The economics of climate change

C 175 - Christian Traeger

Part 5: Risk and Uncertainty

Background reading in our textbooks (very short):

Kolstad, Charles D. (2000), "Environmental Economics", Oxford University Press, New York. Chapter 12.

Varian, Hal R. (any edition…), "Intermediate Microeconomics – a modern approach", W. W. Norton & Company, New York. Edition 6: Chapter 12.

Papers on the topic will be announced at a later point.

Uncertainty

- Where do encounter Uncertainties?
- In every day life?
- In climate change?

Risk, Expected Value, Risk Aversion

Probabilities

- What's a probability?

- 'Something' that
  - gives a weight to events which
  - expresses how likely the event is to occur.
- It also satisfies that the
  - weights (or likelihood) of two events that cannot occur together (disjoint events, e.g. global mean temperature rises by 3°C or by 4°C)
  - add up
    (->Blackboard)
- And it is normalized so that weight *1* means something happens with certainty

Probability

A probability can be *objective* and be derived from

- statistical information (e.g. probability of dying from smoking)
- symmetry reasoning (coin has two sides, dice has six)

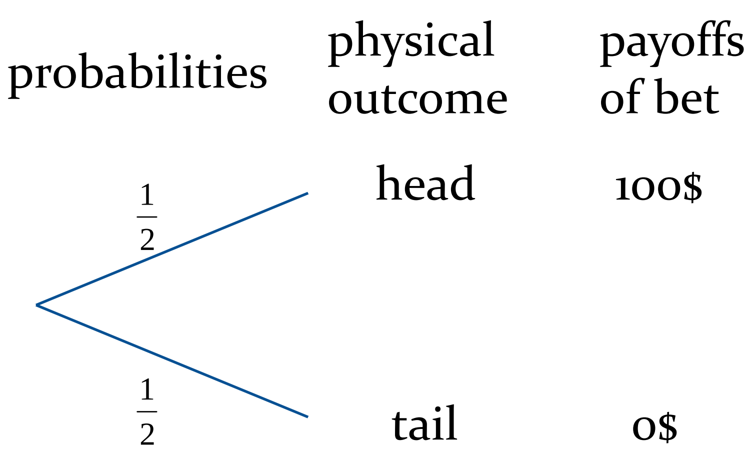
And a probability can be *subjective* and

- express an individual belief,
- there can be different subjective probabilities for the same event

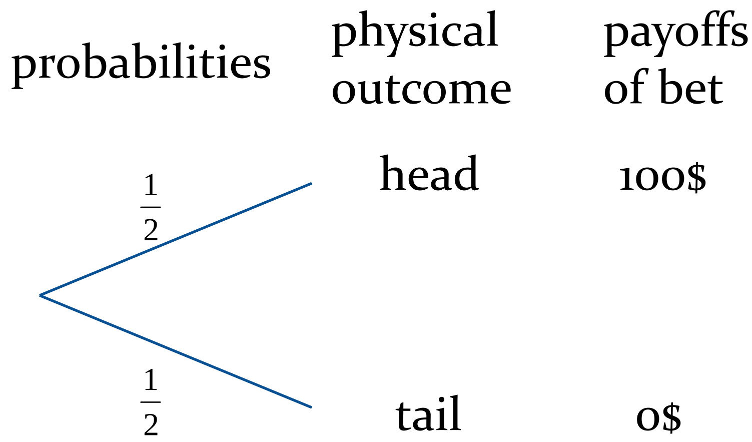
Whether a probability is objective or subjective does not matter for the math!

Risk and Probabilities

- Take a coin toss, bet 100$ on head
- Representation form of a probability tree:



Risk and Probabilities



- Define a variable for the possible payoffs R ("R" for return):

  $R \in \{0{,}100\}$ meaning R being either $R_1 = 0$ *or* $R_2 = 100$

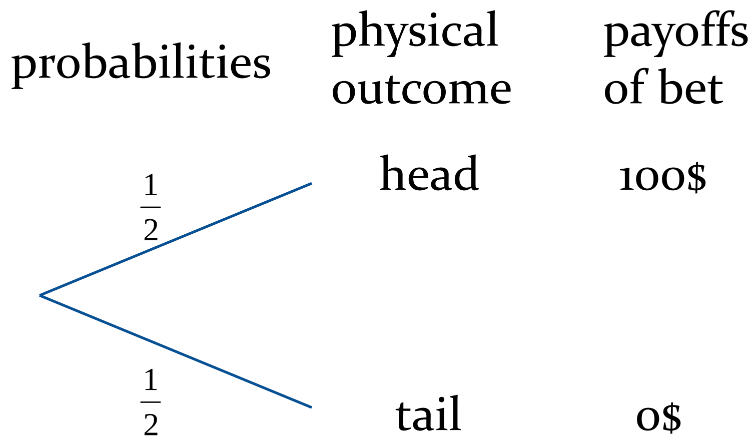
  with probabilities

  $$p_1 = p(R_1) = p(R = 0) = \frac{1}{2} \quad \text{and}$$

  $$p_2 = p(R_2) = p(R = 100) = \frac{1}{2}$$

- R is called a *random variable*

Risk and Probabilities



- Random variable ***R*** with $p(R=0)=\frac{1}{2}$ and $p(R=100)=\frac{1}{2}$
- Expected payoff of bet = *expected value* of the random variable ***R***:

$$\mathrm{E}\ R=\sum_{i=1}^{2}p_i\,R_i=\frac{1}{2}\times 0+\frac{1}{2}\times 100=50$$

