Two-Period Nonrenewable Resource Model with Extraction Costs:

- two periods, t=0 and t=1.
- $B(X_t)$ is the *gross* benefit from using X_t (amt. of the resource in period t)
- let c denote marginal extraction cost.
- *Net Benefit* for period t becomes: $B(X_t) - cX_t$.

We maximize social welfare by solving:

$$\max_{X_0, X_1} NPV[SW(X_0, X_1)] = B(X_0) - c \ X_0 + \frac{1}{1+r}[B(X_1) - c \ X_1]$$

subject to: $S_0 = X_0 + X_1$.

The Lagrangian equation for this problem is:

$$L = B(X_0) - c \ X_0 + \frac{1}{1+r}[B(X_1) - c \ X_1] + \lambda(S_0 - X_0 - X_1)$$

.

The F.O.C.'s are:

(1) $$L_{x_0} = B_X(X_0) - c - \lambda = 0$$

(2) $$L_{x_1} = \frac{B_X(X_1) - c}{1+r} - \lambda = 0.$$

(3) $$L_\lambda = S_0 - X_0 - X_1 = 0$$.

where $B_X(X_t)$ = MB of using X_t and c = MC.

Two-Period Nonrenewable Resource Model with Extraction Costs (cont.):

· Equation (1) => $P_0 = c + \lambda$
the price of the mineral resource equals marginal mining cost plus the shadow cost (user cost) of the resource constraint

· Equation (2) => higher interest rates reduce the user cost
=> use more today and less in the future
=> price will be lower today and higher in the future

· Equations (1)&(2) show that **higher extraction costs** reduce the net MB associated with using the resource in either time period:
- **reduces resource use and raise prices in both periods**.
- **reduces the user cost**, increasing the incentive to shift some consumption from the future to today.

With very high extraction cost, the entire resource stock may not be used up (shadow cost of the resource constraint, **λ = zero**)

Without extraction costs, the price of an optimally-managed nonrenewable resource will grow at the rate of interest:

