

# Learning

## Learning

*In the following:*

- We continue with risk
- We work with a *risk neutral agent*  $U(M)=M$

Justification:

- Still complicated enough
- Shows that value from (anticipated) learning even if no risk aversion

## Learning

- An important characteristic of uncertainty is that it generally resolves over time  
-> We learn
  
  - Two ways to incorporate that we learn:
    1. Naive way:
      - we do not anticipate that we learn
      - we only consider that we learn after new information arrives
    2. Sophisticated way:
      - we anticipate that we will learn
      - we already incorporate in today's plans that we will learn in future
- > How does such an anticipation change today's decisions ?

## 1 - Learning and Option Value

Given is following project:

- Invest USD  $I=60$  now and in the following period receive either USD
  - $R=100$  with  $p(R=100)=.5$  or
  - $R=50$  with  $p(R=50)=.5$
  - Return  $R$  is random variable
- We discount future period with factor  $D < 1$ .

*Find expected return of project:*

- $E -I+D \cdot R = -60 + 0.5 D (100 + 50) = 75 D - 60.$
- Say discount factor  $D=.9 (>.8)$  , then  $E -I+D \cdot R = 7.5$   
-> project has positive expected payoff

*So should we invest?*

## 1 - Learning and Option Value

*Assume we can only do project once.*

(E.g. install a particular new abatement technology in a power plant,  
not sure how much it abates / how much we gain in carbon credits)

Idea:

- *What if uncertainty resolves at beginning of next period?*  
(E.g. we know how well abatement technology works by watching neighbor plant trying the technology)
- *We wait till next period and only invest if  $R=100$*

That can be even better!!

## 1 - Learning and Option Value

- Uncertainty resolves at beginning of next period
- We wait till next period and only invest if  $R=100$
- In the next period(!) we then expect the return:

$$\begin{aligned} E -I+D \cdot R &= .5(-I+D \cdot 100)+.5 \cdot 0 \\ &= .5(-60 + 100 D) = -30+50 D. \end{aligned}$$

*From our present perspective next period payoffs have to be discounted!*

*Thus, expected (net present) value of investing in second period if  $R=100$  is*

$$E -D \cdot I+D^2 \cdot R = (-30+50 D)D.$$

Note that

- $I$  became random variable as well
- Random variable  $R$  changed (pays 100 only in case we invest)

Say  $D=.9$  then  $E -D \cdot I+D^2 \cdot R =13.5$

## 1 - Learning and Option Value

*Thus we either have expected return by investing immediately:*

$$E -I + D \cdot R = 75 D - 60.$$

*and with  $D=.9$  a return of 7.5*

*Or we have expected return by waiting until uncertainty resolves  
and only investing if high payoff:*

$$E -D \cdot I + D^2 \cdot R = (-30 + 50 D) D$$

*and with  $D=.9$  a return of 13.5*

Thus if we can only invest once

- do not invest in present period! (despite expected return positive)
- invest in second period if and only if return is high

## 1 - Learning and Option Value

The different in value between executing project immediately:

$$E -I + D \cdot R = 75 D - 60.$$

And the value from waiting until uncertainty resolves:

$$E -D \cdot I + D^2 \cdot R = (-30 + 50 D) D$$

is called an *option value (OV)*.

(note: not the same as what Kolstad calls option value)

Here:  $OV = (-30 + 50 D) D - [75 D - 60] = 60 - 105 D + 50 D^2$

and with  $D = .9$  we find  $OV = 13.5 - 7.5 = 6$

*OV is the value of having the option to wait for uncertainty to resolve.*

Remark: More precisely it should therefore be defined as  $OV^* = \text{Max}\{0, OV\}$

(the option to invest is only exercised if OV is positive)



## 2 – Learning and Optimal Mitigation Level (Preparation)

Superstylized Climate Change Impact Model (*static warm-up*):

