Example: two period consumption

Suppose Mary owns a resource. Mary would like to consume the resource today. John would like to borrow Mary's resource for one year. Mary agrees to loan John the resource for one year if John will pay Many an amount to compensate her for the cost of delaying consumption for one year. (The amount loaned is called the **Principal.** The payment from John to Mary in compensation for Mary's delayed consumption is called the **Interest** on the loan.)

Suppose Mary's resource is \$100 in cash. Suppose the interest amount agreed to by Mary and John is \$10. Then, at the end of the year of the loan, John repays Mary the principal plus the interest, or \$110:

Principal + Interest = \$100 + \$10 = \$110

The (simple) **interest rate** of the loan, denoted **r**, can be found by solving the following equation for r:

Principal + Interest = (1 + r) Principal

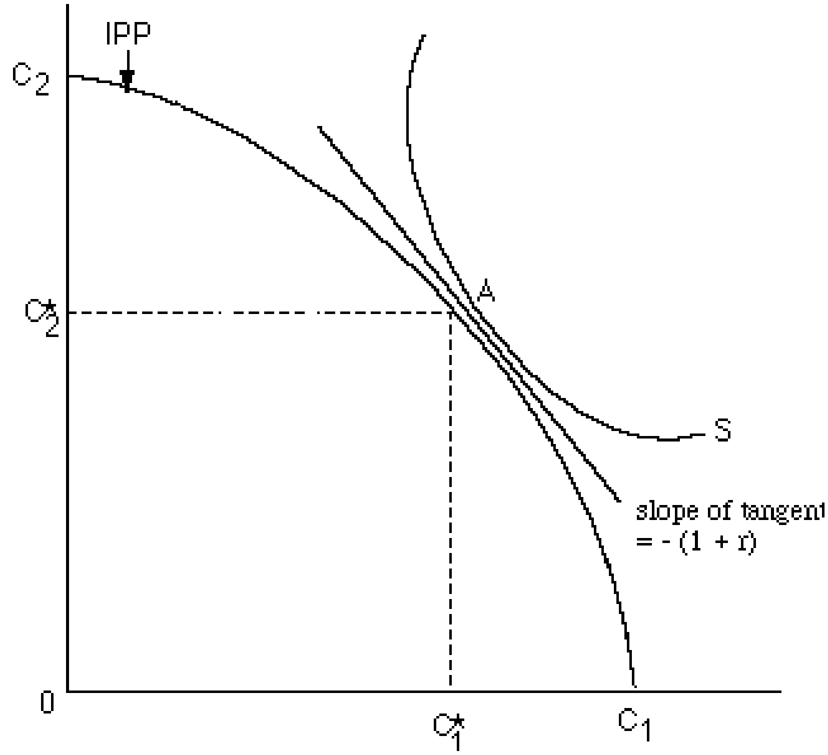
For this example: \$110 = (1 + r) \$100 => r = %10

Generally, we can find the interest rate by noting that:

$$B_1 = B_0 + r\, B_1 = (1+r)\, B_0$$

where:
B_0 = Benefit today
B_1 = Benefit tomorrow

The Interest Rate is an Equilibrium Outcome



C_1 = consumption in period 1,
C_2 = consumption in period 2

Delay of consumption (saving) in period 1 reduces current utility but increases utility in period 2. The **intertemporal production possibilities** curve (IPP) denotes the technological possibilities for trading-off present vs. future consumption.

The **skitovsky contour**, S, is an indifference curve showing preferences between consumption today vs. consumption in the future.

Utility max. occurs at point A, where S is tangent to the IPP.

Two ways to determine equilibrium interest rate, r:

- slope of S at point A = - (1 + r)
- slope of IPP at point A = - (1 + r)

Skitovsky Contour (a mathematical approach)

:

$$\underset{C1,C2}{Max.}\left\{U(C_1, C_2)\right\}$$

$$subject\ to\colon\ I = C_1 + \left(\frac{1}{1+r}\right)C_2$$

which can be written in lagrange form as:

$$L = U(C_1, C_2) + \lambda\left[I - C_1 + \left(\frac{1}{1+r}\right)C_2\right]$$

$FOCS$:

$$\left.\begin{array}{l} U_{C_1} = \lambda \\ U_{C_2} = \dfrac{\lambda}{1+r} \end{array}\right\} \qquad \frac{U_{C_1}}{U_{C_2}} = 1 + r$$

The skitovsky contour is found by setting:

$$U_{C_1} d_{C_1} + U_{C_2} d_{C_2} = 0 \qquad \frac{d_{C_2}}{d_{C_1}} = \frac{-U_{C_1}}{U_{C_2}} = -(1+r)$$

The skitovsky contour simply states that the equilibrium must occur where an individual cannot improve her inter-temporal utility at the margin by changing the amount consumed today and tomorrow.