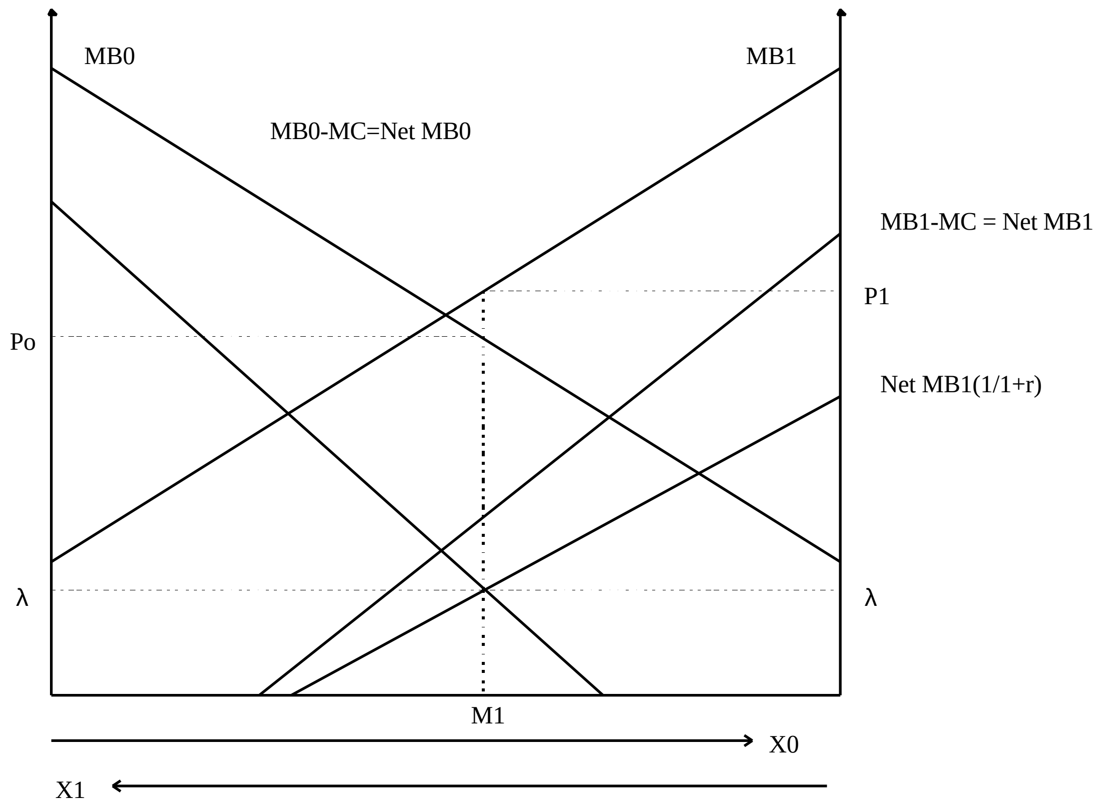
$$\frac{P_1 - P_0}{P_0} = r$$

With extraction costs, the price of an optimally-managed nonrenewable resource minus extraction costs (royalty*)* will grow at the rate of interest:

$$\frac{R_1 - R_0}{R_0} = r\text{, where } R_i = (P_i - c)$$

=> ***with extraction costs*, the price of an optimally-managed natural resource may grow at a rate *less than* the rate of interest.**

Effects of Extraction Costs



In the case of a non-replenishable ground water aquifer, there are additional costs associated with treatment and shipment:

water price = user cost
+ marginal extraction (pumping) cost,
+ marginal shipment (conveyance) cost,
+ marginal treatment cost,

our model predicts that these costs cause the price of water to grow at less than the rate of interest, since royalties will increase at the rate of interest

Royalty = Price - Pumping - Conveyance - Treatment, (per unit).

Two-Period Nonrenewable Resource with Open Access:

- **firms operate as if the user cost of mining the resource is zero**,
- the tradeoff is not between how much to extract in the initial period vs. how much to extract in later periods, but rather how much to extract in the initial period vs. how much do *other* firms extract in the initial period.
- firms extract a marginal unit until MB_0 = MC, as if they were operating in a static model with only a period 0.

- firms enter the industry until price falls to minimum average mining cost (until static profit is driven to zero):

$$\pi = PX - C(X) = 0 \quad \Leftrightarrow \quad P = \frac{C(X)}{X} = AC(X)$$

assume minimum AC = c: $P_0 = B_X(X_0) = c$.

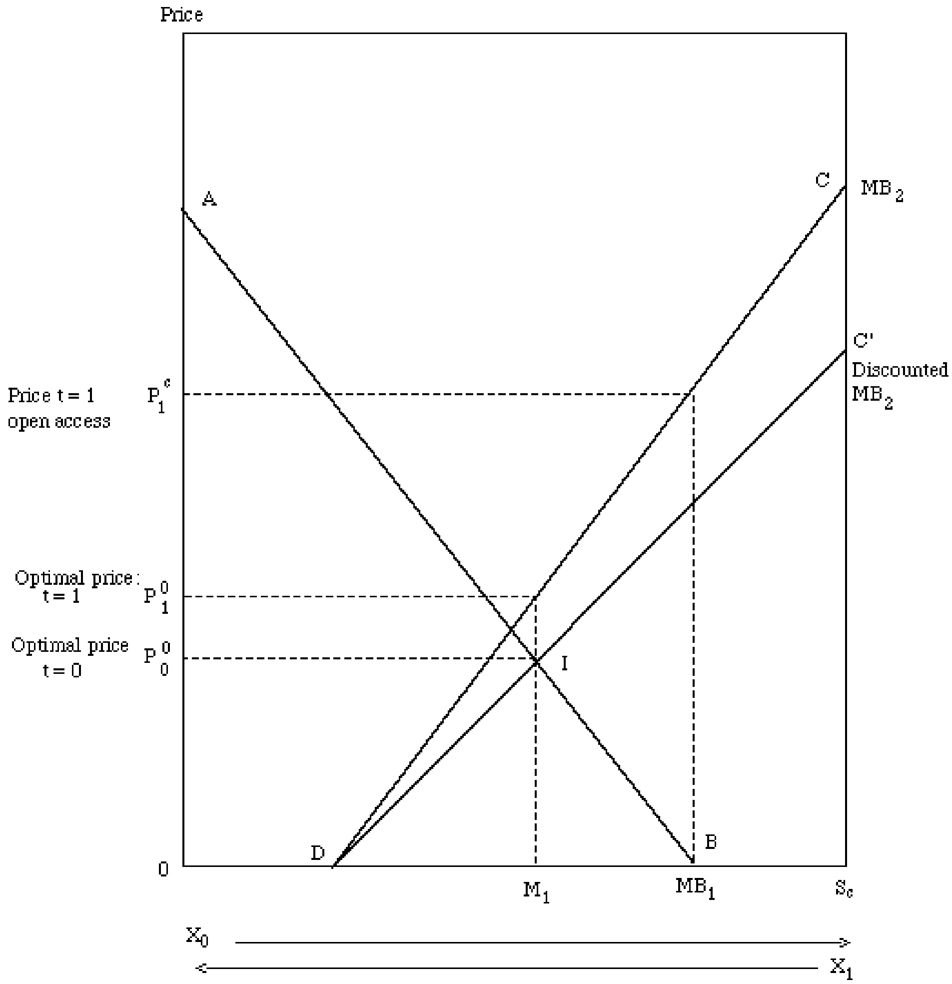
Assuming linear market demand: $B_X(X) = a - bX$.

