From Acid Dip to Thriving Waters
The Impact of Emissions Reductions on Lake Recovery

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Abstract
We develop an optimal control model for the recovery of a representative freshwater lake from acidification. Our objective function is the sum of the disutility from an acidified lake and the cost of emissions abatement by firms. Using emissions as the control variable, the social regulator minimizes the objective function subject to state equations that describe the impact of emissions reductions on the state variables, pH and alkalinity of lake water. We estimate the state equations using a panel data set which monitors the recovery of 43 acidified lakes located in the region surrounding Sudbury, Ontario, Canada over a 24-year period. The results indicate a general upwards trend in both pH and alkalinity, with a decrease in emissions corresponding to an increase in both variables. However, we also find the magnitude of our estimates change as we introduce additional controls, and different assumptions for the specification of acid deposition. This is one of the challenges that must be addressed before our results can be used to solve the control problem for an optimal path of emissions reductions.

1 Introduction

Acid rain, ozone depletion and climate change have been termed the three environmental crises of the twentieth century. While concern over climate change has continued to increase since then, little discussion remains of acid rain and ozone depletion. This is due in large part to the successes of the Montreal Protocol for chlorofluorocarbons (CFCs), and the Convention on Long-Range Transboundary Air Pollution for various pollutants including sulphur dioxide (SO$_2$), nitrogen oxides (NO$_x$) and Volatile Organic Compounds (VOCs). These international agreements created a cooperative effort among countries implementing domestic regulations to control emissions of pollutants. While the problems of ozone depletion and acid rain are no longer intensifying, damage from excessive pollution in the past remains widespread. This damage, and the desire for recovery, are the primary focus of current regulations.

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This paper studies the emissions reductions influencing the recovery of freshwater lakes from acidification in Northern Ontario, Canada. Terrestrial and aquatic acidification in this area is primarily attributable to nickel mining operations centered in the City of Sudbury. At its peak in 1960, total sulphur dioxide emissions from mining operations in Sudbury exceeded 2.56 million tonnes. This represents more than 4% of estimates of total global anthropogenic sulphur dioxide emissions at the time [7, 15]. Estimates indicate over 7,000 lakes located within a 17,000 km$^2$ area of Sudbury have been acidified to pH < 6.0, the point at which significant biological damage starts to occur [14]. Since the 1960s, a series of regulations introducing intensifying emission controls have induced reductions in SO$_2$ emissions of over 90%, and substantial water quality improvements have been observed.

This paper sets up a dynamic programming problem to determine how to optimally implement emissions reductions intended to achieve the recovery of a representative lake in the Sudbury area. A regulator minimizes a social welfare function, defined as the sum of the disutility from lake acidification and the cost of abatement by firms, subject to state equations describing how lake water quality evolves over time. Our objective is to use data on water quality and emissions reductions to estimate the parameters of the state equations. We find water quality is increasing over time, and that decreases in acid depositions will have a positive effect on water quality.

This paper concludes with the estimation of the parameters for the state equations. The next step is to use these estimated parameters to solve the dynamic programming problem for the optimal path of emissions reductions. This introduces a number of further challenges that go beyond the scope of this work, although we provide a brief discussion of them here.

Most significant of the challenges is that we must first parameterize the social welfare function. In past work, this has been done through either formal estimation or simple assumption of the parameter values. For example, in the derivation of the optimal policy for recovery of lakes from eutrophication, Carpenter et al [3] estimate the parameters of the utility function using information on the amount of money lake users are willing to accept to tolerate a given state of the lake, or the amount of money lake users are willing to pay to remove an increment of pollution. They estimate the cost function in a similar way, using information on the amount of money emitters are willing to pay for the pollution they release. In a soil acidification problem,
Kaitala et al. [12] estimate the parameters of the disutility function using two methods; calculation of the lost market value from decreases in annual forest growth, and an indirect, revealed preference approach in which they assume current reductions in sulphur emissions result from rational choices and therefore reveal the implicit marginal cost of forest deterioration. With the abatement cost function, they assume certain parameters values, and estimate the remainder using a linear least squares regression of the annual costs of abatement on yearly abatement levels.

The second challenge is that we formulate our control problem for the recovery of a representative lake. Actual emissions reductions, however, will have an impact on thousands of heterogeneous lakes, and the optimal emissions path must reflect this. Introduction of individual state equations to separately describe the changes in water quality for any significant number of lakes will make the solution of the control problem intractable. Therefore, a reasonable approach is to assume that all lakes in the area are homogeneous, identify a recovery goal that is representative of an average lake in the region, and appropriately parameterize the disutility function so that it is representative of the disutility from widespread acidification. While the homogeneity of lakes is a significant assumption, it is consistent with the previous literature on acidification which assumes homogeneity in the acidification of soil throughout individual countries ([12],[20],[21]).

The final challenge is in identifying the appropriate parameter estimates for the state equations. We find the magnitude of the coefficients change as we add additional explanatory variables to the regression model. A reasonable first approach in this case is to solve the control problem using different sets of the parameter estimates and determine whether there is a significant difference in the optimal path of emissions reductions in each case. If there is a large difference then this suggests additional work should be done on defining the state equations, and understanding the significance of leaving out of the equations certain sources of variation that may impact both the amount of acid deposition in a lake and its rate of recovery.

The remainder of this paper proceeds as follows. Section 2 provides an overview of the economic literature on dynamic ecological models and the scientific literature on acidification dynamics. Section 3 presents the theoretical model for the dynamic programming problem, and the state equations to be estimated. Section 4 describes the data, and Section 5 presents the estimation strategy and results. Finally, Section 6 offers some brief concluding remarks.
2 Literature Review

2.1 The Economics of Dynamic Ecological Systems

Integrated Assessment Models (IAMs) are dynamic models used in environmental economics to capture the interaction between economic objectives and constraints, and dynamic environmental systems. As ecologists recognize, the complex dynamics of environmental systems substantially affect the states of the world in which the economic system operates [1]. IAMs have therefore become an important tool for linking environmental and economic systems.

Past application of IAMs to acidification dynamics focus on the acidification of terrestrial ecosystems. The first major work was by Kaitala et al [12] who consider the optimal regulatory policies of Finland and the Soviet Union within a game-theoretic framework. The objective of both countries is to maximize the net benefits of emissions abatement, which Kaitala et al define as the value of forest growth minus the costs of abatement, subject to an environmental state equation. The environmental state equation describes how soil quality changes over time. It is a function of current soil quality, and current sulphur depositions which Kaitala et al estimate as a linear function of emissions in each region using a sulphur transportation model. Parameters for the equations are either estimated, or drawn from previously published work. With the appropriate identification of the equation parameters, Kaitala et al solve the maximization problem for the optimal cooperative and noncooperative emission rates in Finland and Russia.

Schmieman and van Ierland [20] develop an optimal control model to identify cost effective European abatement policies for the combined reduction of SO$_2$ and NO$_x$. The objective of each country is to minimize the cost of abatement, subject to an environmental state equation and a minimum standard for soil quality. The environmental state equation is based on Kaitala et al, and uses a generalized linear transportation model for estimation of current sulphur depositions which allows for emissions from more than two countries. They solve the optimal control model for a single country and find the optimal path of emissions reductions is higher than those required under the current European protocol scenario.