- GHG emissions  $x$
- Money measured *benefits* from emissions:  $x - \frac{x^2}{2}$   
(cheaper production/saved abatement costs)
- Money measured *damage* from GHG emissions:  $\alpha x^2$
- Damage parameter  $\alpha$  is uncertain (a random variable)
- Interested in finding optimal emissions  $x$
- Assume risk neutrality:  $U(M)=M$  (RRA=? See problem 3.2)

$$\max_x \quad E \left[ x - \frac{x^2}{2} - \alpha x^2 \right]$$

where  $E$  is expectation with respect to the random variable  $\alpha$

## 2 - Learning and Optimal Mitigation Level

- To proceed need assumption with respect to values and likelihood of  $\alpha$
- Assume  $\alpha$  is either  $0$  or  $1$  with equal probability
  - $p(\alpha=0)=.5$  and
  - $p(\alpha=1)=.5$
- Then

$$\begin{aligned} & \max_x E \left[ x - \frac{x^2}{2} - \alpha x^2 \right] \\ & \Leftrightarrow \max_x .5 \left( x - \frac{x^2}{2} \right) + .5 \left( x - \frac{x^2}{2} - x^2 \right) \\ & \Leftrightarrow \max_x x - \frac{x^2}{2} - \frac{x^2}{2} \\ & \Rightarrow x = \frac{1}{2} \end{aligned}$$

## 2 - Learning and Optimal Mitigation Level

- Note that
  - we neglected the underlying wealth  $M$
  - $M$  does not matter under risk neutrality for deciding on  $x$
- That is because:

$$\max_x \quad E M + x - \frac{x^2}{2} - \alpha x^2$$
$$\Leftrightarrow M + \max_x \quad E x - \frac{x^2}{2} - \alpha x^2$$

- So that  $M$  does not matter for the maximization (drops out in first order condition for maximum)

## Variation: Homework!

- Keep other assumptions, but now assume

- $p(\alpha=0) = \frac{1}{3}$  and

- $p(\alpha=.5) = \frac{2}{3}$

- Solve

$$\max_x E x - \frac{x^2}{2} - \alpha x^2$$

and find whether the optimal GHG emission  $x$  is smaller or larger than before?

## 2 - Learning and Optimal Mitigation Level (Dynamic Model)

Model (*dynamic*):

- Assume
  - two periods, *no discounting*
  - in each period benefits  $x_i - \frac{x_i^2}{2}$  where  $i=1,2$
  - *damage only in second period*
  - damage depends on aggregate emissions in both periods (stock)
$$\alpha (x_1 + x_2)^2$$
  - In period 1  $\alpha$  is unknown and  $p(\alpha=0)=.5$  and  $p(\alpha=1)=.5$
- Distinguish two settings:
  1. Also in period 2  $\alpha$  is unknown (*no learning*)
  2. Between period 1 and period 2 value of  $\alpha$  is revealed (*learning*)

## 2 - Learning and Optimal Mitigation Level

1. No learning:
  - Problem symmetric in  $x_1$  and  $x_2$  so we can max  $x=x_1=x_2$

$$\begin{aligned} & \max_x E \left[ 2 \left( x - \frac{x^2}{2} \right) - \alpha (2x)^2 \right] \\ & \Leftrightarrow \max_x \left( x - \frac{x^2}{2} \right) + \left( x - \frac{x^2}{2} - 2x^2 \right) \\ & \Leftrightarrow \max_x 2x - x^2 - 2x^2 \\ & \Rightarrow x = \frac{1}{3} \end{aligned}$$

- Without learning  $x_1 = x_2 = x = 1/3$

## 2 - Learning and Optimal Mitigation Level (Learning, finally!)

### 1. Learning:

- We *learn* true  $\alpha$  at beginning of period 2 (before we make  $x_2$  decision)
- Moreover, we *anticipate* this learning in period 1
- We consider in first period that we will optimally adapt in second period to  $\alpha$  in a way that can depend on first period emissions