The industry *would like to* extract the following open-access amount:

$$a - bX = c \qquad X_{OA} = \frac{a-c}{b}$$

- **If demand is satiated in the initial period** ($X_0^* < S_0$), then X_0^* is *in fact* extracted in the initial period, and $P_0 = c$. The remaining stock ($S_1 = S_0 - X_0^*$) is simply ignored.
- **If demand is nonsatiated in the initial period** ($X_0^* > S_0$), then we must set $X_0^* = S_0$, (impossible to extract more than initial stock). Price falls to $P = a-bS_0$

Open Access Leads to Inefficient Over-extraction



Extraction of nonrenewable resource with satiated demand under open access.

if there are n firms in the industry, each firm would extract X_0^*/n and earn zero economic profits. (**inefficient over-extraction)**

Policies to Correct Open Access Market Failures

- An output tax = λ. (i.e., t* = user cost)
- The government determining the amount to be mined each period by asking competitive producers to bid for mining rights [the per unit price for the right to mine will be λ].
- Establishment of property rights for competitive producers. (Unless extraction costs depend on stock levels, in this case extra tax may be needed.)

Two-Period Nonrenewable Resource Model with Monopoly:

The objective function for the monopoly owner is:

$$\max_{X_0, X_1} \text{NPV}(\pi) = \left[P_0(X_0)\ X_0 - C(X_0)\right] + \frac{\left[P_1(X_1)\ X_1 - C(X_1)\right]}{1+r}.$$

subject to: $S_0 = X_0 + X_1$

Recall that $B_X(X_t) = P_t$. Making this substitution:

$$\max_{X_0, X_1} \text{NPV}(\pi) = \left[B_x(X_0)\ X_0 - C(X_0)\right] + \frac{\left[B_x(X_1)\ X_1 - C(X_1)\right]}{1+r}.$$

subject to: $S_0 = X_0 + X_1$

Introducing a Lagrangian multiplier, λ, the monopoly's problem becomes:

$$\max_{X_0, X_1, \lambda} L = \left[B_x(X_0)\ X_0 - C(X_0)\right] + \frac{\left[B_x(X_1)\ X_1 - C(X_1)\right]}{1+r} + \lambda\left(S_0 - X_0 - X_1\right)$$

F.O.C's:

(1) $dL/dX_0 = B_X(X_0) + B_{XX}(X_0)X_0 - MC(X_0) - \lambda = 0$

(2) $dL/dX_1 = [B_X(X_1) + B_{XX}(X_1)X_1 - MC(X_1)] / [1 + r] - \lambda = 0$

(3) $dL/d\lambda = S_0 - X_0 - X_1 = 0.$

Note that marginal revenue $MR(X_t) = B_x(X_t) + X_t B_{xx}(X_t)$.