Schmieman et al [21] expand upon previous work by considering the interaction between the problems of acidification and tropospheric ozone pollution. They generalize the previous optimal control problem to include damages from both problems as a part of the objective function, and introduce an additional
environmental state equation describing the change in ozone levels over time. The theoretical model is used to calculate efficient abatement strategies for SO$_2$, NO$_x$, and Volatile Organic Compounds (VOCs).

Past applications of IAMs to lake dynamics focus primarily on the problem of eutrophication. Eutrophication occurs when there is excess phosphorous and nitrogen deposition in a lake, typically as a result of runoff from fertilizers used in agriculture. A eutrophic lake can cause significant economic consequences attributable to a decrease in the recreational value of the lake, a decrease in the value of fisheries, and a deterioration of water quality [18], and may be either reversible, hysteretic or irreversible$^1$. Acidified lakes cause similar economic consequences and display similar recovery patterns. Therefore, models of eutrophication are useful in understanding how models of lake acidification can be developed.

Carpenter et al [3] provide a general model of eutrophication. They set up an optimal control model where a regulator maximizes the net benefits from polluting activities and ecosystem services, subject to an environmental state equation which describes how phosphorus levels in the lake evolve over time. They estimate the state equation by an observation error procedure, using data from a single lake, and the control model is solved for the optimal phosphorus input rates under various assumptions regarding certainty over the reversibility of the lake, and the value of the discount rate. While the model is based on deterministic lake dynamics, an important observation of Carpenter et al is that in a more realistic situation where sources of variability exist, reductions in phosphorus input levels should be below the optimal rates the model describes. They note this result extends to other situations where pollution causes nonlinear changes in an ecosystem state, such as acid deposition.

Nævdal [18] focuses on the varying reversibility of lakes and the threshold effects of eutrophication. He sets up an optimal control problem similar to those above, and solves the problem for lakes on either side of the threshold value which determines whether a lake is currently in a state of eutrophy. He finds the optimal path for the reduction of nutrients into the lake is dependent on whether the lake is eutrophying in the initial time period, the number of times it is optimal for the lake to cross the threshold, and whether a eutrophying lake is reversible.

$^1$A reversible lake is one in which eutrophication can be reversed by the reduction of pollution input controls alone. A hysteretic lake is one that can be reversed from its eutrophic state, but requires a perturbation to the lower phosphorous steady state using interventions such as aluminum sulphate treatment or biomanipulation. An irreversible lake is one in which no feasible reduction of pollution input controls or chemical intervention can bring the lake out of its eutrophic state [3].
Hein [10] develops a eutrophication model with threshold effects and two steady states, one corresponding to a eutrophic lake and a second corresponding to an oligotrophic lake. He uses an explicit ecological-economic model in which lake dynamics are modeled by a set of equations obtained through regression analysis of long-term water quality data for a shallow lake ecosystem. He combines this with information on the supply of ecosystem services and the costs of different control measures to determine the optimal control policy. The existence of two steady state has a significant impact on the cost-effectiveness of different policy options, essentially creating two points of maximum efficiency, each corresponding to one of the steady states.

This paper draws upon a number of the above studies in developing an economic-ecological model of lake acidification. We use the acidification models to understand the dynamics of acid deposition, and the eutrophication models to understand the non-linearities of lake response to changes in chemical depositions, and the process of recovery from a polluted state. While we use previous work as a basis for developing our model, this paper makes a number of contributions to the literature. First, this paper models the dynamics of lake acidification, and sets up an optimal control framework for evaluating the implementation of emissions reductions for the recovery of acidified lakes. In addition, while we develop the theoretical model for a representative lake, we estimate the state equations using a panel data set which tracks the water quality of 43 lakes over a 25-year period. Finally, whereas previous work on acidification focuses on preventative measures for the deterioration of soil quality, we develop a model which emphasizes recovery from acidification, with the ultimate goal of returning a lake to its natural state.

2.2 Acidification Dynamics

The primary pollutants responsible for acidification are SO\textsubscript{2} and NO\textsubscript{x}\textsuperscript{2}. These pollutants are derived from a number of sources, with the largest contributors being power stations, industrial plants, and vehicle emissions. Acidification of ecosystems is a result of excessive wet and dry depositions of acid. Wet depositions occur when SO\textsubscript{2} and NO\textsubscript{x} reach the atmosphere, where they react with the moisture and undergo oxidation, resulting in the formation of sulphuric and nitric acids. These acids exist primarily in the clouds and are transported to the ground through rain or snow. Dry depositions, alternatively, occur in a dry atmosphere.

\textsuperscript{2}The science in this section is based on the discussion of acidification in Chapter 5 of Mason [16] and Chapter 1 of Charles [4]
Acid Deposition through a series of complex photochemical reactions in which highly reactive oxidizing agents such as ozone produce sulphuric and nitric acids. Acids from these reactions are transported to the ground in gaseous or particulate form. Wet deposition is relatively easy to quantify, while quantification of dry deposition is more difficult since gases and particulates are more widespread – they enter surface and groundwater basins, are absorbed by vegetation, and are dissolved by precipitation.

In addition to the wet and dry depositions from the atmosphere, freshwater is affected by acidic inputs through indirect atmospheric depositions via run-off in the catchment, and from the generation of acidity within the catchment. Given these two additional sources, acidification of freshwater lakes is most likely to occur in areas with thin soil where there are insufficient base cations freely available to neutralize the deposition of acid to the soil. Similarly, land use also influences the rate of acidification, with acid deposition generally increasing in forested areas.

The process of acidification can be divided into three stages, as shown in Figure 1 (adapted from Mason [16]). It begins when the deposition of sulphate and nitrate ions, which have a negative charge, increases. The lake water responds by an increase in the positive charge, $H^+$, which measures acidity\(^3\). However, this can be matched by a decrease in one of the other negative charges in the water. This is what occurs in the

\[^3\text{pH is defined as the negative logarithm of the concentration of } H^+ \text{ ions in the water, } pH = - \log [H^+]. \text{ As the concentration of } H^+ \text{ ions in the lake water increases, the water becomes acidified, and pH falls.} \]
first stage of Figure 1 when the alkalinity of the water, which has a negative charge, is positive. In this stage, the positive alkalinity acts as a buffer against increases in acid deposition and the concentration of $H^+$ ions$^4$. The end result is a decrease in the negative charge (alkalinity) and no change in the positive charge ($H^+$), so that pH remains at its natural level, and communities of aquatic life remain stable. Some lakes with a high buffering capacity will never move beyond this stage. However, more generally, a lake will move into the second stage where the alkalinity buffer is lost, and continued acid deposition results in increases in $H^+$, large decreases in pH, and the beginning of damages to the biological ecosystem. In the third and final stage, the loss of alkalinity is complete and the pH stabilizes at some low level, typically below 5. In this state the lake is acidic and there are typically increasing levels of metals with positive charges, particularly aluminum. This results in the extermination of fish populations and a decrease in the diversity of other aquatic life.

