AN EMPIRICAL INVESTIGATION OF THE WELFARE EFFECTS OF BANNING WHOLESALE PRICE DISCRIMINATION*

SOFIA BERTO VILLAS-BOAS
(online appendix)

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ABSTRACT
Online Appendix on Deriving uniform wholesale price discrimination margins.

* Department of Agricultural and Resource Economics, University of California at Berkeley; sberto@are.berkeley.edu. Online Appendix on Deriving uniform wholesale price discrimination margins.
1. Deriving Margins under Uniform Pricing

In this appendix I derive the uniform pricing margins to be used for policy simulations. Let me first define a \( N_U \) by \( N \) matrix \( U \) that has as many rows, as many different manufactured products (\( N_U \)), and has \( N \) columns equal to retail level products, and which element \( U(i, j) = 1 \) if product \( i \) is the same manufactured product as product \( j \) and is equal to zero otherwise. For example, assume three products, \( A1, A2 \) and \( B1 \), at the consumer level, where the first two are produced by manufacturer \( A \) and are the same product, and where product \( B \) is sold at retailer 1 and produced by manufacturer \( B \). The matrix \( U \) that describes which manufactured products are in fact the same, for the three sets of products above, is a 2 by 3 matrix

\[
\begin{bmatrix}
1 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

Following this simple example, each of the two retailers, 1 and 2, maximizes the profit function

\[
\pi_1 = [p_{A1} - p_{wA} - c_{A1}] q_{A1}(p) + [p_{B1} - p_{wB} - c_{B1}] q_{B1}(p), \quad \text{and} \quad \pi_2 = [p_{A2} - p_{wA} - c_{A2}] q_{A2}(p),
\]

respectively. Note that the wholesale price for \( A \) is the same for both retailers. Solving for optimal price cost margins yields a system that implicitly defines three retail prices as a function of two wholesale prices. Generally, retailers maximize their profits as given by equation

\[
\pi_r = \sum_{j \in S_r} [p_j - p_{wj} - c_{rj}] q_j(p) \quad \text{for} \quad r = 1, \ldots, R, \quad (1)
\]

where \( S_r \) is the set of products sold by retailer \( r \), but now the same wholesale price is charged for the same manufactured products regardless of retail outlet. If retailers behave as Nash-Bertrand players, then the price-cost margins for all products in vector notation \( m_r \) are

\[
\overbrace{p - p_{w} - c}^{m_r} = -[T_r * \Delta_r]^{-1} q(p). \quad (2)
\]

Manufacturers choose wholesale prices \( p_{w} \) to maximize their profits

\[
\pi_{wt} = \sum_{j \in S_{wt}} [p_{wj} - c_{jt}] q_j(p(p_{wj})), \quad (3)
\]

where \( S_{wt} \) is the set of products sold by manufacturer \( w \) during week \( t \) and \( c_{jt} \) is the marginal cost of the manufacturer that produces product \( j \), knowing that retailers behave according to (2), and subject to \( U \). Manufacturers now are only able to choose wholesale prices for \( N_U \) products, since some manufactured products sold through different retailers are the same and therefore need to be set at the same wholesale price. For example, manufacturers maximize their profits with respect to only two wholesale prices,

\[
\pi_A = [p_{wA} - c_{A}][q_{A1}(p(p_{wA}, p_{wB1})) + q_{A2}(p(p_{wA}, p_{wB1}))] \quad \text{and} \quad \pi_B = [p_{wB1} - c_{B}] q_{B1}(p(p_{wA}, p_{wB1})),
\]

respectively.

Let us now derive the mark-ups for the general case, keeping the notation as in the no uniform wholesale price model. Solving for the first-order conditions from the manufacturers’ profit-maximization problem,
assuming again a pure-strategy Nash equilibrium in wholesale prices and using matrix notation, yields:

\[
(p^w - c^w)_{m^U} = -\left[ \begin{bmatrix} T^U_{m^U, m^U} & \Delta^U_{m^U, m^U} \end{bmatrix}_{N_U \times N_U} \end{bmatrix} \begin{bmatrix} U'_{N_U, m^U} \end{bmatrix}_{N_U \times 1} \right]^{-1} \begin{bmatrix} U \ q(p) \end{bmatrix}_{N_U \times 1},
\]

