Numerical Examples Used in Class for Nonrenewable Resources

Optimal Solutions vs. Monopoly

Suppose demand = $\partial B / \partial x = 10 - x$,
Extraction cost = 0
r = 1, interest rate of 100%

We know that x_0 and x_1 are constrained to be smaller than 10 since, at $x = 0$,

$\frac{\partial B}{\partial x}(x = 10) = 0$.

The Optimal Solution as Function of S_0 if the Constraints Are Binding

$$\frac{\partial B}{\partial x}(x_0) = \frac{1}{1+r}\frac{\partial B}{\partial x}(x_1) = \lambda$$

$$10 - x_0 = \tfrac{1}{2}\left[10 - (S_0 - x_0)\right]$$

which implies $x_0 = \frac{10 + S_0}{3}$ $x_1 = S_0 - x_0$

If $S_0 = 6,\ x_0 = \frac{16}{3},\ x_1 = \frac{2}{3}$

$S_0 = 8,\ x_0 = 6,\ x_1 = 2$

$S_0 = 10,\ x_0 = 6\tfrac{2}{3},\ x_1 = 3\tfrac{1}{3}$

$S_0 = 12,\ x_0 = 7\tfrac{1}{3},\ x_1 = 4\tfrac{2}{3}$

Monopoly Solutions

Note that x cannot exceed 5 since

$$MR(x) = 10 - 2x$$

At $x = 5$, marginal revenue = 0; therefore, production will not exceed 5.

The optimality conditions of monopoly are

$$\lambda_0^M = MR(x_0) = \frac{1}{1+r} MR(x_1) = \frac{1}{1+r} MR(S_0 - x_0)$$

when the resource constraints are binding

$$10 - 2x_0 = \tfrac{1}{2}\left(10 - 2(S_0 - x_0)\right)$$

$$x_0^M = \frac{5 + S_0}{3}.$$

For $S_0 = 6$,

$$x_0^M = \tfrac{11}{3},\; x_1^M = \tfrac{7}{3}$$

when $S_0 = 8$,

$$x_0^M = \tfrac{13}{3},\; x_1^M = \tfrac{11}{3}.$$

In both cases, consumption in period 0 is lower under monopolistic solutions than under optimal solutions.

For $S_0 = 10$,

$$x_0^M = x_1^M = 5$$

For $S_0 = 12$,

$$x_0^M = x_1^M = 5.$$

For cases with $S_0 > 10$, the monopoly solutions does not change. However, optimal solutions will utilize all the resources for $S_0 \leq 20$.