E is the *expectation operator* and stands for the probability weighted sum

Utility of a Lottery

- However:
  - Payoffs yield utility
  - Utility decides about choices

- In general receiving
  - an expected payoff of 50$ or
  - a certain payoff of 50$

  is not the same to us.

- A lottery with expected 50$ payoff generally doesn't give same utility as a certain 50$ payment

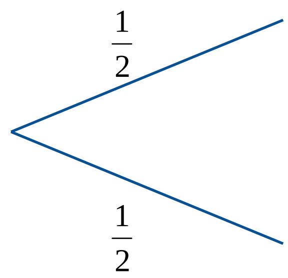
Risk/Lotteries and Utility

- Take a coin toss, bet 100$ on head
- Representation in a probability tree:

probabilities	physical outcome	bet	utility
$\frac{1}{2}$	head	100$	U(M+100)
$\frac{1}{2}$	tail	0$	U(M+0)

- Where M is wealth/monetary value of other consumption.

Expected Utility



probabilities	physical outcome	payoffs of bet	utility
$\frac{1}{2}$	head	100$	U(M+100)
$\frac{1}{2}$	tail	0$	U(M+0)

- Random variable ***R*** with $p(R=0)=\frac{1}{2}$ and $p(R=100)=\frac{1}{2}$
- Say ***U(M)*=ln *M*** and ***M*=1000**
- *Expected utility*:

$$\mathrm{E}\,U(M+R)=\sum_{i=1}^{2} p_i\,U(M+R_i)=\frac{1}{2}\times U(1000)+\frac{1}{2}\times U(1100)$$

$$=\frac{1}{2}\ln 1000+\frac{1}{2}\ln 1100=3.453+3.502=6.955$$

Expected Utility

- In general the *expected utility of a random variable*, here R, is lower than the *utility of the expected value of the random variable*.

- That is because the utility function is concave!
  -> Blackboard

Expected Utility

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Here:

$$\mathrm{E}\,U(M+R)=\frac{1}{2}\times U(1000)+\frac{1}{2}\times U(1100)=6.955 \quad < \quad 6.957=U(1150)$$

Certainty Equivalent

- The certain payment that leaves the agent indifferent to lottery is called *Certainty Equivalent* ***CE :***

$$\mathrm{E}\,U(M+R)=U(M+CE)$$

- Here $\mathrm{E}\,U(M+R)=6.955 \quad = \quad \ln(1049)=U(1049)=U(M+49)$

- So ***CE=49***

- The agent is indifferent between the random variable R
(i.e. the lottery that gives 0$ or 100$ with equal probability)
and the certain payment of 49$

Risk Premium

- The difference between the expected payoff of the lottery and the certainty equivalent payment is called the

  Risk Premium $\boldsymbol{\pi}$: $$\pi = \mathrm{E}\,R - CE$$

  or equivalently $$\mathrm{E}\,U(M+R) = U(M+\mathrm{E}R-\pi)$$

  (***R*** is random and $\boldsymbol{\pi}$ and **E*R*** are certain)

- Here: $\boldsymbol{\pi=50-49=1}$

- or $$\mathrm{E}\,U(M+R) = 6.955 = U(1049) = U(M+50-1)$$

Risk Premium

- Note that the risk premium is small because lottery is relatively small as opposed to baseline consumption:
- Let M=100$:
- Then $\mathrm{E}\,U(M+R)=\frac{1}{2}\ln 100+\frac{1}{2}\ln 200=4.95=\ln 141=U(M+50-9)$
- and certainty equivalent is $CE=41$
- and risk premium is $\pi=9$

Risk Aversion

- A positive risk premium means a decision maker is willing to pay for eliminating the risk
- Such a decision maker is risk averse

- We saw that risk premium is positive if utility is concave
- The 'Arrow-Pratt measure of relative risk aversion'

$$RRA = -\frac{U''(M)}{U'(M)} M$$

measures the concavity of ***U(x)*** and, thus, the degree of risk aversion.

- Here we defined utility on money representing aggregate consumption
- If utility immediately over good: same with ***x*** instead of ***M***
- For U(M)=ln (M) -> see problem 3.2