EEP101/ECON 125 Spring 00
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Suggested Solutions to Problem Set 3

1. There are two graphs for this exercise. One refers to the two types of individual demands. The other refers to the *aggregate* demand for the public good and the marginal costs of providing that public good.

(a) The aggregate marginal benefit curve (AMB) is the vertical sum of all the individual marginal benefit curves. Thus,

$$AMB = 10 \times MB_H + 10 \times MB_L$$
$$AMB = (200 - 10Q) + (80 - 10Q)$$
$$AMB = 280 - 20Q \quad for \quad Q \leq 8$$

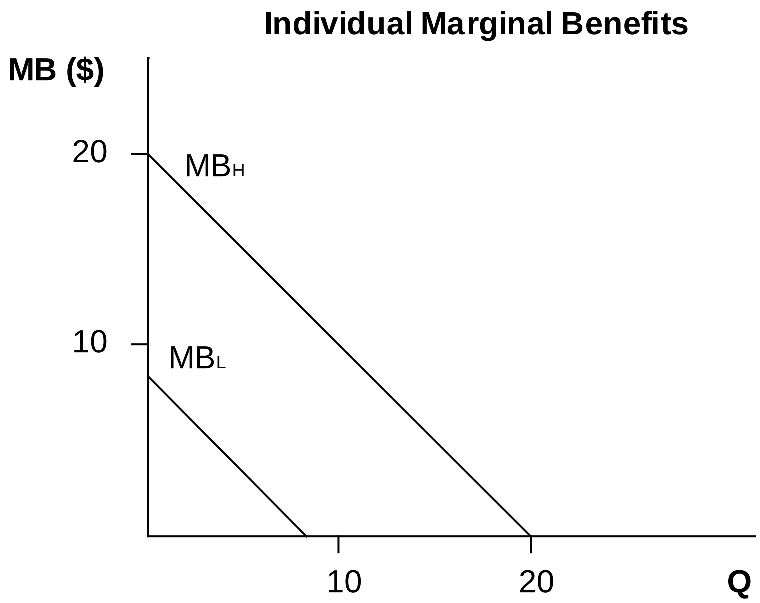
This marginal benefit curve is valid only for Q ≤ 8 because the "low" value businesses aren't willing to pay any additional positive amount for more than 8 lights. That is, there are no marginal benefits to this group from the ninth and subsequent lights, which we take into account in deriving the aggregate marginal benefits. The "high" value businesses are still willing to pay to have additional lights after 8 have already been provided. Therefore, the aggregate marginal benefits are still positive above 8 lights. The region of the aggregate marginal benefit curve above 8 lights is depicted as

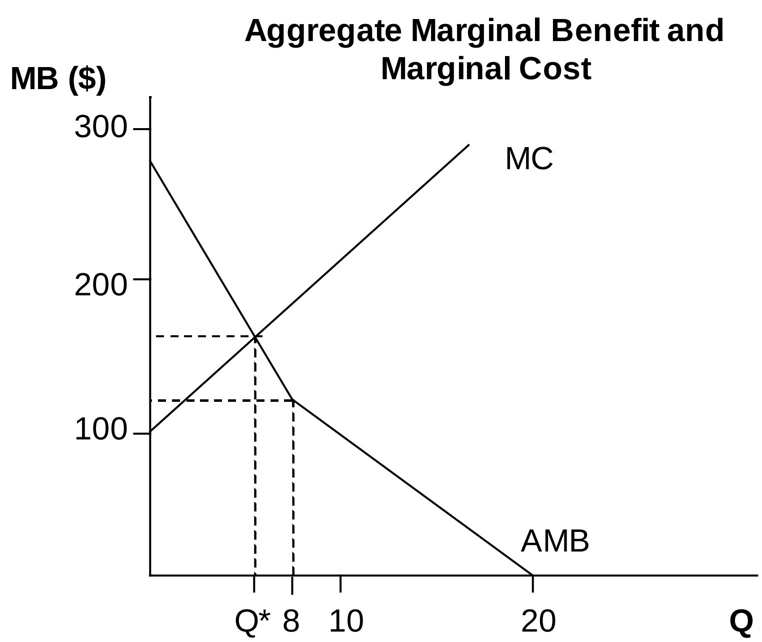
$$AMB = 10 \times 0 + 10 \times MB_H$$
$$AMB = 200 - 10Q \quad for \quad Q > 8$$

The two segments of the aggregate marginal benefit curve are:

$$AMB = 280 - 20Q \quad for \quad Q \leq 8$$
$$AMB = 200 - 10Q \quad for \quad Q > 8$$

The individual business owner's demand curve and the aggregate marginal benefit curve are depicted as follows


