L Karp and C Traeger

Course Outline

ARE 263, Spring 2016

Methods of Dynamic Analysis and Control

- with Applications to Climate Change & Environmental Economics -

Tu Th 11-12:30, 201 Giannini Hall

Instructor Information:
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Prerequisites: Basic background in graduate economics. Contact us for your individual case.

Course Description: Most economic and environmental trade-offs interact over time and today's policies and decisions require dynamic optimization. We introduce some of the common methods of discrete and continuous time dynamic optimization (discrete Euler equation; discrete & continuous time dynamic programming; Pontryagin's maximum principle). Applications are drawn from climate change and resource economics with some connections to macroeconomics and asset pricing. For numeric applications we use Matlab and we encourage (but do not require) students to use Matlab, Julia, or Python. Some very basic knowledge in programming is helpful but not required.

Course Requirements: (i) Half a dozen problem sets and (ii) a final exam. Expected problem sets:

Week 2: Dynamic Programming & Integrated Assessment of Climate Change (discrete time)
Week 5: Maximum Principle (continuous time)
Week 7: Numerical Solution Methods in Discrete and Continuous Time
Week 9: Advanced Topics in Lectures 14-18 (possibility of selection)
Week 12: Estimation of Dynamic Models
Week 14: Continuous Time Dynamic Programming

Text:
A large part of the material will follow our book draft on *Dynamic Methods in Environmental and Resource Economics,* available at https://are.berkeley.edu/~traeger/research_body.html#book. We provide a list of other useful books further below and will reference chapters and articles in class.

Schedule:

Week 1	Discrete Time Dynamic Programming (DP)	Basic Methods & Applications
Week 2	Application: Integrated Assessment of Climate Change	
Week 3	The Linear Quadratic Model (Application taxes versus quantities)	
Week 4	Continuous Time Dynamics (ordinary differential equations ODE)	
Week 5	1. Maximum Principle 2. Fixed Points & Existence (& preparation for value function iteration)	
Week 6	Numeric Methods (function approximation, ODE solution, DP)	
Week 7	1. Numeric Application (hands on) 2. Event Uncertainty (theory)	Advanced Applications Each about 1 lecture
Week 8	1. Non-Convexities & Skiba Points 2. Time Consistency	
Week 9	1. Reactions to Risk & Anticipated Learning 2. Disentangled Risk Aversion (≠ IES, risk sensitive control)	
Week 10	1. Hyperbolic Discounting 2. Overlapping Generations	
Week 11	Estimating Dynamic Models	Advanced Methods
Week 12	1. Application (Estimating Dynamic Models) 2. Continuous Time Dynamic Programming (deterministic)	
Week 13	1. Continuous Time Stochastics & Ito Calculus 2. Continuous Time Stochastic Dynamic Programming (analytic)	
Week 14	Numeric Continuous Time Stochastic DP & Application	

Books:

Applied Computational Economics and Finance, M Miranda and P Fackler, MIT Press, 2002
Economic Dynamics: Theory and Computation, John Stachurski, MIT Press, 2009.
Dynamic Optimization, 2nd ed, M Kamien and N Schwartz, North Holland
Dynamic Programming and Optimal Control, D Bertsekas, Athena Scientific Press (1995)
Recursive Methods in Economic Dynamics, Stokey and Lucas, Harvard University Press, 1989
Mathematical Bioeconomics, C Clark 2nd ed, Wiley
Differential Equations, Stability and Chaos, W Brock and A Malliaris, North Holland, 1989
Numerical Methods in Economics, Kenneth Judd, MIT Press 1999.
Elements of Dynamic Optimization, Alpha Chiang, McGraw Hill.
Differential Games in Economics and Management Science, Dockner, van Long and Sorger, Cambridge University Press 2000, (chapter 3 is a nice introduction to continuous time dynamic programming)
Investment under Uncertainty, Dixit and Pindyck, Princeton University Press, 1994.
Optimal Control Theory and Static Optimization in Economics, Daniel Léonard and Ngo van Long, Cambridge University Press, 1992.
Recursive Macroeconomic Theory, Lars Ljungqvist & Thomas J. Sargent, MIT Press, 2004.

Week 1. Basics of dynamic programming (discrete time). This section introduces the discrete time Euler equation and methods of discrete time dynamic programming for finite and infinite time horizons. We examine examples from resource economics. Background readings include our chapter 1, MF (= Miranda and Fackler) pp 155 - 163, pp 189 – 208, or the more advanced treatment in Stachurski and several of the books listed above. Chapters 5 and 16 of Natural Resources as Capital: Theory and Policy provide introductory treatments of nonrenewable and renewable resources.

Week 2. Application: Integrated Assessment of Climate Change. This section applies basic discrete time dynamics programming to the integrated assessment of climate change. We discuss basics of climate change and combine a simple economic growth model with clean and dirty energy production with a stylized model of the carbon cycle and climate change. The solution method is based on a linear-in-state models. The material is based on Nordhaus, W. (2008), A Question of Balance: Economic Modeling of Global Warming, Yale University Press; Golosov Hassler, Krusell, Tryvinki (2014). "Optimal Taxes on Fossil Fuels in General Equilibrium", Econometrica 82(1), 41–88, Gerlagh, R. & Liski, M. (2012), "Carbon prices for the next thousand years", Cesifo; Traeger (2015), "Analytic Integrated Assessment and Uncertainty", SSRN.