The Components of Interest Rates

Interest rates can be decomposed into several elements:

- Real interest rate r
- Rate of inflation IR
- Transaction costs TC
- Risk factor SR

The interest rate that banks pay to the government (i.e., to the Federal Reserve) = r + IR. This is the **nominal interest rate**.

The interest rate that low-risk firms pay to banks = r + IR + TCm + SRm, where TCm and SRm are minimum transactions costs and risk costs, respectively. This interest rate is called the **Prime Rate**.

Lenders (banks) analyze projects proposed by entrepreneurs before financing them. They do this to assess the riskiness of the projects and to determine SR. Credit rating services and other devices are used by lenders (and borrowers) to lower TC.

Examples:

(1) If the real interest rate is 3% and the inflation rate is 4%,
--then the nominal interest rate is 7%.

(2) If the real interest rate is 3%, the inflation rate is 4% and TC and SR are each 1%,
--then the Prime Rate is 9%.

Discounting

Discounting is a mechanism used to compare streams of net benefits generated by alternative allocations of resources over time.

if \$10 were received at the beginning of the next time period, it would be equivalent to receiving only $\$10/(1 + r)$ at the beginning of the current time period. The value of \$10 received in the next time period is **discounted** by multiplying it by $1/(1+r)$.

· \$10 received *two* periods from now is worth $\$10/(1 + r)^2$.

the value today of \$B received in t periods is $\$B/(1 + r)^t$.

· this is called the **present value** of \$B in the future

Note that if the interest rate increases, the value *today* of an amount received in the future declines. Similarly, if the interest rate increases, then the value *today* of an amount paid in the future declines.

Example: Say you win the Lottery!

- You are awarded after-tax income of \$1M
- However, not all at once, but @ \$100K/year for 10 years
- r = 10%
- NPV = $100K+(1/1.1)100K+(1/1.1)^2 100K + (1/1.1)^3 100K + \ldots + (1/1.1)^9 100K$.
  = \$675,900
- The value of the last payment received is: NPV = $(1/1.1)^9 100K$ = \$42,410.

That is, if you are able to invest money at r = 10%, you would be indifferent between receiving the flow of \$1M over 10 years and \$675,900 today or between receiving a one time payment of \$100K 10 years from now and \$42,410 today.

The Present Value of an Annuity

An **annuity** is a type of financial property that specifies that some individual or firm will pay the owner of the annuity a specified amount of money *at each time period in the future*, ***forever!***

Example:

Jose owns an annuity that specifies that Megafirm will pay him $1000 per year forever. Question: What is the present value (PV) of the annuity? ......Answer: $1000/r.

if r =0.1, then the PV of Jose's annuity is $1000/0.1 = $10,000.

if r decreases, then the PV of the annuity increases
if r increases, then the PV of the annuity decreases

"the **sustainability** of a natural resource" is the ability to maintain a certain level of the natural resource forever

- this is mathematically the same as an annuity

Example:

Suppose society manages a small forest in a sustainable way such that we can harvest a million board-feet of timber per year from the forest forever. Suppose society derives $10 million per year in net benefits from the timber. Suppose the interest rate is 8%. Then the present value of the sustainable timber harvest from the forest is $10 million / 0.08 = $125 million. If society destroys the forest, society loses $125 million in present value timber benefits. Society would need to compare this loss with the gains associated with using the land for other purposes.

Miscellaneous Concepts

The Social Discount Rate is the interest rate used to make decisions regarding public projects. It may be different from the private interest rate:

- Differences between private and public risk preferences—the public overall may be less risk averse than a particular individual due to pooling of individual risk.

- Externalities—In private choices you consider only benefits to yourself; in public choices you consider benefits to everyone in society.

