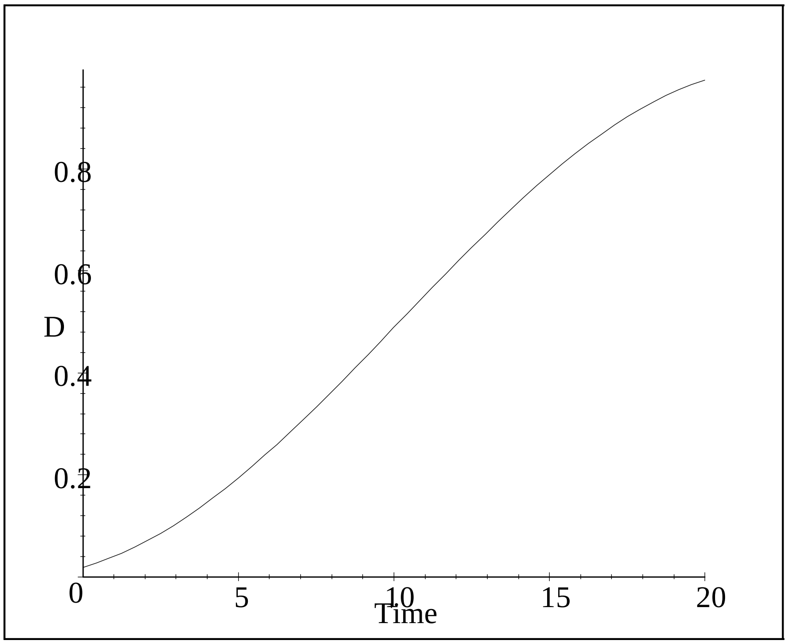
EEP 101/ECON 125 Spring 2001
Solutions to Problem Set 2

1. Equilibrium conditions for a competitive market are $MC(Q,t) = p$. Thus, without any government intervention, at any point in time t, $Q = 500 + 500\cos\left(\frac{450-20t}{157}\right)$.

   (a) There are 1000 units that can be sold. Thus percent diffusion is $\frac{Q(t)}{1000}$, or $\frac{1}{2} + \frac{1}{2}\cos\left(\frac{450-20t}{157}\right)$



   (b) If the government wants 50% diffusion, it must alter the price so that

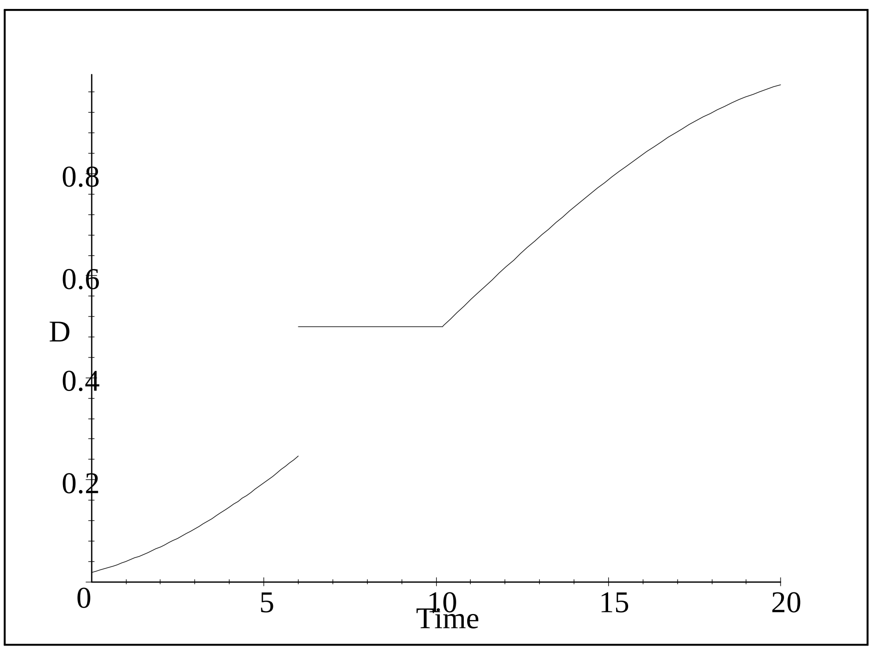
$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{p}{157}\right), \tag{1}$$