Hence, we find that $MR(X_t) = MC(X_t) + \lambda(1 + r)^t$

Two-Period Nonrenewable Resource with Monopoly (cont.):

- recalling the FOCs, we see that in the monopoly equilibrium we equate $MR = MC + \lambda(1+r)^t$

In a static model, we usually set the FOC equal to zero, while in a dynamic model, we now set the static FOC = λ = user cost.

- Perfect Competition: $MB - MC = 0$ $\quad MB_i - MC_i = \lambda(1+r)^i$ in a dynamic model
  - -Now (MB-MC) increases at the rate of interest over time.

- Monopoly: $MR - MC = 0$ $\quad MR_i - MC_i = \lambda(1+r)^i$ in a dynamic model
  - -Now (MR-MC) increases at the rate of interest over time.

- When we also have externalities, the social optimal rate of extraction would be found where $MB_i - MC_i - MEC_i = \lambda(1+r)^i$; that is (MB-MSC) increases at rate = r.

Characteristics of monoplistic extraction:

- prices are initially higher under monopoly but grow at a slower rate over time.
- in the second period, the monopoly provides a greater amount of the resource in order to deplete all of the final stock and meet the constraint.

the period two price is actually lower under a monopoly industry structure than under a competitive structure

Two-Period Nonrenewable Resource with Changing Demand:

Assume $B(X_t) = [(1 + n)^t][a^*(X_t)^{0.5}]$

=> $B_x(X_t) = [(1 + n)^t][0.5^*a^*(X_t)^{-0.5}]$

where **n is the "growth rate of demand"**

The socially optimal extraction (assuming zero extraction costs):

$$\max_{X_0, X_1} \; SW(X_0, X_1) = a(X_0)^{0.5} + \frac{1}{1+r} \left[(1+n) \; a(X_1)^{0.5} \right]$$

subject to: $X_1 = S_0 - X_0$.

From the F.O.C.'s, we find that: $\dfrac{X_0}{X_1} = \left[\dfrac{1+r}{1+n} \right]^2$

