EEP101/ECON 125 Spring 00
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Suggested Solutions to Problem Set 4

Stock of deer = S
Harvest or deer that are killed = X
Growth of the Stock = $g(S) = aS - bS^2$, where $a = 0.8, b = .00002$
Marginal Benefit (or price) per deer = P
Total Cost per deer = $C(X,S) = \frac{KX}{S}$, where $K = 80{,}000$

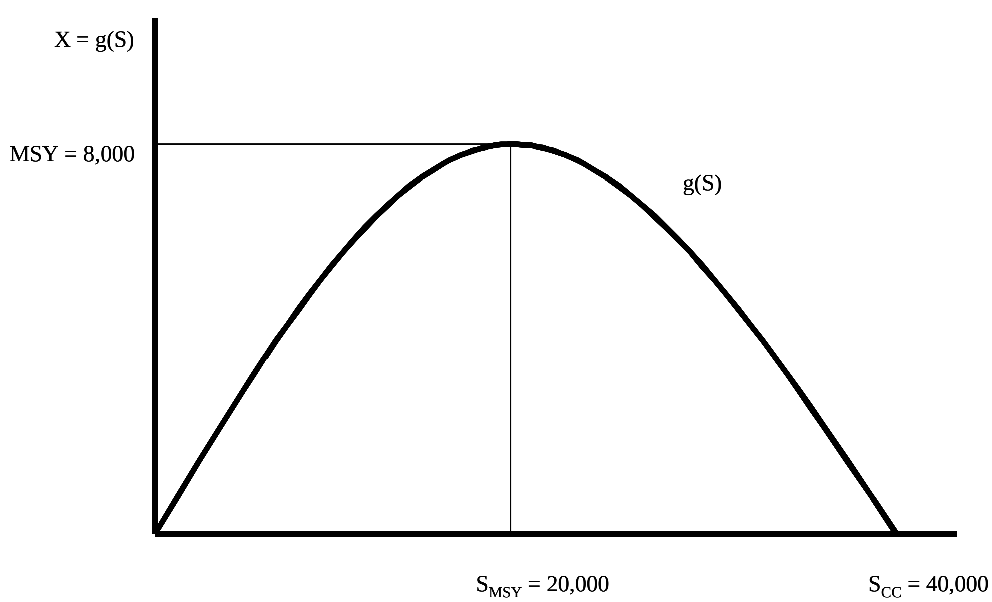
A) A steady state, defined by $X = g(S)$, implies a sustainable harvest level and a constant stock level. The maximum sustainable yield occurs at $g'(S) = 0$. The environmental carrying capacity occurs at $g(S) = 0$.

Carrying Capacity:

$$g(S) = 0$$
$$aS - bS^2 = 0$$
$$a - bS = 0$$
$$Scc = \frac{a}{b} = \frac{0.8}{0.00002} = 40{,}000$$

Maximum Sustainable Yield

$$g'(S) = 0$$
$$a - 2bS = 0$$
$$S_{MSY} = \frac{a}{2b} = \frac{0.8}{0.00004} = 20{,}000$$
$$MSY = g(S_{MSY}) = a(S_{MSY}) - b(S_{MSY})^2 = .8(20{,}000) - .00002(20{,}000)^2 = 8{,}000$$



B) The government charges a hunting fee equal to $l = 5$, and allows anyone who wants to hunt to do so. Therefore, there is an open access situation in which profits are ultimately driven down to zero. If a steady state equilibrium is to be found, then the following conditions must hold:[i]