   or

$$\cos\left(\frac{p}{157}\right) = 0. \tag{2}$$

   From trigonometry (or from our calculator) we know that $\cos\left(\frac{\pi}{2}\right) = 0$. This means that $\frac{p}{157} = \frac{\pi}{2}$, or $p = \frac{157\pi}{2} \approx 246.62$. In year $t = 6$, the price that would occur without intervention is $p(6) = MC(6) = 450 - 20 \times 6 = 330$. Thus the government must subsidize the sale of the product by $s = 330 - 246.62 = 83.38$ per unit sold in year 6. This is just the difference between the price the government wants and the price that occurs without government intervention. We must multiply this subsidy by the number of units sold in year 6. We know 500 will adopt at $t = 6$ (50% of 1000). Thus the number adopting are 500 minus the number who had adopted by year 6, or $500 - 500 - 500\cos\left(\frac{450-20\times 6}{157}\right) = 253.25$ Hence the total expenditures are $83.38 \times 253.25 = 21116$. (You could use year 5 as the base year if you were considering discrete time). If the government uses this policy in year 6, then 500 will have adopted by the end of year 6. Since the problem assumes there is no unadoption, the government need not do anything to maintain at least

50% diffusion. Hence they will employ no policy in year 7 through 15, and the cost will be 0. In other words the government can reach diffusion goals with short term subsidies. When prices go up people seldom sell durable goods, and hence we do not care what happens to prices in years 7 through 15. This means that diffusion is at from year 6 (when the subsidy is put in place) to about year 10, when the prices are again low enough to induce new adoption.



2. Pro t is given by

$$\pi = P_y (hX)^{.5} - P_x X. \tag{3}$$

I will rst calculate input demand, output supply and pro t in the general case (without using numbers). The rst order condition is

$$\frac{\partial \pi}{\partial X} = .5 P_y h (hX)^{-.5} - P_x = 0, \tag{4}$$

which can be rewritten as

$$.5P_y h^{.5} = P_x X^{.5}, \tag{5}$$

or,

$$X = \left(\frac{.5P_y h^{.5}}{P_x}\right)^2 = .25h\left(\frac{P_y}{P_x}\right)^2. \tag{6}$$

This is the input demand for water in the generic case. By substituting 6 back into the production function we find

$$Y = (hX)^{.5} = \left[.25h^2\left(\frac{P_y}{P_x}\right)^2\right]^{.5} = .5h\left(\frac{P_y}{P_x}\right). \tag{7}$$

This is the output supply equation. Profit can be found by substituting 6 into the profit equation, or

$$\pi = P_y\,(hX)^{.5} - P_x X = .5P_y h\left(\frac{P_y}{P_x}\right) - .25P_x h\left(\frac{P_y}{P_x}\right)^2. \tag{8}$$

(a) To calculate demand, supply and profit we need only substitute $h = .75$, $P_Y = 250$, and $P_X = 25$ into (6), (7) and (8). For demand we find

$$X = .25 \times .75\left(\frac{250}{25}\right)^2 = 18.75. \tag{9}$$

For supply we find

$$Y = .5 \times .75\left(\frac{250}{25}\right) = 3.75. \tag{10}$$

For profit we find

$$\pi = .5 \times 250 \times .75\left(\frac{250}{25}\right) - .25 \times 25 \times .75\left(\frac{250}{25}\right)^2 = 468.75. \tag{11}$$

(b) For drip irrigation, we do the same thing, only using $h = .80$.For demand we find

$$X = .25 \times .80\left(\frac{250}{25}\right)^2 = 20.0. \tag{12}$$

For supply we find

$$Y = .5 \times .80\left(\frac{250}{25}\right) = 4.0. \tag{13}$$

For profit we find

$$\pi = .5 \times 250 \times .80\left(\frac{250}{25}\right) - .25 \times 25 \times .80\left(\frac{250}{25}\right)^2 = 500.0. \tag{14}$$

(c) To nd which technology uses more water, we simply compare input demand in each case. $20 > 18.75$, so more water is used with drip irrigation. To nd amount of water wasted we solve $Z_i = (1 - h_i)X$. For ood irrigation this is $.25 \times 18.75 = 4.687\,5$. For drip irrigation this is $.20 \times 20.0 = 4.0$. Thus less water is wasted with drip irrigation.

(d) The maximum price John is willing to pay to switch to drip irrigation is simply the difference in pro t, or $500.0 - 468.75 = 31.\,25$. So he is willing to pay \$31.25 to switch technologies. I was hoping for an intuitive answer as to how the price of water might affect willingness to pay for new technology. To satisfy your curiosity, here is how one might nd it mathematically. First we need the equation of willingness to pay (i.e. $\pi_d - \pi_f$) or

$$\pi_d - \pi_f = .5P_y h_d \left(\frac{P_y}{P_x}\right) - .25P_x h_d \left(\frac{P_y}{P_x}\right)^2 - .5P_y h_f \left(\frac{P_y}{P_x}\right) + .25P_x h_f \left(\frac{P_y}{P_x}\right),^2 \tag{15}$$