$$X_0 = S_0 \frac{(1+r)^2}{(1+n)^2 + (1+r)^2}$$

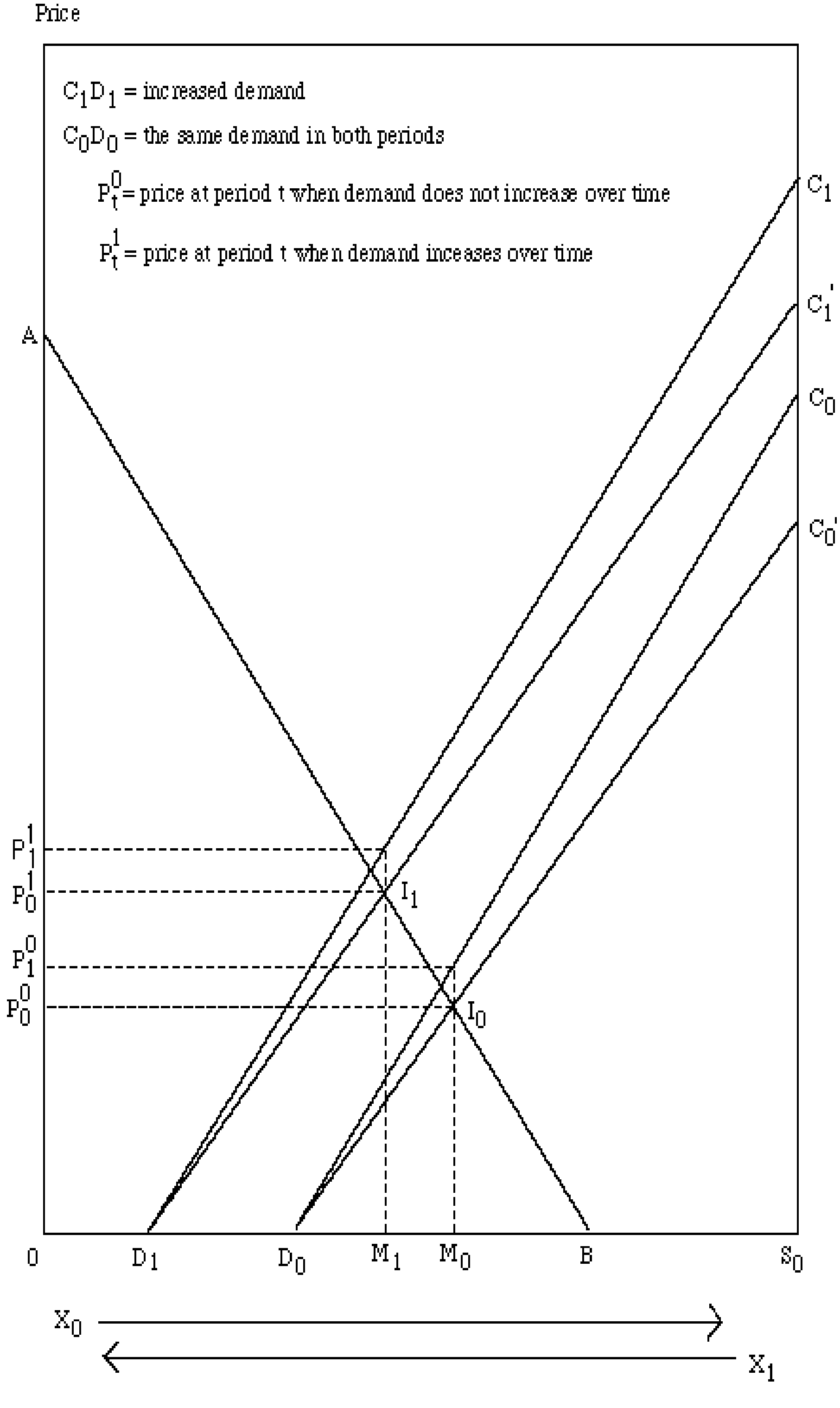
$$X_1 = S_0 \frac{(1+n)^2}{(1+n)^2 + (1+r)^2}$$

$$P_0 = \frac{a}{2(1+r)} \sqrt{\frac{(1+r)^2 + (1+n)^2}{S}} \quad \text{and} \quad P_1 = \frac{a}{2} \sqrt{\frac{(1+r)^2 + (1+n)^2}{S}}$$

- an increase in n will reduce X_0 and increase X_1.
- an increase in n will cause both P_0 and P_1 to "jump" up, but (assuming zero extraction costs) the *rate* of price increase over time will remain:

$$\frac{P_1 - P_0}{P_0} = r$$

Effects of Changing Demand



Optimal management of nonrenewable resources in case with growing demand

Two-Period Nonrenewable Resource (Backstop Technology):

Assume a new technology will make an alternative resource available in the future period (t = 1). Let Z be the output level of the alternative resource, which is a perfect substitute for X. Assume the MC of the alternative resource is a constant, m.

Example: the nonrenewable resource is *fossil fuel and* the backstop technology is *solar power.*

The social optimization problem with a backstop technology:

$$\max_{X_0, X_1} SW(X_0, X_1) = [B(X_0) - C(X_0)] + \frac{1}{1+r}\left[B(X_1 + Z_1) - C(X_1) - m\ Z_1\right]$$

subject to: $S_0 = X_1 + X_0$. (Z is unconstrained)

The Lagrangian problem becomes:

$$\max_{X_1, X_0, Z, \lambda} L = B(X_0) - C(X_0) + \frac{1}{1+r}\left[B(X_1 + Z_1) - C(X_1) - m\ Z_1\right] + \lambda\left[S_0 - X_0 - \right.$$

.

