EEP101/ECON125 Spring 99
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Suggested Solutions to Problem Set 2*

1. All answers referring to graphical details are to be found in figure 1.

a) To find the aggregate marginal benefit curve, solve each of the individual marginal benefit curves for q_i and then add them together. This gives $q_1 = 50 - MB$ for firm 1 and $q_2 = 150 - 1.5\times MB$ for firm 2. Adding these together, we get $Q = q_1 + q_2 = 200 - 2\times MB$ and $MB = 100 - 0.5\times Q$.

b) Firm 1 is more efficient at reducing pollution. It derives less additional benefit from a given increase in pollution. Therefore, its opportunity costs of not polluting are lower. At a given level of pollution, it can reduce its emissions by one unit more cheaply than firm 2. For example, if the two firms were emitting 30 units of pollution, firm 1 has a marginal abatement cost of 40 while firm 2 has a marginal abatement cost of 80. Notice that what matters for efficiency in this context is not so much the slope of the curves, but their relative heights.

c) The optimal level of emissions is where marginal social costs equal marginal social benefits. Equating the marginal benefits found above with marginal social costs leads to the condition: $MB = 100 - 0.5\times Q^* = 40 + Q^* = MSC$. Solving for Q gives $Q^* = 40$. The intersection of these two curves is point **e** in figure 1.

d) The optimal tax, t*, will be the one that induces firms to choose to emit the optimal amount of pollution. Firms will choose to pollute and pay the tax as long as that costs less than abating. They will choose to reduce their pollution if the cost of abating one more unit is cheaper than the cost of paying the tax on one more unit. Thus, firms will abate up to the point where MB = t. Marginal benefits at $Q^* = 40$ are $MB(Q^*) = 100 - 0.5\times 40 = 80$. The optimal level of the tax is $T^* = 80$. We may also find the tax as the marginal social cost at Q* - since marginal benefits and marginal costs are the same at that point. This gives $t^* = MSC(Q^*) = 40 + 40 = 80$, as before.

Firm 1 will pollute up to the point where $MB = 100 - 2q_1 = 80$, implying $q_1^* = 10$. And firm 2 will pollute up to the point where $MB_2 = 100 - (2/3)q_2 = 80$, implying $q_2^* = 30$. Notice that $q_1^* + q_2^* = 10 + 30 = 40 = Q^*$.

e) Because the firms' marginal benefit curves are also their marginal abatement cost curves, the total cost of reducing emissions up to q_1^* and q_2^* is the area under

* Solution to question 1 provided by S. Marceau. Solutions to questions 2 and 3 provided by G. Malick.

the marginal benefit curves of each firm over the units of pollution they are no longer producing. That is, we measure the costs starting from the level of pollution before regulation up to the level of pollution that results from the environmental policy. For firm 1, total abatement costs are $TC_1 = 0.5\times(50 - 10)\times80 = 1600$. For firm 2, total abatement costs are $TC_2 = 0.5\times(150 - 30)\times80 = 4800$. The total social costs of reducing pollution up to Q* under a tax regime are the sum of the abatement costs incurred by the two firms: $TC_{tax} = TC_1 + TC_2 = 1600 + 4800 = 6400$.