There are two methods for reversing the acidification of freshwater lakes. The first is a reduction in emissions, which is typically accomplished by a switch to cleaner production technologies, such as the installation of scrubbers at emitting sources. There has been a significant reduction in acidifying emissions since the 1980s, however, this has yet to lead to recovery of all lakes. While sulphuric and nitric acids in precipitation have declined, a large amount of acid remains deposited in soils and wetlands. Therefore, while direct deposition of acidic inputs is decreasing, depositions via run-off in the catchment and from the generation of acidity within the catchment remain. A second method for reversing acidification is the liming of waters. In liming either pulverized limestone, hydrated lime or quicklime is added to the water to neutralize the acid. This method of recovery has more immediate results, but its effectiveness depends on the retention time of water in the lakes. Lakes with short retention times must be relimed either annually or biannually, and those with longer retention times will generally re-acidify 5 to 10 years after liming. While liming is effective in restoring water chemistry, ecological recovery is not guaranteed, and it is typically an expensive alternative with localized results. Since acidification is a widespread problem, the general consensus is that the causes and not the symptoms of acidification must be addressed, and reductions in emissions should be the main tool used in reversing acidification.

$^4$The major component of alkalinity in most surface waters is bicarbonate, $HCO_3^-$. When alkalinity is positive, bicarbonate is positive, and will combine with the hydrogen ion to form aqueous carbon dioxide and water: $HCO_3^- + H^+ \rightarrow CO_2 + H_2O$, so that the increased acid deposition has no effect on the acidity of the lake water.
3 Theoretical Model of Lake Recovery

We develop an optimal control model for the recovery of a representative lake in the Sudbury area from acidification. Following Nævdal [18], we assume there is a regulator who is concerned about the state of the lake, and define preferences such that any deviation from the lake’s “natural state” causes increasing disutility. We define the state of the lake by the observed pH levels, and assume the natural state is exogenously defined to represent the necessary conditions for biological recovery from acidification. Freshwater lakes in their natural state will typically vary in the biological populations they support, although most will have a pH between 6.0 and 8.0. A convenient definition for the natural state of a lake is a return to its predisturbance state. However, this may not be an accurate definition since recovery will typically take several decades during which time the lake in a healthy state may naturally evolve due to either internal chemical processes or external factors such as climate change. Therefore, a more accurate definition for the natural state is the reference data approach that defines recovery as a return to a state that is typical of the least-disturbed lakes in the area [9]. In a given period, the disutility from the degree of acidification is thus given by:

\[ U(P_t) = \frac{A}{2} (P_t - \bar{P})^2 \]  

(1)

where \( P_t \) is the observed pH of the lake in period \( t \), \( \bar{P} \) is the pH of the lake in its natural state, and \( A \) is a positive parameter. We assume \( P_0 < \bar{P} \), i.e., the starting point for all lakes is in a state of acidification. Since the utility function provides a measure of disutility it is convex, and at the starting point (\( t = 0 \)), \( U'(P_t) < 0, U''(P_t) > 0 \), so the degree of disutility is decreasing as \( P_t \) increases.

The regulator must balance the desire for returning a lake to its natural state with the costs of doing so. We consider only the cost of abatement undertaken by local firms emitting \( \text{SO}_2 \)^5. Following Kaitala et al [12], we assume the cost function defines the minimum cost envelope of the entire range of sulphur abatement options for firm \( j \) in a given time period. In any given period, however, we observe actual emissions by a firm, and not abatement. Therefore, we define the abatement cost function by the following quadratic equation:

\[ C^j(E^j_t) = \psi_1^j \left( \bar{E}^j - E^j_t \right) + \psi_2^j \left( \bar{E}^j - E^j_t \right)^2 + \psi_3^j \]  

(2)

\(^5\)We define local firms as those that are subject to regulation.

\(^6\)From this point forward, we consider only the effects of changes in \( \text{SO}_2 \) emissions on the acidification and recovery of lakes. We ignore the effects of \( \text{NO}_x \) since the local firms under consideration are all industrial plants where the primary pollutant is \( \text{SO}_2 \) and emissions of \( \text{NO}_x \) are minimal.
where $E_j$ are the SO$_2$ emissions of firm $j$ in the absence of regulation, $E_j^t$ are the observed emissions of firm $j$ in period $t$ given regulation, and $\psi_1^j, \psi_2^j, \psi_3^j$ are positive parameters. Actual abatement is therefore given by $\left(\bar{E}^j - E_j^t\right)$. The cost function is convex in emissions, and we require that abatement be non-negative in all periods ($\bar{E}^j - E_j^t \geq 0$) so abatement costs are increasing as actual emissions decrease, $C'(E_j^t) < 0$, $C''(E_j^t) > 0$.

Following the previous literature on acidification ([12], [20], [21]), we assume there is a linear relationship between SO$_2$ emissions and acid deposition at a lake site. Acid deposition will be affected by several stochastic factors beyond actual SO$_2$ emissions, most notably the amount of precipitation a lake receives and the buffering capacity of the land surrounding the lake. Precipitation is exogenous to the model, and monitoring buffering capacity for the land over time requires the introduction of an additional state equation. For simplicity, we disregard these factors and assume the acid deposition equation is a linear transformation of all current emissions impacting the environmental state of the lake. The equation for acid depositions is thus given by:

$$D_t = \sum_m c_m E_m^t$$

where $c_m$ is a lake specific weighting factor relating emissions from firm $m$ to acid depositions at the lake, $m$ is the total number of firms with emissions impacting the lakes, $m \geq j$, and $(m - j)$ is the number of firms not under control of the regulator. We further assume the acid deposition equation is deterministic and the regulator knows the emissions of the $(m - j)$ firms not subject to regulation$^7$. With this assumption, the regulator can accurately predict how changes in the emissions of regulated firms will impact the environmental state of the lake.

The regulator’s control variable is emissions of local firms, $E_j^t$. Following the previous literature, we assume the regulator specifies an emissions cap for each firm in every period, and that observed emissions of firms are exactly equal to their regulated amount. The regulator can therefore precisely determine acid depositions from local firms at the lake site in each period. These acid depositions will impact water quality

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$^7$A more complicated model is found in Kaitala et al [12] who sets up the optimal control problem within a game theoretic framework. He solves the problem for the cooperative solution, where regulators jointly maximize their social welfare functions, and the non-cooperative solution, where regulators consider only the acidification dynamics in their region given a fixed emissions strategy of their opponent. In Kaitala’s framework, our model corresponds to the derivation of a non-cooperative solution to the optimal control problem.
at the lake site, which we measure by the state variables $pH$, $P_t$, and alkalinity, $L_t$.

The state equations for alkalinity and $pH$ describe changes in the environmental state of the lake. Starting from a state of acidification, the desired chemical response to a reduction in acid depositions is an increase in alkalinity and $pH$. If the lake water follows this desired response path, then alkalinity and $pH$ will increase together until the lake returns to its natural state. As alkalinity increases, the effect of changes in acid deposition on $pH$ will begin to decrease. This is the mechanism through which the $pH$ of the water stabilizes to the level corresponding to its natural state. In this state, the lake is able to resist reasonable changes in acid deposition since the restored alkalinity provides a buffering capacity which allows the water to neutralize itself. However, if the acid deposition exceeds some critical threshold then the lake will move out of its natural state and again begin to decline into an acidified state.