where \( U \) is the \((N_U \times N)\) matrix defined above, \( T^U_{m^U, m^U} \) and \( \Delta^U_{m^U, m^U} \) are \((N_U \times N_U)\) matrices to be derived next, and \(*\) represents the element-by-element multiplication of both matrices. The \((N_U \times N_U)\) full rank matrix is inverted. Note that the derived wholesale mark-ups are denoted by the \((N_U \times 1)\) vector \( m^U \) and that \( N - N_U \) products share the same wholesale prices and mark-ups due to uniform wholesale pricing restrictions. If manufacturers behave as Nash-Bertrand players subject to uniform wholesale pricing restrictions, then equation (4) describes their supply relation.

To obtain \( \Delta^U_{m^U, m^U} \), first note that \( \Delta^U_{m^U, m^U} = (\Delta^U_{p^m, p^m})' \Delta_r \), where \( \Delta^U_{p^m, p^m} \) is a matrix of derivatives of all retail prices with respect to all the \( N_U \) independent wholesale prices. To get the expression for \( \Delta^N_{p^m, p^m} \), I start by totally differentiating for a given \( j \) the first order conditions of the retailers with respect to all retail prices \( (dp_k, k = 1, \ldots, N) \), and with respect to a single wholesale price \( p^m_i \), with variation \( dp^m_i \). Note that the first-order conditions, assuming a pure-strategy Nash equilibrium in retail prices, are:

\[
q_j + \sum_{m \epsilon S_r} T_r(m, j) [p_m - p^m_m - c^m_m] \frac{\partial q_m}{\partial p_j} = 0 \quad \text{for} \quad j = 1, \ldots, N,
\]

where matrix \( T_r \) has the general element \( T_r(i, j) = 1 \) if the retailer sells both products \( i \) and \( j \), and is equal to zero otherwise.

Putting all \( j = 1, \ldots, N \) products together, let \( G \) be the matrix with general element \( g(j, k) \), and let \( H_j \) be an \( N \)-dimensional vector with general element \( H(j, f) \), as defined in Villas-Boas (2007). Note now that \( N - N_U \) wholesale price variations are not independent.

In terms of matrix notation, when solving for the derivatives of all retail prices, with respect to the wholesale price \( p^m_j \), the \( f \)-th column of \( \Delta^U_{p^m, p^m} \) is obtained as:

\[
\frac{dp}{dp^m_j} = G^{-1} \begin{bmatrix} H_j + \cdots + H_k \end{bmatrix}_{H^U_{j, f}}, \quad \text{where} \quad j, \ldots, k = f, \quad \text{are restricted to be the same in} \ U.
\]

Stacking all \( N - N_U \) independent-wholesale-price-corresponding columns together, \( \Delta^U_{p^m, p^m} = G^{-1} H^U_{p^m} \) reflects the derivatives of all \( N \) retail prices with respect to \( N_U \) wholesale prices, where the general element of \( \Delta^U_{p^m, p^m} \) is \((i, j) = \frac{\partial m_i}{\partial p_j}\).

For the simple example, (4) corresponds to

\[
\begin{bmatrix} m^U_{A1} \\ m^U_{B1} \end{bmatrix} = -\begin{bmatrix} \frac{\partial (q_{A1} + q_{A2})}{\partial p_{A}} & 0 \\ \frac{\partial q_{B1}}{\partial p_{A}} & \frac{\partial q_{B1}}{\partial p_{B1}} \end{bmatrix}_{2 \times 2}^{-1} \begin{bmatrix} q_{A1} + q_{A2} \\ q_{B1} \end{bmatrix},
\]

where \( \frac{\partial m_{A1}}{\partial p_{A}} = \sum_k (\frac{\partial q_{A1}}{\partial p_k} \frac{\partial p_k}{\partial p_{A}}) \), \( k = A1, A2, B1 \), and now \( \Delta^U_{p^m, p^m} \) is \( 3 \times 2 \) and gives the responses of the three
retail prices with respect to changes in the two upstream wholesale prices. This matrix is constructed by totally differentiating the system of three first order conditions (for the three retail prices) of the two retailers, subject to common wholesale price for the products $A_1$ and $A_2$.