Determinants of Social Discount Rate, general case

- The resulting equation

  $$r = \rho + \theta\, g$$

  is known as the "Ramsey equation" after Frank Ramsey (1928)
- The equation states that in an optimal intertemporal allocation:
- the productivity of capital (interest rate) = the return on investment is the sum of
  - The rate of pure time preference (describing impatience)
  - And the product of
    - the consumption elasticity of marginal utility θ (describing how fast marginal consumption decreases in consumption)
    - the growth rate g (describing how fast consumption increases)

The Economics of Climate Change

C 175 - Christian Traeger

Part 4: Discounting

continued

Lecture 17

Background still Cameron Hepburn's (2006), "Discounting climate change damages: Working note for the Stern review".

Review

Problem:

- How to compare costs and benefits that occur at different points in time?

Economic solution concept involves:

- **Discounting:** Describes the valuation in present day terms of future outcomes (damages, costs, benefits, utility values)
- **Cost Benefit analysis:**
  - Assess costs and benefits in monetary units
  - Express all benefits and costs in present value terms
- **Net present value *NPV*:**
  Support a project if present value benefits exceed present value costs

$$NPV = \sum_{t=0}^{T} \frac{B_t - C_t}{(1+r)^t}$$

Answer to Homework

Example II Cost Benefit analysis:

- Consider the two modified projects and a discount rate of 5%.

	Benefits (in $)			
Year	0	1	2	3
Project A	-30	20	10	10
Project B	-30	10	20	20

Assume you only have $30 that you can invest in the first period.

Which project would you invest in?

- NPV_A=-\$30+\$20/(1,05) +\$10/(1,05)^2 +\$10/(1,05)^3 = ***\$6.76***
- NPV_B =-\$30+\$10/(1,05) +\$20/(1,05)^2 +\$20/(1,05)^3 = ***\$14.94***

Project B because $NPV_B > NPV_A$

Review: How to find the discount rate?

- Recall importance of discounting for long time horizons: At a
  - 10% discount rate $ 1 Mio in 150 years have present value of
  - 1% discount rate $ 1 Mio in 150 years have present value of $ 225 000
- High discount rate implies
  - A dollar today is much more valuable than a dollar tomorrow
  - Hard to justify climate policy where costs occur today but benefits (abated damages) accrue later
- How do we find the "correct" discount rate?
  - *Market Intertest Ra*te -> BUT: might not exist for long time horizons or not represent "correct" information (market failures, responsibilities for future generations)
  - Identify components of the *Social Discount Rate*

Review: The Social Discount Rate

We have derived how the optimal real interest rate should be composed if markets were perfect and represented all information:

- Then: MRS = MRT
  (Marginal rate of substitution = Marginal rate of transformation)
- Where:
  - Good 1 = consumption today
  - Good 2 = consumption tomorrow
- Result is the Ramsey equation: $\boldsymbol{r = \rho + \theta\, g}$
- We derived the equation for a two period setting, but holds in general, also in continuous time:
  $\boldsymbol{r(t) = \rho + \theta(x(t))\ g(t)}$

Remark: $\boldsymbol{\theta(x(t))}$ is determined by the utility function. If utility is a power function (i.e. $\boldsymbol{x}^{\alpha}$) then $\boldsymbol{\theta = \alpha}$ independent of $\boldsymbol{x(t)}$.

Review: Determinants of Social Discount Rate

- Ramsey equation: $r(t) = \rho + \theta\, g(t)$

  Optimal productivity of capital (***r***) equals
  - The rate of pure time preference (***ρ***) (describing impatience)
  - And the product of
    - the consumption elasticity of marginal utility ***θ***
      (describing how fast marginal utility decreases in consumption)

$$\theta = -\frac{U''(X)X}{U'(X)} = -\frac{\frac{dU'(X)}{U'(X)}}{\frac{dX}{X}}$$

Relative change of marginal utility under a relative change of consumption.

or

By how many percent does marginal utility change if consumption increases by one percent

- the growth rate ***g***
  (describing how fast consumption increases)

So what's the rate? - Normative vs Descriptive

- Two approaches
  - *Descriptive*:
    Observe $r(t)$ and $g(t)$. Either in market or in experiments. Different combinations of ρ and θ are generally compatible with these observations.
    Determining θ by observation is harder but would identify corresponding ρ.
  - *Prescriptive*:
    Start with deciding on ρ and θ. Then growth rate $g(t)$ determines the corresponding real interest rate $r(t)$. Arguments for picking ρ and θ generally rely on ethical reasoning.