The F.O.C.'s are:

1. $\frac{\partial L}{\partial X_0} = B_X(X_0) - C_X(X_0) - \lambda = 0.$

2. $\frac{\partial L}{\partial X_1} = \frac{1}{1+r}\left[B_X(X_1 + Z_1) - C_X(X_1)\right] - \lambda = 0.$

3. $\frac{\partial L}{\partial Z} = \frac{1}{1+r}\left[B_Z(X_1 + Z_1) - m\right] = 0$.

4. $\frac{\partial L}{\partial \lambda} = S_0 - X_0 - X_1 = 0$

Two-Period Nonrenewable Resource (Backstop Technology):

· FOC (1) states that, at the optimum:

$$P_0 = B_X(X_0) = C_X(X_0) + \lambda.$$

P= MB = MC Extraction + User Cost of Consum. in Period 0.

· FOC (2) states that, at the optimum:

$$P_1 = B_x(X_1 + Z) = C_x(X_1) + (1 + r)\,\lambda.$$

P = MB = MC Extraction + User Cost of Consum. on Period 1.

· FOC (3) states that, at the optimum: $P_1 = m$.

The price of X in period 1 equals the price of Z in period 1.

· From equations (5) and (6), we find

$P_1 - C_x(X_1) = (1 + r)\,(P_0 - C_x(X_0))$.

$P_0 = C_x(X_0) + \frac{1}{1+r}\left[m - C_x(X_1)\right]$.

User Cost $\lambda = P_0 - C_x(X_0)$.

For $C_x = 0$:

· If $X_1 > 0$, $P_0 = \frac{m}{1+r}$., (i.e., the price in the first period is smaller than the cost of the backstop technology).
· As m becomes lower, X_0 increases, X_1 declines, and Z increases.
· If m is sufficiently low, the resource will be used only at the first period, with $P_0 > m$. In this case some of the exhaustible resource may be left unused.

Sketch of An "n-Period" Model of Nonrenewable Resources

- Assume zero costs; assume T time periods.
- Assume competitive market for nonrenewable resource.

Objective function:

$$\max_{X_0, X_1, \ldots X_T} NPV(X_0, X_1, \ldots X_T) = B(X_0) + \frac{B(X_1)}{1+r} + \frac{B(X_2)}{(1+r)^2} + \ldots + \frac{B(X_T)}{(1+r)^T}$$

Equation of motion constraints: $S_{t+1} - S_t = X_t, \qquad t = 0, T-1$.

We can combine equation of motion contraints into a single constraint:

$$X_1 + X_2 + \ldots + X_T = S_0.$$

Lagrangian problem becomes:

$$\max_{X_0, X_1, X_t, \lambda} L = B(X_0) + \frac{1}{1+r} B(X_1) + \left(\frac{1}{1+r}\right)^2 B(X_2) + \ldots + \left(\frac{1}{1+r}\right)^T B(X_T)$$
$$+ \lambda\left(S_0 - X_0 - X_1 - \ldots - X_{T-1}\right)$$

or,

$$L = \underset{X_t\ \Omega}{Max} \left\{ \sum_{t=0}^{T} \left[\left(\frac{1}{1+r}\right)^t B(X_t) + \lambda \left[S_0 - \sum_{t=0}^{T} X_t \right] \right] \right\}$$