The chemical recovery of a lake is a dynamic process, and is dependent on the lake’s chemical history. Therefore, we include the lagged value of $pH$ in its state equation. With the inclusion of lagged $pH$ we capture the entire history of the effects of acid deposition on $pH$. Any measured influence of acid deposition in the current period is therefore conditioned on this history, and will represent only the effect of new information [8]. The next term in the state equation for $pH$ is the lagged value of acid deposition, which captures the direct effect of acid deposition from the previous period on the current period’s $pH$. We also include an interaction term between the lagged values of alkalinity and $pH$. This term captures the non-linearity of $pH$ response to changes in emissions. In particular, we expect the coefficient on depositions to be negative, and the coefficient on the interaction term to be positive so that as alkalinity increases, the effect of further changes in acid depositions on $pH$ goes to zero. Finally, we include an error term with mean zero and variance $\sigma^2$. This term accounts for the stochastic factors impacting changes in $pH$ each period. The state equation for $pH$ is therefore given by:

$$ P_{t+1} = \beta_0 + \beta_1 P_t + \beta_2 \left( \sum_m c^m E^m_t \right) + \beta_3 L_t \left( \sum_m c^m E^m_t \right) + \epsilon_{t+1} $$

We define the state equation for alkalinity analogously to the state equation for $pH$. In this case we do not include the last interaction term since the level of $pH$ does not effect how alkalinity responds to changes
in acid deposition. Therefore, the state equation for alkalinity is given by:

\[ L_{t+1} = \gamma_0 + \gamma_1 L_t + \gamma_2 \left( \sum_m c^m E^m_t \right) + \nu_{t+1} \]  

(5)

Combining the above equations, we get the following discrete time optimal control problem:

\[
\min_{E^j_t} \sum_{t=0}^{\infty} \delta^t \left( \frac{A}{2} (P_t - \bar{P})^2 + \sum_j \left( \psi^j_1 (\bar{E}^j_t - E^j_t) + \psi^j_2 (\bar{E}^j_t - E^j_t)^2 + \psi^j_3 \right) \right)
\]

s.t. 

\[ P_{t+1} = \beta_0 + \beta_1 P_t + \beta_2 \left( \sum_m c^m E^m_t \right) + \beta_3 L_t \left( \sum_m c^m E^m_t \right) \]  

(7)

\[ L_{t+1} = \gamma_0 + \gamma_1 L_t + \gamma_2 \left( \sum_m c^m E^m_t \right) \]  

(8)

\[ E^j_t \geq 0 \quad \forall \quad j \]  

(9)

where \( \delta \) is the appropriate discount factor.

The discrete time dynamic programming equation corresponding to (6) is:

\[
J(P_t, L_t) = \max_{E^j_t} - \left( \frac{A}{2} (P_t - \bar{P})^2 + \sum_j \left( \psi^j_1 (\bar{E}^j_t - E^j_t) + \psi^j_2 (\bar{E}^j_t - E^j_t)^2 + \psi^j_3 \right) \right) + \delta J(P_{t+1}, L_{t+1})
\]

s.t. Equations (7), (8), (9)

(10)

For simplicity in notation, from this point forward we express the utility function as \( U(P_t) \), the cost function as \( C(E^j_t) \), the state equation for pH as \( f(P_t, L_t, E^m_t) \), and the state equation for alkalinity as \( g(P_t, L_t, E^m_t) \). With this formulation of the problem, we cannot exclude the possibility that the non-negativity constraint on emissions will be binding in certain states. Therefore, along the optimal solution path, emissions in each period must be chosen so that the following Euler equilibrium conditions are satisfied:

\[
-C E^j_t(t) + \delta [J_F(t+1) f_{E^j}(t) + J_L(t+1) g_{E^j}(t)] = \mu^j_t
\]

(11)

\[
J_F(t) = -U_F(t) + \delta [J_F(t+1) f_F(t)]
\]

(12)

\[
J_L(t) = \delta [J_F(t+1) f_L(t) + J_L(t+1) g_L(t)]
\]

(13)

\[ E^j_t \geq 0, \quad \mu^j_t \geq 0, \quad E^j_t \cdot \mu^j_t = 0 \]

(14)

where \( \mu^j_t \) measures the current and expected future reward from a marginal decrease in emissions by firm \( j \) in period \( t \) [17]. The conditions along the optimal path require that in every period, each local firm reduces its emissions until either the long-run marginal reward from further decreasing emissions, or emissions...
themselves, are zero. In addition to satisfying the above Euler conditions, the steady state to the problem must also satisfy the following state stationarity conditions:

\[ P^* = f(P^*, L^*, E^{j*}) \]  \hspace{1cm} (15)

\[ L^* = g(P^*, L^*, E^{j*}) \]  \hspace{1cm} (16)

So in the steady state, pH, alkalinity and regulated emissions are all constant from one period to the next.

4 Data Summary

The data used in this paper are from the Sudbury Environmental Study (SES) Extensive Monitoring Programme. The SES Programme is conducted by the Ontario Ministry of the Environment, through the Freshwater Ecology Unit at Laurentian University in Sudbury, Ontario. It began as a chemistry survey of 209 lakes from 1974–1976. This survey revealed significant acidification, and loss and depression of fish populations in a 5,300 km\(^2\) area around Sudbury, which included 650 km\(^2\) of lake surface area [5]. A second chemistry survey of 250 lakes was subsequently conducted from 1981–1983. In 1983, 44 lakes which had an observed pH of less than 5.5 in at least one of the previous surveys were chosen for continued monitoring. These lakes have been sampled once per year, during the summer stratified period, from 1981–2006. This paper uses annual observations of pH and alkalinity recorded from 1981–2004 for 43 of these lakes\(^8\). Summary statistics for these data are provided in Table 1, and Figure 2 shows the location of the lakes in relation to Sudbury\(^9\).

Lake water samples are taken either from a location near the lake centre, or near the centre of a main basin on a very large lake. From 1981 to 1994 lake water samples were collected as non-volume weighted tygon tube composites through the two, upper stratified layers of the lake. If the lake was too shallow for thermal stratification then the sample was collected to 1 metre above the lake bottom. Beginning in 1995, sampling methods changed to the use of a four-litre plastic jug immersed by hand to completely below the

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\(^8\)For the majority of years in the sample, the laboratory value of alkalinity is reported as the total inflection point. The total inflection point is routinely measured by titration of the water with strong acid or base until the inflection point is reached. At this point, the acid neutralizing capacity of the water is zero. A positive alkalinity indicates a net strong base in the water, and a negative alkalinity indicates a net strong acid. The exception to measuring by total inflection point is 1995 and 1996 where the laboratory value of alkalinity is the fixed-endpoint alkalinity value. For these years, we use inflection point alkalinity values calculated by the Freshwater Ecology Unit, and reported in their 2006 data report [13].

