Suggested Solutions to Problem Set 3*

1.

(a) The firm's profits are given by the difference between revenues and costs,

$$\Pi = P_y Y - P_x X \tag{1}$$

Using the equation for the production function of Y,

$$\Pi = P_y (hX)^{\beta} - P_x X \tag{2}$$

If the firm's objective is to maximize profits, then the firm's problem is stated as,

$$\text{Max}_x \ \Pi = P_y (hX)^{\beta} - P_x X \tag{3}$$

The first order conditions (FOC) for this problem are given by,

$$\frac{\partial \Pi}{\partial X} = 0$$

$$\beta h P_y (hX)^{\beta - 1} - P_x = 0$$

or

$$\beta h P_y (hX)^{\beta - 1} = P_x \tag{4}$$

The value of the marginal product of applied input X should be equal to its marginal cost P_x.

Solving for X in equation (4) gives us the *profit maximizing input demand function* for X.

$$X^* = \left(\frac{h^{\beta}\beta P_Y}{P_X}\right)^{\frac{1}{1-\beta}} \tag{5}$$

* Solution to question 1 provided by G. Malick. Solution to question 2 provided by S. Marceau.

(b) With $\beta = .75$, $P_y = 500$, and $P_x = 80$, the demand function for X is

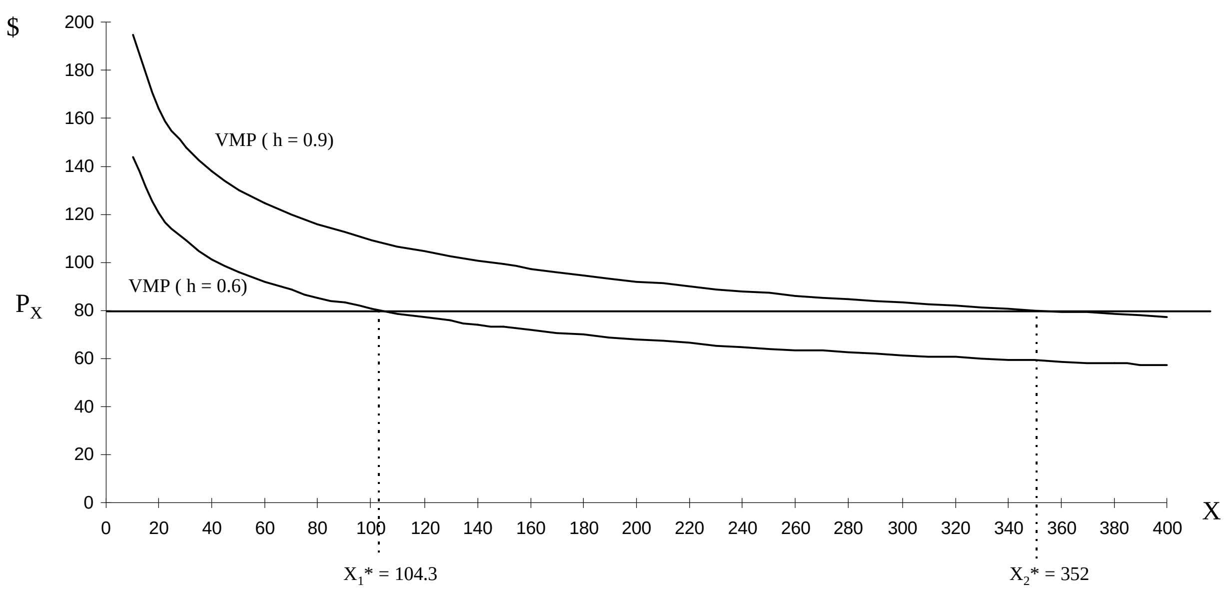
$$X^* = \left(\frac{h^{.75}(.75)(500)}{80}\right)^{\frac{1}{1-.75}} = 482.8\, h^3 \tag{6}$$

With this result, we can create the following table:

h	$X^* = 482.8\, h^3$	$Y^* = (hX^*)^{0.75}$	$\Pi = 500Y^* - 80X^*$
0.6	104.3	22.2	2781
0.9	352.0	75.1	9386

(c) The grower will change to drip irrigation because the profits increase by $9,386 - $2,781 = $6,605, while the capital costs of adopting such technology are only $5,000.

(d) Yes. The increment in the productivity is so high, that we will end up producing more, earning more profits, and using more of X. In this particular example, switching to drip irrigation will increase input use from $X_1^* = 104.3$ to $X_2^* = 352$. Look at the following graph.



2.

There are two graphs for this exercise. One refers to the three types of individual demands. The other refers to the aggregate demand for the public good and the marginal costs of providing that public good.

(a) The most an individual is willing to pay for 1 garden is given by the area under their marginal benefit curve up to 1 unit of the public good. This is just the integral of the individual marginal benefit curves from 0 to 1.