1. Total Revenues = Total Costs (Zero Profits)
2. $X = g(S)$ (Sustainability or steady state)

At steady state,

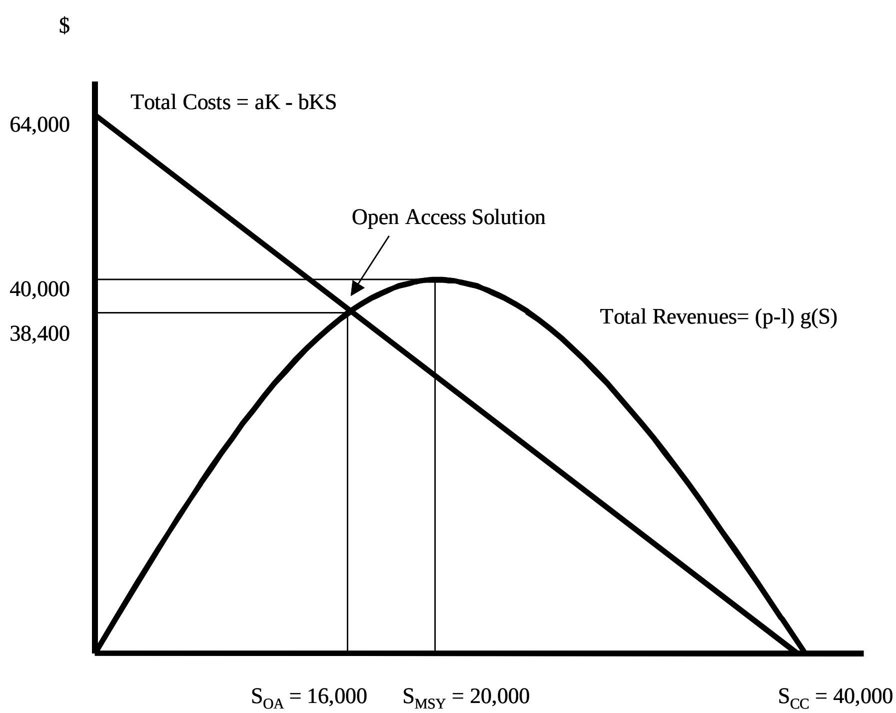
Total Revenues = $TR = (p-l)X = (p-l)g(S) = (p-l)(aS - bS^2)$

Total Costs = $TC = \frac{KX}{S} = \frac{Kg(S)}{S} = aK - bKS$

Open access equilibrium is found by setting TR =TC

$$TR = TC$$
$$(p-l)(aS - bS^2) = aK - bKS$$
$$(p-l)S = K$$
$$S_{oa} = \frac{K}{P-l} = \frac{80{,}000}{10-5} = 16{,}000$$

Since $X = g(S)$, then the open access level of hunting is $X_{oa} = g(S_{oa}) = 7{,}680$. The government revenues are $lX_{oa} = 5(7{,}680) = 38{,}400$



D) The owner of the private reserve seeks to maximize profits for the group of hunters *over time*. This suggests a steady state equilibrium. In this case, a private owner is able to impose limits on hunters' catch. Since we express profits in terms of the stock *S*, those profits are maximized when the derivative with respect to *S* is set to zero. Therefore, the two equilibrium conditions for profit maximization are:

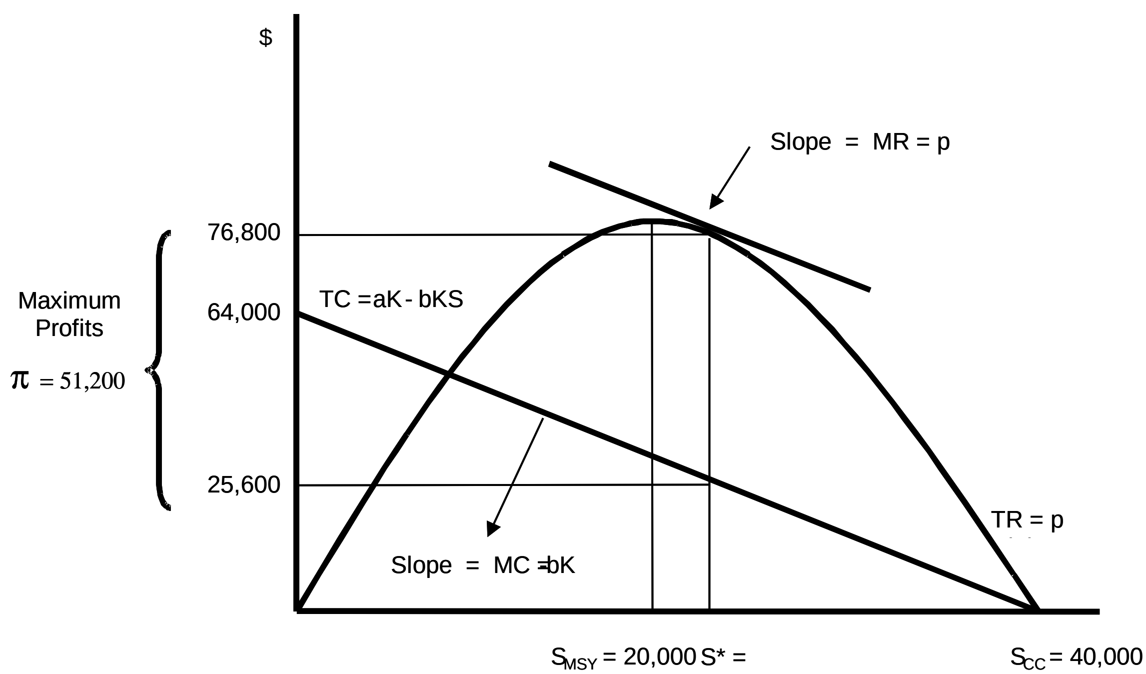
1. MR=MC (Maximize Profits)
2. $X = g(S)$ (Steady State)