\(^9\)The map of the location of lakes is provided by the Freshwater Ecology Unit. Note that Whitson Lake, immediately North-East of Sudbury, is the 44\(^{\text{th}}\) lake in the SES study for which we do not have data.
lake surface. In its 2006 data report [13], the Freshwater Ecology Unit conducts an analysis of the difference between water samples collected using the collected tube composite and surface grab sampling methods. Of 22 chemical variables, they find significant differences in results for 8 of the variables, including pH, which is significantly lower in tube composite samples. Since we are interested in determining how emissions reductions influence the increase of pH over time, we recognize this change in sampling methods may cause an upwards bias in our results.

Due to outliers in the measured chemical values, and some years in which alkalinity and pH were not measured, there are 10 lakes for which one or both of the observations on pH and alkalinity are missing for a single year. We choose to drop these years from the data set, thereby creating an unbalanced panel. Since

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10 The analysis of water samples is conducted using data from 15 Ontario Ministry of Environment long-term monitoring lakes in Northeastern Ontario where the two collection methods were simultaneously used on sampling dates in the summer stratified period. Comparisons between results from the mean grab and mean tube composite samples were conducted using paired t-tests.

11 While the change in sampling methods does create a potential bias, we hope it will be somewhat mitigated by the fact that the majority of emissions reductions are observed prior to 1995. Total Sudbury emissions are reduced by 66.4% between 1981 and 1995, whereas the reduction between 1995 and 2004 is a more modest 14.6%.
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th>Water Quality Variables: 1981</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Min</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH</td>
<td>4.87</td>
<td>0.35</td>
<td>4.13</td>
<td>5.76</td>
<td>43</td>
</tr>
<tr>
<td>Alkalinity</td>
<td>-0.91</td>
<td>0.89</td>
<td>-4.40</td>
<td>0.46</td>
<td>42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Water Quality Variables: 2004</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Min</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH</td>
<td>5.57</td>
<td>0.52</td>
<td>4.66</td>
<td>6.47</td>
<td>43</td>
</tr>
<tr>
<td>Alkalinity</td>
<td>0.29</td>
<td>0.75</td>
<td>-1.06</td>
<td>2.14</td>
<td>43</td>
</tr>
</tbody>
</table>

| Environmental Variables: 1981 |                       |
| Current Regulatory Cap (kt of SO₂) | 1082.23 | - | - | - | - |
| Future Regulatory Cap (kt of SO₂)   | 882 | - | - | - | - |
| Total Emissions (kt of SO₂)         | 837 | - | - | - | - |
| Lake Site Acid Deposition           | 21.4 | 20.6 | 6.54 | 104.63 | 43 |

| Environmental Variables: 2004 |                       |
| Current Regulatory Cap (kt of SO₂) | 365 | - | - | - | - |
| Future Regulatory Cap (kt of SO₂)   | 241 | - | - | - | - |
| Total Emissions (kt of SO₂)         | 240 | - | - | - | - |
| Lake Site Acid Deposition           | 6.11 | 5.85 | 1.88 | 30.0 | 43 |

| Lake Characteristics |                       |
| Distance from Sudbury (km) | 59.6 | 30 | 8 | 128 | 43 |
| Direction from Sudbury (=1 Downwind) | 0.58 | 0.5 | 0 | 1 | 43 |
| Lake Area (hectares) | 273.7 | 305.8 | 14.54 | 1316.45 | 43 |
| Elevation (m)       | 300.4 | 73.5 | 189 | 486 | 43 |
| Shoreline Length (km) | 19.8 | 20.8 | 2.6 | 89.3 | 43 |
| Maximum Depth (m)   | 31.1 | 17.4 | 8.0 | 90.3 | 40 |
| Mean Depth (m)      | 9.3 | 4.8 | 3.8 | 24.1 | 39 |
| Volume (x 10⁴ m³)   | 3125.3 | 4032.4 | 83.0 | 17,621.0 | 36 |
| Road Access (=1 Access) | 0.35 | 0.48 | 0 | 1 | 43 |
| No. Observations    | 1022 |       |     |     |    |

our estimation equations are dynamic, in addition to losing the observations from the year in which the measurements are not taken, we must also drop from our sample the observations from the year immediately following. This is because for those years we do not have data for the value of the lagged dependent variable in the state equations. The result is that we drop from our sample 19 periods of observations\textsuperscript{12}.

As noted previously, the acidification of Sudbury area lakes occurred primarily as a result of sulphur dioxide emissions from nickel mining operations in the Sudbury area. The goal of the SES Programme was to assess the impacts of emissions reductions from these operations on lake water quality, and to provide

\textsuperscript{12} We drop only 19 observations because for one lake, the data is missing for the first period of observation, which is not included in our estimation for any of the lakes. Therefore, for this lake, we only drop the observation for the following year.
ongoing documentation of the recovery of lakes from acidification. Damage from acidification typically begins to occur when the pH of a lake drops below 6.0. However, an observed pH of 5.5 or less was used to identify lakes for inclusion in the SES study as this is the approximate threshold at which damage to acid-sensitive sport fish begins to occur [13]. None of the lakes included in the SES Study were part of the region’s Experimental Liming Program [23], so observed improvements in lake water quality over the study period can be strictly attributed to emissions reductions. While not a random sample of all lakes impacted by SO$_2$ emissions, the data are representative of those lakes that suffered significant biological damage, and which are the primary targets of emissions reductions introduced to aid in the recovery of lakes from acidification.

The two major mining facilities in the Sudbury area are INCO and Falconbridge. Historically, INCO has been the largest point source of SO$_2$ emissions in North America, with peak emissions in the 1960s of over 2200 kt of SO$_2$ per year [22]. While substantially smaller than INCO, Falconbridge is still one of the main pollution sources in Ontario, with peak emissions in the 1960s of around 300 kt per year. The first environmental regulations introducing caps on the emissions of INCO and Falconbridge came into effect in the early 1970s. From 1970−1980, INCO’s emissions were reduced by 59% and Falconbridge by 64%. The major program introduced during the study period, however, was the Countdown Acid Rain Program, which began in 1985. Relative to 1980 emission levels, it required both INCO and Falconbridge to achieve a 60% reduction in their emission levels by 1994 [22]. These 1994 emission levels remained the standard through to the end of 2005. Regulation in 2000, 2002 and 2004 set new caps that came into effect in 2006, and provides annual emissions limits for both firms through 2015 ([19],[6],[11]). A summary of the current and future emissions caps faced by INCO and Falconbridge at the start and end of the study period is provided in Table 1.[13]

Currently we only have data on annual sulphur dioxide emissions for INCO and Falconbridge. These data are available from the Ontario Ministry of the Environment, and were provided by the Freshwater Ecology