Maximum willingness to pay of a tourist:

$$WTP_t = 10Q - \frac{5Q^2}{2}\bigg|_0^1 = 10 - \frac{5}{2} = 7.5$$

Maximum willingness to pay of a “high” value local:

$$WTP_h = \frac{20Q}{3} - \frac{4Q^2}{3\times 2}\bigg|_0^1 = \frac{20}{3} - \frac{2}{3} = 6$$

Maximum willingness to pay of a “low” value local:

$$WTP_l = \frac{10Q}{3} - \frac{2Q^2}{3\times 2}\bigg|_0^1 = \frac{10}{3} - \frac{1}{3} = 3$$

(b) The aggregate marginal benefit curve is the sum of all the individual marginal benefit curves. Thus,

$$MB_{agg} = 45\times MB_t + 15\times MB_h + 15\times MB_l$$
$$MB_{agg} = (450 - 225Q) + (100 - 20Q) + (50 - 10Q)$$
$$MB_{agg} = 600 - 255Q \;\; for\ Q \leq 2$$

The qualification that this marginal benefit curve is valid only for Q ≤ 2 arises because the tourists aren’t willing to pay any additional positive amount for more than two gardens. That is, their marginal benefits are zero beyond 2 gardens, and we need to take that into account when deriving the aggregate marginal benefits. The locals are still willing to pay to have additional gardens. Therefore, the aggregate marginal benefits are still positive. Yet, they only reflect the positive valuation of the locals. Thus, we also have

$$MB_{agg} = 45 \times 0 + 15 \times MB_h + 15 \times MB_l$$
$$MB_{agg} = 150 - 30Q \;\; for\ Q > 2$$

The two segments of the aggregate marginal benefit curve are:

$$MB_{agg} = 600 - 255Q \;\; for\ Q \le 2$$
$$MB_{agg} = 150 - 30Q \;\; for\ Q > 2$$

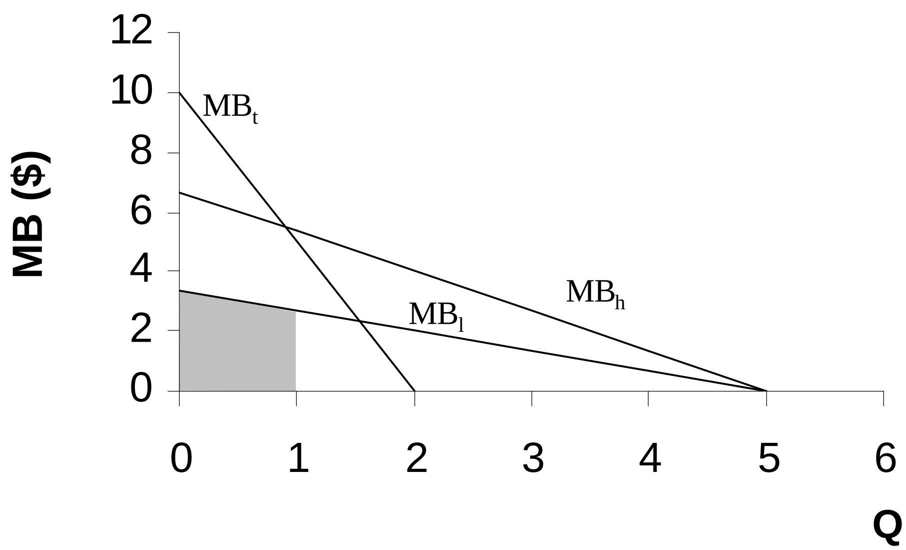
(c) Social welfare is maximized when the marginal cost of providing one more garden equals the benefit of one more garden, i.e., where marginal cost equals marginal benefit. Thus, MC = 30 + 315Q = 600 - 255Q = MB_{agg}. Solving for Q gives Q* = 1. Since this is less than two, we know that we have used the relevant section of the aggregate marginal benefit curve. The total cost of Q* = 1 is the integral under the marginal cost curve between 0 and 1 (or the area of the parallelogram).

Total cost = $30Q + \left.\frac{315Q^2}{2}\right|_0^1 = 30 + \frac{315}{2} = 187.5$.

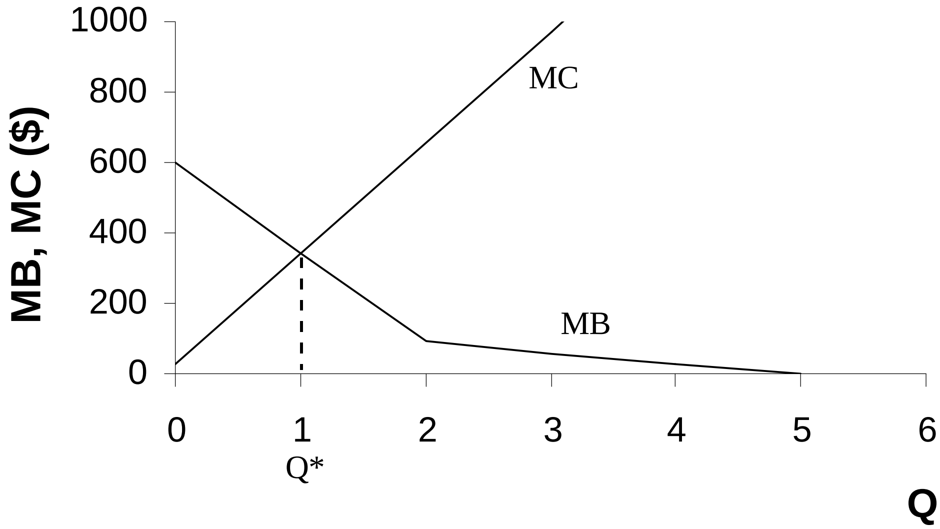
(d) We can see from our calculations in (a) that all types are willing to pay more than \$2.50. All types will visit the garden, implying 75 visitors. Total revenues are TR = fee$\times$number of visitors = 2.50$\times$75 = 187.5. Total costs were 187.5, implying profits = 187.5 - 187.5 = 0.