We therefore start reasoning about the

- *Second period:*

$$\max_{x_2} \quad E \left( x_2 - \frac{x_2^2}{2} \right) - \alpha (x_1 + x_2)^2$$

given

- $x_1$  (already chosen in first period)
- $\alpha$  (uncertainty has resolved)

## 2 - Learning and Optimal Mitigation Level

Second period:  $\max_{x_2} \left( x_2 - \frac{x_2^2}{2} \right) - \alpha (x_1 + x_2)^2$  given  $x_1$  and  $\alpha$

- For  $\alpha=0$ :  $\max_{x_2} x_2 - \frac{x_2^2}{2}$   
 $\Rightarrow x_2 = 1$

- For  $\alpha=1$ :  $\max_{x_2} \left( x_2 - \frac{x_2^2}{2} \right) - (x_1 + x_2)^2$   
 $\Leftrightarrow \max_{x_2} x_2 - \frac{x_2^2}{2} - (x_1^2 + 2x_1x_2 + x_2^2)$   
 $\Leftrightarrow \max_{x_2} x_2(1 - 2x_1) - \frac{3}{2}x_2^2 - x_1^2$   
 $\Rightarrow 1 - 2x_1 - 3x_2 = 0 \quad \Rightarrow x_2 = \frac{1}{3} - \frac{2}{3}x_1$



## 2 - Learning and Optimal Mitigation Level

Second period:  $\max_{x_2} \left( x_2 - \frac{x_2^2}{2} \right) - \alpha (x_1 + x_2)^2$  given  $x_1, \alpha$

- For  $\alpha=0$ :  $\max_{x_2} x_2 - \frac{x_2^2}{2}$  and for  $\alpha=1$ :  $\max_{x_2} x_2(1-2x_1) - \frac{3}{2}x_2^2 - x_1^2$   
 $\Rightarrow x_2 = 1$   $\Rightarrow x_2 = \frac{1}{3} - \frac{2}{3}x_1$

- Thus expected value for second period payoffs  $V$  is  
 (that is .5 times  $\alpha=0$  case and .5 times  $\alpha=1$  case substituting in optimal  $x_2$ )

$$\Rightarrow V^{\text{second period}}(x_1) = .5 \cdot \left( 1 - \frac{1}{2} \right) + .5 \cdot \left( \left[ \frac{1}{3} - \frac{2}{3}x_1 \right] (1 - 2x_1) - \frac{3}{2} \left[ \frac{1}{3} - \frac{2}{3}x_1 \right]^2 - x_1^2 \right)$$

## 2 - Learning and Optimal Mitigation Level

- Expected value for second period payoffs is

$$V^{\text{second period}}(x_1) = .5 \cdot \left(1 - \frac{1}{2}\right) + .5 \cdot \left( \left[ \frac{1}{3} - \frac{2}{3}x_1 \right] (1 - 2x_1) - \frac{3}{2} \left[ \frac{1}{3} - \frac{2}{3}x_1 \right]^2 - x_1^2 \right)$$

- Anticipating the above we optimize in first period

$$\max_{x_1} x_1 - \frac{x_1^2}{2} + V^{\text{second period}}(x_1)$$

$$\Leftrightarrow \max_{x_1} x_1 - \frac{x_1^2}{2} + .5 \cdot \left(1 - \frac{1}{2}\right) + .5 \cdot \left( \left[ \frac{1}{3} - \frac{2}{3}x_1 \right] (1 - 2x_1) - \frac{3}{2} \left[ \frac{1}{3} - \frac{2}{3}x_1 \right]^2 - x_1^2 \right)$$

$$\Rightarrow x_1 = \frac{1}{2}$$

CHECK THIS!