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13Our theoretical model assumes the regulator sets an annual emissions cap for each firm and that firms’ emissions are exactly equal to this cap. This does not accurately reflect the regulatory environment in Sudbury where INCO and Falconbridge face a single, current emissions cap extending between 2 and 10 years, and in most cases, a future emissions cap which they work towards meeting during this time period. As a result, we often observe actual emissions that are significantly lower than the current cap. To accurately model this scenario emissions from each firm must be defined as a state variable, where the state equation describes observed emissions as a function of the regulator’s control variables, the current and future emissions caps. We choose to follow previous literature in assuming emissions of firms are the control variable for the regulator. This is for simplicity in solving the optimal control model, and is also due to the endogeneity problem that arises when estimating observed emissions as a function of the current and future emissions caps.
Unit. While INCO is much larger than Falconbridge, both firms had comparable emissions reductions of approximately 73% between 1981 and 2004. We do not have specific information on the annual acid deposition at each lake site resulting from sulphur dioxide emissions. To estimate depositions, we weight the annual total emissions from INCO and Falconbridge by the inverse of the lake’s distance from Sudbury. Relating this estimation method to the equation for depositions, equation 3, we are assuming the local regulated firms are the only firms with emissions impacting the lakes, $j = m^{14}$. We also assume $c^m$ is the same for both our local firms, and is equal to the inverse of the distance from each lake to Sudbury. The distance measurement is provided by the Freshwater Ecology Unit. Summary statistics for actual sulphur dioxide emissions, and for the estimates of depositions at each lake site, are provided in Table 1. Figure 3 provides a comparison between actual sulphur dioxide emissions and their regulated amount\textsuperscript{15}.

Data on time-invariant physical characteristics of the lakes are also provided by the Freshwater Ecology Unit. While INCO is much larger than Falconbridge, both firms had comparable emissions reductions of approximately 73% between 1981 and 2004. We do not have specific information on the annual acid deposition at each lake site resulting from sulphur dioxide emissions. To estimate depositions, we weight the annual total emissions from INCO and Falconbridge by the inverse of the lake’s distance from Sudbury. Relating this estimation method to the equation for depositions, equation 3, we are assuming the local regulated firms are the only firms with emissions impacting the lakes, $j = m^{14}$. We also assume $c^m$ is the same for both our local firms, and is equal to the inverse of the distance from each lake to Sudbury. The distance measurement is provided by the Freshwater Ecology Unit. Summary statistics for actual sulphur dioxide emissions, and for the estimates of depositions at each lake site, are provided in Table 1. Figure 3 provides a comparison between actual sulphur dioxide emissions and their regulated amount\textsuperscript{15}.

\textsuperscript{14}There are additional firms located in other parts of Ontario, Michigan and the Ohio Valley with SO$_2$ emissions impacting the lakes around Sudbury. As a result of a number cross-border agreements for the reduction of acid depositions, many of these firms will have undertaken similar emissions reductions to INCO and Falconbridge during our study period. The exclusion of these firms’ emissions from our dataset will therefore result in an upwards bias in our estimation of the impact of a decrease in depositions on pH.

\textsuperscript{15}The downwards spike in emissions observed in 1982 is the result of a prolonged shutdown of the INCO and Falconbridge smelters from June 1982 until March 1983. This was the combined result of a labor dispute at INCO, high energy prices and low nickel prices.
Unit. Characteristics available for all lakes are the direction of the lake from Sudbury (upwind or downwind), lake area, elevation, shoreline length, and whether there is road access. For a subset of lakes, data is also available for mean depth, maximum depth, and volume. The lake area and shoreline length was calculated by the Freshwater Ecology Unit using the mapping software MapInfo, while the other data was collected from a variety of available sources [13]. Summary statistics for data describing the physical characteristics of the lakes are provided in Table 1.

5 Estimation & Results

Our objective is to estimate the parameters of the state equations for pH and alkalinity. We begin by assuming the error components of each state equation are made up of two terms; $\epsilon_{it}$ and $\nu_{it}$ which are iid over $i$ and $t$, and $\alpha_i$ and $\eta_i$, which are random variables that capture unobserved heterogeneity among the lakes. We further assume strict exogeneity of the error terms, $E[\epsilon_{it}|\alpha_i, x_{i1}, \ldots, x_{iT}] = 0$, $E[\nu_{it}|\eta_i, y_{i1}, \ldots, y_{iT}] = 0$ where we let $x_{it}$ and $y_{it}$ be the vectors of right hand side variables in the state equations for pH and alkalinity respectively. We expect the random variables, $\alpha_i$ and $\eta_i$, will be correlated with the observed regressors $x_{it}$ and $y_{it}$, particularly the lagged values of the dependent variables in each state equation. This is because certain time invariant characteristics of the lake, such as the types of soil or vegetation found on surrounding land, may impact both the natural state of the lake, and the rate at which it recovers from acidification.

Therefore, the first model we estimate is a fixed effects model of the following form:

$$P_{it} = \alpha_i + \beta_0 + \beta_1 P_{i,t-1} + \beta_2 D_{i,t-1} + \beta_3 L_{i,t-1} D_{i,t-1} + \epsilon_{it} \quad (17)$$

$$L_{it} = \eta_i + \gamma_0 + \gamma_1 L_{i,t-1} + \gamma_2 D_{i,t-1} + \nu_{it} \quad (18)$$

Results from the estimation of equations 17 and 18, using the within estimator, are provided in column (1) of Tables 2 and 3 respectively. The results from the estimation of both equations are mostly as expected. The coefficients on the lagged dependent variables are positive, significant and less than 1, indicating the state equations for pH and alkalinity are stable, and there is a general trend of improving water quality over time. The coefficients on depositions are negative and significant, indicating that current values of pH and alkalinity are increasing as lagged depositions decrease. Finally, the coefficient on the interaction between
depositions and alkalinity in the state equation is also negative and significant, although only at the 10% level. This is not the result we expect, since it suggests the greater the alkalinity, the greater will be the increase in pH attributable to a decrease in depositions. One explanation, however, is that the lakes are surveyed early in the recovery process, during which time alkalinity can be more responsive than pH to changes in depositions. In this case, pH only starts responding to changes in deposition after alkalinity has already been increasing, which is consistent with the negative sign on the interaction between depositions and alkalinity.