(e) The “low” value locals are willing to pay at most \$3 to visit the garden. At \$3.50, they will choose not to go and the garden will get only 60 visitors. The highest fee that could be charged that doesn’t exclude anyone is \$3 - the most the low-value locals are willing to pay. At \$3.00, total revenues would be TR = 3$\times$75 = 225. At \$3.50, total revenues are 3.5$\times$60 = 210. Total profits are 225 - 187.5 = \$37.50 when the fee is \$3. Profits are 210 - 187.5 = \$22.50 when the fee is \$3.50. The Garden would make more money at the lower price because the increase in the fee per person is not enough to compensate the lower number of people visiting the garden.

Individual Marginal Benefits



Aggregate Marginal Benefits and Marginal Cost



(f) The monopoly concessionaire will charge a fee that will maximize profits. Since the costs of maintaining the garden do not depend on the number of users in this example, maximizing profits is equivalent to maximizing revenues. The monopolist will want to extract as much consumer surplus as possible. As there are three types of consumers and the monopolist must set a single fee, it has three options:

option 1: charge a fee equal to the most the "low" types will pay.
option 2:charge a fee equal to the most the "high" types will pay, thereby excluding the "low" types.
option 3: charge a fee equal to the most the tourists will pay, thereby exclusing the locals.

We saw that total revenues under option 1 are $225. Under option 2, the fee is $6 (calculated in part (a)) and the number of visitors is 60. Total revenues are $6\times60 = 360$. Under option 2, the fee is $7.50 and there are 45 visitors. Total revenues are $7.5\times45 = 337.5$.

Profits are highest under option 2 and a fee of $6. Although the tourists are willing to pay the most, the maximum increase in the fee is not enough to outweigh the fact that fewer people are buying tickets to enter the garden.

(g)Yes, the Garden can make positive profits even if only the tourists pay. In fact, the RPC doesn't even have to charge the tourists their full willingness to pay. They only need to charge them the amount that will cover their costs, i.e., 187.5/45 = $4.17. Logistically, the locals would only need to show proof of residence (e.g., ID, driver's license, phone bill, etc.) to get in free.

3.

Some visitors to the Japanese Tea Garden have expressed the opinion that the residents of San Francisco should not have to pay to enjoy one of the more treasured amenities in Golden Gate Park. If locals were the only users of the Garden, free access at the gate would not necessarily mean free enjoyment. The Garden's operating expenses would need to be paid somehow. One alternative is to use public funds, i.e., taxes, to finance the daily operations of the Garden. Indeed, this is the solution for many public goods, including some of the other amenities in the park.

Is this the only solution? No, government support may be justified in the case of a pure public good, but the excludable nature of a park can make raising revenues from individuals feasible. No one may be prevented from using a pure public good, and one person's use does not detract from someone else's use of that good. Under these circumstances, it is difficult to exact payment for the good once it is provided since people know they will be able to enjoy it even without paying. People may not be able to enjoy a garden if there is a fence around it, and getting through the fence means paying a fee. This is the case with the JTG. It is not a pure public good.

Moreover, the residents of San Francisco are not the only people who derive benefit from visiting the Garden. Each year, a large number of tourists willingly pay the entry fee. In our numerical exercise, the tourists' willingness to pay to visit the Garden outweighed the cost of maintaining it. If we considered a policy in which only the tourists paid a cost-recovery-based fee with a policy in which everyone was asked to pay a cost-recovery-based fee, we would find that net benefits were the same. And they would be as high as they possibly could be.

Consider the costs and benefits of these two policies. First of all, note that the costs are the same regardless of the level of the entry fee. If we set the fee so that all costs will be paid and no more, then the producer's surplus will be zero. In order to determine social welfare, we only need to consider the consumers' surplus. The locals' surplus is lower by 2.5×30 = \$75 under the uniform fee compared to the free policy. When the locals are granted free entry, this is the increase in the amount the tourists must pay. Thus, their surplus is \$75 lower under the tourists-only-pay fee. Under the TOP policy, tourist surplus is \$75 lower and locals' surplus is \$75 higher. Under the universal fee, the locals' surplus is \$75 lower and the toursists' surplus is \$75 higher. The losses and gains are shifted around so that the net effect is zero. The same number of people are visiting the Garden under both policies and the net benefits are the same under both policies. We cannot say which policy is better using only the efficiency criteria and without bringing in other considerations. We can say that both policies are feasible.