(Inter-)Subjective Risk, Confidence, and Ambiguity

Christian Traeger

Department of Agricultural & Resource Economics,
UC Berkeley

- ▷ Motivation & Preview
- ▷ A normative representation & evaluation
  (setting, axioms, representation)
- ▷ Characterizing uncertainty aversion
  (The 2 types of risk aversion,
  Aversion to the lack of confidence,)
- ▷ Conclusions

Probabilistic reasoning 1)

- Bet on a fair dice revealing a number ≥ 4.

Probabilistic reasoning 2)

- Bet on a randomly drawn American male being more than one inch taller than a randomly drawn German male.

Probabilistic reasoning 1)

- ▷ Bet on a fair dice revealing a number ≥ 4.

You know the probability distribution from either: Symmetry, A convincing high school teacher, long series of observation...

Probabilistic reasoning 2)

- ▷ Bet on a randomly drawn American male being more than one inch taller than a randomly drawn German male.

Two situations:

1. You have no data access: *You might guess normal distributions and make up some expected values and variances.*
2. With data: *Both pools known to be normally distributed. For both mean and variance are known. Close to objective probability.*

Probabilistic reasoning 2)

- Bet on a randomly drawn American male being more than one inch taller than a randomly drawn German male.

2. With data: *Both pools known to be normally distributed. For both mean and variance are known. Close to objective probability.*

Probabilistic reasoning 2)

- ▷ Bet on a randomly drawn American male being more than one inch taller than a randomly drawn German male.

2. With data: *Both pools known to be normally distributed. For both mean and variance are known. Close to objective probability.*

Probabilistic reasoning 3)

- ▷ Bet on global economic growth rate between 2136 and 2137 being larger than rate between 2013 and 2014.

Might well assign normal distribution, but type/moments involve some amount of crystal ball reading...more so for the further future...

Probabilistic reasoning 2)

- ▷ Bet on a randomly drawn American male being more than one inch taller than a randomly drawn German male.

2. With data: *Both pools known to be normally distributed. For both mean and variance are known. Close to objective probability.*

Probabilistic reasoning 3)

- ▷ Bet on global economic growth rate between 2136 and 2137 being larger than rate between 2013 and 2014.

Might well assign normal distribution, but type/moments involve some amount of crystal ball reading...more so for the further future...

Related behavioral phenomenon: Ellsberg paradox

- ▷ People prefer to bet on known probabilities

Models of ambiguity including

- Rank dependent utility, Choquet expected utility
- Multiple prior models, Second order probabilities

and "source" models generally

Models of ambiguity including

- Rank dependent utility, Choquet expected utility
- Multiple prior models, Second order probabilities

and “source” models generally

- capture Ellsberg-paradox style behavior
- model agents that behave as if there exists some non-unique probability or capacity
- relax normatively desirable axioms including
  - time consistency
  - independence

This presentation is

- ▷ NOT focussed on individual behavior
- ▷ NOT about "there exists a distribution" and "behaves as if"

This presentation is

- NOT focussed on individual behavior
- NOT about "there exists a distribution" and "behaves as if"

Instead this presentation

- takes intersubjective probabilities as inputs
  - Probabilities that are not only existing in one individual's mind, but still are not necessarily objective
  - E.g.: Probabilistic estimates derived by expert groups based on limited data, simplified models, or expert judgements
- derives rules for deliberate/rational/normative decision-making
  - of e.g. a policy-maker, an expert panel, or a sophisticated portfolio manager

Example: The IPCC distinguishes on scientific side

Table 1. A simple typology of uncertainties

Type	Indicative examples of sources	Typical approaches or considerations
Unpredictability	Projections of human behaviour not easily amenable to prediction (e.g. evolution of political systems). Chaotic components of complex systems.	Use of scenarios spanning a plausible range, clearly stating assumptions, limits considered, and subjective judgments. Ranges from ensembles of model runs.
Structural uncertainty	Inadequate models, incomplete or competing conceptual frameworks, lack of agreement on model structure, ambiguous system boundaries or definitions, significant processes or relationships wrongly specified or not considered.	Specify assumptions and system definitions clearly, compare models with observations for a range of conditions, assess maturity of the underlying science and degree to which understanding is based on fundamental concepts tested in other areas.
Value uncertainty	Missing, inaccurate or non-representative data, inappropriate spatial or temporal resolution, poorly known or changing model parameters.	Analysis of statistical properties of sets of values (observations, model ensemble results, etc); bootstrap and hierarchical statistical tests; comparison of models with observations.

Not taken up on economic side!

Example: The IPCC distinguishes on scientific side

Table 1. A simple typology of uncertainties

Type	Indicative examples of sources	Typical approaches or considerations
Unpredictability	Projections of human behaviour not easily amenable to prediction (e.g. evolution of political systems). Chaotic components of complex systems.	Use of scenarios spanning a plausible range, clearly stating assumptions, limits considered, and subjective judgments. Ranges from ensembles of model runs.
Structural uncertainty	Inadequate models, incomplete or competing conceptual frameworks, lack of agreement on model structure, ambiguous system boundaries or definitions, significant processes or relationships wrongly specified or not considered.	Specify assumptions and system definitions clearly, compare models with observations for a range of conditions, assess maturity of the underlying science and degree to which understanding is based on fundamental concepts tested in other areas.
Value uncertainty	Missing, inaccurate or non-representative data, inappropriate spatial or temporal resolution, poorly known or changing model parameters.	Analysis of statistical properties of sets of values (observations, model ensemble results, etc); bootstrap and hierarchical statistical tests; comparison of models with observations.

Not taken up on economic side! Should it? And if so how?

The simple idea:

- ▷ Classify probabilities by their intersubjective degree of confidence
- ▷ Reduce compound probabilities ("treat the same") only if same confidence
- ▷ Otherwise standard axioms (von Neumann-Morgenstern, certainty separability, time consistency)

The simple idea:

- ▷ Classify probabilities by their intersubjective degree of confidence
- ▷ Reduce compound probabilities ("treat the same") only if same confidence
- ▷ Otherwise standard axioms (von Neumann-Morgenstern, certainty separability, time consistency)

Results in:

- ▷ Decision support framework taking account of confidence
- ▷ A concept of aversion to the lack of confidence
- ▷ unified framework of Epstein-Zin and (intersubjective) KMM model (smooth ambigutiy aversion
- ▷ nested model of Arrow-Hurwitz, Gilboa-Schmeidler, KMM where particular decision criteria depends on degree of confidence

I.1 Setting

- ▷ Representing (general) uncertainty trees
- ▷ Reducing uncertainty trees
- ▷ Mixing uncertainty trees

I.2 Assumptions

- ▷ on reduction
- ▷ on mixing

I.3 Representation of preferences

Later:

II.1 Characterizing intrinsic risk aversion

II.2 Characterizing aversion to the lack of confidence

III.1 Application: Discounting

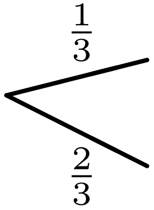
[III.2 Application: Implied choice restrictions]

PART I

Decision-Making and

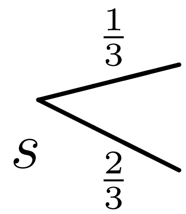
Intersubjective uncertainty classes

Representing 1 layer of uncertainty



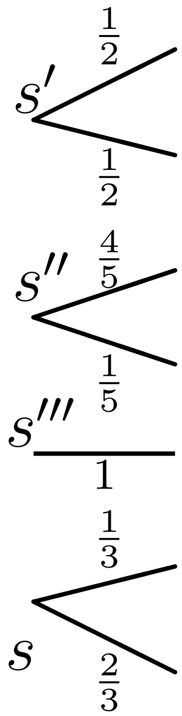
Representing 1 layer of uncertainty

s: degree of intersubjective confidence (short: subjectivity)



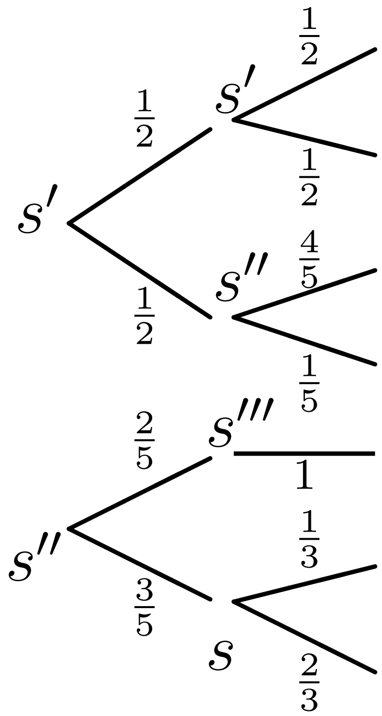
Representing 1 layer of uncertainty

s: degree of intersubjective confidence (short: subjectivity)