Individual Marginal Benefits
MB ($)
20
10
MBH
MBL
10
20
Q


Aggregate Marginal Benefit and Marginal Cost
MB ($)
300
200
100
MC
AMB
Q*
8
10
20
Q

The most a "high" value business owner is willing to pay for 2 streetlights is

$$\int_0^2 MB_H\,(Q)\;dQ$$

$$= 20Q - \frac{Q^2}{2}\Bigg|_0^2 = 40 - 2 = 38$$

This is equivalent to the area under the MB_H curve from zero to two:

$$\frac{1}{2}(20+18)\times 2 = 38$$

The maximum willingness to pay for the "low" value business for two streetlights is

$$\int_0^2 MB_L\,(Q)\;dQ$$

$$= 8Q - \frac{Q^2}{2}\Bigg|_0^2 = 16 - 2 = 14$$

(b) Social welfare is maximized when the marginal cost of providing one more light equals the benefit of one more light, i.e., where marginal cost equals marginal benefit. Thus, MC = 100 + 10Q = 280 - 20Q = AMB. Solving for Q gives Q* = 6. Since this is less than eight, we know that we have used the relevant region of the aggregate marginal benefit curve.

The total cost of providing Q* is the integral (area) under the marginal cost curve between 0 and 6.

$$\text{Total cost} = 100Q + \frac{10Q^2}{2}\Bigg|_0^6 = 600 + \frac{360}{2} = 780\,.$$

(c) Streetlights can be considered a *pure* public good, that is, a good that is both non-rival and non-excludable. Since this good is non-excludable, it is difficult (and indeed wasteful of resources) to exclude consumers, and it is possible for a non-paying business to free-ride. If this were provided in competitive markets, less than the socially optimal quantity would be provided.

One way that city government can finance the provision of streetlights is to require all 20 businesses to pay a portion of the cost of providing the public good. In other words, the government would provide Q* and levies a uniform tax on each business where the uniform tax is simply

$$\frac{TC(Q^*)}{n} = \frac{780}{20} = 39$$

(d) If it were possible to prevent some businesses from "consuming" lights so that those who wish to do so must pay a fee (what we've been calling an "entry" fee). Now suppose this fee (E_g) is $39 (ie, the uniform tax we found in c)). If the business owner's maximum willingness to pay for Q* is larger than E_g , then the business owner will choose to pay the fee. Conversely, if the maximum willingness to pay is smaller than E_g, then the business owner will choose not to consume the public good.

The maximum amount "high" value business is willing to pay for Q* is determined as follows:

$$\begin{aligned} WTP_H &= 20Q - \frac{Q^2}{2}\Big|_0^6 \\ &= 120 - 18 \\ &= 102 \end{aligned}$$

(which is the area under the curve, as seen in part *a* calculations). Similarly, maximum willingness to pay for the "low" value firm is

$$\begin{aligned} WTP_L &= 8Q - \frac{Q^2}{2}\Big|_0^6 \\ &= 48 - 18 \\ &= 30 \end{aligned}$$

Notice that $WTP_H = 102 > 39 = E_g$, so each business within "high" value business group will pay the $39 fee; the total benefit of the 6 streetlights to each of these businesses is greater than the fee. On the other hand , "low" value businesses chooses not to consume since the fee exceeds the total benefit of consuming. Therefore, the government cannot continue to impose the same uniform tax since it's not possible to cover the costs. Notice here, we are making fee payment voluntary, whereas in the previous case, the government imposes a uniform tax on everyone. Think of real world examples of pure public goods – national defense, for instance. One often hears a lot of grumbling about having to pay "too much" for these types of goods, ie the uniform tax is higher than many people are willing to pay, yet are required to do so.

One possible strategy is to allow a monopolist to provide the public good and charge a fee. A possible fee would be one which is exactly equal to the "high" value business owner's maximum willingness to pay for 6 lights, which would exclude the "low" value businesses, but would allow the provider to cover the costs of providing the public good, yet still earn a profit. Alternatively, the monopolist could charge a fee that is exactly equal to the maximum willingness to pay for the low value business, so that the consumer surplus of this group is zero, but the high value businesses have a positive consumer surplus. Of course, if she were able to price discriminate, she would charge each business its own maximum willing to pay. (although in this case, we would not necessarily have $Q^* = Q^m$)

Another alternative for a discriminating public supplier is Lindahl pricing, charging each business the marginal value on the sixth light for all units: $MB_H = 20 - 6 = 14$, and $MB_L = 8 - 6 = 2$. As is the case when Lindahl pricing is used where marginal costs are increasing, total revenue would exceed total costs:
[TC(6) = (10*6*2) + (10*6*14) = 960 > 720]

2. (a) The firm's profits are given by the difference between revenues and costs,

$$\Pi = P_B B - P_T T$$

Inserting the equation for the production function of B,

$$\Pi = P_B (g_i T)^{1/2} - P_T T$$

where g_i is a constant representing the applied input efficiency under each technology i, (note g_i is a simplification of the "h" function in the lecture notes, and was unfortunately reduced to an underline "_" in the printed problem set), i = old, new.

