Supplement to Vertical Contracts between Manufacturers and Retailers: An Empirical Analysis*

Simple 2 by 2 Model

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ABSTRACT

This supplement presents the expressions for the retailers' and manufacturers' price-cost margins in a "simple" model for the supply side with two manufacturers who sell two products to two retailers. This is done given a Logit demand model and then given the random coefficients model for demand. In particular, I focus on the supply model of double marginalization since the price-cost margins for the other supply scenarios considered in this paper can be derived from the formulae of the double marginalization price-cost margins by changing the ownership matrices accordingly. Expressions for the crucial matrices for obtaining the retailers' and the manufacturers' price-cost margins are provided.

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1. Solution for a Simple 2 by 2 by 2 Model for Supply

This supplement presents the expressions for the retailers' and manufacturers' price-cost margins in a "simple" model for the supply side with two manufacturers who sell two products to two retailers. This is done given a Logit demand model and then given the random coefficients model for demand. In particular, I focus on the supply model of double marginalization since the price-cost margins for the other supply scenarios considered in this paper can be derived from the formulae of the double marginalization price-cost margins by changing the ownership matrices accordingly. Expressions for the crucial matrices for obtaining the retailers' and the manufacturers' price-cost margins are provided. These crucial matrices are:

1. The retail and manufacturer ownership matrices (T_r and T_w), with general elements $T_r(j,k) = 1$ if both products j and k are sold at the same retailer (and zero otherwise) and $T_w(j,k) = 1$ if both products j and k are produced by the same manufacturer (and zero otherwise).

2. The retailer's response matrix Δ_r containing the derivatives of the shares with respect to all retail prices.

3. The manufacturer's response matrix Δ_w containing the derivatives of the shares with respect to all wholesale prices.

The third matrix is very complicated to obtain, while the others are very straightforward. A procedure to obtain Δ_w is described and intermediary expressions handy for computer coding are provided.

The rest of this supplement is organized as follows. The ownership matrices are derived first and then the expressions for the price-cost margins in the double marginalization model are obtained. A short illustration is provided next on how all the price-cost margins in the other supply models considered are derived. Finally, expressions for the response matrices Δ_r and Δ_w are given.

1.1. The Ownership Matrices

The case of two manufacturers a and b producing one good each, which they sell to two retailers 1 and 2 is considered. Furthermore, and for the purpose of illustrating the price-cost margins for scenario 3, it is assumed that the good produced by manufacturer a is a private label of retailer 1. This implies that there are three products in this model. Let product a_1 be the private label produced by manufacturer a and sold to retailer 1. Product b_1 is produced by manufacturer b and sold to retailer 1 and product b_2 is produced by manufacturer b and sold to retailer 2. This implies that the retailer's (T_r) and the wholesaler's

(T_w) ownership matrices are given by

$$T_r = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } T_w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

1.2. Double Marginalization Model

In this model the manufacturers set their prices first and then the retailers follow. Each one of the two retailers maximizes his profit function given by

$$\pi_{rt} = \sum_{j \epsilon S_{rt}} \left[p_{jt} - p_{jt}^w - c_{jt}^r \right] s_{jt}(p) \quad \text{for} \quad r = 1, 2 \tag{1}$$

where S_{rt} is the set of products sold by retailer r in week t, p_{jt} is the retail price, p_{jt}^w is the wholesale price he pays, c_j^r is the retailer's marginal cost and $s_{jt}(p)$ is the product's market share. The first order conditions, assuming a pure strategy Nash-equilibrium in prices, are

$$s_{jt} + \sum_{m=a_1,b_1,b_2} T_r(m,j) \left[p_{mt} - p_{mt}^w - c_{mt}^r \right] \frac{\partial s_{mt}}{\partial p_{jt}} = 0 \quad \text{for} \quad j = a_1, b_1, b_2 \tag{2}$$

or in matrix notation

$$\begin{bmatrix} \frac{\partial s_{a1}}{\partial p_{a1}} & \frac{\partial s_{b1}}{\partial p_{a1}} & 0 \\ \frac{\partial s_{a1}}{\partial p_{b1}} & \frac{\partial s_{b1}}{\partial p_{b1}} & 0 \\ 0 & 0 & \frac{\partial s_{b2}}{\partial p_{b2}} \end{bmatrix} \begin{bmatrix} p_{a1} - p_{a1}^w - c_{a1}^r \\ p_{b1} - p_{b1}^w - c_{b1}^r \\ p_{b2} - p_{b2}^w - c_{b2}^r \end{bmatrix} = - \begin{bmatrix} s_{a1} \\ s_{b1} \\ s_{b2} \end{bmatrix}.$$

