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EEP 101
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Lecture 18

Nonrenewable Resources

(1) Outcomes with Extraction Costs

$B(x)$ = benefit function = area under demand curve.

For period n, $B_{x_n} = \frac{\partial B}{\partial x}(x_n) = P_n$, and this is the market price at period n.

Per unit extraction cost is c.

For a two-period model, social optimization is

$$\max_{x_0, x_1} B(x_0) - cx_0 + \frac{1}{1+r}\left[B(x_1) - cx_1\right]$$

subject to $x_1 + x_0 \leq S_0$.

$$L = B(x_0) - cx_0 + \frac{1}{1+r}\left[B(x_1) - cx_1\right] + \lambda\left[S_0 - x_1 - x_0\right]$$

λ = user cost = shadow price of the resource stock.

Optimal decision rules:

$$L_{x_0} = B_{x_0} - c - \lambda = 0$$

$$L_{x_1} = \frac{B_{x_1} - c}{1+r} - \lambda = 0$$

$$B_{x_0} = P_0^* = \text{optimal price of } x_0$$

$$B_{x_1} = P_1^* = \text{optimal price of } x_1^*$$

$$P_0^* = c + \lambda^*$$

The optimal price is the sum of extraction cost and user cost.

$$P_1^* = c + \lambda^*(1 + r).$$

If there are *n* periods,

$$P_n^* = c + \lambda^*(1 + r)^n.$$

Optimal price is the sum of extraction cost and the adjusted user cost.

<u>Examples</u>:

The solution for a two-period model for $B_x = a - b_x$ where the resource constraint is binding is

$$S_0 = x_1 + x_0.$$

Under these assumptions,

$$a - bx_0^* - c = \frac{a - b\left(S_0 - x_0^*\right) - c}{1 + r}$$

$$x_0^* = \frac{bS_0 + (a - c)r}{b(2 + r)} \quad x_1^* = \frac{bS_0(1 + r) - (a - c)r}{b(2 + r)}$$

$$P_0^* = a - bx_0^* \qquad P_1^* = a - bx_1^*$$

$$= \frac{2a + rc - bS_0}{2 + r} \qquad = \frac{(1 + r)\left(2a - bS_0\right) - rc}{2 + r}$$

$$\lambda^* = a - bx_0^* - c$$
$$= \frac{2(a-c) - bS_0}{2+r}$$

Our analysis is applied to the cases where all the resources are utilized. If $S_0 > \frac{2(a-c)}{b}$, $\lambda^* = 0$, not all the resources are extracted. In these cases,

$$P_0 = P_1 = c, x_0^* = x_1^* = \frac{a-c}{b}.$$

(2) Outcomes under Open Access

At period 1,

$$P_0^0 = c; \text{ for } B_x = a - b_x$$

$$x_0^0 = \frac{a-c}{b}$$

$$x_1^0 = S_0 - \frac{a-c}{b}$$

$$P_1^0 = 2a - c - bS_0.$$

Note that, if $S^0 < \frac{a-c}{b} = x_0^0 = S^0, x_1^0 = 0$. All the resources will be exhausted in the initial period.

Numerical example:

Suppose

$$S_0 = 15 \quad B_x = 11 - x, \qquad a = 11, \; b = 1, c = 1, r = .1$$

$$x_0^* = \frac{15+1}{2.1} = \frac{16}{2.1} = 7.62 \qquad x_1^* = 7.38$$

$$P_0 = 11 - 7.62 = 3.38$$

$$P_1 = 11 - 7.38 = 3.62.$$

To check over answers, $\lambda^* = P_0^* - c = 2.38$

$$2.38 \times 1.1 = 2.62 + 1 = 3.62 = P_1^*$$

If $S_0 = 18$

$$x_0^* = \frac{18+1}{2.1} = 9.05 \quad x_1^* = 8.95$$

$$P_0^* = 11 - 9.04 = 1.95 \quad P_1^* = 2.05,$$

under open access,

$$S_0 = 15 \qquad x_0^0 = 10 \qquad x_1^1 = 5 \qquad P_1^0 = 1 \qquad P_1^0 = 6$$

$$S_0 = 18 \qquad x_0^0 = 10 \qquad x_1^1 = 8 \qquad P_0^0 = 1 \qquad P_1^0 = 3.$$

