Solutions to Problem Set 1

Part A: Numerical Problems

1. The numerical results for question 1 are given in Table 1. Their derivation is explained below.

Table 1

	Social optimum (question 1a)	Monopoly (question 1b)	Monopoly with tax (questions 1c and d)	Competitive pricing (question 1e)
P	380/3 = 126.67	110	380/3 = 126.67	70
Q	65/3 = 21.67	30	65/3 = 21.67	50
CS	4225/9 = 469.44*	900	4225/9 = 469.44	2500
PS	37375/18 = 2076.39*	2250	21125/18 = 1173.61	1250
TEC	6825/6 = 1137.5	1950	6825/6 = 1137.5	4750
GS	0*	0	8125/9 = 902.78	0
W	4225/3 = 1408.33	1200	4225/3 = 1408.33	-1000
DWL	0	625/3 = 208.33	0	7225/3 = 2408.33
tax/unit	0	0	125/3 = 41.67	0

The answers for CS, PS* and GS* are consistent with Q* being achieved through a quota or a standard on output. If the economy gets to Q* using another policy instrument, the *distribution* of the surplus between consumers, producers and government could be different. The *total* surplus will remain unchanged.

a) The socially optimal level of output is the one where marginal social cost is equal to marginal social benefit. The marginal social cost is the sum of marginal private cost and marginal external cost: $MSC = MPC + MEC = 40 + 4Q$.

Marginal social benefits are given by the demand curve: $MSB = P = 170 - 2Q$.

Equating the two and solving for Q gives:

$$MSC = 40 + 4Q^* = 170 - 2Q^* = MSB$$
$$130 = 6Q^*$$
$$\Rightarrow Q^* = 130/6 = 65/3 = 21.67$$

This solution is illustrated in Figure 1. It is possible to find total external cost, consumer surplus and producer surplus geometrically. Recall that consumer surplus is the difference between total benefits from consumption and what consumers pay for the good consumed. If the quantity Q* is consumed, then an equilibrium price will be the one given by the demand curve for that quantity. ($P^* = 170 - 2Q^* = 170 - 2*65/3 = 380/3 = 126.67$). Consumer surplus is the area below the demand curve and above this price. In Figure 1, it is the shaded triangle labeled CS*. Producer surplus is the difference between total revenues and variable costs. Since variable costs are the sum of the marginal costs for all units produced, producer surplus is the area below the price and above the marginal cost curve. It is the parallelogram with a thick border[1] labeled PS* in Figure 1. The total external cost is the sum of the marginal external costs over all units produced and is given by the area below the marginal external cost curve. It is the shaded parallelogram labeled TEC*.

To calculate these areas, we need to know price, marginal private cost and marginal external cost at Q*. To find them, simply plug the value of Q* back into the demand MPC and MEC functions:

$$P^* = 170 - 2*65/3 = 380/3 = 126.67$$
$$MPC = 20 + 65/3 = 125/3 = 41.67$$
$$MEC = 20 + 3*65/3 = 85$$

Calculate TEC* as the area of the parallelogram:

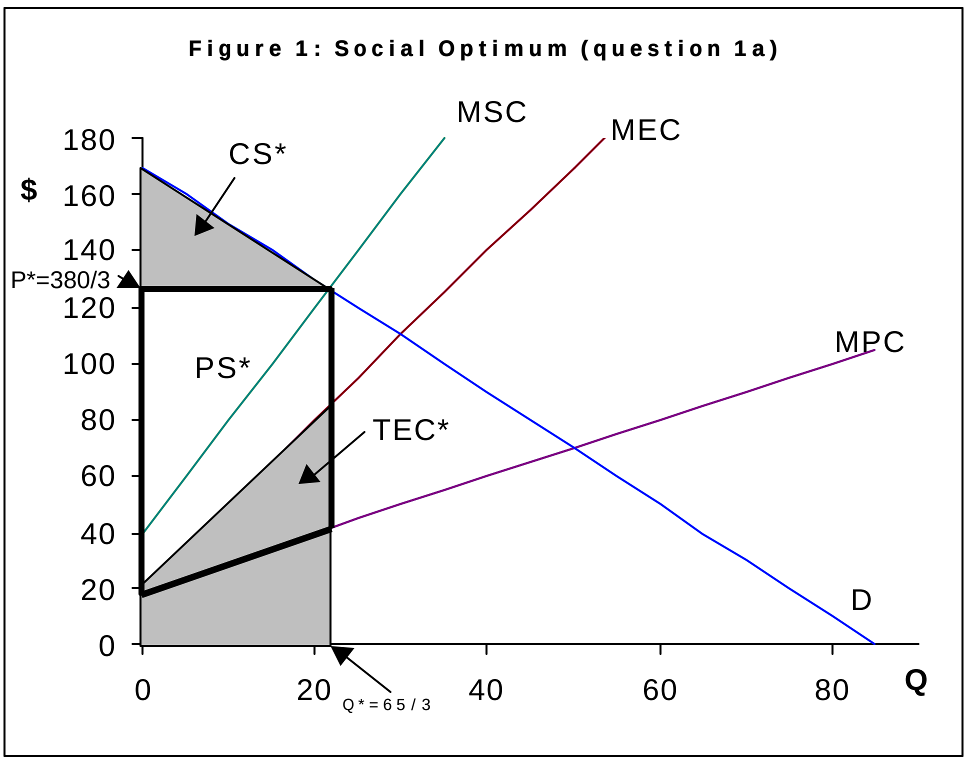
$$TEC^* = \frac{1}{2}(20 + MEC^*)Q^* = \frac{1}{2}(20+85)*65/3 = 6825/6 = 1137.5$$

Calculate CS* as the area of the triangle:

$$CS^* = \frac{1}{2}(170 - P^*)Q^* = \frac{1}{2}(170 - 380/3)*65/3 = 8450/18 = 469.44$$

Calculate PS* as the area of the parallelogram:

$$PS^* = \frac{1}{2}\left((P^* - 20) + (P^* - MPC^*)\right)Q^* = \frac{1}{2}\left(2*380/3 - 20 - 125/3\right)*65/3 = 37375/18 = 2076.39$$



[1] These thick lines may not be visible when you print the document. You should be able to see them on your computer screen, however.

Social welfare is the sum of consumer surplus, and producer surplus, minus total external cost:

$$W^* = CS^* + PS^* - TEC^* = 8450/18 + 37375/18 - 6825/6 = 25350/18 = 1408.33$$

We can confirm that this is the right answer by separately calculating social welfare as the difference between social benefits and social costs. This is the triangle formed by the demand and the marginal social cost curve. This gives $W^* = \frac{1}{2}(170-40)65/3 = 8450/6 = 1408.33$, which is what we found before.

b) When a monopolist chooses the price for its product, it sets it at the level that sell the profit-maximizing quantity. This profit-maximizing quantity is the one where marginal revenue equals marginal private cost. [The monopolist does not account for the external costs.] Marginal revenue depends on the demand curve for the output. If the demand curve is linear, then the marginal revenue curve has the same vertical intercept, but twice the slope:

$$P = 170 - 2Q \Rightarrow MR = 170 - 4Q$$

Equating marginal revenue and marginal private cost, and solving for the monopolist's quantity gives:

$$170 - 4Q_m = 20 + Q_m$$
$$150 = 5Q_m$$
$$\Rightarrow Q_m = 150/5 = 30$$

When selling this quantity, the monopolist will charge whatever price the market will bear. Plug this quantity back into the demand curve to get the highest price the monopolist can charge for this quantity:

$$P_m = 170 - 2Q_m = 170 - 2*30 = 110$$

Figure 2 illustrates this solution. Since the monopolist's price is lower than he socially optimal price, we expect consumer surplus to be higher than in the previous case. We also expect the total external cost to be higher since the output is higher than socially optimal. Because the monopolist's output is not the socially optimal output in the case, the social welfare will be less than the maximum social welfare, implying a deadweight loss.