Week 3. The linear-quadratic model. We discuss a second class of models with analytic solutions, the linear-quadratic (LQ) model. The section discusses some of its variations and examines the dynamic version of the "*taxes versus quantities*" comparison: should we regulate stock pollutants by a tax, a cap and trade system, or something else? The material is based on Weitzman (1974), Prices vs. Quantities, Review of Economic Studies 41: 477-491; Hoel and Karp (2002), Taxes versus quotas for a stock pollutant, Resource and Energy Economcis 24: 367-384; Newell and Pizer (2003), Regulating stock externalities under uncertainty, Journal of Environmental Economics and Management 45: 416-432; and some of our recent work.

Week 4. Introduction to continuous time dynamics. This section discusses the basics of Ordinary Differential Equations (ODEs) and Phase Plane Analysis. Readings include our chapter 8, Clark chapter 6; Kamien and Schwartz appendix. Applications include Krugman (1991) History versus Expectations QJE 651 - 67 and Brander and Taylor (1998) The Simple Economics of Easter Island: a Ricardo-Malthus Model of Renewable Resource Use AER, vol 88 pp 119 - 138

Week 5.1. The Maximum Principle. The section introduces Pontryagin's maximum principle, a continuous time dynamic optimization method using the Hamiltonian to translate the dynamic optimization problem into a boundary value problem of solving ODEs. After introducing the general method, we discuss the analysis of a one-state variable optimal control problem, emphasizing comparative statics and comparative dynamics. Readings include our chapter 9, Kamien and Schwartz, part II, sections 1 - 9; or Chiang, chapters 7 - 9.

Weeks 5.2 Fixed Points & Existence. The section discusses the assumptions and proofs the convergence of infinite time horizon dynamic programming problems. Using contraction mappings, the section is a preparation for the numeric solution approach of function iteration for discrete time dynamic programming problems. Readings include Appendix A or our book, as well as part I of Stokey and Lucas.

Week 6-7.1. Numeric Methods (function approximation, ODE solution, DP). The section introduces some numeric methods that are useful for solving continuous time and discrete time dynamic optimization problems. We approximate functions by Chebychev polynomials (or other basis functions) and use these approximations in solving ordinary differential equations and dynamic programming problems using function and modified policy iteration. Reading include our chapter 7 and MF Chapter 6, Chapter 7 pp 163 – 182, Chapter 9. You will implement our theory on your computer with our support.

Weeks 7.2-10. Advanced Applications. We plan to cover the following advanced applications. Event uncertainty treats regime shifts happening with exogenous or endogenous probability; our chapter 12, Kamien and Schwartz pp 61 – 63 and 190 – 193. Non-convex systems give rise to multiple steady states and we discuss how Skiba Points help us to identify our optimal trajectories; chapter 13 of our book, Managing Systems with Non-convex Positive Feedback, Environmental and Resource Economics 26: 575–602. The lecture on time inconsistency treats situations where a policy maker has (foreseeable) incentives to deviate from his announced plans in the future; our chapter 17, Karp and Newbery (1993) "Intertemporal Consistency Issues in Depletable Resources." In Handbook of Natural Resources, Vol. III, edited by Allen Kneese and James Sweeny, pp. 881-930; Karp and Lee (2003) "Time Consistent Policies" Journal of Economic Theory 112: 353- 64.

The lecture on reactions to risk and anticipated learning analyzes how optimal policy responds to future risk and learning; our chapters 4-6, Journal of Public Economics; Kelly and Kolstad (1999), Bayesian learning, growth, and pollution, Journal of Economic Dynamics and Control, 23: 491-518. The lecture on disentanglement explains how to treat risk aversion separately from consumption smoothing and preference satiation (and relates to risk sensitive control); our chapter 18, Epstein and Zin (1989), Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns – A Theoretical Framework, Econometrica 57: 937-969, Traeger (2015), Capturing Intrinsic Risk Aversion. Hyperbolic discounting discusses optimal decision-making under non-constant discount rates; Barro, R (1999) "Ramsey meets Laibson in the neoclassical growth model" Quarterly Journal of Economics 14: 1125 – 52. Karp (2005), Global warming and hyperbolic discounting, Journal of Public Economics 89: 261–282. The section on overlapping generations introduces generational structure and its relation to hyperbolic discounting; Ekeland, I and A Lazrak (2010) "The Golden Rule when Preferences are Time Inconsistent" Mathematical and Financial Economics 4: 29 – 55, Karp, L (2015, forth) "Provision of a public good with multiple dynasties" Economic Journal.

Weeks 11-12.1. Estimating Dynamic Models. To be decided.

Weeks 12.1-14. Continuous Time Dynamic Programming. We derives the continuous time dynamic programming equation and discusses its relation to the Maximum Principle. We introduce continuous time stochastic calculus (Ito calculus) because standard derivatives are no longer informative (or finite) in the continuous time stochastic world. Then, we combine the material to derive some of the rules of stochastic dynamic programming in continuous time. The section is based on chapter 11 of our book, Dixit and Pindyck 1994, chapters 3-5, and Darrell Duffie's Dynamic Asset Pricing Theory, Princeton University Press: 2001.