(3) <u>Outcome under Monopoly</u>

Recall that price = $B_x(x)$ and at each period the monopoly's profits are

$x(b_x(x) - c)$. The monopoly's optimization problem is

$$L = \max_{x_0, x_1, \lambda} \; x_0\left[B_x(x_0) - c\right] + \frac{1}{1+r}\left[x_1\left(B_x(x_1) - c\right)\right] + \lambda^M\left[S_0 - x_1 - x_0\right]$$

$$L_{x_0} = B_x(x_0) + B_{xx}(x_0)x_0 - c - \lambda^M = 0$$

$$L_{x_1} = \frac{B_x(x_1) + B_{xx}(x_1)x_1 - c}{1+r} - \lambda^M = 0$$

Note that

$$B_x(x) + xB_{xx}(x) = MR(x) = \text{marginal revenue}$$

and the optimization conditions become

$$MR(x_0) - c = \lambda^M$$

$$\frac{MR(x_1) - c}{1 + r} = \lambda$$

$$\lambda^M = \text{user cost of monopoly}$$

for $B_x(x) = a - b_x$

$$a - 2bx_0 - c = \frac{a - 2bx_1 - c}{1 + r}$$

when $x_1 = S_0 - x_0$

$$a - 2bx_0 - c = \frac{a - 2b(s - x_0) - c}{1 + r}$$

$$x_0^M = \frac{bS_0 + r / 2(a - c)}{b(2 + r)} \qquad x_1^M = \frac{bS_0(1 + r) - r / 2(a - c)}{b(2 + r)}$$

compares to

$$x_0^* = \frac{bS_0 + a - c}{b(2 + r)}$$

$$x_0^M < x_0^*.$$

$$\lambda^M = a - 2bx_0^M - c = \frac{2(a - c - bS_0)}{2 + r}.$$

Note that when

$$S_0 > \frac{a-c}{b}, \lambda^M = 0.$$

The monopoly utilizes all the resource only when $S_0 < \frac{a-c}{b}$ and when

$$S_0 > x_0^M = x_1^M = \frac{a-c}{2b}.$$

Note that, if $\frac{2(a-b)}{b} > S_0 > \frac{a-c}{b}$, all S_0 will be extracted under optimal solution but

not under monopoly.

For $B_x = 11 - 1x,\ \ c = 1,\ \ r = .1.$

If $S_0 = 15 > \frac{11-1}{1} = \lambda^M = 0$,

$$x_1^M = x_0^M = 5$$

$$P_1^M = P_0^M = 6.$$

Not all the resources are used in contrast to the optimal outcome analyzed above.

For $S_0 = 8$,

$$x_0^M = \frac{8+.5}{2.1} = 4.05 \qquad x_1^M = 3.95$$

$$x_0^* = \frac{8+1}{2.1} = 4.29 \qquad x_1^* = 3.71$$

when all the resource is utilized under both solutions, monopoly utilizes fewer resources

in the first period and less in the second period.

Open access leads to more use of resources in the first period than optimal solution and that is greater than under the monopoly.

How to Correct Open Access Problems

1. Tax = user cost – the tax increase over time; thus it is $\lambda(1+r)$.
2. The government determines the quantity to be mined and introduces mining permits.
3. Private ownership of resources.

(4) Outcomes under Growing Demand

Benefit at period 0, $B^0(x)$.

Benefit at period 1, $B^1(x)$.

$$B^1(x) = (1+\eta)\,B^0(x)$$

$$B_x^1 = \frac{\partial B^1}{\partial x}(x) = (1+\eta)\frac{\partial B^0}{\partial x}(x) = (1+\eta)\,B_x^0.$$

Social optimization:

$$L = \max_{x_0, x_1, \lambda} B^0(x_0) - cx_0 + \frac{1}{1+r}\left[B^1(x_1) - c(x_1)\right] + \lambda\left[S_0 - x_1 - x_0\right]$$

$$L_{x_0} = B_x^0(x_0) - c - \lambda = 0$$

$$L_{x_1} = \frac{B_x^1(x_1) - c}{1+r} - \lambda = 0$$

$$B_x^0(x_0) - c = \frac{(1-\eta)B_x^0(x_1) - c}{1+r}.$$

Suppose $B_x(x_0) = a - bx$,

$$a - bx_0 - c = \frac{(1+\eta)(a - bx_1) - c}{1+r}$$

$$a - bx_0 - c = \frac{(1+\eta)\left(a - b(S_0 - x_0)\right) - c}{1 + r}$$

$$x_0^* = \frac{b(1+\eta)S_0 + r(a-c) - a\eta}{b(2+\eta+r)}$$

$$x_1 = S_0 - x_0 .$$

For the case b = 1, a = 11, c = 1, r = .1, $\eta = .05$, $S_0 = 15$.