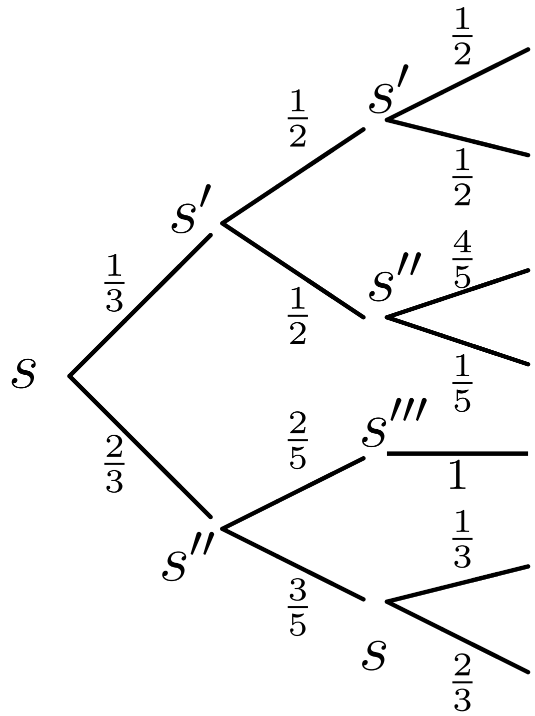
Representing 2 layers of uncertainty

s: degree of intersubjective confidence (short: subjectivity)

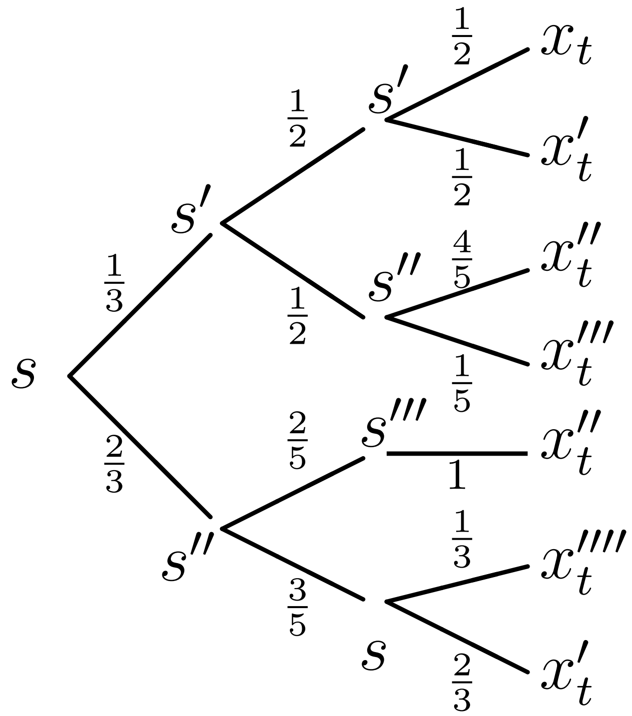


Representing 3 layers of uncertainty

s: degree of intersubjective confidence (short: subjectivity)

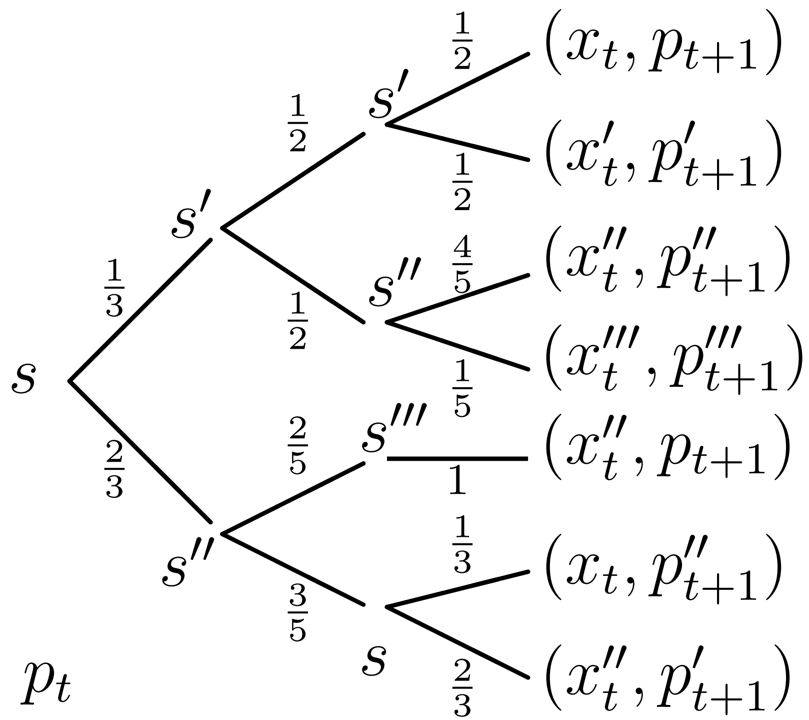


Representing 3 layers of uncertainty



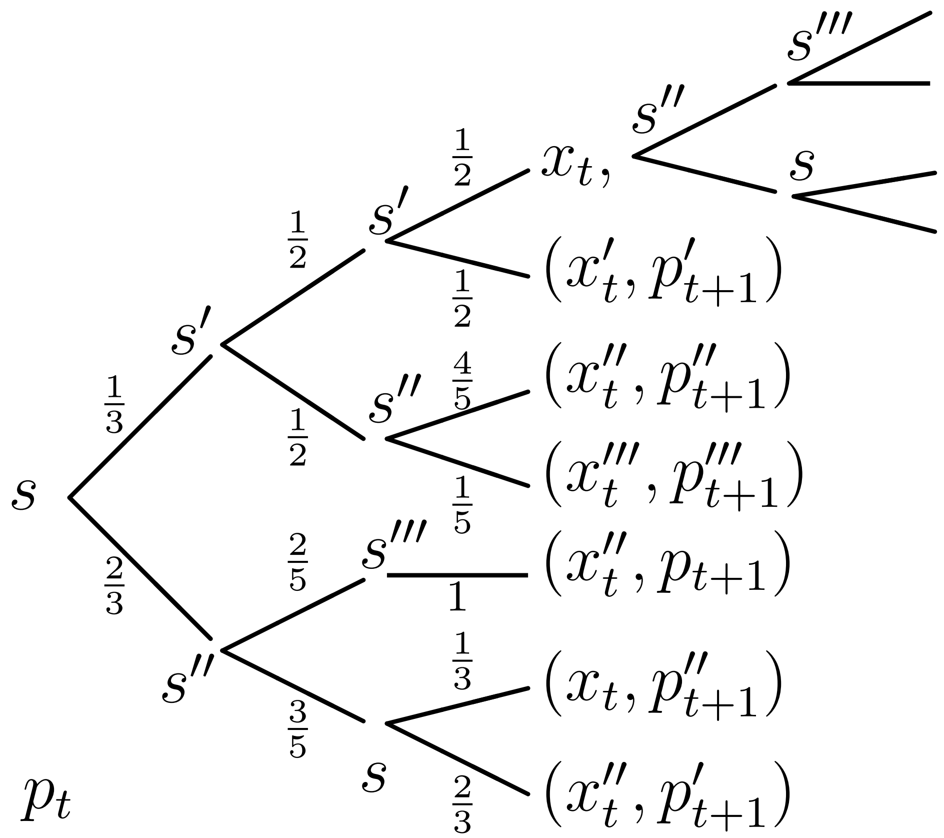
One period future

Representing 3 layers of uncertainty



Multi-period setting

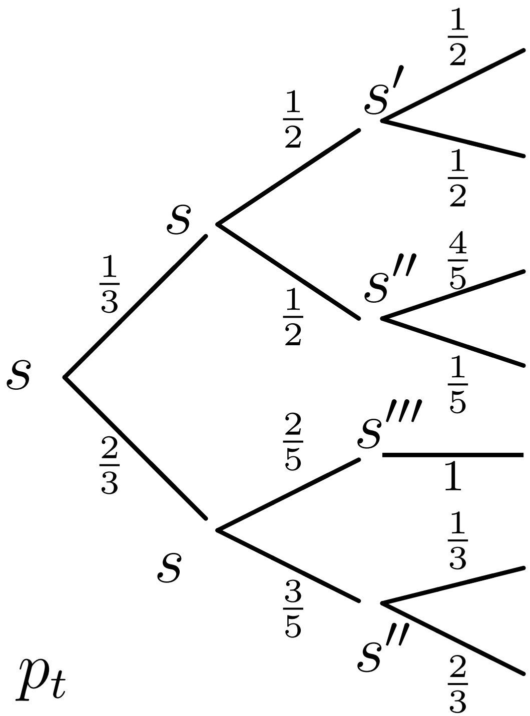
Representing 3 layers of uncertainty



Multi-period setting

Denote

- ▷ P_t^s: Subset of P_t with first node of degree of subjectivity s
- ▷ $\hat{s}(p_t) = s$ iff $p_t \in P_t^s$