For the profit-maximizingfirm, the objective is to

$$\underset{T}{Max} \quad \Pi = P_B (g_i T)^{1/2} - P_T T$$

We can find the profit maximizing applied input demand function for T by maximizing the objective function with respect to T. The first order condition for this problem is

$$\frac{\partial \Pi}{\partial T} = \frac{1}{2} g_i P_B (g_i T)^{-1/2} - P_T = 0 \quad \Rightarrow \quad \frac{1}{2} g_i P_B (g_i T)^{-1/2} = P_T$$

In other words, the MC (P_T) of the applied input is equal to the marginal revenue product of the applied input. This condition implicitly defines the profit maximizing level of applied input, T*. Substitute the given parameters (except for g_i since we want to compare profits under each technology) into this condition and solve for T, to find T*(g).

$$5 = \frac{1}{2}(g_i)(100)(g_i T)^{-1/2} \quad \Rightarrow \quad T^* = 100 g_i$$

To find the profit level under the old technology as well as under new technology, T* can be substituted back into the profit equation. Under the old technology, since $g_{old} = 0.8$,

$$\begin{aligned}\Pi_{old} &= 100[g_{old}T^*)]^{1/2} - 5T^* \\ &= 100 \ \left[0.8\left(100(0.8)\right)\right]^{1/2} - \ 5 \ \left(100(0.8)\right) \\ &= \ 800 - 400 = 400\end{aligned}$$

Similarly, the profit level using the new technology is found to be

$$\begin{aligned}\Pi_{new} &= 100[g_{new}T^*)]^{1/2} - 5T^* \\ &= 100 \ \left[0.9\left(100(0.9)\right)\right]^{1/2} - \ 5 \ \left(100(0.9)\right) \\ &= \ 900 - 450 = 450\end{aligned}$$

The gain in profit of adopting the new technology is Π_{new} - Π_{old} = 450 - 400 = 50. However, since the cost of converting to this new technology (k=70) exceeds the gain in profit, the producer would choose to keep the older technology.

(b) Government has four ways (actually there are more, but those are the ones you have seen in lectures and notes) to intervene and change the relative profits of the two technologies. It can

- subsidize output: P_B becomes P_B+s,
- tax/subsidize inputs: P_T becomes P_T-w,
- impose a tax “v” on waste: subtract v(1-g_i)T from profits,
- subsidize fixed costs of adoption.

The latter option is easy to calculate: just give $50.01 to producers to adopt. Otherwise, policies apply to output, inputs and waste independently of the technology chosen by producers. Because the new technology necessarily produces more output, subsidizing the price of boards increase profits with the new technology relatively more than with the older one. To calculate the minimum level of *output subsidy*, go back to the beginning of the problem, and rewrite the profits to be maximized as:

$$\Pi = P_B (g_i T)^{1/2} - P_T T - v(1 - g_i)T$$

Leaving the symbols in the equation gives a profit-maximizing value of input of

$$T^* = \frac{g_i}{4}\left(\frac{P_B}{P_T + v(1-g_i)}\right)^2$$

You can check this is correct by substituting in the correct values for g_i, P_T, P_B, and v (which was zero in part *a* since we did not have a tax). If you plug that back into the definition of profits, you get (after a lot of substitution):

$$\Pi^* = \frac{g_i P_B^2}{4(P_T + v(1-g_i))}$$

The trick now is to replace P_B by P_B+s (and P_B=100, P_T=5 and v=0) in the equation above (so producers find it in their interest to adopt and pay the fixed fee of 70):

$$\Pi^*_{new} - \Pi^*_{old} = \frac{.9(100+s)^2}{4(5)} - \frac{.8(100+s)^2}{4(5)} > 70 \Rightarrow s > 18.3$$

So government must subsidize boards by at least 18.3 per unit to incite producers to adopt. You can do similar calculations for the case of a subsidy on inputs, and get w > 1.43 (if producers pay no more than P_T = 5 – 1.43 = 3.57, they will find it profitable to adopt.) In the case of a tax, you should find that a tax v > 3.6 per unit of timber wasted, (1-g_i)T, is sufficient to incite adoption. (Ask your friendly GSI if you really want detailed calculations)

2. Essay

The government could use a variety of techniques to assess the monetary benefits of preservation, some of which are more applicable than others to valuing the amenity at hand. The most relevant are probably the interview and the travel cost methods. These and others are listed and discussed below.