- In the first period emits  $x_1 = .5$  as opposed to  $x_1 = 1/3$  without learning!
- In the second period emits  $x_2 = 1$  in case of no damage and  $x_2 = 0$  in case of damage

## 2 - Learning and Optimal Mitigation Level

- Anticipation of learning can increase current emissions
  - Intuition: We can react in second period and reduce emissions in case damage turns out high
- Note that also expected overall emissions go up:
  - No learning:  $E x_1 + x_2 = x_1 + x_2 = 1/3 + 1/3 = 2/3$
  - Anticipated learning:  $E x_1 + x_2 = 1/2 + .5 \cdot 0 + .5 \cdot 1 = 1$

Further remarks:

- Both results do not always hold
  - It can happen that anticipated learning decreases emissions
  - Crucial determinant is the curvature of marginal(!) utility

## Let us use what we learned for discussing...

### The Precautionary Principle

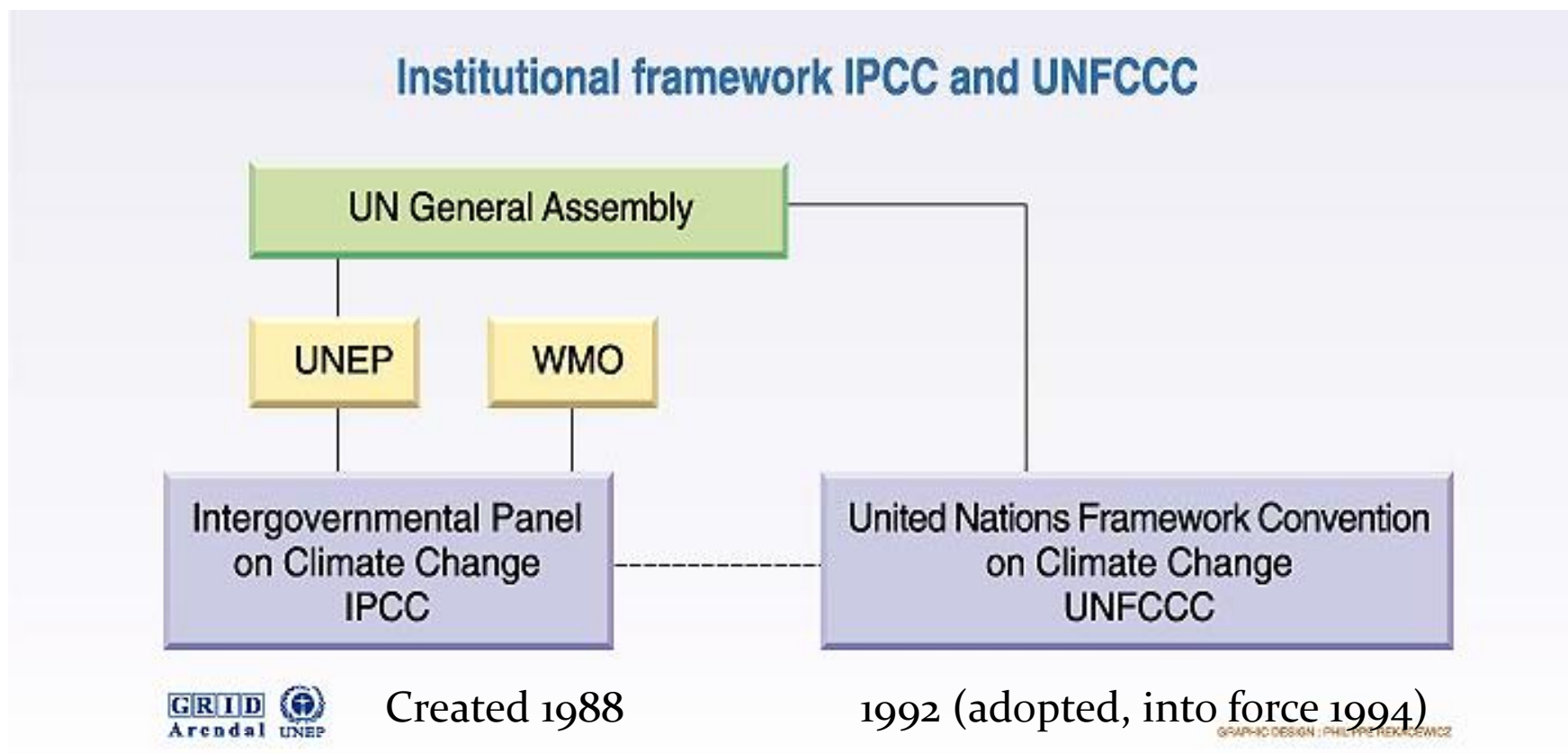
- The ‘common sense’ versions:
  - ‘Better safe than sorry’
  - ‘An ounce of prevention is worth a pound of cure’
- United Nations Framework Convention on Climate Change:

*“Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation.”*

## Aside: What is the UNFCCC

- International Environmental Treaty signed on the 1992 ‘Earth Summit’ (United Nations Conference on Environment and Development - UNCED),
- At the same time name of the secretariat supporting the convention
- *“The Convention on Climate Change sets an overall **framework for intergovernmental efforts to tackle the challenge posed by climate change**. It recognizes that the climate system is a shared resource whose stability can be affected by industrial and other emissions of carbon dioxide and other greenhouse gases. The Convention enjoys near universal membership, with 192 countries having ratified.”*  
([http://unfccc.int/essential\\_background/convention/items/2627.php](http://unfccc.int/essential_background/convention/items/2627.php))
- Organizes conferences of the parties (‘COP’s’)
  - COP-3: 1997 in Kyoto -> adopted the Kyoto protocol
  - COP-15: This December in Copenhagen -> hopes for post-Kyoto

## Aside: UNFCCC and IPCC



Source: United Nations framework convention on climate change (UNFCCC).

UNEP=United Nations Environment Programme (Nairobi)

WMO=World Meteorological Organization (Geneva)

IPCC: Established to provide objective source of information about climate change

## The Precautionary Principle

Stronger formulation than in UNFCCC:

- Third North Sea Conference (1990), particularly strong version:  
*“...apply the precautionary principle, that is to take action to **avoid potentially damaging impacts** of substances that are persistent, toxic, and liable to bioaccumulate **even where there is no scientific evidence to prove a causal link between emissions and effects...**”*

Importance:

- III-233 of draft Treaty establishing a constitution for Europe stipulates:  
*“Union policy on the environment [...] shall be based on the precautionary principle and on the principles that preventive action should be taken...”*
- Hahn & Sunstein (2005) predict that “over the coming decades, the increasingly popular ‘precautionary principle’ is likely to have a significant impact on policies all over the world”.

## Precautionary Principle

- Floor open for discussion!!

If you want to read over the statements again...

- Third North Sea Conference (1990):  
*“...apply the precautionary principle, that is to take action to avoid potentially damaging impacts of substances that are persistent, toxic, and liable to bioaccumulate even where there is no scientific evidence to prove a causal link between emissions and effects...”*
- United Nations Framework Convention on Climate Change (1992):  
*“Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation.”*

Remark: Take notes and contemplate once more at home! !!!



## Uncertainty & the Precautionary Principle

Relating what you learned on decision making under uncertainty to

### The Precautionary Principle

- United Nations Framework Convention on Climate Change (1992):  
*“Where there are **threats** of serious or irreversible damage, **lack of full scientific certainty shall not be used** as a reason for **postponing cost-effective measures to prevent** environmental degradation.”*
- Third North Sea Conference (1990):  
*“...apply the precautionary principle, that is to **take action** to avoid **potentially damaging** impacts of substances that are persistent, toxic, and liable to bioaccumulate even where there is **no scientific evidence** to prove a causal link between emissions and effects...”*

## Uncertainty & the Precautionary Principle

What's characteristic of the situation?

- Intertemporal (costly action today, possible payoff tomorrow)
- Uncertainty (uncertain damage)

What did you learn to do in order to decide upon taking action?