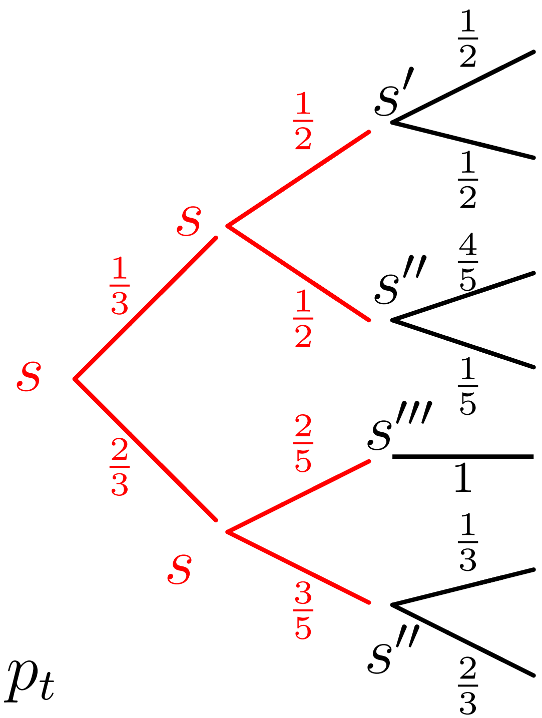
p_t

$p_t \in P_t^s$

$\hat{s}(p_t) = s$

Denote

▷ P_t^{ss}: Subset of P_t with first two uncertainty layers of degree s



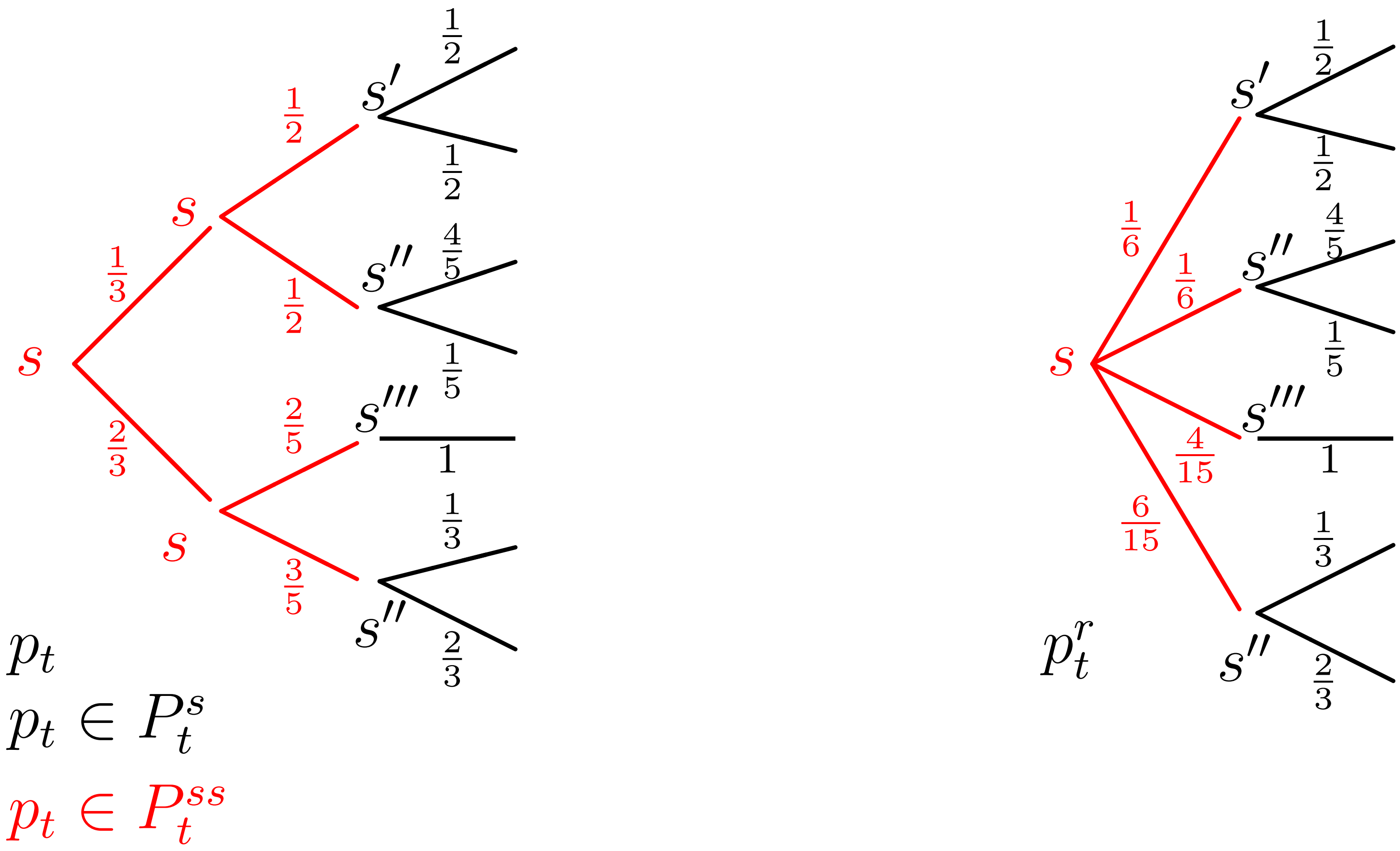
p_t

$p_t \in P_t^s$

$p_t \in P_t^{ss}$

Denote

- P_t^{ss}: Subset of P_t with first two uncertainty layers of degree s
- p_t^r: Reduction of $p_t \in P_t^{ss}$ obtained by collapsing first two layers

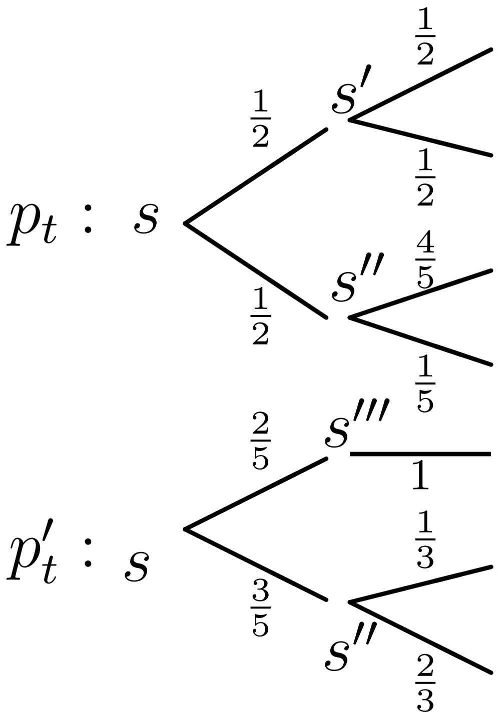


Definitions:

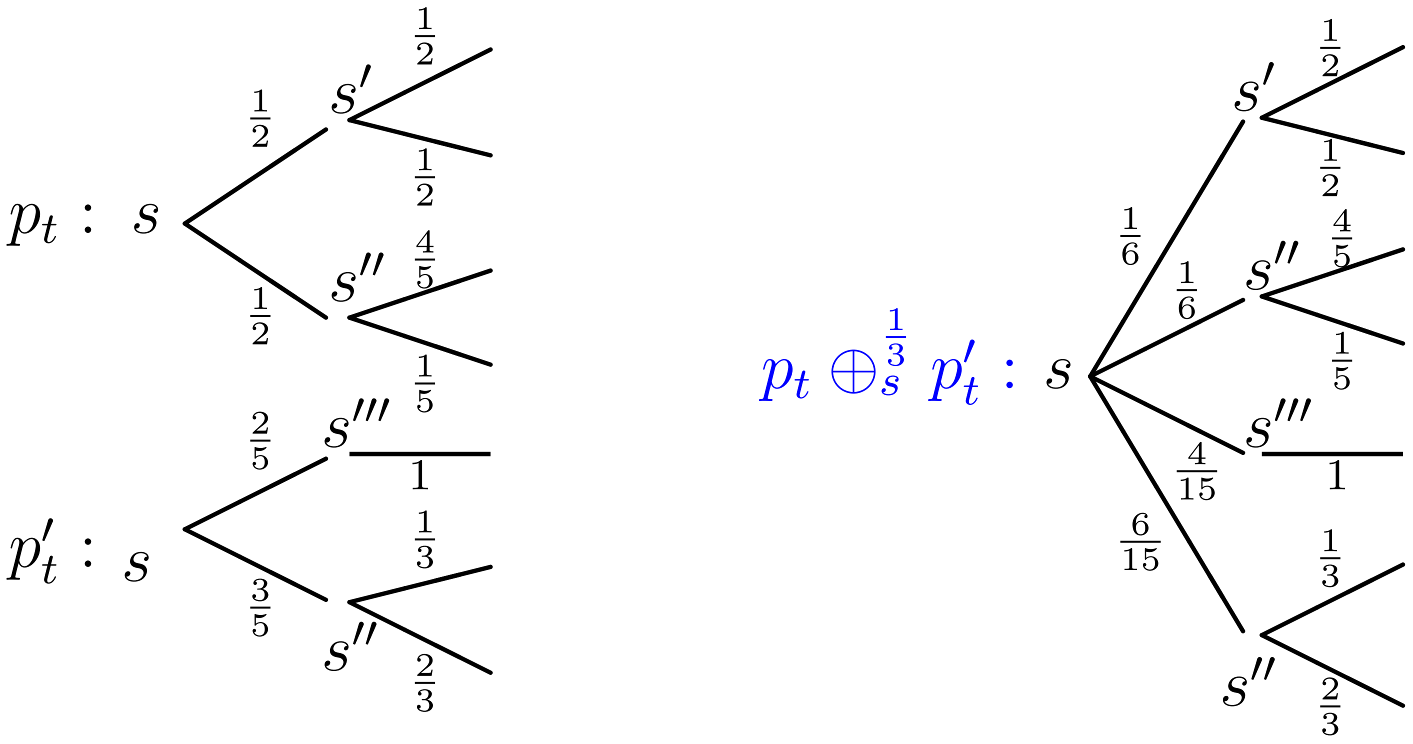
- Mixing of lotteries:
  For $p_t, p'_t \in P^s_t$ define for $\alpha \in [0,1]$ and $s \in S$ the mixture

  $p_t \oplus^\alpha_s p'_t$ as lottery in P^s_t yielding
  - p_t with probability α and
  - p'_t with probability $1-\alpha$ with
  - degree of subjectivity s

Example: Mixing of lotteries:



Example: Mixing of lotteries:



Axioms

- Indifference to reduction of same degree of subjectivity lotteries:
  For all $t \in \{0, ..., T\}, s \in S$, and $p_t \in \cup_{s \in S} P_t^{ss}$: $\quad p_t^r \sim_t p_t$

Axioms

- Indifference to reduction of same degree of subjectivity lotteries:

  For all $t \in \{0, ..., T\}$, $s \in S$, and $p_t \in \cup_{s \in S} P_t^{ss}$: $\quad p_t^r \sim_t p_t$

- Independence:

  For all $t \in \{0, ..., T\}, s \in S, \alpha \in [0,1]$ and $p_t, p'_t, p''_t \in P_t^s$

$$p_t \succeq_t p'_t \quad \Leftrightarrow \quad p_t \oplus_s^\alpha p''_t \succeq_t p'_t \oplus_s^\alpha p''_t$$

Axioms

- Indifference to reduction of same degree of subjectivity lotteries:

  For all $t \in \{0, ..., T\}, s \in S$, and $p_t \in \cup_{s \in S} P_t^{ss}$: $\quad p_t^r \sim_t p_t$

- Independence:

  For all $t \in \{0, ..., T\}, s \in S, \alpha \in [0,1]$ and $p_t, p_t', p_t'' \in P_t^s$

  $$p_t \succeq_t p_t' \quad \Leftrightarrow \quad p_t \oplus_s^\alpha p_t'' \succeq_t p_t' \oplus_s^\alpha p_t''$$

- Standard axioms: weak order, continuity, certainty separability, time consistency

shortcut

The representation uses:

- A generalized mean for uncertainty aggregation
  - For f strictly increasing define: $\mathcal{M}_p^f \, z \equiv f^{-1} \left[\mathsf{E}_p \, f(z) \right]$
  - Note: For f concave $\mathcal{M}_p^f \, z < \mathsf{E}_p \, z$

The representation uses:

- A generalized mean for uncertainty aggregation
  - For f strictly increasing define: $\mathcal{M}_p^f \, z \equiv f^{-1} \left[\mathsf{E}_p \, f(z) \right]$
  - Note: For f concave $\mathcal{M}_p^f \, z < \mathsf{E}_p \, z$
- Sequence $\hat{f}_t = (f_t^s)_{s \in S}$ of functions $f_t^s : \mathbb{R} \to \mathbb{R}$

The representation uses:

- A generalized mean for uncertainty aggregation
  - For f strictly increasing define: $\mathcal{M}_p^f \, z \equiv f^{-1} \left[\mathsf{E}_p \, f(z) \right]$
  - Note: For f concave $\mathcal{M}_p^f \, z < \mathsf{E}_p \, z$
- Sequence $\hat{f}_t = (f_t^s)_{s \in S}$ of functions $f_t^s : \mathbb{R} \to \mathbb{R}$

The representation - informal:

- An uncertainty node of degree of subjectivity s is evaluated using the generalized mean $\mathcal{M}_{p^s}^{f^s}$ that is characterized by the function f_t^s

The representation uses:

- A generalized mean for uncertainty aggregation
  - For f strictly increasing define: $\mathcal{M}_p^f \, z \equiv f^{-1} \left[\mathsf{E}_p \, f(z) \right]$
  - Note: For f concave $\mathcal{M}_p^f \, z < \mathsf{E}_p \, z$
- Sequence $\hat{f}_t = (f_t^s)_{s \in S}$ of functions $f_t^s : \mathbb{R} \to \mathbb{R}$

The representation - informal:

- An uncertainty node of degree of subjectivity s is evaluated using the generalized mean $\mathcal{M}_{p^s}^{f^s}$ that is characterized by the function f_t^s
- Aggregates recursively over all uncertainty layers in a period
- Aggregates recursively over time periods

The representation uses:

- A generalized mean for uncertainty aggregation
  - For f strictly increasing define: $\mathcal{M}_p^f\, z \equiv f^{-1}\left[\mathsf{E}_p\, f(z)\right]$
  - Note: For f concave $\mathcal{M}_p^f\, z < \mathsf{E}_p\, z$
- Sequence $\hat{f}_t = (f_t^s)_{s \in S}$ of functions $f_t^s : \mathbb{R} \rightarrow \mathbb{R}$

The representation uses:

- A generalized mean for uncertainty aggregation
  - For f strictly increasing define: $\mathcal{M}_p^f \, z \equiv f^{-1} \left[\mathsf{E}_p \, f(z) \right]$
  - Note: For f concave $\mathcal{M}_p^f \, z < \mathsf{E}_p \, z$
- Sequence $\hat{f}_t = (f_t^s)_{s \in S}$ of functions $f_t^s : \mathbb{R} \to \mathbb{R}$

Formally: Define generalized uncertainty aggregator:

Let p_t be lottery p_t^1 over lotteries p_t^2 over ... over p_t^N over (x_t^*, p_{t+1})

$$\mathcal{M}_{p_t}^{\hat{f}_t} \, W_t(x_t^*, p_{t+1}) = \prod_{i=1}^N \mathcal{M}_{p_t^i}^{f^{\hat{s}(p_t^i)}} \, W_t(x_t^*, p_{t+1})$$
$$= \mathcal{M}_{p_t^1}^{f^{\hat{s}(p_t^1)}} \cdots \mathcal{M}_{p_t^N}^{f^{\hat{s}(p_t^N)}} \, W_t(x_t^*, p_{t+1})$$

Theorem:

The sequence of preference relations $(\succeq_t)_{t\in T}$ satisfies the axioms if, and only if, for all $t \in \{0, ..., T\}$ there exist

- a set of strictly increasing and continuous functions $\hat{f}_t = (f_t^s)_{s\in S}$, $f_t^s : \mathbb{R} \to \mathbb{R}$
- a continuous and bounded function $u_t : X^* \to \mathbb{R}$

Theorem:

The sequence of preference relations $(\succeq_t)_{t\in T}$ satisfies the axioms if, and only if, for all $t \in \{0, ..., T\}$ there exist

- a set of strictly increasing and continuous functions $\hat{f}_t = (f_t^s)_{s\in S}$, $f_t^s : \mathbb{R} \to \mathbb{R}$
- a continuous and bounded function $u_t : X^* \to \mathbb{R}$

such that by defining recursively the functions $W_T = u_T$ and

- $W_{t-1} : X^* \times P_t \to \mathbb{R}$ by

$$W_{t-1}(x_{t-1}, p_t) = u_{t-1}(x_{t-1}) \; + \; \mathcal{M}_{p_t}^{\hat{f}_t} W_t(x_t, p_{t+1})$$

Theorem:

The sequence of preference relations $(\succeq_t)_{t\in T}$ satisfies the axioms if, and only if, for all $t \in \{0, ..., T\}$ there exist

- a set of strictly increasing and continuous functions $\hat{f}_t = (f_t^s)_{s\in S},\ f_t^s : \mathbb{R} \to \mathbb{R}$
- a continuous and bounded function $u_t : X^* \to \mathbb{R}$

such that by defining recursively the functions $W_T = u_T$ and

- $W_{t-1} : X^* \times P_t \to \mathbb{R}$ by

$$W_{t-1}(x_{t-1}, p_t) = u_{t-1}(x_{t-1}) \ + \ \mathcal{M}_{p_t}^{\hat{f}_t} W_t(x_t, p_{t+1})$$

it holds for all $t \in T$ and all $p_t, p'_t \in P_t$

$$p_t \ \succeq_t \ p'_t \quad \Leftrightarrow \quad \mathcal{M}_{p_t}^{\hat{f}_t} W_t(x_t, p_{t+1}) \ \geq \ \mathcal{M}_{p'_t}^{\hat{f}_t} W_t(x_t, p_{t+1})$$

Example $\hookrightarrow$

PART II

- Characterizing intrinsic risk aversion
- Characterizing aversion to the lack of confidence

$$W_{t-1}(x_{t-1}, p_t) = u_{t-1}(x_{t-1}) \; + \; \mathcal{M}_{p_t}^{\hat{f}_t} W_t(x_t, p_{t+1})$$

Function u measures aversion to intertemporal subst.

There are 2 effects of risk:

i) Generates fluctuations over time
$\rightarrow$ Disliked by agents who prefer smooth consumption over time
$\rightarrow$ Measured by u

$$W_{t-1}(x_{t-1},p_t) = u_{t-1}(x_{t-1}) \; + \; \mathcal{M}_{p_t}^{\hat{f}_t} W_t(x_t,p_{t+1})$$

Function u measures aversion to intertemporal subst.

There are 2 effects of risk:

i) Generates fluctuations over time
$\rightarrow$ Disliked by agents who prefer smooth consumption over time
$\rightarrow$ Measured by u

ii) Makes agent unsure about their future
$\rightarrow$ Disliked by agents with intrinsic aversion to risk
$\rightarrow$ Measured by f [depends on confidence]

Interpretation

$$W_{t-1}(x_{t-1}, p_t) = u_{t-1}(x_{t-1}) \; + \; \mathcal{M}_{p_t}^{\hat{f}_t} W_t(x_t, p_{t+1})$$

Function u measures aversion to intertemporal subst.

There are 2 effects of risk:

i) Generates fluctuations over time
 $\rightarrow$ Disliked by agents who prefer smooth consumption over time
 $\rightarrow$ Measured by u

ii) Makes agent unsure about their future
 $\rightarrow$ Disliked by agents with intrinsic aversion to risk
 $\rightarrow$ Measured by f [depends on confidence]

Alternative measures for risk aversion in 1 commodity setting

u as above & introduce function measuring i and ii jointly
$\rightarrow$ Epstein-Zin's measure of Arrow-Pratt risk aversion

$\hookrightarrow$

Let x, x' be two consumption paths of length T.

Example, $T = 4$:

$$x = (\quad , \quad , \quad , \quad)$$

$$x' = (\quad , \quad , \quad , \quad)$$

Let $x \succ x'$ denote a strict preference for x over x'.
Let $\sim$ denote indifference.

Let x, x' be two consumption paths of length T.

Example, $T = 4$:

x = (☺, ☹, ☺, ☹)

x' = (☹, ☺, ☹, ☺)

Let $x \succ x'$ denote a strict preference for x over x'.
Let $\sim$ denote indifference.

Let x, x' be two consumption paths of length T.

Example, $T = 4$:

x =(☺,☹,☺,☹)

x' =(☹,☺,☹,☺)

Let $x \succ x'$ denote a strict preference for x over x'.
Let $\sim$ denote indifference.

Define for x and x' the consumption paths

- ▷ x^{high}: collects better outcomes of every period
- ▷ x^{low}: collects inferior outcomes of every period

Let x, x' be two consumption paths of length T.

Example, $T = 4$:

$$x = (\smiley, \frownie, \smiley, \frownie) \qquad x^{\text{high}} = (\smiley, \smiley, \smiley, \smiley)$$

$$x' = (\frownie, \smiley, \frownie, \smiley) \qquad x^{\text{low}} = (\frownie, \frownie, \frownie, \frownie)$$

Let $x \succ x'$ denote a strict preference for x over x'.
Let $\sim$ denote indifference.

Define for x and x' the consumption paths

- x^{high}: collects better outcomes of every period
- x^{low}: collects inferior outcomes of every period

A Question of Preference

Assume you'd be indifferent between

$$(☺, ☹, ☺, ☹) \sim (☹, ☺, ☹, ☺)$$

Assume you'd be indifferent between

$$(\smiley,\frownie,\smiley,\frownie) \sim (\frownie,\smiley,{\color{red}\frownie},\smiley)$$

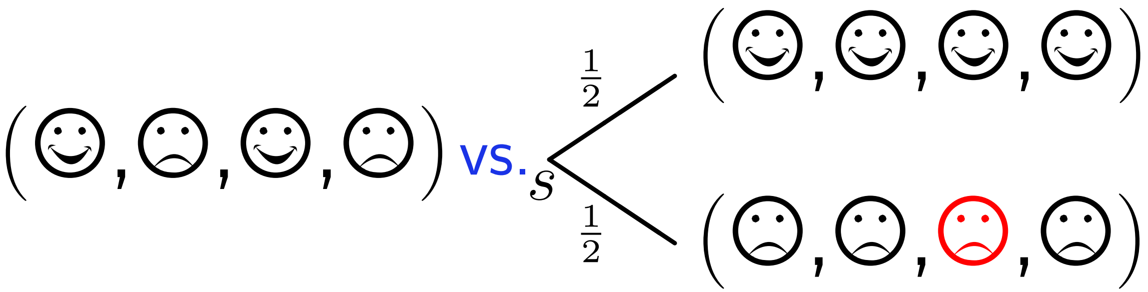
If not, please mentally adjust the corners of the mouth of the red frowny ${\color{red}\frownie}$ to reach indifference.

Assume you'd be indifferent between

(☺,☹,☺,☹) $\sim$ (☹,☺,☹,☺)

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?