Calculate consumer surplus, producer surplus and total external cost for this monopoly case in the same way you did when finding these values for the socially optimal case. You need to know the marginal private cost and the marginal external cost at the monopolist's level of output. These are:

$$MPC_m = 20 + Q_m = 20 + 30 = 50$$
$$MEC_m = 20 + 3Q_m = 20 + 3*30 = 110$$

The surpluses and external costs are:

$$TEC_m = \frac{1}{2}(20 + MEC_m)Q_m = \frac{1}{2}(20+110)*30 = 1950$$

$$CS_m = \frac{1}{2}(170 - P_m)Q_m = \frac{1}{2}(170-110)*30 = 900$$

$$PS_m = \frac{1}{2}((P_m - 20) + (P_m - MPC_m))Q_m = \frac{1}{2}(2*110-20-50)*30 = 2250$$

The deadweight loss is the difference between social welfare at the monopoly level of output and the socially optimal level of output. Welfare at the socially optimal level of output was W* = 408.33. Social welfare under the monopoly is:

$$W_m = CS_m + PS_m - TEC_m = 900 + 2250 - 1950 = 1200$$

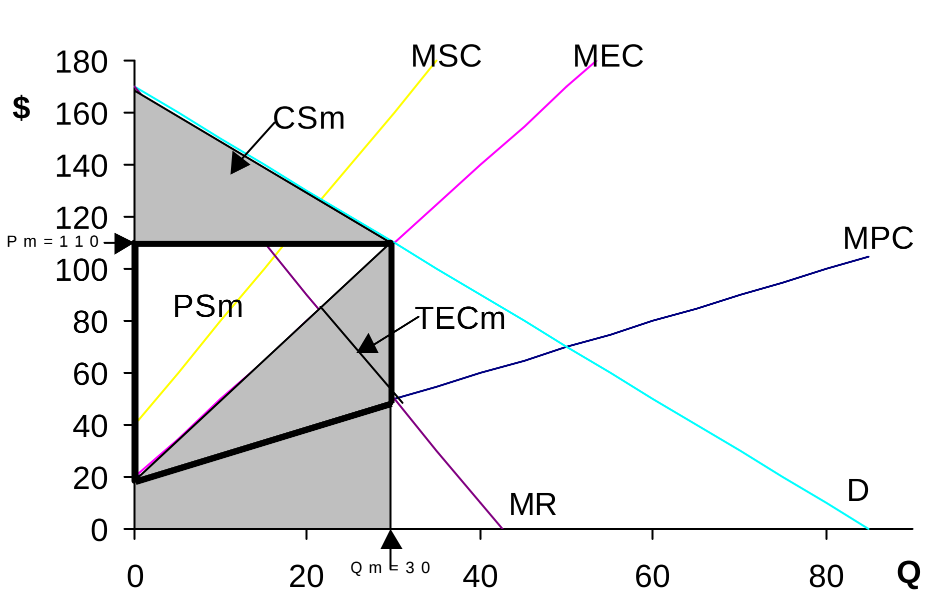
The difference between this and W* is:

$$DWL_m = W* - W_m = 4225/3 - 1200 = 625/3 = 208.33.$$

Confirm this by calculating it separately. First, note that the problem with this monopoly is too much output. The costs of extra units of output outweigh the benefits from those extra units. The deadweight loss triangle is going to be to the right of the intersection between marginal social cost and marginal benefit. The area of the triangle labeled DWL_m is:

$$DWL_m = \frac{1}{2}(MSC_m - P_m)(Q_m - Q*) = \frac{1}{2}((40+4*30)-110)(30-65/3) = 1250/6 = 208.33$$