Total revenues at steady state are equal to $TR = PX = Pg(S) = P(aS - bS^2)$, and "marginal revenue" is equal to $MR = P(a - 2bS)$. (we are using the term "MR" in a different way than usual, since it is generally meant to indicate change in total cost for incremental increases in *output X*, rather than in *stock S*, as is the case here)

Total costs at steady state are equal to $C(g(S), S) = \frac{Kg(S)}{S} = \frac{K(aS - bS^2)}{S} = aK - bKS$, and "marginal cost" is equal to $MC = -bK$. (same remark with respect to MC)

Equating marginal revenue with marginal cost, and then solving for S,

$$MR = MC$$
$$P(a - 2bS) = -bK$$
$$S^* = \frac{aP + bK}{2bP} = \frac{0.8(10) + 0.00002(80{,}000)}{2(.00002)(10)} = 24{,}000$$
$$X^* = g(S^*) = 7{,}680$$



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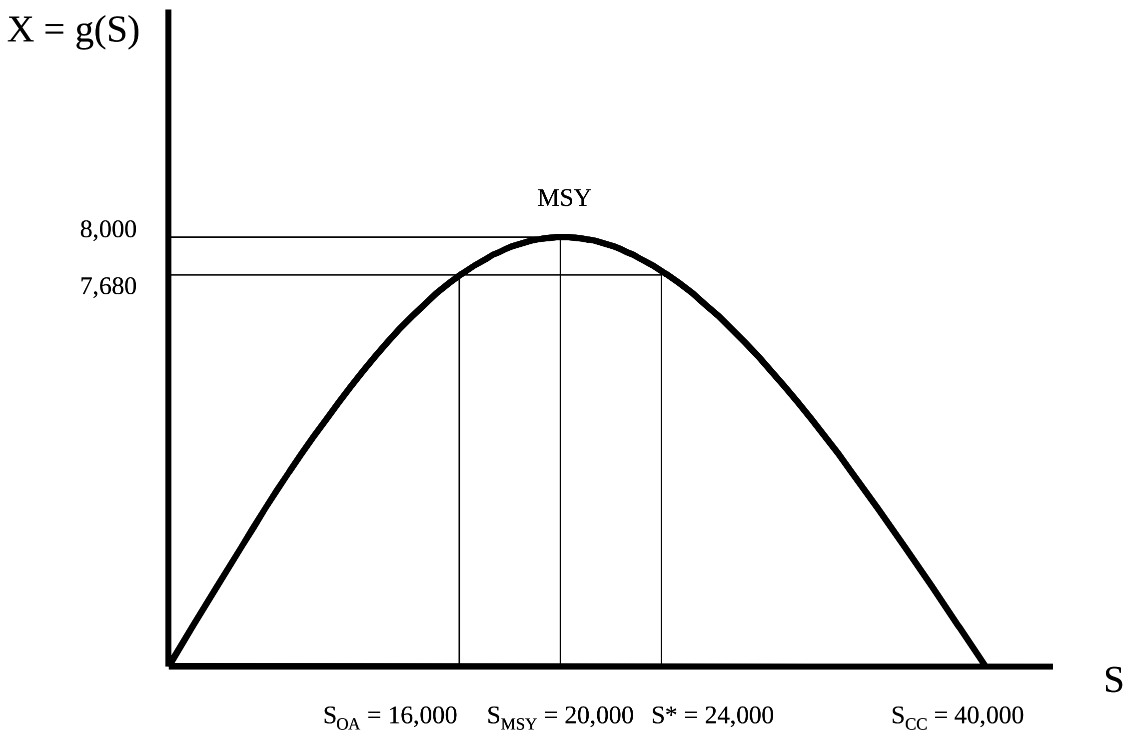


