EEP101/ECON125 Spring 99
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TA's: Malick/Marceau

PROBLEM SET 5

Due Monday May 10, in your TA's mailbox

1. Controlled dehorning of rhinos has been proposed as a way of reducing poaching and raising income for wildlife management programs in Africa. Milner-Gulland et al. (1992)[1] calculated the optimal frequency at which to crop African rhinos. This exercise is a simplified version of their model. They found that the growth of a rhinos horn obeys the equation $Q(t) = 3(1 - 0.8e^{-0.87t})^3$ where Q(t) is the mass in kg after t years. It costs \$960 to remove a 3 kg horn and the price is \$750 per kg. Interest rates in African countries tend to be quite high, around 10% - 30%.
   (a) Using the single rotation forestry model (no replanting) and an interest rate of 10%, determine how long a rhino horn should be allowed to grow before removing it.
   (b) The authors actually argue that "the optimum rotation period will not usually exceed 3 years".
      (i) What factor(s) would shorten the optimal rotation time to something less than what you calculated in part (a)? (Hint: Rhino horn will regrow rather quickly once it is cut.)
      (ii) What factor(s) might explain more frequent dehorning in reality?

2. Consider two landowners in very different northern US states. One is an old reclusive coot who enjoys hunting and talks only to other hunters. The other is the government. They both hold the same amount of land on which a population of deer roam. The deer population (*S*) grows according to the following relationship: $g(S) = 0.4S - 0.00002S^2$. Hunters derive a constant marginal benefit of \$12 from each deer successful killed[2]. However, the cost of hunting reflects primarily a combination of foregone wages and time spent in the activity. The costs per deer are $\frac{C(x,S)}{x} = \frac{60000}{S}$. In addition to this, hunters will incur any license fees charged by the landowner.

   (a) What is the maximum carrying capacity of the stock? Compute the size of the stock and the annual harvest at maximum sustainable yield.

   (b) The government, ceding to the argument that public forests belong to everyone and everyone should be allowed to use them, sells hunting licenses for a nominal fee of \$4 per deer to anyone willing to buy. The fee is used to finance a program that monitors the deer population. Calculate how much money will be raised for this program each year in a steady state. To do this, you will need to determine the harvest level (*x*).

[1] E.J. Milner-Gulland, J.R. Beddington and N. Leader-Williams (1992), "Dehorning African Rhinos: A Model of Optimal Frequency and Profitability", Proc. R. Soc. Lond. B, 249: 83-87.
[2] This is like saying the hunter will receive \$12 for each deer killed.

(c) What is the population of deer on the government-owned land when hunting takes place? Is this more or less than the maximum sustainable yield? On your graph from part (a), indicate the harvest and population levels on the government's land.

(d) The old coot cannot bring himself to charge his fellows for hunting. However, he limits the number of deer he allows them shoot. If he chooses the number of deer "rights" so as to maximize the net benefits of the hunter community over time, how many deer will he allow to be harvested? (The solution to a quadratic equation $aZ^2 + bZ + c = 0$ can be found with $Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and bearing in mind that S ≥ 0.)

(e) Will the deer population on the old coot's land be larger or smaller than the deer population on the State-owned land? Explain briefly why. On your graph from part (a), indicate the harvest and population levels on the private land.

(f) Suppose the old coot asks each hunter to contribute $1 to help buy bales of hay that will be left in the woods for the deer during the harsh winter. How might this affect the growth curve? Illustrate graphically. How might this enable hunters to take more deer without compromising the population?

3. What fee would the government have to charge in order to <u>maximize steady state annual revenues</u> for its deer-monitoring program when hunters treat the resource as an open access one? To answer this, go through the following steps:
(a) Let *l* denote the fee. What is the objective function?
(b) What are the two constraints (Hint: steady state, open access)?
(c) In setting the fee, the government also sets the harvest and stock levels. What are the three control variables in this problem?
(d) Write out the Lagrangian.
(e) Find the first order conditions.
(f) Use substitutions to rearrange the first order conditions to eliminate two of the control variables (*x* and *l*). What is the one remaining equation?
(g) Using the information given in question 2, solve for the revenue maximizing level of *S*, *x* and *l*.