Figure 2: Monopoly (question 1b)



c) Because the monopolist is producing too much, the optimal price-based mechanism is one that would lead to lower output. A specific tax on output will effectively do the job by increasing the firm's marginal costs of production.. In order to fix the externality problem, we need to set the tax at the precise level that will induce the monopolist to produce only the socially optimal level of output. Since a monopolist's decision rule is to sell that level of output where marginal revenue equals marginal cost, set the tax such that MR = MC + tax at the socially optimal level of output. That is,

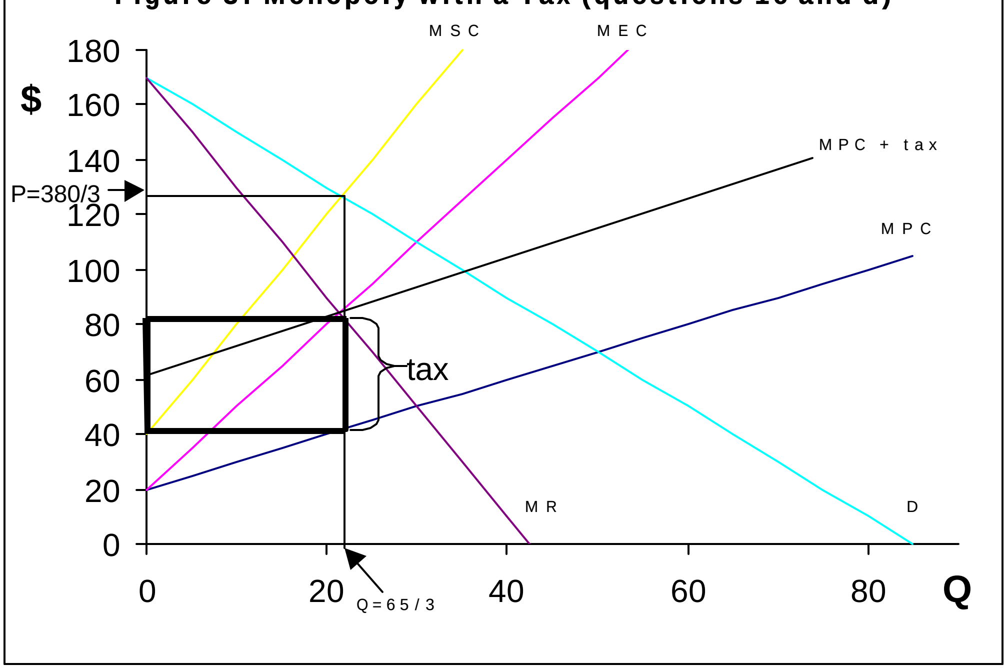
$$MR = MPC + tax$$
$$170 - 4Q_m^* = 20 + Q_m^* + tax$$

Set $Q_m^* = Q^* = 65/3$ in the previous equation and solve:

$$170 - 4*65/3 = 20 + 65/3 + tax$$
$$\Rightarrow tax = 150 - 5*65/3 = 125/3 = 41.67$$

Figure 3 shows how the tax shifts up the marginal private cost curve until it intersects the marginal revenue curve at Q*.

Figure 3: Monopoly with a Tax (questions 1c and d)



d) The government is now earning some revenue. The firm must pay the per unit tax for each unit of output it produces. Thus tax revenues are the per unit tax (41.67) times the quantity produced (65/3).

$$\text{tax revenue} = tax \cdot Q^* = 125/3 * 65/3 = 8125/9 = 902.78$$

To find the change in consumer surplus, producer surplus and total external we need to identify these magnitudes under the tax policy.

