PROBLEM SET 4
Due Thursday, April 27, 2000 in class
(Late assignments will not be graded)

1. Consider two landowners in very different northern US states. One is an old reclusive coot that enjoys hunting and talks only to other hunters. The other is the government. They both hold the same amount of land and each landowner's land contains a population of deer. The deer population (*S*) grows according to the following relationship: $g(S) = 0.8S - 0.00002S^2$. Hunters derive a constant marginal benefit of $10 from each deer successful killed[1]. However, the cost of hunting reflects primarily a combination of foregone wages and time spent in the activity. The costs per deer are $\frac{C(x,S)}{x} = \frac{80000}{S}$. In addition to this, hunters will incur any license fees charged by the landowner.

   a) What is the maximum carrying capacity of the stock? Compute the size of the stock and the annual harvest at maximum sustainable yield.

   b) The government, ceding to the argument that public forests belong to everyone and everyone should be allowed to use them, sells hunting licenses for a nominal fee of $5 per deer to anyone willing to buy. The fee is used to finance a program that monitors the deer population. Calculate how much money will be raised for this program each year in a steady state. To do this, you will need to determine the harvest level (*x*).

   c) What is the population of deer on the government-owned land when hunting takes place? Is this more or less than the maximum sustainable yield? On your graph from part (a), indicate the harvest and population levels on the government's land.

   d) The old coot cannot bring himself to charge his fellows for hunting. However, he limits the number of deer he allows them to shoot. If he chooses the number of deer "rights" so as to maximize the net benefits of the hunter community over time, how many deer will he allow to be harvested? (The solution to a quadratic equation $aZ^2 + bZ + c = 0$ can be found with $Z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and bearing in mind that S ≥ 0.)

[1] This is like saying the hunter will receive $10 for each deer killed.

e) Will the deer population on the old coot's land be larger or smaller than the deer population on the State-owned land? Explain briefly why. On your graph from part (a), indicate the harvest and population levels on the private land.

f) Suppose the old coot asks each hunter to contribute $1 to help buy bales of hay that will be left in the woods for the deer during the harsh winter. How might this affect the growth curve? Illustrate graphically. How might this enable hunters to take more deer without compromising the population?

2. What fee would the government have to charge in order to maximize annual revenues for its deer-monitoring program? To answer this, go through the following steps:

   a) Let l denote the fee. What is the objective function?

   b) What are the two constraints?

   c) In setting the fee, the government also sets the harvest and stock levels. What are the three control variables in this problem?

   d) Write out the Lagrangian.

   e) Find the first order conditions.

   f) Rearrange the first order conditions to get a single equality in S.

   g) Solve numerically for the revenue maximizing level of S, x and l.