the FOC's imply: $B_X(x_0) = \lambda$

$B_X(x_1) = (1 + r)\lambda$

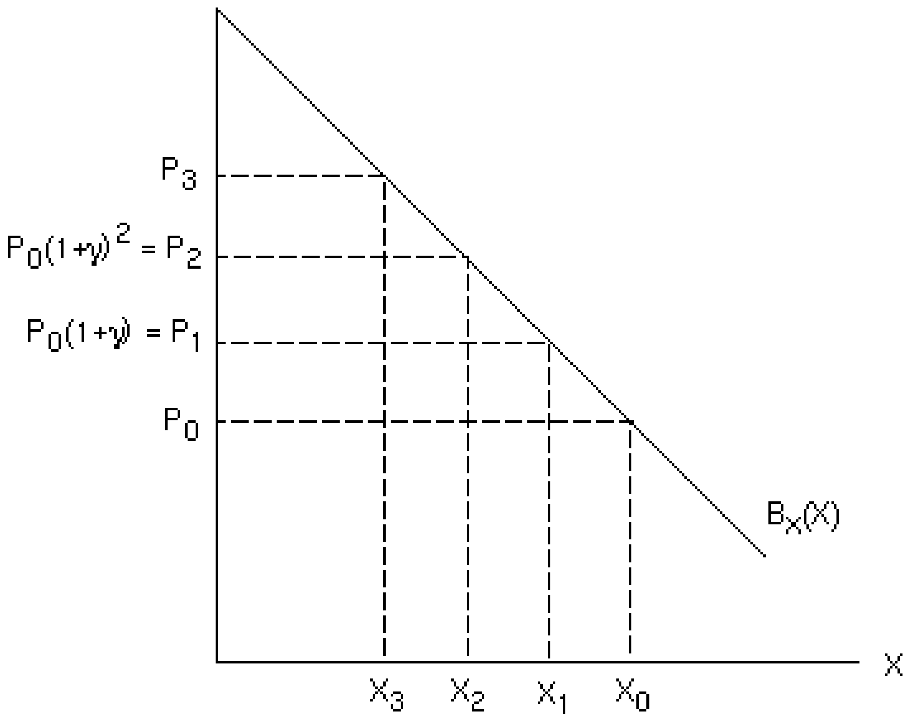
$B_X(x_2) = (1 + r)^2\lambda$

:

$B_x(x_t) = (1 + r)^t\lambda$

where $X_0 + X_1 + \ldots + X_T = S_0$ by the FOC of the constraint

Price Rises at the Rate of Interest and Extraction Decreases Over Time



- Recall that MB = P for a competitive industry.
- substituting price for Bx(Xt) in the optimal decision rules =>

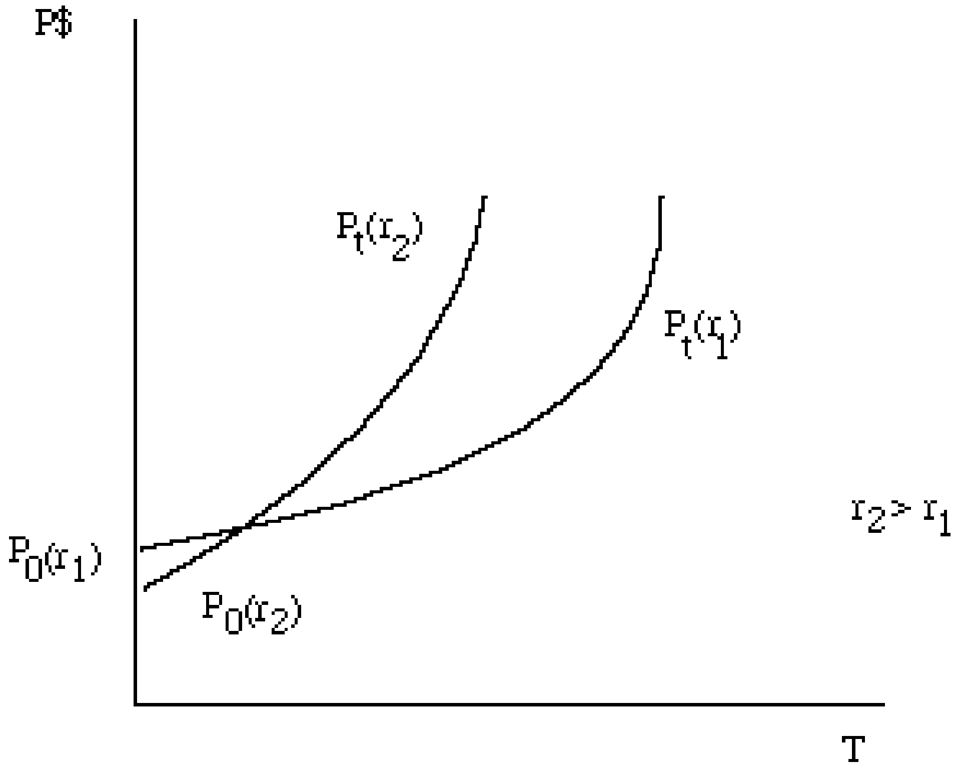
$$P_t = P_0(1 + r)^t ,$$

or, $$\frac{P_t - P_{t-1}}{P_{t-1}} = r, \quad t \; ,$$

- the price rises at the rate of interest

- as price, P_t , rises over time, the amount of the resource that is extracted in each period, X_t , will decline over time accordingly.