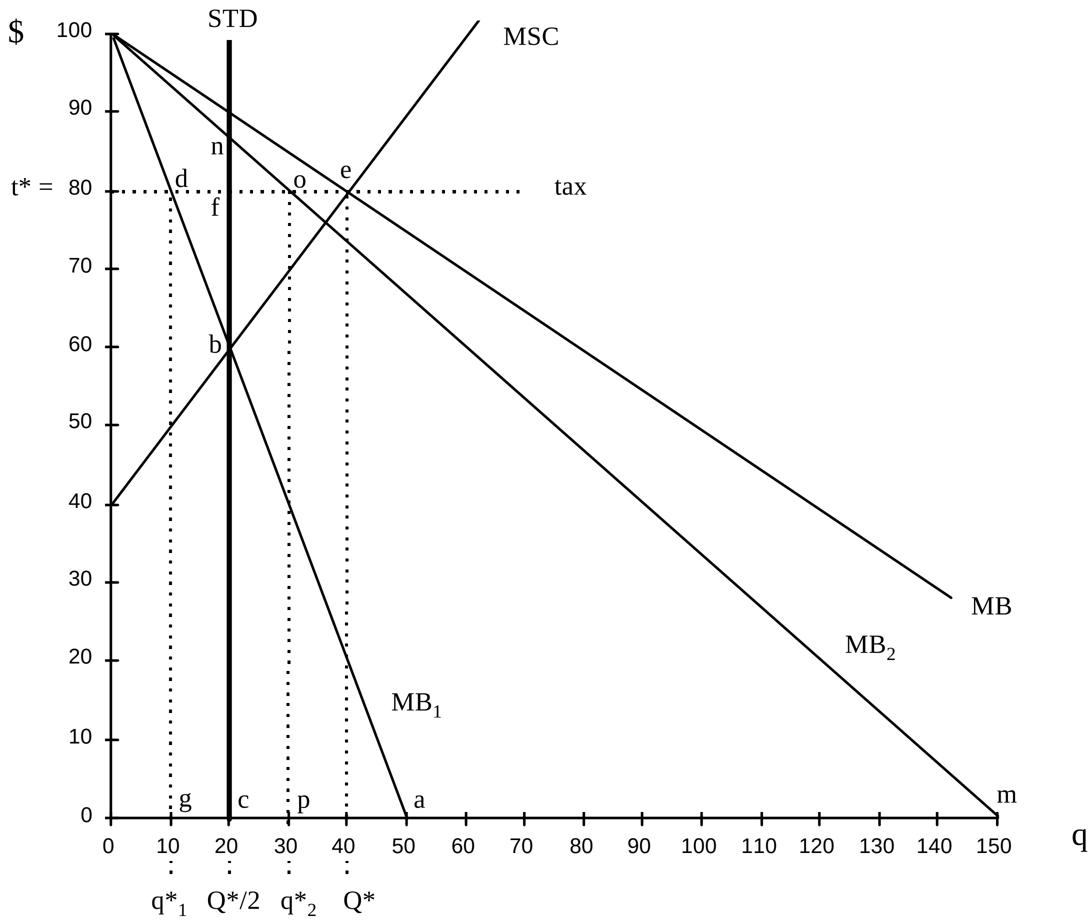
f) When the government sets a uniform standard equal to Q*/2 for each firm, it limits each firm's emissions to Q*/2 = 40/2 = 20 units. Thus, each firm produces 20 units of pollution.

g) Under the uniform standard, firm 1 is producing ten units more pollution than it did under the tax. Firm 2 is producing ten using less than it did under the tax. One would like to see the firm that benefits most from additional pollution be allowed to pollute more than a firm that benefits less. Equivalently, one would like to see the firm that suffers less from foregoing pollution (has lower marginal abatement costs) pollute less than a firm that suffers more. This does not arise under the uniform standard, but it does under the tax.

h) To calculate the total costs of reducing pollution under the standard, we need to know the value of the marginal at q = 20. For firm 1, $MB_1 = 100 - 2\times20 = 60$. For firm 2, $MB_2 = 100 - (2/3)\times20 = 86.67$. The total costs of reducing pollution to 20 units are $TC_1 = 0.5\times(50 - 20)\times60 = 900$ for firm 1 and $TC_2 = 0.5\times(150 - 20)\times86.67 = 5633.33$ for firm 2. Total social costs of reducing pollution to Q* = 40 under the uniform standard are $TC_{std} = TC_1 + TC_2 = 900 + 5633.33 = 6533.33$. This is larger than the costs of reducing pollution to the optimal level under the tax scheme. Thus, the tax is more efficient since it minimizes the cost of reducing pollution to a given level. The tax is more efficient than the standard because it achieves the optimal distribution of pollution among firms within the industry in addition to achieving the optimal level of pollution for the industry as a whole. The tax ensures that the firm that benefits the least from producing additional pollution will not emit as much as a firm that benefits more from producing additional pollution. The uniform standard ignores the differences between firms.

i) There are two conditions under which a tax or a standard would be equally efficient. First, if firms have the same marginal benefit curves, then they will all choose the same amount of pollution under a tax. This coincides with a standard since all firms produce the same amount of pollution, as they do under a uniform standard. Second, If firms are allowed to trade their permits to pollute, they will exchange their pollution allocations up to the point where their marginal benefits are equal. When firms have different marginal benefits/marginal abatement costs at a given level of pollution, it is in the high-cost firm's interest to offer to buy permits from the low-cost firm. The low-cost firm will demand a price no lower than their own marginal abatement cost and the high-cost firm will not agree to pay a price higher than their own marginal benefit. When there is no longer any difference between marginal benefits/marginal abatement costs between firms, trading will cease. At this point, marginal benefits will de equalized for all firms. This is the same condition we have under a tax scheme. If the total amount of emissions is the same in both cases - taxes and tradable permits - then, when marginal benefits are equated in both cases, firms will each be producing the same amount of pollution in both cases.