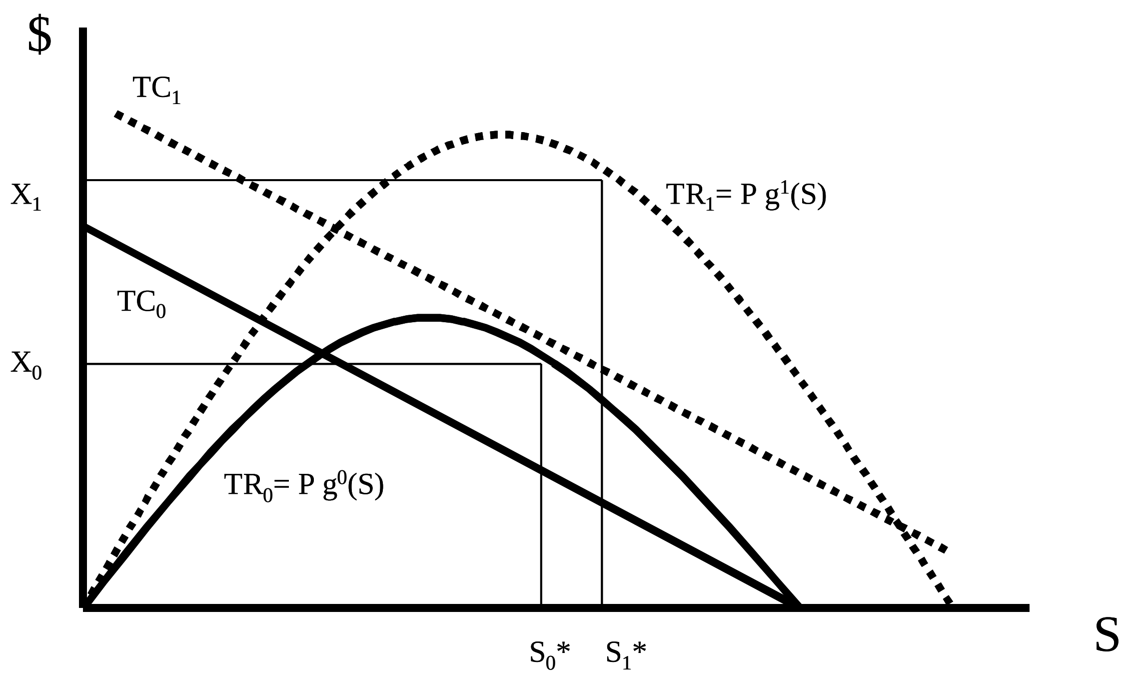
The maximum sustainable yield (MSY) criterion focuses on maximizing the yield or revenues and does not consider harvesting costs. The MSY occurs at g'(S)= 0.

The economic efficiency criterion focuses on maximizing profits, which consider both revenues and costs. The larger the stock level, the lower the costs. Economic optimal occurs at a higher stock level compared to the MSY. If there were no costs, then the MSY would be economically efficient. The larger the marginal costs, the more stock you want to keep and the lower the harvest level.

Under open access, profits are driven down to zero by excessive harvesting, which lowers the stock. Hence, in general (but not always), the open access solution occurs to the left of the MSY. If the total cost curve crosses the total revenue curve at the MSY, then the MSY would also be the open access solution.

A harvest level of X = 7,680, at a stock level of 24,000, will maximize the net value of the resource. On the other hand, the same harvest level, but at a stock level of 16,000, will eliminate the net value of the resource.

F) More available food will enhance the growth of the deer population. The growth function will shift from $g^0(S)$ to $g^1(S)$. Let's assume for the sake of simplicity that P=1, so that total revenues are equal to $X = g(S)$. The extra cost of $1 per hunter will increase the total costs of hunting. Conceivably, we could have more hunting without compromising the stock level. As a matter of fact, and as the graph shows, we could have more hunting and a larger stock than before.



2) The government charges a hunting fee of t dollars per deer killed. The government's objective is to maximize the revenues, tX, received from this fee.