- Interview

Asks people directly or indirectly how much they would be willing to pay to preserve the Headwaters area. This method may not reveal true willingness to pay or accept for a variety of reasons. The questions are subject to interpretation and the respondent may answer differently than anticipated. Different questions designed to elicit the same values may get different answers. People may be unfamiliar with the good and not have well-formed preferences. They may answer strategically rather than truthfully (If they think they will have to pay the amount of their answer for a public good, they could understate their preferences; if they think they won't have to pay, they could overstate them.)

- Travel cost

  It is reasonable to assume that people would not visit a site if the opportunity costs (travel and time) of getting there exceeds the benefits they would derive from being there. Thus, the travel cost, in some sense, reflects the price of the visit. We may construct a demand curve - higher travel costs imply a high price and should get fewer visits, and so forth. The problem is that by attributing all costs to the site, the method assumes the individual does not benefit from the journey itself. If there are such benefits, we will overestimate the value. On the other hand, people may have been willing to spend more than what they actually had to in order to get there. We would then be underestimating the value. Nor does this method assess the value of the benefits that do not require a trip to the site, such as bequest, existence, option etc. (You could get these values from interviews, which is an important point favoring of that technique.)

- Hedonic

  Uses traded commodities that have the amenity in question as an attribute that affects the market price of the commodity. Through statistical regression analysis, we can determine how much the price of that commodity changes in the presence or absence of the attribute under study. That price change reflects the utility derived from the attribute and can be used to impute a value to it. The difficulty here will be finding appropriate market goods. One could think of comparing the prices of houses near the forest and those further away. Yet, there may not be any houses close enough to the forest to capitalize its benefits into their price since the surrounding area is undeveloped. Moreover, people like to be close to certain man-made amenities (shopping, work, public transportation, etc.) and are willing to pay more for the convenience. Houses around Headwaters would be far from these services, and this valuation technique may actually attribute a negative value to the forest.

- Donations

  One could examine how much money people have donated to groups like Headwaters Earth First! that lobbied not to have the forest preserved. We could compare these sums to the amounts donated to other causes to get a measure of the relative value of Headwaters. These sums may not reflect the particular forest in question, however, as people may simply be doing what they think is the “right” thing to do an give money to save the environment. Donations may also be affected by free-riding, misinformation, media attention and the personal zeal of the lobbyists trying to sell their agenda.

- Entry fee/ marketing/ experiment

  We could charge an entry fee and calculate how many people visit. By varying the entry fee a few times we could find a demand curve, as in a marketing experiment. The problem is that people may not have to pay the entry fee if the forest is not excludable. It is not feasible to build a fence around 11 000 acres of land in order to prevent people from using the “park” unless they pay. This entry fee would only be part of the actual cost to the visitors as it does not include travel costs. Under the travel cost method, the fee would be included as a cost of visiting the park. So, the “entry fee: method is not as reliable as the travel cost technique.

- Engineering/restoration

  Claims that the cost of restoring a degraded environment to its natural state can be used as a measure of the value of the restored state. Simply put, this is absurd. As economists, we know that there is no relationship between the total costs of producing a good and the total benefits people get from consuming that good. There is one possible exception: if we observe private citizens voluntarily restoring part of a river, a landscape, etc., then we can conclude that their private valuation is at least as large as the costs they incurred. This could not give a complete values for Headwaters since the primary reasons for preserving it are its uniqueness and unreproducibility (species cannot be brought back from extinction and it would take centuries to grow a comparable forest.)

- Voting

  Like the interview technique, this asks people about their preferences. For the method to be useful, people have to be asked in a referendum to approve or disapprove of a policy that will have predictable costs to them (e.g., a $5 tax increase). The difference is that people are likely to take this question seriously and reveal their true preference. Overstating your preference will only help pass the measure, in which case you will have to deal with the consequence of your lie, i.e., an increase in taxes you aren’t really willing to take on. Understating your preference will only help defeat the measure, in which case you will have to live with the consequence of not having something you were actually willing to pay for. The drawback of this method is that you don’t actually get the full value to people. You get a lower bound for those people who voted. Moreover, an election is likely to be more costly than a contingent valuation survey.

- Timber

  The logging company’s profits from selling the timber derived from Headwaters Grove is not a measure of the benefits from preserving the forest. It is a measure of the benefits from cutting it down. Reciprocally, it is a cost of preserving the forest.