So what's the rate? - Pure Time Preference

- **ρ:** Some opinions that it should be zero:
  - Ramsey (1928) describes placing different weights upon the utility of different generations, as '*ethically indefensible*'.
  - Harrod (1948) stated that discounting utility represented a '*polite expression for rapacity and the conquest of reason by passion*'.
  - …
  - Stern Review (2007) – to be encountered in integrated assessment part

  -> These are mostly ethical arguments and thus prescriptive
- Remark:
  - Ramsey equation including (undetermined!) parameter ρ can be derived from axioms, i.e. assumptions on behavior rather than starting with our utility model
  - Changing the axioms to include risk attitude in a more comprehensive way can eliminate ρ, that is, make it zero starting from assumptions on rational and consistent behavior rather than for ethical reasons (paper on my homepage)

So what's the rate? - Descriptive

- Descriptive take on ρ: By reverse engineering from observations economists often take values between *2%* and *3%*.
- Similarly θ is engineered to be between 1 and 4.
  Remark:
  The high (& even higher) values of θ are obtained from risk studies where θ also represents risk aversion rather than just reduction in marginal utility from growth over time.
- The growth rate g is observed and predicted to be approximately in the range *1%* to *3%*
- The real interest rate employed as observed rate ranges approximately from **2%** to **7%**

A convenient example of somewhat reasonable values (Weitzman 2007)

ρ=2%, θ=2, g=2%, r=6%

Other examples - Stern review (2007), Nordhaus (2007) – on problem set.

So what's the rate? - Prescriptive

- ρ: possibly zero for ethical reasons as proposed on earlier slide
- θ: Normative/ethical information conveyed:

Equalizing consumption across generations

  - If $\theta = 0$, then more or less consumption in the future does not change willingness to invest (r independent of growth rate g)
  - If θ is large, then with *positive growth -> not willing* to invest in future (future generations have more anyhow)
  - If θ is large, then with *negative growth -> very willing* to invest in future (future generations are poorer than today's)
  - Proposed range for θ ranges from *1* to *4*

Remark: Willing to invest in future = low real interest rate in optimal allocation.

Distributive Justice

- Excursion: Rawls theory of justice applied to intertemporal distribution
  - Would set $\boldsymbol{\rho}$ to zero and increase $\boldsymbol{\theta}$ to ∞

- More generally, an egalitarian perspective with respect to
  - **time** yields $\boldsymbol{\rho}$ small and thus $\boldsymbol{r}$ small
  - **distribution across generations** (stripping off the time dimension) yields $\boldsymbol{\theta}$ large and thus $\boldsymbol{r}$ large if positive growth

Sustainability

is another normative value theory that relates to the discount rate

- Definitions:
  - "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (WCED 1987).
  - Common denominator of sustainability theories is the acknowledgment of the "*long-run mutual dependence of environmental quality and resource availability* on the one hand, *and economic development* on the other hand". Van den Bergh & Hofkes (1998, 11)
- One distinguishes:
  - **weak sustainability**: preservation of a *non-decreasing overall welfare*. To this end, a *substitution between environmental and man-made capital is permitted*.
  - **strong sustainability**: requires a *non-declining value or physical amount of natural capital and its service flows. Substitution* possibilities between man-made goods and natural resources and service flows are either *limited or ethically indefensible*.

Sustainability – A Formalization

- To formalize this idea, we need to model
  - environmental goods $\boldsymbol{x^E}$ AND
  - produced goods $\boldsymbol{x^P}$ AND
  - the fact that both goods are not perfect substitutes
- Assume that substitutability between environmental goods and produced goods is of Cobb Douglas form.
- Assume that the decrease of marginal utility in overall consumption is is parameterized by a power function (parameter $\boldsymbol{\theta}$, see problem 3.3)

$$W = U(x_1^E, x_1^P) + \frac{1}{1+\rho} U(x_2^E, x_2^P)$$

$$= \frac{\left[(x_0^E)^{.5}(x_0^P)^{.5}\right]^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{\left[(x_1^E)^{.5}(x_1^P)^{.5}\right]^{1-\theta}}{1-\theta}$$

Sustainability & Discounting

- In general growth rates of environmental goods g^E lie below the growth rates of produced goods g^P : $g^E < g^P$
- Now we can derive two discount rates:
  - A discount rate for environmental goods r^E and
  - A discount rate for produced goods r^P
- A similar but slightly more complicated derivation as earlier yields:
  - $r^E = \rho + \theta\,(.5\, g^E + .5\, g^P) - (.5\, g^P - .5\, g^E)$
  - $r^P = \rho + \theta\,(.5\, g^E + .5\, g^P) + (.5\, g^P - .5\, g^E)$
- For coinciding growth rates $g^E = g^P = g$ we have same formulas as earlier
- For $g^E < g^P$ however, we find that environmental goods have to be discounted at a lower than average rate and produced goods have to be discounted at a higher than average rate.