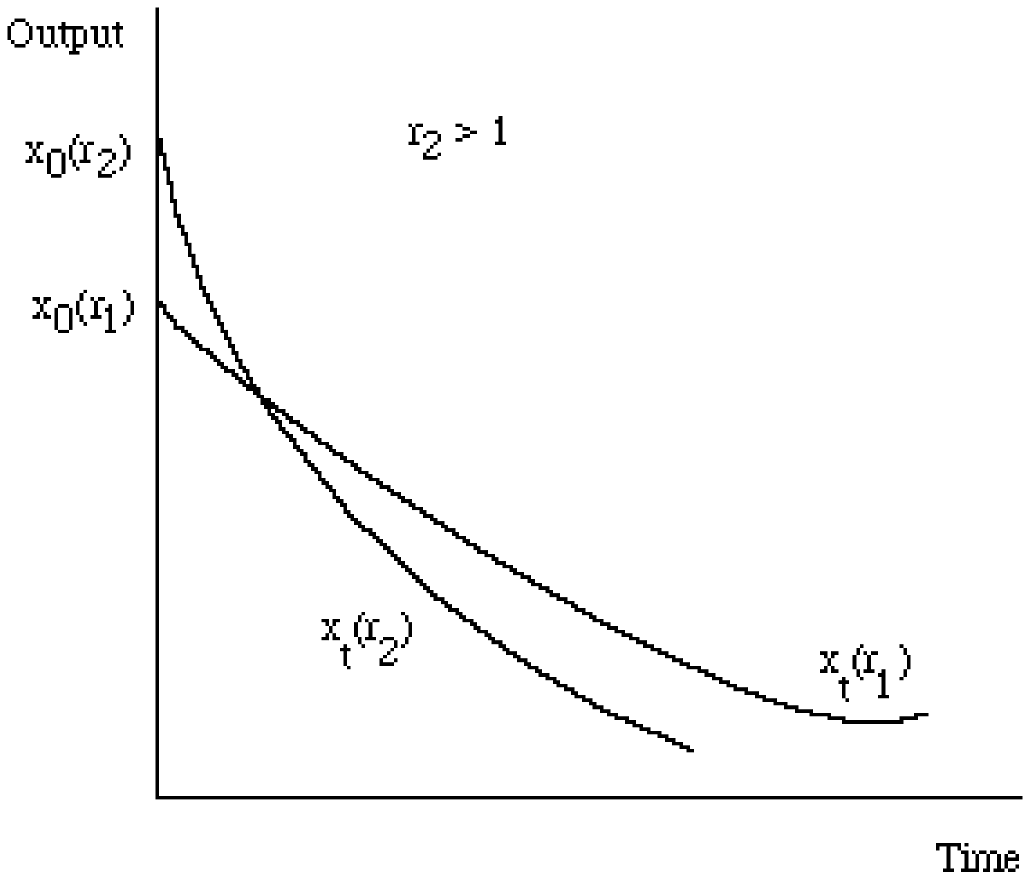
Higher interest rates lead to faster price increases but lower initial prices



· When r is larger, more is extracted in earlier time periods and less is extracted in later time periods.

=> prices rise faster over time, but the *initial price is lower* because the *initial level of extraction is larger*.

Higher interest rates lead to "faster exploitation" of resource stock



Summary of Nonrenewable Resource Model Results

Present price of exhaustible resource (P_0):

- Declines with r.
- Declines with extraction cost.
- Increases as demand increases.
- Decreases as new stocks are discovered.
- Declines as new extraction technologies are developed.
- Declines as backstop technologies are developed.
- Increases as industry gets more monopolistic.
- Declines as alternative products get cheaper.