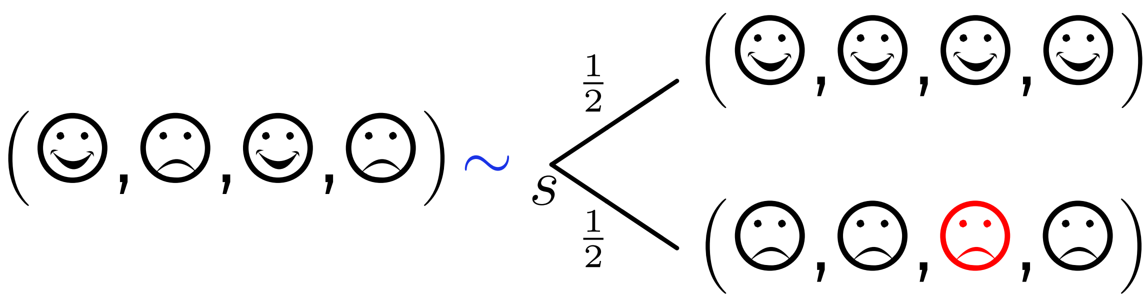
certain path   coin toss if s=obj

Assume you'd be indifferent between

(☺,☹,☺,☹) ∼ (☹,☺,☹,☺)

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?



certain path　　　coin toss if s=obj

Assume you'd be indifferent between

$$(\smiley, \frownie, \smiley, \frownie) \sim (\frownie, \smiley, \textcolor{red}{\frownie}, \smiley)$$

If not, please mentally adjust the corners of the mouth of the red frowny $\textcolor{red}{\frownie}$ to reach indifference.

What preference do you have in the following choice?

$$(\smiley, \frownie, \smiley, \frownie) \textcolor{blue}{\succ} \; s \begin{cases} \frac{1}{2} & (\smiley, \smiley, \smiley, \smiley) \\ \frac{1}{2} & (\frownie, \frownie, \textcolor{red}{\frownie}, \frownie) \end{cases}$$

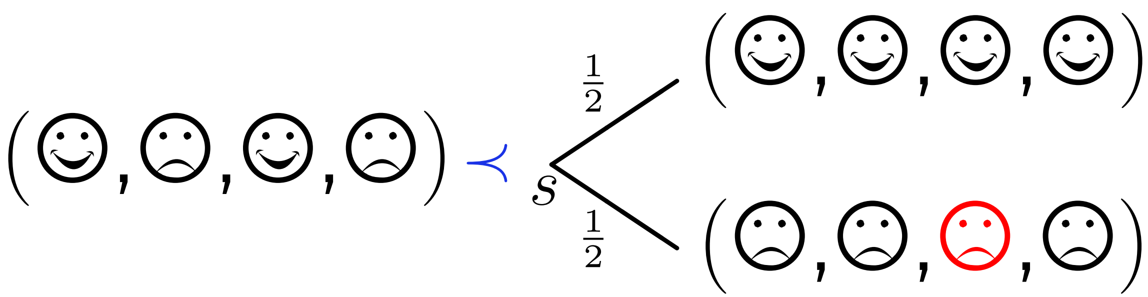
certain path — coin toss if s=obj

Assume you'd be indifferent between

(☺,☹,☺,☹) $\sim$ (☹,☺,☹,☺)

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?



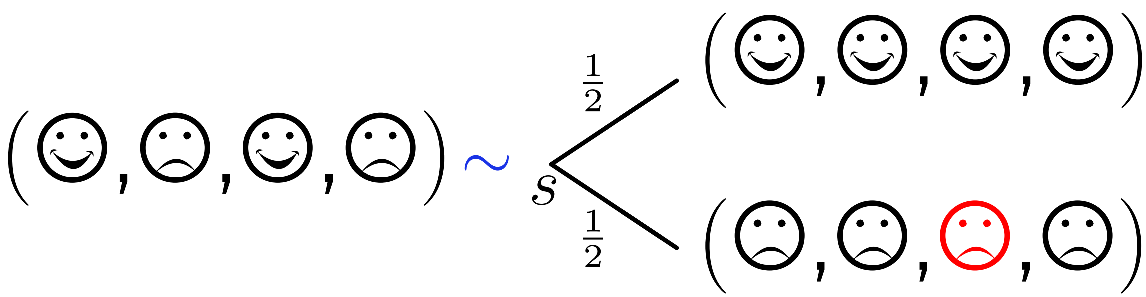
certain path coin toss if s=obj

Assume you'd be indifferent between

(☺,☹,☺,☹) ∼ (☹,☺,☹,☺)

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?



certain path   coin toss if s=obj

$\forall\, s \Rightarrow$ STANDARD MODEL $\mathrm{E}\sum_t \beta^t u(x_t)$

Assume you'd be indifferent between

$$(☺,☹,☺,☹) \sim (☹,☺,☹,☺)$$

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?

$$(☺,☹,☺,☹) \succ s \begin{cases} \frac{1}{2} & (☺,☺,☺,☺) \\ \frac{1}{2} & (☹,☹,☹,☹) \end{cases}$$

certain path — coin toss if s=obj

INTERTEMPORAL RISK AVERSE

with respect to degree of subjectivity s

For any two consumption paths x, x' define composed paths

- $x^{\text{high}}(x, x')$ collecting better outcomes of every period
- $x^{\text{low}}(x, x')$ collecting inferior outcomes of every period

For any two consumption paths x, x' define composed paths

- $x^{\text{high}}(x, x')$ collecting better outcomes of every period
- $x^{\text{low}}(x, x')$ collecting inferior outcomes of every period

Definition 1:

A decision maker is intertemporal risk averse w.r.t. to lotteries of degree of subjectivity s in period t

- iff for all certain consumption paths x and x'

$$x \sim_t x' \quad \Rightarrow \quad x \ \succeq_t \ x^{\text{high}}(x, x') \oplus_s^{\frac{1}{2}} x^{\text{low}}(x, x')$$

For any two consumption paths x, x' define composed paths

- $x^{\text{high}}(x, x')$ collecting better outcomes of every period
- $x^{\text{low}}(x, x')$ collecting inferior outcomes of every period

Characterization of f:

A decision maker is intertemporal risk averse w.r.t. to lotteries of degree of subjectivity s in period t

- iff for all certain consumption paths x and x'

$$x \sim_t x' \quad \Rightarrow \quad x \ \succeq_t \ x^{\text{high}}(x, x') \oplus_s^{\frac{1}{2}} x^{\text{low}}(x, x')$$

- iff f_t^s in the representation is concave

For any two consumption paths x, x' define composed paths

- $x^{\text{high}}(x, x')$ collecting better outcomes of every period
- $x^{\text{low}}(x, x')$ collecting inferior outcomes of every period

Characterization of f:

A decision maker is intertemporal risk averse w.r.t. to lotteries of degree of subjectivity s in period t

- iff for all certain consumption paths x and x'

$$x \sim_t x' \quad \Rightarrow \quad x \;\succeq_t\; x^{\text{high}}(x, x') \oplus_s^{\frac{1}{2}} x^{\text{low}}(x, x')$$

- iff f_t^s in the representation is concave

Note on relation to one-commodity Epstein-Zin (1989) model:
f_t^{obj} measures the difference between Arrow Pratt aversion to objective risk and aversion to intertemporal substitution

Getting more seriously at “confidence” I now add more structure:

Assume

- Decision maker has order relation for degree of subjectivity $s \triangleright s'$: lottery labeled s is more subjective than lottery labeled s'.

Getting more seriously at "confidence" I now add more structure:

Assume

- ▷ Decision maker has order relation for degree of subjectivity $s \rhd s'$: lottery labeled s is more subjective than lottery labeled s'.

Definition 2:

A decision maker is (strictly) averse to subjectivity of belief iff for all $x, x' \in X^t$ and $s, s' \in S$:

$$s \rhd s' \quad \Rightarrow \quad x \oplus_{s'}^{\frac{1}{2}} x' \;\succeq_t\; (\succ_t)\; x \oplus_{s}^{\frac{1}{2}} x' \; .$$

Getting more seriously at "confidence" I now add more structure:

Assume

- Decision maker has order relation for degree of subjectivity $s \triangleright s'$: lottery labeled s is more subjective than lottery labeled s'.

Definition 2:

A decision maker is (strictly) averse to subjectivity of belief iff for all $\mathrm{x}, \mathrm{x}' \in \mathrm{X}^t$ and $s, s' \in S$:

$$s \triangleright s' \quad \Rightarrow \quad \mathrm{x} \oplus^{\frac{1}{2}}_{s'} \mathrm{x}' \;\succeq_t\; (\;\succ_t\;)\; \mathrm{x} \oplus^{\frac{1}{2}}_{s} \mathrm{x}' \;.$$

Or: s is less confident than lottery labeled s'.