The problem with the above estimation is that due to the lagged dependent variable in both state equations, the strict exogeneity assumption is violated. Considering first the estimation of the state equation for pH, the within estimator regresses \( P_{i,t-1} \) on \( (P_{i,t-1} - \bar{P}_t) \) and \( (x_{it} - \bar{x}_t) \). This generates an error term \( (\epsilon_{it} - \bar{\epsilon}_t) \). By equation 17, \( P_{it} \) is correlated with \( \epsilon_{it} \), so \( P_{i,t-1} \) is correlated with \( \epsilon_{i,t-1} \), and therefore \( P_{i,t-1} \)
Table 3: State Equation for Alkalinity, Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>(1) OLS</th>
<th>(2) AB</th>
<th>(3) AB</th>
<th>(4) AB</th>
<th>(5) AB</th>
<th>(6) AB</th>
<th>(7) AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{i,t-1}$</td>
<td>0.4858</td>
<td>0.4003</td>
<td>0.3858</td>
<td>0.3836</td>
<td>0.3783</td>
<td>0.3578</td>
<td>0.3447</td>
</tr>
<tr>
<td></td>
<td>(0.0959)**</td>
<td>(0.1217)**</td>
<td>(0.1158)**</td>
<td>(0.1252)**</td>
<td>(0.1323)**</td>
<td>(0.1241)**</td>
<td>(0.1353)**</td>
</tr>
<tr>
<td>$D_{i,t-1}$</td>
<td>-0.0214</td>
<td>-0.0258</td>
<td>-0.0540</td>
<td>-0.0504</td>
<td>-0.2214</td>
<td>-0.4894</td>
<td>-1.0308</td>
</tr>
<tr>
<td></td>
<td>(0.0029)**</td>
<td>(0.0041)**</td>
<td>(0.0065)**</td>
<td>(0.0305)*</td>
<td>(0.0542)**</td>
<td>(0.0749)**</td>
<td>(0.5522)*</td>
</tr>
<tr>
<td>$D_{i,t-1} \cdot Precipitation_{i,t-1}$</td>
<td>3.1e-05</td>
<td>(5.13e-06)**</td>
<td>(3.55e-04)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{i,t-1} \cdot Lake Area_i$</td>
<td>0.0001</td>
<td>(0.0001)</td>
<td>(0.0024)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{i,t-1} \cdot Direction_i$</td>
<td>0.0054</td>
<td>(0.0206)</td>
<td>(0.4913)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{i,t-1} \cdot Elevation_i$</td>
<td>0.0001</td>
<td>(0.0001)</td>
<td>(0.0041)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{i,t-1} \cdot Shoreline Length_i$</td>
<td>-0.0014</td>
<td>-0.0279</td>
<td>(0.0016)</td>
<td>(0.0382)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_{i,t-1} \cdot Road Access_i$</td>
<td>0.0005</td>
<td>(0.0162)</td>
<td>(0.3009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.2195</td>
<td>0.0059</td>
<td>0.0038</td>
<td>0.0105</td>
<td>0.0182</td>
<td>0.0187</td>
<td>0.0220</td>
</tr>
<tr>
<td></td>
<td>(0.0575)**</td>
<td>(0.0051)</td>
<td>(0.0051)</td>
<td>(0.0115)</td>
<td>(0.0066)**</td>
<td>(0.0063)**</td>
<td>(0.0083)**</td>
</tr>
<tr>
<td>Year Dummies</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Emissions Weighting Factor</td>
<td>$\frac{1}{D_{it1}}$</td>
<td>$\frac{1}{D_{it2}}$</td>
<td>$\frac{1}{D_{it1}}$</td>
<td>$\frac{1}{D_{it1}}$</td>
<td>$\frac{1}{D_{it1}}$</td>
<td>$\frac{1}{D_{it1}}$</td>
<td>$\frac{1}{D_{it1}}$</td>
</tr>
<tr>
<td>Observations</td>
<td>970</td>
<td>920</td>
<td>920</td>
<td>920</td>
<td>920</td>
<td>920</td>
<td>920</td>
</tr>
<tr>
<td>Number of Lakes</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses, standard errors for the OLS regression are clustered by watershed.

\* *, **, and *** represent significance at the 10%, 5%; and 1% level respectively.

is also correlated with $\bar{\epsilon}_i$, implying the regressor $(P_{i,t-1} - \bar{P}_i)$ is correlated with the error term $(\epsilon_{it} - \bar{\epsilon}_i)$. In this case, consistency of the parameter estimates requires that $T \to \infty$ since then the effect of $\epsilon_{i,t-1}$ on $\bar{\epsilon}_i$ becomes very small. However, in our dataset $T$ is only equal to 25, so OLS estimation of the model leads to inconsistent parameter estimates [2]. The same result holds for the state equation for alkalinity.

To obtain consistent estimates of the parameter values, we use the one-step Arellano-Bond estimator, which is an IV variant of the first-differences estimator. The first differences estimator leads to the following regression model for the state equations:

\[
(P_{it} - P_{i,t-1}) = \beta_1 (P_{i,t-1} - P_{i,t-2}) + \beta_2 (D_{i,t-1} - D_{i,t-2}) + \\
\beta_3 (D_{i,t-1} L_{i,t-1} - D_{i,t-2} L_{i,t-2}) + (\epsilon_{it} - \epsilon_{i,t-1})
\]

(19)

\[
(L_{it} - L_{i,t-1}) = \gamma_1 (L_{i,t-1} - L_{i,t-2}) + \gamma_2 (D_{i,t-1} - D_{i,t-2}) + (\nu_{it} - \nu_{i,t-1})
\]

(20)

As with the within estimator, OLS estimation of these models will be inconsistent because the lagged dependent variables are correlated with the error terms. However, if the error terms are not serially correlated
then efficient estimation is possible using additional lags of the dependent variables as instruments [2]. Again considering the state equation for pH, in any period $t$, the vector of instruments is defined by

$$z_{it} = [P_{i,t-2}, P_{i,t-3}, \ldots, P_{i1}, (D_{i,t-1} - D_{i,t-2}), (L_{i,t-1}D_{i,t-1} - L_{i,t-2}D_{i,t-2})]$$

(21)

Then the matrix of instruments is given by:

$$Z_i = 
\begin{pmatrix}
  z'_{i3} & 0 & \ldots & 0 \\
  0 & z'_{i4} & 0 \\
  \vdots & \ddots & \ddots & 0 \\
  0 & \ldots & 0 & z'_{iT}
\end{pmatrix}
$$

(22)

and the one-step Arellano-Bond estimator is defined by:

$$\hat{\beta}_{AB} = 
\left( \sum_{i=1}^{N} \tilde{X}_i'Z_i \right) \left( \sum_{i=1}^{N} Z_i'Z_i \right)^{-1} 
\left( \sum_{i=1}^{N} \tilde{X}_i'Z_i \right) \left( \sum_{i=1}^{N} Z_i'Z_i \right)^{-1} 
\left( \sum_{i=1}^{N} Z_i'\tilde{P}_i \right)
$$

(23)

where $\tilde{X}_i$ is a $(T-2) \times 3$ matrix with $t^{th}$ row $((P_{i,t-1} - P_{i,t-2}), (D_{i,t-1} - D_{i,t-2}), (L_{i,t-1}D_{i,t-1} - L_{i,t-2}D_{i,t-2}))$, $t = 3, \ldots, T$ and $\tilde{P}_i$ is a $(T-2) \times 1$ vector with $t^{th}$ row $(P_{i,t} - P_{i,t-1}), t = 3, \ldots, T$ [2]. The one-step Arellano-Bond estimator for the state equation for alkalinity can be derived in an analogous fashion.

The results from the estimations of equations 19 and 20 using the one-step Arellano-Bond estimator are provided in column (2) of Tables 2 and 3 respectively. In both state equations the coefficient on the lagged dependent variable decreases, while the absolute value of the coefficient on depositions increases. We therefore observe that in any period, a decrease in depositions will have a larger impact on lake water recovery than previously indicated. In the state equation for pH, the coefficient on the interaction term between alkalinity and depositions remains negative and significant at the 10% level, but its absolute value increases. This further suggests the early data is capturing a beginning stage in the recovery process during which pH is not readily responding to emissions reductions. In both estimations we cannot reject the hypothesis that the average autocovariance in residuals of order 1 is zero, and we reject the hypothesis that the average autocovariance in residuals of order 2 is zero only in the state equation for alkalinity. This indicates that although our results are favorable, they must be interpreted with care as there is likely still an endogeneity problem and therefore some bias in the estimated coefficients.