Sustainability & Discounting

- $r^E = \rho + \theta\,(.5\,g^E + .5\,g^P) - (.5\,g^P - .5\,g^E)$
- $r^P = \rho + \theta\,(.5\,g^E + .5\,g^P) + (.5\,g^P - .5\,g^E)$

- For $g^E < g^P$ however:
  - environmental goods have to be discounted at a lower than average rate
  - produced goods have to be discounted at a higher than average rate
- In general it can be shown that
  - The more substitutable both classes of goods, the smaller the reduction of r^E and the smaller the increase in r^P
  - The less substitutable both classes of goods, the larger the reduction of r^E and the larger the increase in r^P

Remark: The reasoning in terms of lower discount rates for environmental goods is equivalent to arguing in an aggregate one commodity model that due to increasing relative scarcity the monetary value of environmental goods increases in the future.

Hyperbolic Discounting I

- It has been suggested from a normative perspective:
- to discount the close future at higher/observed rates
- to discount the far future at relatively lower rates
- Analogy:
  - I care more for myself than for someone close (e.g. my close relatives).
  - I care more for someone close than for someone I hardly know (e.g. more for my close relatives than for my far relatives).
  - But I really don't make a big difference between hardly knowing and not knowing (or I really don't care whether it's my cousin of degree 100 or 101).
- Formally (Version 1): $W = \sum_{t=0}^{T} \frac{1}{(1+\rho^a{}_t)^t} U(x_t)$ where $\boldsymbol{\rho^a{}_{t+1} < \rho^a{}_t}$

Here $\boldsymbol{\rho^a{}_t}$ is the *average discount rate* between $\boldsymbol{t}$ and the present

Hyperbolic Discounting II

- Alternatively (Version 2):
- Can express the welfare evaluation in terms of *per period discount rates.*
- Here $\boldsymbol{\rho_t}$ is the rate that discounts period $\boldsymbol{t}$ welfare into period $\boldsymbol{t-1}$ values (also called *instantaneous discount rate*).
- The corresponding welfare function writes as

$$W = \sum_{t=0}^{T} \frac{1}{\prod_{\tau=0}^{t}(1+\rho_\tau)} U(x_t)$$

$$= U(x_0) + \frac{1}{(1+\rho_1)} U(x_1) + \frac{1}{(1+\rho_1)(1+\rho_2)} U(x_2) + \ldots + \frac{1}{(1+\rho_1)\ldots(1+\rho_T)} U(x_T)$$

- again with $\boldsymbol{\rho_{t+1} < \rho_t}$

Hyperbolic Discounting III

- Both forms are equivalent for modeling hyperbolic discounting
- The second form looks more complicated, but turns out to be preferable in many occasions.
- A similar discounting problem of form 2 appears in problem 3.2, however, in terms of real interest discounting consumption rather than pure time preference discounting utility.
- In that problem you will learn that hyperbolic (=falling) discount rates can make a project worth investing in the present not worth inversing anymore in the future.
- This dependence of the worthiness of a project on the period in which it is evaluated is referred to as time inconsistency.
- Time inconsistencies can lead to a continuous reversion of a (thought to be optimal) project and lead to dynamic inefficiencies.

Heterogeneity

So far we have not looked at distribution and heterogeneity!

- Distribution: -> see problem 3.3 and Integrated Assessment section

- Example of heterogeneity:
  - What happens if individuals have different rates of pure time preference?
  - Aggregating individual utility functions yields:
  - The further into the future, the more important become the smaller time preferences in the aggregate welfare function (yielding hyperbolic discounting)
  - In the long-run limit only the lowest discount rate counts

So what's the rate? - Asking the experts:

- Weitzman (2001) surveyed 2160 economists asking:

  "Taking all relevant considerations into account, what real interest rate do you think should be used to discount over time the (expected) benefits and (expected) costs of projects being proposed to mitigate the possible effects of global climate change?"

