An Empirical Investigation of the Welfare Effects of Banning Wholesale Price Discrimination*

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(online appendix)

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ABSTRACT

Online Appendix on Deriving uniform wholesale price discrimination margins.

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1. Deriving Margins under Uniform Pricing

In this appendix I derive the uniform pricing margins to be used for policy simulations. Let me first define a N_U by N matrix U that has as many rows, as many different manufactured products (N_U), and has N columns equal to retail level products, and which element $U(i,j) = 1$ if product i is the same manufactured product as product j and is equal to zero otherwise. For example, assume three products,$A1$, $A2$ and $B1$, at the consumer level, where the first two are produced by manufacturer A and are the same product, and where product B is sold at retailer 1 and produced by manufacturer B. The matrix U that describes which manufactured products are in fact the same, for the three sets of products above, is a 2 by 3 matrix $U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Following this simple example, each of the two retailers, 1 and 2, maximizes the profit function $\pi_1 = [p_{A1} - p_A^w - c_{A1}^r]\, q_{A1}(p) + [p_{B1} - p_B^w - c_{B1}^r]\, q_{B1}(p)$, and $\pi_2 = [p_{A2} - p_A^w - c_{A2}^r]\, q_{A2}(p)$, respectively. Note that the wholesale price for A is the same for both retailers. Solving for optimal price cost margins yields a system that implicitly defines three retail prices as a function of two wholesale prices. Generally, retailers maximize their profits as given by equation

$$\pi_r = \sum_{j \epsilon S_r} \left[p_j - p_j^w - c_j^r\right] q_j(p) \quad \text{for} \quad r = 1, ...R, \tag{1}$$

where S_r is the set of products sold by retailer r, but now the same wholesale price is charged for the same manufactured products regardless of retail outlet. If retailers behave as Nash-Bertrand players, then the price-cost margins for all products in vector notation m_r are

$$\underbrace{p - p^w - c^r}_{m^r} = -[T_r * \Delta_r]^{-1} q(p). \tag{2}$$

Manufacturers choose wholesale prices p^w to maximize their profits

$$\pi_{wt} = \sum_{j \epsilon S_{wt}} [p_{jt}^w - c_{jt}^w]\; q_{jt}(p(p^w)), \tag{3}$$

where S_{wt} is the set of products sold by manufacturer w during week t and c_{jt}^w is the marginal cost of the manufacturer that produces product j, knowing that retailers behave according to (2), and subject to U. Manufacturers now are only able to choose wholesale prices for N_U products, since some manufactured products sold through different retailers are the same and therefore need to be set at the same wholesale price. For example, manufacturers maximize their profits with respect to only two wholesale prices, $\pi_A = [p_A^w - c_A^w]\,[q_{A1}(p(p_A^w, p_{B1}^w)) + q_{A2}(p(p_A^w, p_{B1}^w))]$ and $\pi_B = [p_{B1}^w - c_B^w]\, q_{B1}(p(p_A^w, p_{B1}^w))$, respectively.

Let us now derive the mark-ups for the general case, keeping the notation as in the no uniform wholesale price model. Solving for the first-order conditions from the manufacturers' profit-maximization problem, assuming again a pure-strategy Nash equilibrium in wholesale prices and using matrix notation, yields:

$$\underbrace{(p^w - c^w)}_{m_U^w} = - \underbrace{\left[\left(\underbrace{T_w^{Up}}_{N_U \text{ by } N} * \underbrace{\Delta_w^{Up}}_{N_U \text{ by } N} \right) \underbrace{U'}_{N \text{ by } N_U} \right]^{-1}}_{(N_U \text{ by } N_U)} \underbrace{[U\ q(p)]}_{(N_U \text{ by } 1)}, \tag{4}$$

where U is the (N_U by N) matrix defined above, T_w^{Up} and Δ_w^{Up} are (N_U by N) matrices to be derived next, and $*$ represents the element-by-element multiplication of both matrices. The (N_U by N_U) full rank matrix is inverted. Note that the derived wholesale mark-ups are denoted by the (N_U by 1) vector m_U^w and that $N - N_U$ products share the same wholesale prices and mark-ups due to uniform wholesale pricing restrictions. If manufacturers behave as Nash-Bertrand players subject to uniform wholesale pricing restrictions, then equation (4) describes their supply relation.

To obtain Δ_w^{Up}, first note that $\Delta_w^{Up} = \underbrace{(\Delta_p^{Up})'}_{(N_U \text{ by } N)} \Delta_r$, where Δ_p^{Up} is a matrix of derivatives of all retail prices with respect to all the N_U independent wholesale prices. To get the expression for $\Delta_p^{N_U}$, I start by totally differentiating for a given j the first order conditions of the retailers with respect to all retail prices ($dp_k, k = 1, \cdots, N$), and with respect to a single wholesale price p_f^w, with variation dp_f^w. Note that the first-order conditions, assuming a pure-strategy Nash equilibrium in retail prices, are:

$$q_j + \sum_{m \epsilon S_r} T_r(m,j)\,[p_m - p_m^w - c_m^r]\,\frac{\partial q_m}{\partial p_j} = 0 \text{ for } j = 1, \ldots N, \tag{5}$$

where matrix T_r has the general element $T_r(i,j) = 1$ if the retailer sells both products i and j, and is equal to zero otherwise.

Putting all $j = 1$,...N products together, let G be the matrix with general element $g(j,k)$, and let H_f be an N-dimensional vector with general element $H(j,f)$, as defined in Villas-Boas (2007). Note now that $N - N_U$ wholesale price variations are not independent.

In terms of matrix notation, when solving for the derivatives of all retail prices, with respect to the wholesale price p_f^w, the f-th column of Δ_p^{Up} is obtained as:

$$\frac{dp}{dp_f^w} = G^{-1} \underbrace{[H_j + \ldots + H_k]}_{H_f^{Up}}, \text{ where } j, \ldots, k = f, \text{ are restricted to be the same in } U. \tag{6}$$

Stacking all $N - N_U$ independent-wholesale-price-corresponding columns together, $\Delta_p^{Up} = G^{-1} H^{Up}$ reflects the derivatives of all N retail prices with respect to N_U wholesale prices, where the general element of Δ_p^{Up} is $(i,j) = \frac{\partial p_i}{\partial p_j^w}$.

For the simple example, (4) corresponds to $\begin{bmatrix} m_A^w \\ m_{B1}^w \end{bmatrix} = - \begin{bmatrix} \frac{\partial(q_{A1}+q_{A2})}{\partial p_A^w} & 0 \\ 0 & \frac{\partial q_{B1}}{\partial p_{B1}^w} \end{bmatrix}^{-1} \begin{bmatrix} q_{A1} + q_{A2} \\ q_{B1} \end{bmatrix}$, where $\frac{\partial q_i}{\partial p_f^w} = \sum_k (\frac{\partial (q_i}{\partial p_k} \frac{\partial p_k}{\partial p_f^w})$, $k = A1, A2, B1$, and now Δ_p^{Up} is 3 by 2 and gives the responses of the three retail prices with respect to changes in the two upstream wholesale prices. This matrix is constructed by totally differentiating the system of three first order conditions (for the three retail prices) of the two retailers, subject to common wholesale price for the products $A1$ and $A2$.

Villas-Boas, S. B. 2007. "Vertical Relationships Between Manufacturers and Retailers: Inference With Limited Data," *The Review of Economic Studies*, 74 (2): 625-52.