As discussed when previously deriving our theoretical model, we choose to ignore certain variable factors, such as precipitation, in order to simplify the estimates of acid deposition, and thereby the solution of the
optimal control model. We now check the impact of this simplifying assumption by adding to both regression equations an interaction term for lagged precipitation and depositions. We use data on annual precipitation which includes total rain and snowfall. Due to the remoteness of many of the lakes, we only have data from two weather stations, one which is representative of the lakes located South (upwind) of Sudbury, and a second that is representative of the lakes located North (downwind) of Sudbury. For each lake, we interact the lagged annual precipitation from the appropriate monitoring site with the estimate of lagged depositions at that lake. The results from estimating our regressions with this added term are given in column (3) of Tables 2 and 3. In the estimation of both state equations, the direct effect of precipitation on the impact of depositions is negligible, with the coefficient on the interaction term virtually equal to zero, although not significant in the state equation for pH. The inclusion of the additional term, however, does alter the previous results. In the state equation for pH, the coefficient on lagged pH slightly decreases, while the coefficient on depositions decreases. We observe the same results in the state equation for alkalinity, although the effects are larger, with the coefficient on depositions almost doubling with the inclusion of the precipitation term.

Similar to lakes that are recovering from eutrophication, a lake that is recovering from acidification may be either reversible, hysteretic, or irreversible. In addition, some lakes may be acidified in their natural state, in which case we would not expect to observe a significant recovery over time. We do not directly observe which of the study lakes fall into each of these categories, however, there are large differences in the recovery rates of lakes. The pH of the most improved lake increases by 1.97 between 1981 and 2004, while the pH of the least improved increases by only 0.14. To better understand these differences, and the impact of specific lake characteristics on water quality recovery, we add to both regressions interaction terms for lagged depositions and a set of time invariant lake characteristics\(^\text{16}\). In the results presented in column (4) of Tables 2 and 3, we include characteristics available for all lakes – direction from Sudbury, lake area, shoreline length, elevation, and whether there is road access. In the state equation for alkalinity, addition of these variables has only a small effect. None are significant, and the impact on estimation of the variables from the original state equation is almost identical to that from adding the precipitation term. We observe more significant

\(^{16}\text{With eutrophication of lakes, a significant variable in this context is depth. Often referred to as the shallow lakes problem, shallow lakes are more likely to be either hysteretic or irreversible as they tend to have higher rates of phosphorous recycling, making them unresponsive to phosphorous input controls [3]. We are interested in seeing if there are any similar characteristics of acidified lakes that are significant in determining how and whether they recover from acidification.}\n
\[\text{[text continues...]}\]
results in the state equation for pH. The coefficient on direction from Sudbury is negative and significant, indicating that as expected, a decrease in depositions has a greater effect on the pH of lakes that are downwind of Sudbury. The coefficients on elevation, and whether there is road access are also characteristics that significantly change the impact of depositions on pH. With inclusion of these extra variables, the coefficient on estimated depositions more than triples, while the coefficient on the interaction between alkalinity and weighted emissions is no longer significant. As this coefficient was previously only significant at the 10% level and we have gotten an unexpected negative sign in all our regression results, this suggests the interaction between pH, alkalinity and depositions is not well identified in our data. A contributing factor to this is likely that the recovery of lakes from acidification will often take much longer than 25 years. Therefore, our time period of observations may be too short to accurately identify the relationship between pH, alkalinity and depositions as the lake converges to its natural state.

We also estimate the equations using the full set of characteristics available for only a subset of the lakes. However, we do not report these results here as they are not readily interpretable. In the state equation for pH, the only significant variable is the coefficient on lagged pH and the interaction between depositions and road access, while in the state equation for alkalinity the only significant variable is the coefficient on lagged alkalinity. We suspect the lack of results in this estimation may be due to the reduced sample size since we use observations for only 36 lakes. To check this, we re-estimate the regressions reported in column (4) of Tables 2 and 3 using only data from the 36 lakes for which all characteristics are available. We again find the only significant parameters are the lagged dependent variables, and the interaction of road access with depositions in the state equation for pH. This suggests the smaller sample size is driving the lack of results when we include the additional lake characteristics as dependent variables.

As discussed in our data section, we do not have accurate information on annual acid depositions at each lake site, and therefore do not have a good estimate of \( c_m \). We use as a rough estimate the inverse of the lake’s distance from Sudbury. To see how our results change using an alternative estimate for \( c_m \), we re-estimate our three Arellano-Bond regressions using the inverse of the lake’s squared distance from Sudbury as the estimate for \( c_m \). The results from these regressions are reported in columns (5) through (7) of Tables 2 and 3. They are qualitatively similar to those previously found. The coefficient on the lagged dependent variable
is positive, significant, and less than one in all three specifications. The coefficient on depositions is negative, and increases as additional terms are added to the state equations. It is also significant in all estimations except the final specification of the state equation for pH. The coefficient on the interaction term in the state equation for pH remains negative, although in this case it is only significant in the specification where the interaction of lagged depositions and precipitation is included. Finally, we again find the coefficient on the interaction term of lagged depositions and precipitation is virtually zero, while the coefficients on the interaction terms of lagged depositions and lake characteristics are generally not significant.

With the alternative estimate of $c^m$ we do find a large difference in the estimated marginal effect of a decrease in emissions on pH and alkalinity. Using the inverse of the distance from Sudbury as the estimate of $c^m$, for a lake that is located 60 km from Sudbury, the original state equation indicates a 10 kt decrease in emissions will increase pH by 0.02, and alkalinity by 0.043. With the inverse of the squared distance from Sudbury as the estimate of $c^m$, however, the estimate of the increase in pH is only 0.0023, and for alkalinity is 0.0062. On average, for the three specifications of the state equations, using the inverse of distance rather than distance squared as the estimate of $c^m$ increases the estimate of the marginal effect of depositions on improvements in pH and alkalinity by approximately six times. This suggests that more work must be done in either identifying a more accurate method for estimating $c^m$, or in testing to see which of our current estimates is most appropriate.

6 Conclusion

The acidification of freshwater lakes continues to be a significant environmental problem in many areas of the world. The problem of how to optimally implement emissions reductions that will lead to their recovery therefore remains a relevant policy question. We develop a framework for an optimal control problem that seeks to answer this question. A define a social welfare function that is maximized by a social regulator who must balance the cost of imposing emissions reductions on firms against the social disutility resulting from the existence of acidified lakes that are unable to support aquatic life. The change in water quality over time is described by two state equations which measure how the pH and alkalinity of the lake water respond to emissions reductions by firms. The parameters of the state equations are estimated using the Arellano-Bond
estimator, and the results are mostly consistent with the theoretical model of lake recovery.

The main contribution of this paper is in the development of a model that considers the acidification dynamics of freshwater lakes, and the role of emissions reductions in the recovery of lakes from acidification. As outlined above, however, there remain numerous research questions that we must address before we can derive a solution to the optimal control model. While we do not attempt to address these questions here, we do provide a reasonable starting framework for further work.
References