j) Taxes provide a greater incentive. Suppose the two marginal benefit curves above give the marginal abatement costs associated with two technologies instead of two firms. Firm 2 here, represents the "dirtier" technology. When a firm adopts a cleaner technology, it lowers its costs of abiding by a given environmental policy. Not only will its costs of reducing pollution be lower, it will be producing less pollution under a tax. Hence, the cleaner technology allows the firm to reduce its tax payment as well. It must incur the fixed costs of the cleaner technology, however. If the reduction in the firm's costs outweigh these fixed costs, then the firm will want to adopt the cleaner technology. Consider the exercise above. A firm with the "dirtier" technology would spend $4800 + 80\times30 = \$7200$ when there is a tax. A firm with the cleaner technology would spend $900 + 80\times10 = \$1700$ when there is a tax. Thus, under a tax, adopting the cleaner technology saves $7200 - 1700 = \$5500$. Let the standard be set at the same level that would prevail under the tax with the dirty technology, that is, $q = 30$. A firm would spend \$4800 to comply if they had that technology, but only $0.5\times(50 - 30)\times40 = \$400$ if they had the cleaner technology. Under a standard, adopting the cleaner technology saves only \$4400. Since the savings are larger under the tax, a firm is more likely to adopt under a tax scheme than under a standards scheme. If the cost of the technology were \$4000, then a firm will want to adopt the cleaner technology under either a tax or a standard. If the cost of the technology is \$5000, then a firm will want to adopt it only under a tax policy. Even though both policies provide an incentive for adopting cleaner technologies, the tax provides a larger incentive. Essentially this is because the clean technology enables the firm to lower its abatement costs and its tax bill. The reduction in the tax bill adds to the reduction in pollution-control costs to increase the potential savings.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2. Please refer to Table 1 and Figure 1.

a) Firm J has the lowest input per output rate (x = 0.21) and the lowest pollution per output rate (z = 0.11).

b) Case 1. With (P,W,V) = (10,10, 8), firms A and D will have negative profits, so they will not be able to compete. The total output of the surviving firms is 330, and the total pollution is 80.

c) Case 2. With (P,W,V) = (10,12, 8), firms A, D, and H will have negative profits, so they will not be able to compete. The total output of the surviving firms is 305, and the total pollution is 70.

d) Case 3. We now eliminate the tax, which gives us (P,W,V) = (10,12, 0). In this case, firms A and H will not be able to compete. As it was determined in part (a), total output is 330 and total pollution level is 80. Since we want to have the same level of pollution as we had with the tax (70 units of pollution), we need to eliminate 10 units of pollution, which is the pollution output of one firm.

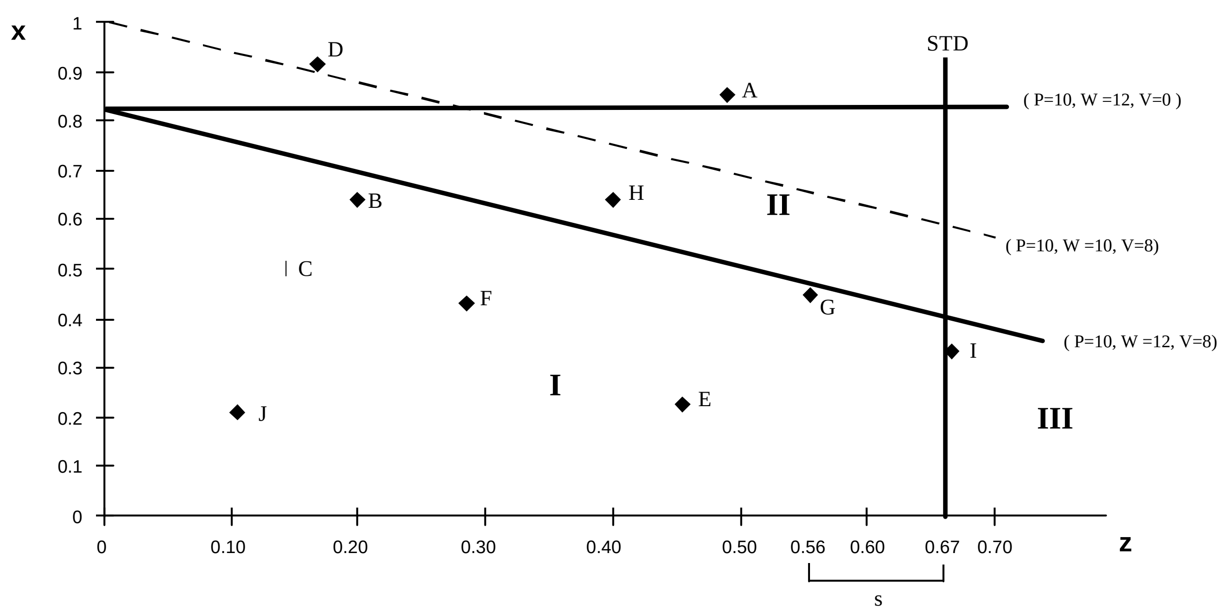
Therefore, we introduce a standard restricting the maximum pollution per output to be allowed per firm so that we eliminate only one firm; the dirtiest firm (firm I). Any standard belonging to the open interval s = (0.56 , 0.67) will be sufficient to eliminate just firm I. Thus, the maximum standard will be a little bit less than 0.67 emissions per output.