CS: Since the quantity being sold is the same as in the socially optimal situation, consumers will pay the same price. Thus consumer surplus is the same as in part (a).

$$CS_m^* = CS^* = 4225/9 = 469.44$$

TEC: Since the quantity of output is the same, the amount of pollution will be the same as at the social optimum. Thus, the total external cost is the same as in part (a).

$$TEC_m^* = TEC^* = 6825/6 = 1137.5$$

PS: Price and quantity being the same as in part (a) implies revenues and actual production costs are the same as in part (a). However, the firm must now pay the tax for all units produced. Producer surplus is therefore the amount in part (a) minus the total tax payment.

$$PS_m^* = PS^* - tax \cdot Q_m^* = 37375/18 - 125/3 * 65/3 = 21125/18 = 1173.61$$

The changes in consumer surplus, producer surplus and external cost are:

$$\Delta CS = CS_m^* - CS_m = 4225/9 - 900 = -3875/9 = -430.56$$
$$\Delta PS = PS_m^* - PS_m = 21125/18 - 2250 = -19375/18 = -1076.39$$
$$\Delta TEC = TEC_m^* - TEC_m = 6825/6 - 1950 = -4875 = -812.5$$

The decreases in CS and PS are losses whereas the decrease in TEC is a gain. The increase in government revenues is also a gain. The net effect of these changes is:

$$\Delta W = \Delta CS + \Delta PS - \Delta TEC + \Delta GR = -3875/9 - 19375/18 + 4875/6 + 8125/9 = 3750/18 = 208.33$$

Social welfare has increased by the amount of the deadweight loss caused by the unregulated monopoly. With the optimal tax, there is no longer any deadweight loss.

e) The competitive price will prevail if the quantity on the market is the one where marginal private costs equal marginal private benefits. Equating demand and marginal private costs gives:

$$P = 170 - 2Q_c = 20 + Q_c = MPC$$
$$150 = 3Q_c$$
$$\Rightarrow Q_c = 150/3 = 50$$

At this quantity, the market price has to be $P_c = 70 - 2Q_c = 170 - 2*50 = 70$

If a monopolist could not charge a price higher than this competitive price, this price effectively becomes the firms marginal revenue as long as people are willing to pay that price. Thus, when the monopolist maximizes profits and sets MR = MC, the result is:

$$MR = 70 = 20 + Q_m^c = MPC$$
$$\Rightarrow Q_m^c = 50$$

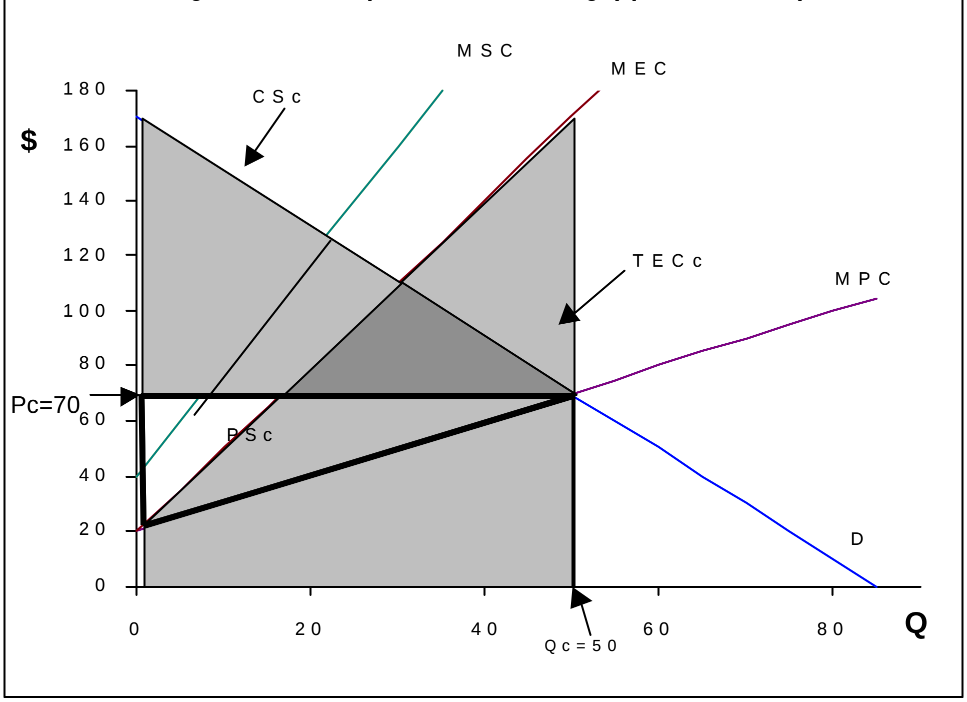
That is, the monopolist produces the competitive market quantity. Figure 4 illustrates this situation. The shaded triangle formed by the vertical axis, the demand curve and the price represents consumer surplus. The parallelogram under marginal external cost represents the total external cost. The triangle with a thick border, below the price and above the marginal private cost, is the producer surplus. Calculating the areas of each gives:

$$TEC_c = \frac{1}{2}\left(20 + MEC^c\right)Q_c = \frac{1}{2}\left(20 + (20 + 3 * 50)\right) * 50 = 4750$$
$$CS_c = \frac{1}{2}(170 - P_c)Q_c = \frac{1}{2}(170 - 70) * 50 = 2500$$
$$PS_c = \frac{1}{2}(P_c - 20)Q_c = \frac{1}{2}(70 - 20) * 50 = 1250$$

Since the competitive price is lower than both the socially optimal price and the monopoly price, consumer surplus is larger in this case than it was in either of those cases. In addition, the higher level of output implies a higher total external cost.

Social welfare is $W^c = CS^c + PS^c - TEC^c = 2500 + 1250 - 4750 = -1000$.

Figure 4: Competitive Pricing (question 1e)



The measure of social welfare is actually negative. This tells us that at that level of output, costs outweigh benefits. The deadweight loss is the difference between social welfare at this level of output and social welfare at the optimal level of output:

$$DWL^c = W^* - W^c = 4225/3 - (-1000) = 7225/3 = 2408.33$$

f) If the market were in fact competitive, the optimal tax would be equal to the marginal external cost at the socially optimal level of output.($tax_c = 20 + 3Q^* = 20 + 3*65/3 = 85$.

Before proceeding, we should point out that the price regulation can be specified in two ways. First, the monopolist may not be allowed to charge a price higher than P_c, meaning consumers will always pay the price $P_c = 70$. Second, the monopolist may not be allowed to receive a price higher than P_c, meaning consumers may pay a price higher than that.

If we impose the competitive level tax on the monopolist, it will reduce output too much. If this tax is imposed and the monopolist continues to charge the competitive price, it will not earn any profit and will prefer to not operate. Alternatively, you could allow the monopolist to increase its price by 85, leading to a price for consumers of 85 + 70 = 155. This will reduce output to 15/2 = 7.5, a level far below the social optimum. These results suggest that the appropriate tax for e price-regulated monopolist should be less than the competitive tax.

What is the appropriate tax to set when the monopolist also faces price regulation? If the regulation specifies that consumers shall not pay a price higher than P_c, then the tax is the one that gets the producer to reduce its output wile still receiving a marginal revenue of $P_c = 70$:

$$MR = 70 = 20 + Q^* + tax = MPC + tax$$
$$\Rightarrow tax = 50 - 65/3 = 85/3 = 28.33$$

If the regulation specifies that the monopolist receive price P_c, then the appropriate tax will be the one that increases the consumers' price up to the socially optimal price, thereby staving demand. In this case, $tax = P^* - P_c = 380/3 - 70 = 170/3 = 56.67$.

As expected, both of these monopoly taxes are less than the competitive tax of 85. Notice, finally, that the two monopoly taxes are not equal.

2. A summary of the numerical answers is given in Table 2:

Table 2	
Preferred X from the city-folks P.O.V. = X_c	500
Preferred X from the original residents' P.O.V. = X_r	0
Socially optimal X = X*	300
DWL at X_c = 500	1000
DWL at X_r = 0	2250
Maximum possible gains from trade at X_c	1000
Maximum possible gains from trade at X_r	2250

a) As long as additional housing units bring additional benefits, the city-folks will want more housing units. Only when the marginal benefits of additional housing are zero will they no longer demand anymore. Marginal benefits are zero where:

$$MB = 15 - 0.03X = 0$$
$$\Rightarrow X_c = 15/0.03 = 500$$

Thus, the optimal number of housing units from the city-folks' point-of-view is 500.

