Willingness to Pay for a Risk Reduction

Back to Risk

We will mostly treat the category of risk/likelihood/probabilities:

- Easiest to capture mathematically
- Sufficiently profound to derive interesting insights

Willingness to Pay for a Risk Reduction

Model:

- Two possible outcomes
- Probability of damage (e.g. Great Barrier reef dead by 2050)
  - $D=d=5$ with probability $p=\Pr(D=5)=.5$
  - $D=0$ with probability $(1-p)=\Pr(D=0)=.5$
  - D is random variable and d is magnitude of possible damage (in $)
- Baseline consumption $M=9$

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Expected utility of 'lottery'

$$\mathrm{E}\,U(M-D)=(1-p)U(M)+pU(M-d)$$

Example of Risk neutrality where U(M)=M:

$$\mathrm{E}\,M-D=\frac{1}{2}9+\frac{1}{2}4=\frac{13}{2}$$

Willingness to Pay for a Risk Reduction

Utility of 'lottery' $$E_p U(M-D) = (1-p)U(M) + pU(M-d)$$

SMALL Change in risk: ***p-> p*=p+Δp***

$$E_{p*} U(M-D) = (1-p*)U(M) + p*U(M-d)$$
$$= (1-[p+\Delta p])U(M) + [p+\Delta p]U(M-d)$$
$$= E_p U(M-D) - \Delta p\left[U(M) - U(M-d)\right]$$

So the change ***Δp*** causes a welfare change:

$$\Delta U^{\Delta p} = E_{p*} U(M-D) - E_p U(M-D) = -\Delta p\left[U(M) - U(M-d)\right]$$

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Utility of 'lottery' $\mathrm{E}_p\, U(M-D) = (1-p)U(M) + pU(M-d)$

Similarly :

SMALL Change in baseline consumption/money: ***M*->*M***=*M*+ *ΔM***

$$\begin{aligned}\mathrm{E}_p\, U(M^*-D) &= (1-p)\,U(M^*) + p\,U(M^*-d) \\ &= (1-p)\,U(M+\Delta M) + p\,U(M+\Delta M-d) \\ &\approx \mathrm{E}_p\, U(M-D) + \mathrm{E}_p\, U'(M-D)\,\Delta M\end{aligned}$$

With ≈ step intuition: small amount of consumption change ***ΔM*** times expected marginal utility derived from ***ΔM***

So the change ***ΔM*** causes a welfare change:

$$\Delta U^{\Delta M} = \mathrm{E}_p\, U(M^*-D) - \mathrm{E}_p\, U(M-D) \approx \mathrm{E}_p\, U'(M-D)\,\Delta M$$

Willingness to Pay for a Risk Reduction

Question: How much ***ΔM*** willing to spend (at most) to reduce risk by ***Δp***?

Answer:

- Welfare change $\Delta U^{\Delta M}$ caused by ***ΔM*** together with
- welfare change $\Delta U^{\Delta p}$ caused by ***Δp***
- should leave agent indifferent to no change

Hence:

$$\Delta U^{\Delta p} + \Delta U^{\Delta M} = 0$$

$$\Leftrightarrow \quad \Delta p \left[U(M) - U(M-d) \right] \approx \mathrm{E}_p\, U'(M-D)\, \Delta M$$

From which follows:

$$\frac{\Delta M}{\Delta p} \approx \frac{U(M) - U(M-d)}{\mathrm{E}_p\, U'(M-D)}$$

Willingness to Pay for a Risk Reduction

$$\frac{\Delta M}{\Delta p} \approx \frac{U(M) - U(M-d)}{\mathrm{E}_p\, U'(M-D)}$$

Interpretation:

The willingness to pay for a risk reduction

- Increases in the utility loss caused by the damage
- Decreases in the expected value of money (which agent has to give up)

Willingness to Pay for a Risk Reduction: Example 1

$$\frac{\Delta M}{\Delta p} \approx \frac{U(M) - U(M-d)}{\mathrm{E}_p\, U'(M-D)}$$

