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EEP 101
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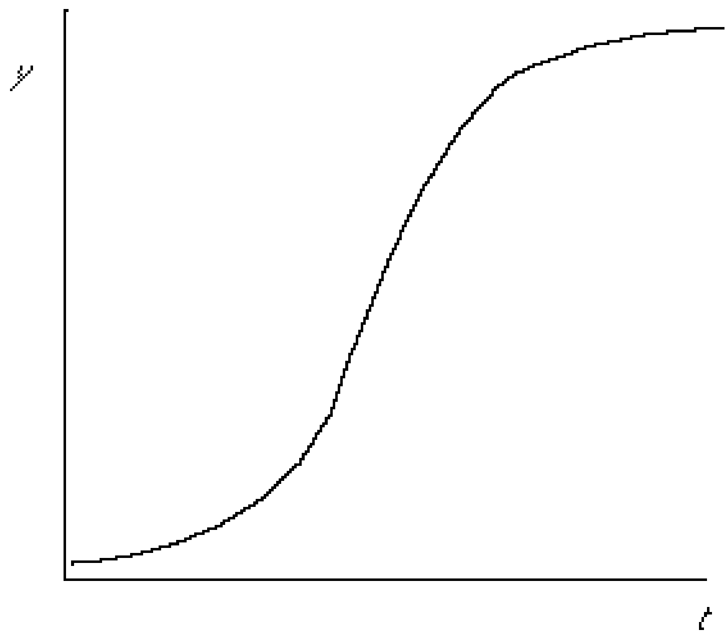
Lecture 20

Renewable Resources Forestry

Classic Forestry Problems (Faustmann)

At what age is it optimal to cut a tree? Output of a tree depends on age,

$$y = f(t)$$



At the economic region

$$y(t) \text{ is such that } \frac{\partial y}{\partial \gamma} = y_t > 0$$

$$\partial_y^2 / \partial t^2 < 0.$$

The optimization problem is to determine the cutting age that maximizes net present value of the tree.

Assume the unit price of wood is *P* and the net present value is a continuous function of *t*.

$$NP(t) = e^{-rt}Py(t)$$

The optimal age is determined solving,

$$\frac{\partial}{\partial t}\left[Pe^{-rt}y(t)\right] = 0.$$

The optimal condition is

$$\max_{t} \frac{Pe^{-rt}y(t)}{\text{optimal condition}}$$

since

$$\frac{\partial}{\partial t}\left[a(t)\ \ b(t)\right]$$

$$= \quad \frac{\partial a(t)}{\partial t}b(t) + a(t)\frac{\partial b(t)}{\partial t}$$

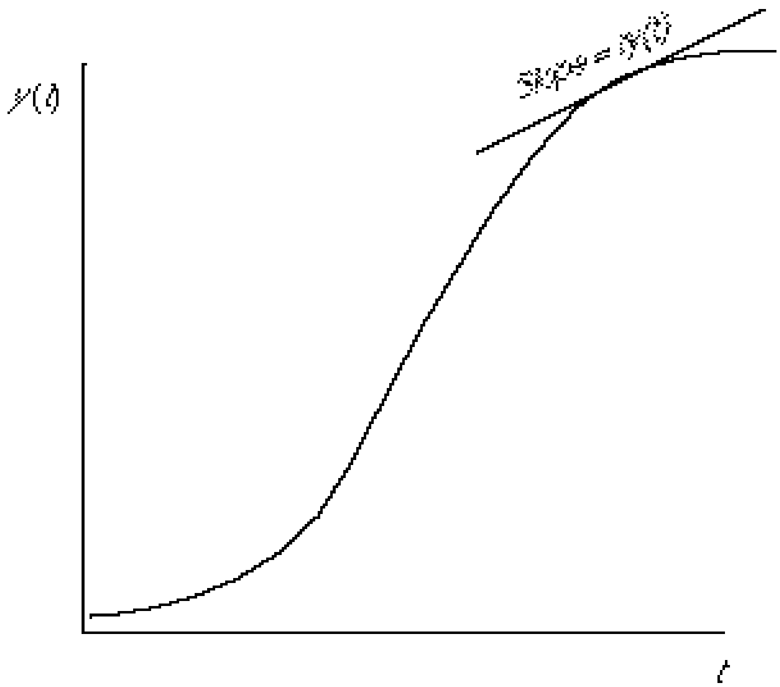
$$\frac{\partial NP(t)}{\partial t} = \quad P\left[-re^{-rt}y(t) + e^{-rt}\frac{\partial y}{\partial t}(t)\right] = 0$$

$$=> \quad \frac{\partial y/\partial t}{y(t)} = r.$$

The optimal cutting age is when the rate of the growth of the tree,

$$\frac{\partial y}{\partial t}\Big/ y(t) \quad \left(\text{or} \quad \frac{\partial y}{y}\Big/ \partial t\right),$$

is equal to the discount rate.



<u>Implication</u>

(1) We do not cut at harvest (because it hasn't yet aged) with maximum yield when $\partial y / \partial t = 0$, but we do cut at a younger age, when the slope is positive.

(2) Higher *r* leads to lower *t*, since the slope of *y* declines.

Examples

$$y = At^{\alpha} \qquad 0 < \alpha < 1$$

$$\frac{\partial y}{\partial t} = \qquad \alpha A t^{\alpha - 1} \qquad = \alpha \frac{y}{t}$$

Optimal t is when

$$r = \frac{\alpha y}{ty} = \frac{\alpha}{t}$$

$$t = \alpha / r$$

If $\alpha = .5 \qquad r = .02 \Rightarrow t^{\alpha} = 25$ years

If $\alpha = .9 \qquad r = .01 \Rightarrow t^{\alpha} = 90$ years

A more realistic model is when the region of decreasing marginal growth occurs after age t_0.

$$y = A(t - t_0)^{\alpha}$$

$$\frac{\partial y}{\partial t} = \frac{\alpha y}{t - t_0}$$

The optimal rule is

$$t^* = t_0 + \frac{r}{\alpha}.$$

An open access problem exists here. If individuals are not sure of ownership, their r is very high, and they will cut trees earlier than socially optimal. That is one reason for deforestation.

Problem of Model

(1) Simplistic: This is a model for one tree. In a forest we may have many trees of different ages that will rotate t^* years.

A more complete analysis needs to be incorporated:

(1) Extraction cost

(2) Demand

(3) Alternative use of land.

Within our framework, a higher demand will increase the acreage of trees but will not change the optimal age.

In essence, we have a t^x year rotation, but the acreage in each parcel increases with higher demand.

Other Issues

(1) Multiple use (in a narrow sense). Trees are used for pulp and furniture. One may use different species for different purposes, and different uses imply different rotations. Younger trees are better for pulp than for housing and furniture.

(2) Multiple use (in a broad sense). Trees provide recreational and wildlife benefits, so rotation adjusts to it. Cutting cost may emphasize a forest with trees of uniform species and age. That may not be optimal if other considerations are incorporated.

(3) Nutrition and fertilization.

$$y = f(t_1 \ a)$$

where a is input (water fertilizer, etc.). Forests can be cultivated agronomically, thus increasing yield and reducing pressure on primary forests.