e) Under the tax alternative, only region I and region III firms will survive. These firms are {B, C, F, G, E J, I}. The total output of these firms is 305. Under the standard alternative, only firms in region I and region II will survive. These firms are {B, C, F, G, E J, H}, and the total output is 315. The relevant comparison between the two policies is the total output of firms in region III under a tax, versus the output of firms in region II under the standard. The only firm in region III is firm I with an output of 15 units, and the only firm in region II is firm H with an output of 25 units. Therefore, this explains the difference of 10 units in total output (305 under the tax versus 315 under the standard). We see in this example that although the two policies achieve the same pollution level, the tax policy reduces more the aggregate output than the standard policy does.

Table 1

						Case: 1			Case: 2			Case: 3		
						P=10, W=10, V=8			P=10, W=12, V=8			P=10, W=12, V=0		
Firm	**Q**	**L**	**Z**	***z = Z/Q***	***x = L/Q***	**Profits**	**Z**	**Q**	**Profits**	**Z**	**Q**	**Profits**	**Z**	**Q**
A	21	18	10	0.48	0.86	-50	0	0	-86	0	0	-6	0	0
B	50	32	10	0.20	0.64	100	10	50	36	10	50	116	10	50
C	70	35	10	0.14	0.50	270	10	70	200	10	70	280	10	70
D	60	55	10	0.17	0.92	-30	0	0	-140	0	0	-60	0	0
E	22	5	10	0.45	0.23	90	10	22	80	10	22	160	10	22
F	35	15	10	0.29	0.43	120	10	35	90	10	35	170	10	35
G	18	8	10	0.56	0.44	20	10	18	4	10	18	84	10	18
H	25	16	10	0.40	0.64	10	10	25	-22	0	0	58	10	25
I	15	5	10	0.67	0.33	20	10	15	10	10	15	90	10	15
J	95	20	10	0.11	0.21	670	10	95	630	10	95	710	10	95
Total	**411**		**100**				**80**	**330**		**70**	**305**		**80**	**330**

Figure 1



3) Please refer to Figure 2.

a) The deadweight loss due to the use of a standard (d1 = 40) is less than that of a tax (d2 =90). Therefore, the standard is preferred to the tax.

b) You set the standard or the tax according to your estimated equilibrium at point e, (q = 40). However, this is not the true social optimum because the marginal benefits have been overestimated. The true social optimum is at point e*, (q = 32).

<u>Deadweight loss of the tax</u>

$$d2 = \int_{q_{tax}}^{q^*} [MB(q)_{true} - MSC(q)]dq = \int_{20}^{32} [40 - 1.25q]dq = 40q - (.625)q^2\Big|_{20}^{32} = 90$$

<u>Deadweight loss of the standard</u>

$$d1 = \int_{q^*}^{q_{std}} [MSC(q) - MB(q)_{true}]dq = \int_{32}^{40} [-40 + 1.25q]dq = -40q + (.625)q^2\Big|_{32}^{40} = 40$$

c) Yes. Now the deadweight loss of the tax will be lower than the deadweight loss of the standard. In this case, the deadweight loss of the tax is d2 = 9 , while the deadweight loss of the standard is d1 = 25. Thus, in this case, the tax would be preferred to the standard.

d) Yes, now a tax and a standard will yield the same deadweight loss d1 = d2 = 50.

The choice of one instrument over the other will depend on the <u>relative slopes</u> of the MB and MSC curves. In general,

i. if $\left|\frac{\partial MB}{\partial q}\right| > \left|\frac{\partial MSC}{\partial q}\right|$, then a tax is preferred over the standard. (part c)

ii. if $\left|\frac{\partial MB}{\partial q}\right| = \left|\frac{\partial MSC}{\partial q}\right|$, then a tax is indifferent to the standard. (part d)

iii. if $\left|\frac{\partial MB}{\partial q}\right| < \left|\frac{\partial MSC}{\partial q}\right|$, then a standard is preferred over the tax. (part b)

e) No. the uncertainty about the MSC curve is irrelevant because the firms' behavior is determined by the MB curve. In this case, both the standard and the tax will have the same deadweight loss because the quantity produced under both schemes is exactly the same.

Figure 2