Risk neutral agent ***U(M)=M***:

$$\frac{\Delta M}{\Delta p} \approx \frac{M - M + d}{\mathrm{E}_p\, 1} = d = 5$$

Willingness to Pay for a Risk Reduction: Example 2

$$\frac{\Delta M}{\Delta p} \approx \frac{U(M) - U(M-d)}{\mathrm{E}_p\, U'(M-D)}$$

Risk neutral agent ***U(M)=M***:

$$\frac{\Delta M}{\Delta p} \approx \frac{M - M + d}{\mathrm{E}_p\, 1} = d = 5$$

Risk averse agent ***U(M)***= $M^{\frac{1}{2}}$:

$$\frac{\Delta M}{\Delta p} \approx \frac{\sqrt{M} - \sqrt{M-d}}{\mathrm{E}_p\, \frac{1}{2}(M+D)^{-\frac{1}{2}}} = \frac{\sqrt{9} - \sqrt{4}}{\frac{1}{2}\frac{1}{2}\frac{1}{\sqrt{9}} + \frac{1}{2}\frac{1}{2}\frac{1}{\sqrt{4}}} = \frac{1}{\frac{2+3}{24}} = \frac{24}{5} < 5$$

Willingness to Pay for a Risk Reduction: Homework

$$\frac{\Delta M}{\Delta p} \approx \frac{U(M) - U(M-d)}{\mathrm{E}_p\, U'(M-D)}$$

Assume ***p=Pr(D=5)=.1***

Risk neutral agent ***U(M)=M***:

$$\frac{\Delta M}{\Delta p} \approx$$

Risk averse agent ***U(M)***= $M^{\frac{1}{2}}$:

$$\frac{\Delta M}{\Delta p} \approx$$

Willingness to Pay for a Risk Reduction: Homework

$$\frac{\Delta M}{\Delta p} \approx \frac{U(M)-U(M-d)}{E_p\, U'(M-D)}$$

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$$\frac{\Delta M}{\Delta p} \approx \frac{\sqrt{M}-\sqrt{M-d}}{E_p \frac{1}{2}(M+D)^{-\frac{1}{2}}} = \frac{\sqrt{9}-\sqrt{4}}{\frac{9}{10}\frac{1}{2}\frac{1}{\sqrt{9}}+\frac{1}{10}\frac{1}{2}\frac{1}{\sqrt{4}}} = \frac{1}{\frac{18+3}{120}} = \frac{120}{21} \approx 5.7$$

Intuition Risk Neutral vs. Risk Averse

$$\frac{\Delta M}{\Delta p} \approx \frac{U(M) - U(M-d)}{\mathrm{E}_p\, U'(M-D)}$$

Without loss of generality: (if you wonder why that is 'without loss…' -> office hour)

- Assume that U(M) and U(M-d) coincide for risk averse and risk neutral agent

Then denominator decides:

- Risk neutral agent has same *marginal* utility at ***M*** and ***M-d***

Intuition Risk Neutral vs. Risk Averse

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- Assume that U(M) and U(M-d) coincide for risk averse and risk neutral agent

Then denominator decides:

- Risk neutral agent has same *marginal* utility at ***M*** and ***M-d***
- Risk averse agent has
  - *Higher* marginal utility at ***M-d*** than risk neutral agent
  - *Lower* marginal utility at ***M*** than risk neutral agent

-> *More weight on high* outcome (***M***) *decreases expected marginal utility* that risk averse agent derives from the dollar he has to give up to reduce ***p***

-> *More weight on low* outcome (***M-d***) *increases expected marginal utility* that risk averse agent derives from the dollar he has to give up to reduce ***p***

Intuition Risk Neutral vs. Risk Averse

Risk averse decision maker relative to risk neutral decision maker:

In the ***bad state (M-d)*** *the risk averse decision maker is* ***hurt relatively more*** *by the money he gives up for the risk reduction*

(which makes him even worse off than in the former bad state)

In the ***good state (M)*** *the risk averse decision maker is* ***hurt relatively less*** *by the money he gives up for the risk reduction*

(concave utility -> prefers less in good state in order to avoid bad state)

It depends on

- probability weight of good vs. bad state
- magnitude of marginal utility difference in good vs. bad state with respect to risk neutral agent

whether risk averse agent is willing to pay more or less for risk reduction