Decision maker is averse to the lack of confidence.

Characterization:

A decision maker is (strictly) averse to subjectivity of belief iff for all $\mathrm{x}, \mathrm{x}' \in \mathrm{X}^t$ and $s, s' \in S$:

$$s \rhd s' \quad \Rightarrow \quad \mathrm{x} \oplus_{s'}^{\frac{1}{2}} \mathrm{x}' \;\succeq_t \; (\succ_t) \; \mathrm{x} \oplus_{s}^{\frac{1}{2}} \mathrm{x}' \; .$$

which is equivalent to

$$s \rhd s' \quad \Leftrightarrow \quad f_t^s \circ (f_t^{s'})^{-1} \text{ (strictly) concave} \quad \forall s, s' \in S \; .$$

Interpretation:

- More averse to more subjective lottery labeled s than to less subjective lottery labeled s'

Conclusion:

- Standard model does not capture *confidence* of belief
- *von Neumann-Morgenstern* setting is easily extended to do so keeping main, normatively desirable axioms

Conclusion:

- Standard model does not capture *confidence* of belief
- *von Neumann-Morgenstern* setting is easily extended to do so keeping main, normatively desirable axioms
- Confidence in probabilistic description and aversion to the lack of confidence/subjectivity become **relevant for evaluation**!

Conclusion:

- Standard model does not capture *confidence* of belief
- *von Neumann-Morgenstern* setting is easily extended to do so keeping main, normatively desirable axioms
- Confidence in probabilistic description and aversion to the lack of confidence/subjectivity become **relevant for evaluation**!
- Evaluation nests, depending on degree of confidence and aversion:
  - decision making under ignorance by Arrow Hurwitz;
  - maxi-min expected utility by Gilboa Schmeidler;
  - (intersubjective) smooth ambiguity aversion by KMM

Three restrictions make representation an intersubjective von Neumann-Morgenstern version of KMM's smooth ambiguity aversion:

- ▷ only 2 layers of uncertainty (in every period)
- ▷ only subjective ($subj$) over objective (obj) lotteries
- ▷ f_t^{obj} is identity (absent)

Three restrictions make representation an intersubjective von Neumann-Morgenstern version of KMM's smooth ambiguity aversion:

- only 2 layers of uncertainty (in every period)
- only subjective ($subj$) over objective (obj) lotteries
- f_t^{obj} is identity (absent)

Most important features:

- f_t^{subj} is corresponds to (an intersubjective version of) KMM's smooth ambiguity aversion
- KMM do not allow for intertemporal risk aversion with respect to objective lotteries
- If real world agents would be more averse to risk than to deterministic intertemporal change, it would "conflate" the ambiguity aversion measure.

For general agents:

I suggest defining (intersubjective) smooth ambiguity aversion as

- ▷ the binary special case of aversion to the lack of confidence:

$$f_t^{amb} = f_t^{subj} \circ (f_t^{obj})^{-1}.$$

For general agents:

I suggest defining (intersubjective) smooth ambiguity aversion as

- ▷ the binary special case of aversion to the lack of confidence:

  $f_t^{amb} = f_t^{subj} \circ (f_t^{obj})^{-1}$.

In the case where indeed $f_t^{obj} = \text{id}$ we have

- ▷ $f_t^{amb} = f_t^{subj} \circ (f_t^{obj})^{-1} = f_t^{subj} \circ \text{id}^{-1} = f_t^{subj}$.

For general agents:

I suggest defining (intersubjective) smooth ambiguity aversion as

- the binary special case of aversion to the lack of confidence:

  $$f_t^{amb} = f_t^{subj} \circ (f_t^{obj})^{-1}.$$

In the case where indeed $f_t^{obj} = \text{id}$ we have

- $f_t^{amb} = f_t^{subj} \circ (f_t^{obj})^{-1} = f_t^{subj} \circ \text{id}^{-1} = f_t^{subj}.$

In the general case f_t^{amb}

- extracts the aversion due to subjectivity (lack of confidence),
- filters out intrinsic risk aversion (aversion to risk dominating propensity to smooth intertemporally)

Relation to Arrow Pratt risk aversion:

For the one-commodity setting (with utility str. increasing)

- u_t characterize aversion to intertemporal substitution

Relation to Arrow Pratt risk aversion:

For the one-commodity setting (with utility str. increasing)

- u_t characterize aversion to intertemporal substitution

Define:

- $g_t^{obj} \equiv f_t^{obj} \circ u_t^{-1}$: measures Arrow Pratt risk aversion with respect to objective risk
- $g_t^{subj} \equiv f_t^{subj} \circ u_t^{-1}$: measures Arrow Pratt risk aversion with respect to subjective risk
- Then $f_t^{amb} = g_t^{subj} \circ (g_t^{obj})^{-1}$

Relation to Arrow Pratt risk aversion:

For the one-commodity setting (with utility str. increasing)

- u_t characterize aversion to intertemporal substitution

Define:

- $g_t^{obj} \equiv f_t^{obj} \circ u_t^{-1}$: measures Arrow Pratt risk aversion with respect to objective risk
- $g_t^{subj} \equiv f_t^{subj} \circ u_t^{-1}$: measures Arrow Pratt risk aversion with respect to subjective risk
- Then $f_t^{amb} = g_t^{subj} \circ (g_t^{obj})^{-1}$

Here, my suggested refinement of smooth ambiguity aversion is equivalent to being

- more Arrrow Pratt risk averse to subjective than to objective risk.

Climate change: Another Implication

Distinguish confidence in probabilistic predictions of climate change

Possible conjecture:

Probabilistic climate information is

- ▷ most confidently known under current climate
- ▷ less confidently known the more we perturb the system

Climate change: Another Implication

Distinguish confidence in probabilistic predictions of climate change

Possible conjecture:

Probabilistic climate information is

- ▷ most confidently known under current climate
- ▷ less confidently known the more we perturb the system

↪ Scenario evaluations that

- ▷ employ aversion to subjectivity of belief
  (aversion to lack of conficence in probabilistic predictions)

will give relatively less value to high GHG scenarios

- ▷ than evaluation based on the standard model

↪ "Preference" for perturbing the system less

Climate change, how would we build a model (illustration):

A more sophisticated model can incorporate anticipated learning

1. *Given* a certain *climate* we experience objective risk of damage
2. *Given* a climate *model* we *learn* about future *climate*
3. *Confidence* into the *model* in question as well

Climate change, how would we build a model (illustration):

A more sophisticated model can incorporate anticipated learning

1. *Given* a certain *climate* we experience objective risk of damage
2. *Given* a climate *model* we *learn* about future *climate*
3. *Confidence* into the *model* in question as well

Formally let overall probability of some damage event x be given by

- $p(x) = \int \int l(x|\theta_1, \theta_2) d\mu_1(\theta_1|\theta_2) d\mu_2(\theta_2)$
- l given θ_1, θ_2: remaining stochasticity if we knew everything that is to be learned
- θ_1, μ_1: parameters/uncertainty we learn about *in* climate change *models*
- θ_2, μ_2: capture learning *how well* climate models do

- $p(x) = \int \int l(x|\theta_1, \theta_2) d\mu_1(\theta_1|\theta_2) d\mu_2(\theta_2)$

For example

- $l \in \Delta_s(X)$
- $\mu_1 \in \Delta_{s'}(\Delta_s(X))$
- $\mu_2 \in \Delta_{s''}(\Delta_{s'}(\Delta_s(X)))$
- $s'' \triangleright s' \triangleright s$ and aversion to the subjectivity of belief

- $p(x) = \int \int l(x|\theta_1, \theta_2) d\mu_1(\theta_1|\theta_2) d\mu_2(\theta_2)$

For example

- $l \in \Delta_s(X)$
- $\mu_1 \in \Delta_{s'}(\Delta_s(X))$
- $\mu_2 \in \Delta_{s''}(\Delta_{s'}(\Delta_s(X)))$
- $s'' \triangleright s' \triangleright s$ and aversion to the subjectivity of belief