So what's the rate? - Asking the experts:

- The result:

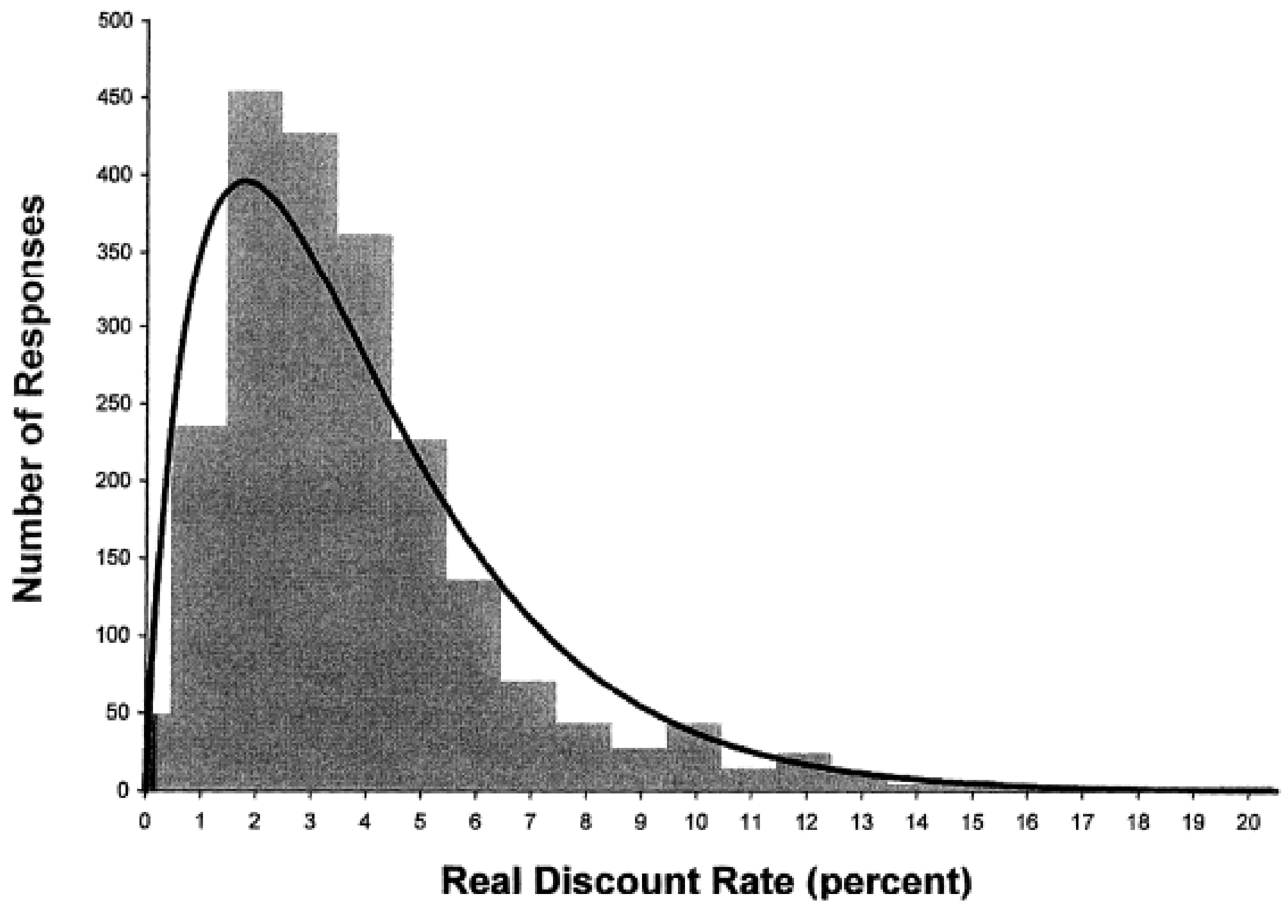

FIGURE 1. ACTUAL (HISTOGRAM) AND FITTED (CURVE) FREQUENCY DISTIRBUTIONS

Final Remarks

- Note that discount rates changes, if growth rate changes.
  - In the extreme: discount rates could be negative if negative growth.
- The "correct" discount rate has not yet been agreed upon
- The parameters $\boldsymbol{\rho}$ and $\boldsymbol{\theta}$ can be associated with *individual trade-offs* over time as well as with *intergenerational trade-offs*.
  - Building a model that depicts both as *different parameters* is likely to help in the dispute between the normative and the descriptive interpretations of $\boldsymbol{\rho}$ and $\boldsymbol{\theta}$
  - Related thought:
    - Assume we give less weight to generations born at a later date.
    - Those generations overlap with generations born earlier.
    - Thus, we effectively discriminate between welfare of different generations alive at the same time. Ethically correct?