or

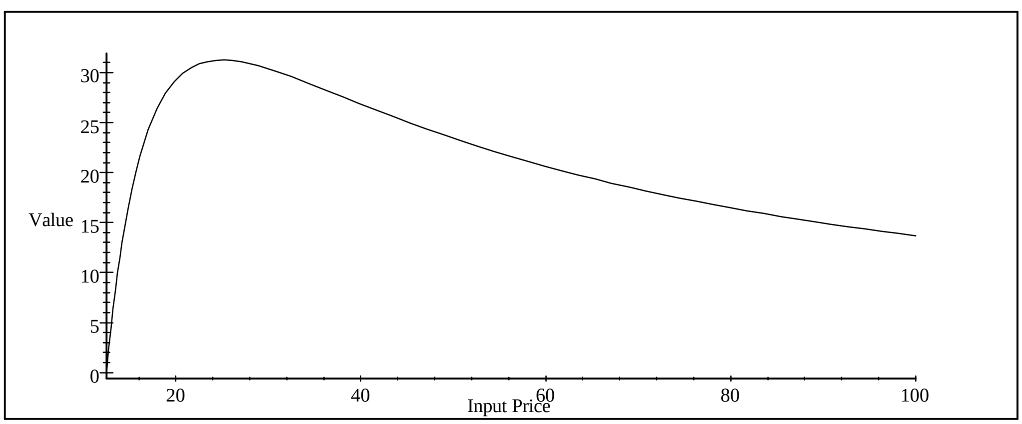
$$\pi_d - \pi_f = .5P_y \left(h_d - h_f\right) \left(\frac{P_y}{P_x}\right) - .25P_x \left(h_d - h_f\right) \left(\frac{P_y}{P_x}\right)^2. \tag{16}$$

Substituting in for $h_d = .80$, $h_f = .75$, $P_y = 250$, we nd

$$\pi_d - \pi_f = .5 \times 250\,(.80 - .75) \left(\frac{250}{P_x}\right) - .25 \times 25\,(.80 - .75) \left(\frac{250}{P_x}\right)^2, \tag{17}$$

or,

$$\pi_d - \pi_f = \frac{1562.5}{P_x} - \frac{19531.25}{P_x^2}. \tag{18}$$



Differentiating with respect to P_x will tell us the direction of change in willingness to pay when we increase input price.

$$\frac{\partial\left(\pi_d - \pi_f\right)}{\partial P_x} = \frac{-1562.5}{P_x^2} + \frac{39062.5}{P_x^3}. \tag{19}$$

The sign of the change (i.e. whether he is willing to pay more or less when P_x is increased) will depend on the size of P_x. At $P_x = 25$, 19 will be $\frac{-1562.5}{25^2} + \frac{39062.5}{25^3} = 0$. This means that willingness to pay to adopt the new technology is not changing at the margin (we happen to be at the maximum for). If we were to change input price much in either direction, John s willingness to pay would decrease (see the graph) because pro ts for the new technology would decline relative to the old.

The Essay

The essay was an attempt to force you to think about issues of adoption. There were several ways you could have approached the issue. Here are some ideas I think are interesting

- Localized constraints are easily violated. For instance, if I lived in an urban area of Utah, and the car I purchased happened to fail the urban restrictions, I might nd a way around the local restriction. If I had a friend or relative living in a more rural area of Utah, I could sign my car over to them (on paper) and register my car under the lower restrictions, continuing to drive my car in the urban area. In other words, this style of legislation may just push paperwork out of the urban areas rather than polluting cars. Restrictions like this may not have all that much affect on whether clean or dirty cars are adopted in urban areas.

- Lighter restrictions on cars that pollute more may not make much sense from a Pareto perspective. Some have argued that SUV s have greater capacity and thus may pollute less per person traveling. This is only true if the average number traveling in an SUV is actually larger than that in other vehicles, and by a greater proportion than the difference in pollution emitted. Greater capacity for passengers does not lower pollution per person unless the capacity is used. In the case of older cars, allowing older cars to pollute more may make sense as some sort of income transfer. Older cars tend to be driven by poorer individuals, and hence any tax (or restriction) on the pollution of older cars may act as a tax on the poor. This same argument doesn t work for SUV s that tend to be driven by the more well to do. The government might want to introduce tighter restrictions on SUV s to discourage pollution.

- Coase might think something like the following is possible: suppose there is an organization of people who are affected by air quality. They may be able to negotiate with and pay car makers to lower prices on the less polluting vehicles, or even to introduce cleaner technology. The possible barriers to such an action are the costs of organizing the group and negotiating with major automakers. Transactions costs seem to justify government intervention in this case.