For the original residents, any amount of new development entails some cost without corresponding benefits for them. Their preferred amount of additional housing is therefore

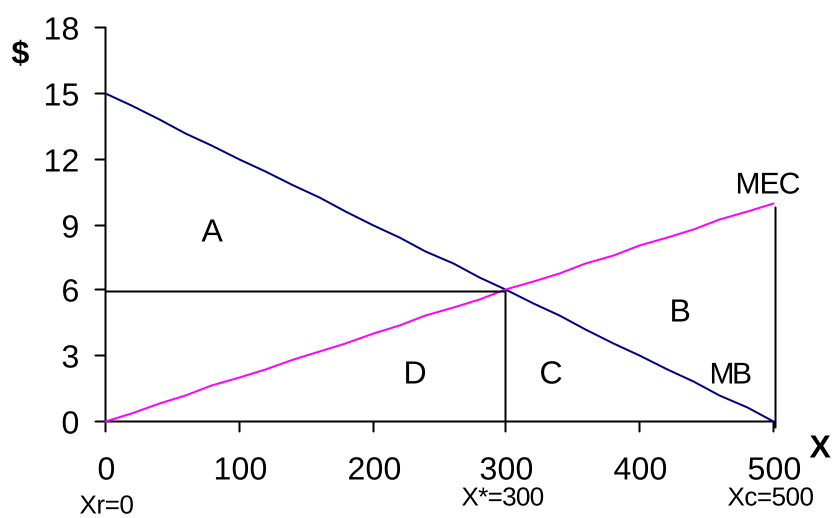
$$X_r = 0.$$

We have seen that the socially optimal amount of a good is the amount where marginal costs equal marginal benefits for society as a whole. We want to weigh the benefits of the city-folks against the external cost for the existing residents. The social optimum is given by:

$$MB = 15 - 0.03X^* = 0.02X^* = MEC$$
$$\Rightarrow X^* = 15/0.05 = 300$$

This being the "right" answer depends at least in part of whether we have captured all the costs and benefits. If there are additional external costs because non-residents also appreciate the open space in this area, the MEC will underestimate the social costs. Crowding in the city may be serious enough to affect people's quality of life. Then, the benefit to the people who move to the rural area underestimates social benefits since it does not account for the benefits to the people who remain in the now less-crowded city.

Figure 5: question 2



b) (i) Start with the case of unlimited housing development. Social welfare at X_c = 500 is total benefits to the city-folks minus total costs to the existing residents. The total benefits are represented by the area under the marginal benefit curve. The total costs are represented by the area under the marginal external cost curve. These areas are calculated as follows:

$$TB_c = \frac{1}{2} * 15 * X_c = 15 * 500 / 2 = 3750$$
$$TEC_c = \frac{1}{2} MEC_c * X_c = \frac{1}{2}(0.02 * 500) * 500 = 2500$$

Social welfare is $W_c = TB_c - TEC_c = 3750 - 2500 = 1250$

Total benefits, total costs and social welfare at the optimum are:

$$TB^* = \frac{1}{2}(15 + MB^*)X^* = \frac{1}{2}(15 + (15 - 0.03 * 300)) * 300 = 3150$$
$$TC^* = \frac{1}{2} MEC^* \cdot X^* = \frac{1}{2} * (0.02 * 300) * 300 = 900$$
$$W^* = TB^* - TC^* = 3600 - 900 = 2250$$

Finally, calculating deadweight loss at $X_c = 500$:

$$DWL_c = W^* - W_c = 2250 - 1250 = 1000$$

Graphically, this is area B in Figure 5. If we calculate that area directly, we get:

$$DWL_c = \frac{1}{2}(X_c - X^*)MEC_c = \frac{1}{2}(500 - 300) * (0.02 * 500) = 1000$$