Max tX Subject to $x = g(S)$ and $(p\text{-}t)X = C(X,S)$

Max $L = tX + l_0[X - g(S)] + l_1[(p-t)X - C(X,S)]$

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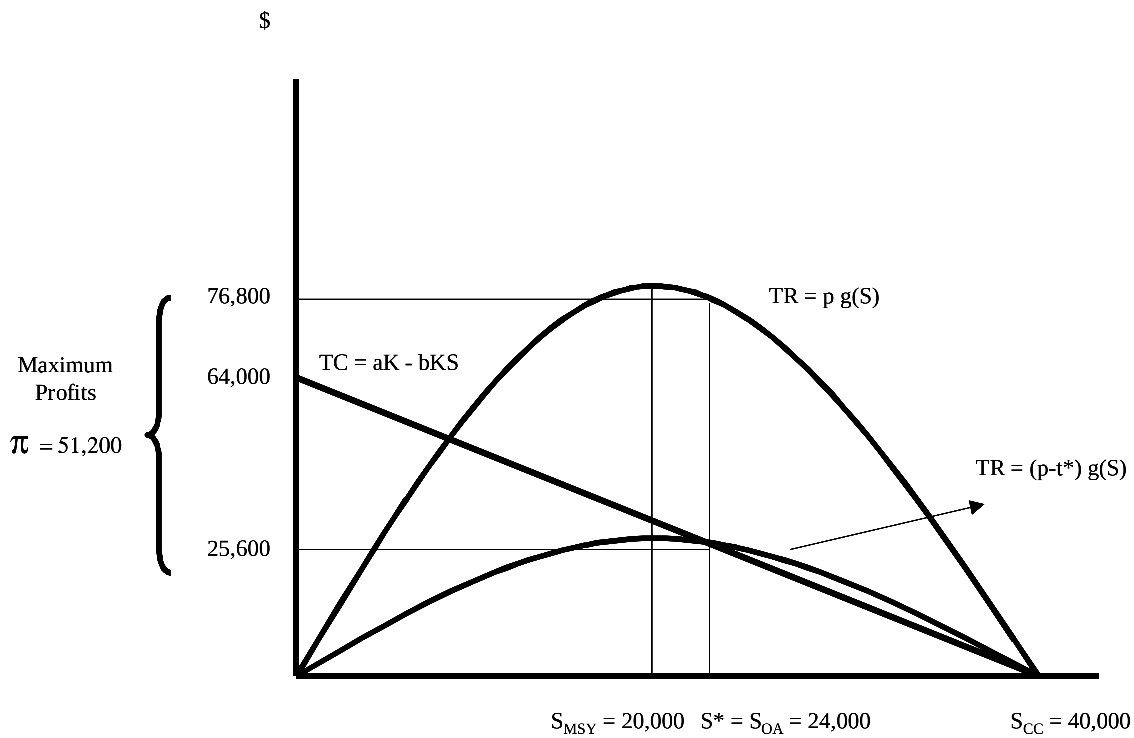
1. $\dfrac{\partial L}{\partial t} = X - l_1 X = 0$
2. $\dfrac{\partial L}{\partial X} = t + l_0 + l_1(p-t) - l_1 C_X = 0$
3. $\dfrac{\partial L}{\partial S} = -l_0 g'(S) - l_1 C_S = 0$
4. $\dfrac{\partial L}{\partial l_0} = X - g(S) = 0$
5. $\dfrac{\partial L}{\partial l_1} = (p-t)X - C(X,S) = 0$

From 1 we find that $l_1 = 1$. Substitute this into 2 and solve for l_0. Then substitute l_0 into 3 and we get the following

$$p - C_X - \frac{C_S}{g'(S)} = 0$$
$$p - \frac{K}{S} + \frac{KX}{S^2 g'(S)} = 0$$
$$p - \frac{K}{S} + \frac{Kg(S)}{S^2 g'(S)} = 0$$
$$P - \frac{K}{S} + \frac{K[aS - bS^2]}{S^2[a - 2bS]} = 0$$
$$S^* = \frac{aP + Kb}{2bP} = 24{,}000$$
$$X^* = 7{,}680$$
$$t^* = P - \frac{C(X^*, S^*)}{X^*} = 10 - 3.33 = 6.6\bar{6}$$

Government Revenues = $GR = t^* X^* = 6.6\bar{6}(7{,}680) = 51{,}200$. Note that the government revenues are equal to the maximized profits under private ownership. (Can you tell why this must be so?)



[i] A different outcome might result under open access if we assumed a discrete timeline (as opposed to continuous). Think about the situation facing the hunters in the initial period.

Total Cost in period 0 is given by $TC = \frac{KX_0}{S_0}$, where X_0 is the choice variable and S_0 is the given level of the stock. Individual hunters will kill deer as long as profits are positive or zero. (total revenue equal or greater than total cost) There will be no hunting otherwise. Total revenue in period 0 is given by $TR = (p-l)X_0$.

Open access equilibrium is characterized by zero profits, and there are two possible paths:

1. If $S_0 \geq \frac{K}{p-l} = 16{,}000$ then TR >TC for *any* X_0, so hunters would shoot the entire population of deer ($X_0 = S_0$) in period zero. Therefore, $S_{OA} = 0$ and stays there.

2. If instead $S_0 < \frac{K}{p-l} = 16{,}000$ then TR <TC for *any* X_0, so there would be no profits to be made (only losses) by hunting and harvest would be zero. In such a case, the stock would eventually grow to $S_t = \frac{K}{p-l}$, at which point situation 1 (see above) occurs, and the stock is driven down to zero.

The problem with this interpretation is that it does not offer a realistic description of costs. It assumes that costs do not adjust as the stock is depleted. In reality we would expect costs to increase as deer get more and more scarce, until total costs just equal total revenues, profits are zero, and there is no more incentive for new hunters to come in. Eventually, hunting takes place at a positive steady state level of stock, rather than the total depletion outcome suggested by the discrete case.