Could

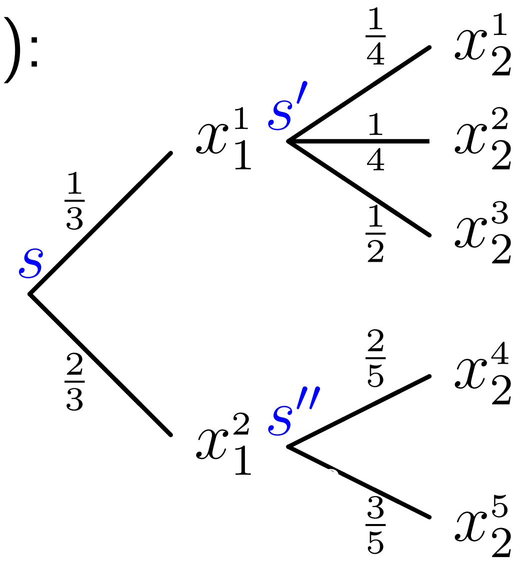
- use standard Bayesian learning within each subjectivity dimension

or

- think about how degree of subjectivity might change as we learn (interesting and much harder)

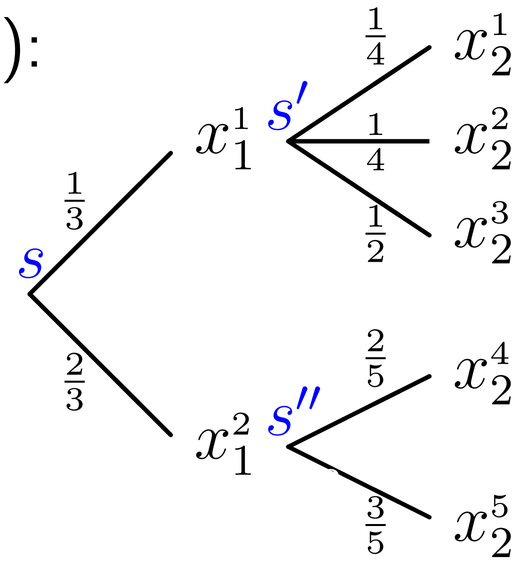
Example ($T = 2$):

Evaluate

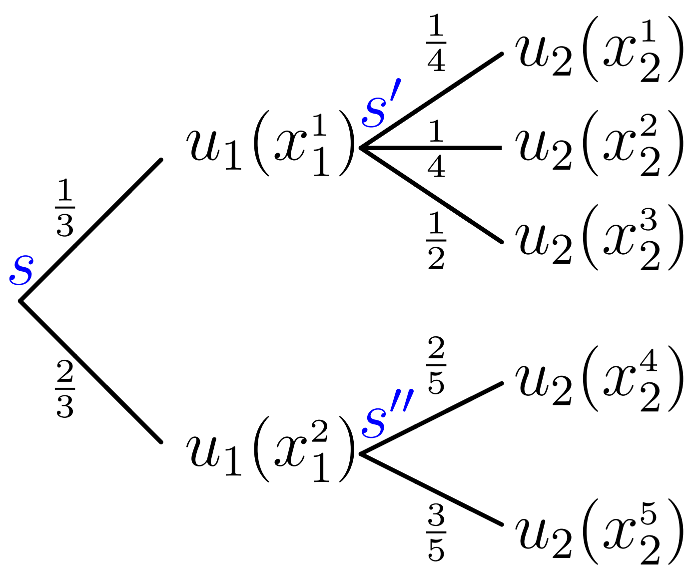


Example ($T = 2$):

Evaluate

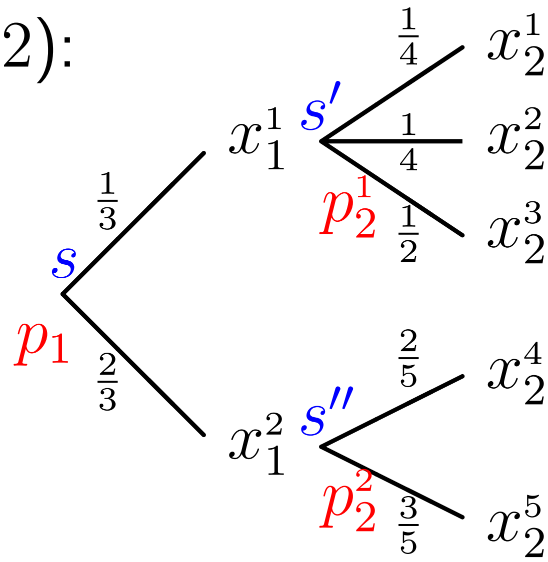


by

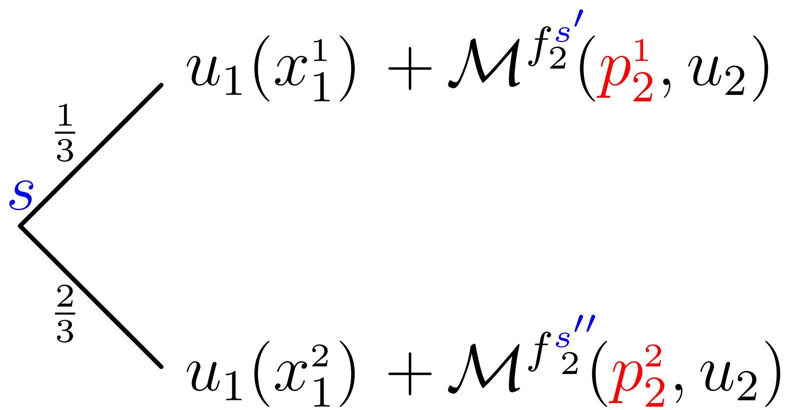


Example ($T = 2$):

Evaluate

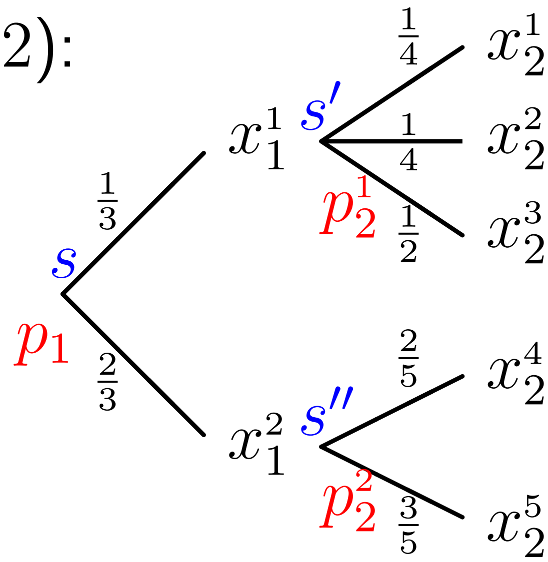


by

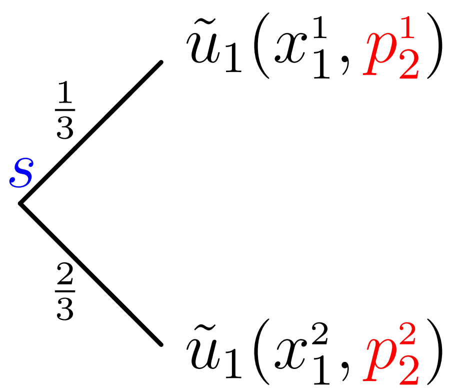


Example ($T = 2$):

Evaluate

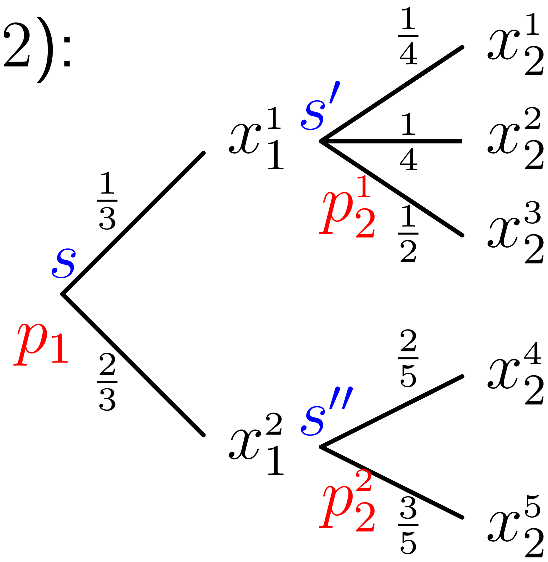


by



Example ($T = 2$):

Evaluate



by $\mathcal{M}^{f_1^s}(p_1, \tilde{u}_1)$