In the case of risk:

- Assess possible costs and benefits
- Discount future payoffs and assess Net Present Value
- Take expectations with respect to uncertainty and consider whether you can postpone action and let uncertainty resolve
- Adjust current decisions for anticipating that you can react to resolved uncertainty in the future (we: mitigation levels)

## Uncertainty & the Precautionary Principle

Back to the Precautionary Principle:

- Strong version:

*“... **take action** to avoid **potentially damaging** impacts of substances that are persistent, toxic, and liable to bioaccumulate even where there is **no scientific evidence** to prove a causal link between emissions and effects...”*

- No quantification of possible damage
- No reasoning on future vs. present trade-off (probably to be interpreted as no discounting)
- No expectations
- No space for learning before acting

## Uncertainty & the Precautionary Principle

Back to the Precautionary Principle:

- United Nations Framework Convention on Climate Change:

*“Where there are **threats** of serious or irreversible damage, **lack of full scientific certainty shall not be used** as a reason for **postponing cost-effective measures to prevent** environmental degradation.”*

- Cost-effectiveness suggests
  - some quantitative assessment of costs and damages
  - Intertemporal aggregation/comparison of costs and damages

Related Question: What's discount rate?

- to take expectations

Related Question: What's the risk aversion?

## Uncertainty & the Precautionary Principle

Back to the Precautionary Principle (PP):

- United Nations Framework Convention on Climate Change:

*“Where there are **threats** of serious or irreversible damage, **lack of full scientific certainty shall not be used** as a reason for **postponing cost-effective measures to prevent environmental degradation.**”*

- Lack of knowledge & not postponing
    - We know that wait & learn can(!) raise expected payoffs (Option Value)
    - Knowing that we learn can make us mitigate less today
- > Can go against the precautionary principle as not postponing action

Remark: Wait & learn can also mean to preserve environment today, but here PP is about not postponing measures for environment

## Uncertainty & the Precautionary Principle

PP as a political statement and tool:

- Precautionary Principle as an answer to  
*“Cause or effect are not certain -> we do not until complete knowledge”*  
-> An argument that is deprived of a scientific foundation but is still encountered way too frequently  
(instead: Acknowledge what can happen, add probabilities...)
- Precautionary Principle is ‘not symmetric’ in application:  
It asks for a high willingness to give up consumption today to protect the environment
  - That can be because of a particular vulnerability of environmental goods, e.g. because damages can be irreversible
  - That can also be a simple preference for the environment dressed as a principle

## The Precautionary Principle

- Hahn & Sunstein (2005)  
“the precautionary principle **does not help individuals or nations make difficult choices in a non-arbitrary** way. Taken seriously, it can be **paralyzing**, providing no direction at all”. The authors continue that  
“In contrast, balancing costs against benefits can offer the foundation of a principled approach for making difficult decisions”.

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“In contrast, balancing costs against benefits can offer the foundation of a principled approach for making difficult decisions”.

I'd say

- Also standard cost benefit analysis does not acknowledge:
  - Learning
  - Knightian Uncertainty and unforeseen contingencies
- ‘Principled approach’: Yes! But be aware that not always easy...



## Precautionary Principle

Final Remark (present research):

- Recall that curvature of utility...  
(You calculated consumption elasticity of marginal utility  $\theta$  as a measure)
  - ...in certainty context specifies preference for equality (or ‘smoothness’) of consumption over time (discounting discussion)
  - ... in uncertain context specifies risk aversion

## Precautionary Principle

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- Using the standard utility model for intertemporal uncertainty where

$$W(x_1, x_2) = E U(x_1) + DU(x_2)$$

is overall welfare -> U describes both at the same time

## Precautionary Principle

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is overall welfare -> U describes both at the same time

- One can show that preferences generally don't coincide
  - > Often people more averse to risk than to fluctuations over time
  - > Then more willing to invest today into a risk reduction tomorrow (than in the standard model also used for climate change evaluation)