Uncertainty and interest rates

- borrowers may go bankrupt and not be able to repay the loan
- lenders may take several types of actions:
  - -- They Limit the size of loans
  - -- Demand collateral or co-signers
  - -- Charge high-risk borrowers higher interest rates

Risk - Yield Tradeoffs

Investments vary in their degree of risk. Generally, higher risk investments also tend to entail high expected benefits (i.e., high yields). If they did not, no one would invest money in the high risk investments. For this reason, lenders often charge higher interest rates on loans to high risk borrowers, while large firms can borrow at the Prime rate.

Evaluating Allocations of Resources Over Time

Net Present Value (NPV) = Sum of the present values of the net benefits accruing from an investment or project. Net benefit is time period t is B_t - C_t, where Bt is the Total Benefit in time period t and Ct is the Total Cost in time period t.

· Discrete time formula for N time periods with r = a constant:

$$NPV = \sum_{t=0}^{N} \frac{(B_t - C_t)}{(1+r)^t}.$$

Net Future Value (NFV): Sum of compounded differences between project benefits and project costs.

· Discrete time formula for N time periods with r = a constant:

$$NFV = \sum_{t=0}^{N} (B_t - C_t)\ (1+r)^{N-t}$$

Internal Rate of Return (IRR) = The IRR is the interest rate that is associated with zero NPV of a project. IRR = the x that solves the equation:

$$0 = \sum_{t=0}^{N} \frac{(B_t - C_t)}{(1+x)^t}$$

The Relationship Between IRR and NPV:

- <u>If r < IRR</u> then the project has a <u>positive NPV</u>
- <u>If r > IRR</u> then the project has a <u>negative NPV</u>

Project Investments (Examples):

- consider an investment which costs you $100 now but which will pay you $150 next year.

  If r = 10%, then the NPV is: -100 + 150/1.1 = $36.36
  If r = 20%, then the NPV is: -100 + 150/1.2 = $25
  If r = 50%, then the NPV is: -100 + 150/1.5 = $0

- Consider the "stream" of net benefits from the following:

Time Period:

	0	1	2
$B_t - C_t$:	-100	66	60.5

The NPV for this investment is:

$$\text{NPV} = -100 + \frac{66}{(1+0.1)^1} + \frac{60.5}{(1+0.1)^2} = 10$$

The IRR for this investment is the value of x that solves:

$$0 = -100 + \frac{66}{1+x} + \frac{60.5}{(1+x)^2} \implies 100 = \frac{66}{1+x} + \frac{60.5}{(1+x)^2}$$

$$100x^2 + 134x - 26.5 = 0. \implies x = \frac{-134 \pm \sqrt{(134)^2 + 400\ \ 26.5}}{200}$$

$$\implies x = \frac{-134+169}{200} = .175$$

if the investor has a discount rate of r < .175, then invest

Problem with Internal Rate of Return:

You may not get a unique answer!

· For example:

You pay "a" in period 0, receive "b" in period 1, and pay "c" in period 2. To derive internal rate of return z, solve:

$$-a + \frac{b}{(1+z)} - \frac{c}{(1+z)^2} = 0.$$

· Suppose a = 10, b = 30, c = 20, then:

$$-10(1+z)^2 + 30(1+z) - 20 = 0$$

Using the quadratic formula:

$$1+z = \frac{b \pm \sqrt{b^2 - 4ac}}{-2a} = \frac{30 \pm \sqrt{100}}{20} = \frac{30 \pm 10}{20}$$

(1 + z) = 1 or 2 ==> z = 0 or 1

the IRR calculation does not give us a unique answer.

Benefit-Cost Analysis

- **Step 1**: Estimate the economic impacts (costs and benefits) that will occur in the current time period and in each future time period.

- **Step 2**: Use interest rate to compute NPV or compute IRR of the project/investment. Use IRR only in cases in which net benefits switches sign once. Namely, investment cost occurs first and investment benefits return later. Use NPV in most cases.

- a project with a positive NPV has the potential to improve welfare

Issues in benefit-cost analysis:

- Discount rates affect outcomes of benefit-cost analysis.
- When discount rates are low, more investments are likely to be justified.
- Does public rate of discount = private rate of discount?
- How should nonmarket environmental benefits in benefit-cost analysis be incorporated in cost-benefit analysis?
- How should price changes because of market interaction be incorporated in benefit-cost analysis?
- How should uncertainty considerations be incorporated in benefit-cost analysis?