$$x_0 = \frac{(1.05)\ \ 15 + 1 - .55}{2.15} = 7.53 \quad x_1 = 7.47$$

$$P_0 = 3.47 \qquad P_1 = 1.05\,(11 - 7.47) = 3.71.$$

Growth in demand reduces consumption in period 1 and increases price in both periods (with constant demand, $P_0 = 3.38, P_1 = 3.62$). It may lead to a reversal of patterns and an increase in consumption over time if η is big enough. For example, under the parameter of our example, if we change η so

$$\eta = .15$$
$$x_0 = \frac{1.15\ \ 15 + 1 - 1.65}{2.25} = 7.38$$
$$x_1 = 7.62$$
$$P_0 = 3.62$$
$$P_1 = 3.89$$

Case with Backstop Technology

Suppose that at period 1 a new technology is available, and it will produce the resource at cost m. We now need to separate three quantities:

Quantity consumed	call it y
Quantity mined	call it x
Quantity produced	call it z.

The benefit function is $B(y)$ at period 0.

$$\underbrace{y_0}_{\text{quantity consumed}} = \underbrace{x_0}_{\text{quantity mined}} .$$

At period 1,

$$\underbrace{y_1}_{\text{quantity consumed}} = \underbrace{x_1}_{\text{quantity mined}} + \underbrace{z_1}_{\text{quantity produced}} .$$

Optimization problem

$$L = \max_{x_0, x_1, z_1 \lambda} \underset{\substack{\uparrow \\ \text{benefit of} \\ y_0 = x_0}}{B(x_0)} - \underset{\substack{\uparrow \\ \text{mining} \\ \text{cost}}}{cx_0} + \frac{1}{1+r}\Big[\underset{\substack{\uparrow \\ \text{benefit of} \\ y_1 = x_1 + z_1}}{B(x_1 + z_1)} - \underset{\substack{\uparrow \\ \text{mining} \\ \text{cost}}}{cx_1} - \underset{\substack{\uparrow \\ \text{production} \\ \text{cost}}}{mz_1}\Big] + \lambda \underset{\substack{\uparrow \\ \text{resource} \\ \text{constraint}}}{\big[S_0 - x_1 - x_0\big]}$$

$$L_{x_0} = B_y(x_0) - c - \lambda = 0$$

$$L_{x_1} = \frac{1}{1+r}\left[B_y(x_1 + z_1) - c\right] - \lambda = 0$$

$$L_{z_1} = \frac{1}{1+r} B_y(x_1 + z_1) - m = 0$$

$$L_x = S_0 - x_1 - x_0 = 0.$$

These conditions show that the cost of the backup will effect mining patterns, consumption, and resource use. They can be rewritten

(1) $$P_0^* = B_y(x_0) = c + \lambda$$

(2) $$P_1^* = B_y(x_1 + z_1) = c + \lambda\,(1 + r)$$

(3) $$P_1^* = B_y(x_1 + z_1) = m.$$

Condition (3) suggests that the price at period 1 is equal to the production cost of backup technology. Conditions (2) and (3) suggest that

$$m = c + (1 + r)\lambda$$
$$\lambda = \frac{m - c}{1 + r}.$$

Condition (1) suggests

$$P_0^* = \frac{m + rc}{1 + r}.$$

Lower m will reduce λ and P_0^* and increase consumption at both periods.

If $m < c$, no mining will occur at period 1.

<u>Example</u>:

$$B_y = 11 - y, \quad c = 1, \quad r = .1 \quad m = 3$$
$$P_1^* = 3$$
$$\lambda = \frac{3 - 1}{1.1} = 1.9$$
$$P_0^* = 3 - .1 = 2$$
$$Y_0^* = x_0^* = 11 - 2.9 = 8.1$$

$x_1^* = 15 - 8.1 = 7.9$ amount mined in period1

$y_1^* = 11 - 3 = 8$ amount consumed in period 1

$z_1^* = 8 - 7.9 = .1$ amount produced in period1

If

$$m = 2$$

$$P_1^* = 2$$

$$\lambda = \frac{2-1}{1.1 = .9}$$

$$P_0^* = 1.9$$

$$y_0^* = x_0^* = 9.1$$

$x_1^* = 15 - 9.1 = 5.9$ amount mined

$y_1^* = 11 - 2 = 9$ amount consumed

$z_1^* = 9 - 5.9 = 3.1$ amount produced.

Summary

When resources are renewable:

(1) Optimal resource use equal price of extraction cost plus time adjustment user cost.

With open access, user cost is zero if too much is mined early. With monopoly, there is undermining at all periods.

Higher interest rates will reduce λ and P_0 but increase x_0.

New discoveries will reduce λ and P_0 and increase x_0.

Backstop technology will reduce λ and P_0 but increase x_0.

Increase in future demand will increase λ and P_0 and reduce x_0.