(ii) Now suppose no additional housing development is permitted. The total benefits and total costs are zero. Thus, the deadweight loss is given by the foregone net benefits from the socially optimal level of housing, i.e.,

$$DWL_r = 2250\,.$$

Graphically, this is area A in Figure 5. Calculating that area directly gives:

$$DWL_r = \frac{1}{2} * 15 * X^* = \frac{1}{2} * 15 * 300 = 2250$$

c) If property rights are given to new residents, they have the right to develop to the extent X_c. They will accept to develop only to the point X* if they are duly compensated for all foregone benefits. These foregone benefits are represented by area C in Figure 5 and are:

$$area\ C = \frac{1}{2}(X_c - X^*)MB^* = \frac{1}{2}(500 - 300)*(15 - 0.03*300) = 600$$

When development is X* instead of X_c, the original residents face fewer external costs. These residents will be willing to pay for these lower costs. The most they will pay is the reduction in costs they expect when the town moves to X* instead of X_c. The costs they do not incur are the two areas B and C. These are:

$$areas\ B\ and\ C = \frac{1}{2}(MEC^* + MEC_c)(X_c - X^*) = \frac{1}{2}(0.02*300 + 0.02*500)(500 - 300) = 1600$$

With the existing residents are willing to pay at most 1600 and the new residents willing to accept no less than 600, the maximum net gains from trading are 1000. (For example, suppose, the old residents pay 1200. They are still better off by 400, while the city-folks are better off by 600. Together, the two groups are better off by 1000).

Note that these gains are exactly equal to the deadweight loss at X_c. This isn't a coincidence. The deadweight loss tells us what the gains are from moving to the social optimum. The two parties involved can then distribute these gains amongst themselves.

d) If the property rights go the existing home-owners, the maximum possible gains from trade are will be the same as the deadweight loss at $X_r = 0$. Thus, the maximum possible gains from trade are 2250. (The city-folk will have to pay the existing residents. The city-folk are willing to pay at most areas A and D to get to X*, while the original residents require area D to accept that much housing development.)

e) The main obstacle to achieving X* is practice is high transactions costs. Reaching X* without government intervention will require negotiations by both parties (potential and existing residents). Since there are many individuals in each group, it is reasonable to expect difficulties when trying to obtain an agreement. The fact that the new residents aren't known before they actually move in also makes negotiations difficult. That is, it isn't obvious with whom the existing residents should be negotiating. Basically any violation of one of the assumptions behind the Coase theorem (full information, well-defined and enforceable property rights, low transactions costs) will make a market resolution less likely in real life.

Part B: Essay Question

Your essay should have mentioned the following:

a) Smokers are inflecting an externality on others through second-hand smoke. Although it is not terribly difficult to find a bar wherein someone is smoking, the amount of smoking is dramatically less than it was before the ban. The law isn't being flouted.

b) Difficulties arise when the individuals attempt to negotiate a solution because of the large number of individuals involved and because property rights to clean air/right to smoke were not well defined. Hence, the government intervention.

c) The rationality of smokers is a matter of dispute. Smokers derive benefits from smoking. If they properly weigh the costs of smoking against these benefits before deciding to smoke, and choose to smoke only if the benefits outweigh the costs, they are being rational. Some feel, however, that the benefits from smoking cannot possibly outweigh the possible health costs. That many smokers try desperately to quit suggests they may not be maximizing their welfare (or that they are making time inconsistent decisions). That many smokers say they can "quit whenever I want to" may suggest an element of cognitive dissonance as well. (They are aware of the risk, but don't believe it applies to them.).

d) The ban is consistent with non-smokers' right to clean air.

e) Some alternative policies include higher cigarette taxes, public education against smoking, subsidies to help people quit (they pay for quitting programs, nicotine gum, etc.), clean air standards in bars, right-to-smoke fees in public places and segregation of bars (not just sections) into smoking and non-smoking.