (Development Benefits less the opportunity cost of foregone preservation) will be known with absolute certainty.

- It does not matter how far into the future information is revealed
- **important element is that the distribution of future NB is known**
 - If we don't know it, we use our best estimate.

Option Values In a More General Context (cont.)

Say we know that the distribution of future net benefit at time T will be:

- $B(X_T) \sim N(B(X_0), \sigma^2)$

That is; NB_T will be, on average, equal to NB today, but will be subject to some random variation given by σ^2 .

When development is non-optimal for all $NB_T < B(X_1)$, then development will not occur for any realization under the shaded portion of the distribution.

We can now re-calculate the mean of the truncated distribution of NB_T , which only considers the cases in which development will occur. (mean of all possible values outside the shaded region)

This gives us our Expected Benefit of development if we waited until time period T

- This **threshold value** is known as **the hurdle rate**

Intuitively, it is optimal to wait in every period in which a positive Option Value exists (**i.e., wait in period t whenever Option Value = $NPV_{OV} - NPV_t > 0$**).

The Optimal Investment Time:

- optimal time to develop is when NB reaches the hurdle rate.
- Under $B(X_t)$, the optimal time to develop the resource is in period t^* .
- For another possible realization of NB such as $\hat{B}(X_t)$, development will never be optimal, even if the expected NB_0 is positive under Cost-Benefit calculations.