Define $[A * B]$ as the element by element multiplication of the two matrices of the same dimensions A and B. Then using the ownership matrix notation and solving for the price-cost margins yields

$$\begin{bmatrix} p_{a1} - p_{a1}^w - c_{a1}^r \\ p_{b1} - p_{b1}^w - c_{b1}^r \\ p_{b2} - p_{b2}^w - c_{b2}^r \end{bmatrix} = - \left[\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \frac{\partial s_{a1}}{\partial p_{a1}} & \frac{\partial s_{b1}}{\partial p_{a1}} & \frac{\partial s_{b2}}{\partial p_{a1}} \\ \frac{\partial s_{a1}}{\partial p_{b1}} & \frac{\partial s_{b1}}{\partial p_{b1}} & \frac{\partial s_{b2}}{\partial p_{b1}} \\ \frac{\partial s_{a1}}{\partial p_{b2}} & \frac{\partial s_{b1}}{\partial p_{b2}} & \frac{\partial s_{b2}}{\partial p_{b2}} \end{bmatrix} \right]^{-1} \begin{bmatrix} s_{a1} \\ s_{b1} \\ s_{b2} \end{bmatrix}. \tag{3}$$

Writing the price-cost margins for all products in vector notation gives the implied price-cost margins for the retailers

$$p - p^w - c^r = -[T_r * \Delta_r]^{-1} s(p), \tag{4}$$

which is a system of three implicit functions that expresses the three retail prices as a function of the wholesale prices.

Looking now at the manufacturers, they maximize their profits choosing the wholesale prices p^w knowing that the retailers behave according to (4). The first-order conditions are, assuming again pure strategy Nash-Equilibrium in the wholesale prices,

$$\begin{bmatrix} \frac{\partial s_{a1}}{\partial p^w_{a1}} & 0 & 0 \\ 0 & \frac{\partial s_{b1}}{\partial p^w_{b1}} & \frac{\partial s_{b2}}{\partial p^w_{b1}} \\ 0 & \frac{\partial s_{b1}}{\partial p^w_{b2}} & \frac{\partial s_{b2}}{\partial p^w_{b2}} \end{bmatrix} \begin{bmatrix} p^w_{a1} - c^w_{a1} \\ p^w_{b1} - c^w_{b1} \\ p^w_{b2} - c^w_{b2} \end{bmatrix} = - \begin{bmatrix} s_{a1} \\ s_{b1} \\ s_{b2} \end{bmatrix}. \tag{5}$$

Solving for the price-cost margins of the manufacturers and using the ownership matrix notation yields

$$\begin{bmatrix} p^w_{a1} - c^w_{a1} \\ p^w_{b1} - c^w_{b1} \\ p^w_{b2} - c^w_{b2} \end{bmatrix} = - \left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} * \begin{bmatrix} \frac{\partial s_{a1}}{\partial p^w_{a1}} & \frac{\partial s_{b1}}{\partial p^w_{a1}} & \frac{\partial s_{b2}}{\partial p^w_{a1}} \\ \frac{\partial s_{a1}}{\partial p^w_{b1}} & \frac{\partial s_{b1}}{\partial p^w_{b1}} & \frac{\partial s_{b2}}{\partial p^w_{b1}} \\ \frac{\partial s_{a1}}{\partial p^w_{b2}} & \frac{\partial s_{b1}}{\partial p^w_{b2}} & \frac{\partial s_{b2}}{\partial p^w_{b2}} \end{bmatrix} \right]^{-1} \begin{bmatrix} s_{a1} \\ s_{b1} \\ s_{b2} \end{bmatrix} \tag{6}$$

or short

$$(p^w_t - c^w_t) = -[T_w * \Delta_w]^{-1} s_t(p), \tag{7}$$

where c^w is the manufacturer's marginal cost.

1.3. The Other Supply Models

The price-cost margins of the other supply models considered can be obtained from equations (3) and (6) by changing the ownership matrices accordingly. First, for the model that assumes zero manufacturer margins and retailers having the pricing decisions (Scenario 2 Case 1), the price-cost margins for the manufacturers are zero for all products, since $p^w = c^w$ and the price-cost margins for the retailers are given by (3).

Second, for the model that assumes zero retail margin and in which the manufacturers have the pricing decision (Scenario 2 Case 2), the price-cost margins for the retailers are zero for all products since $p = c^r + p^w$. The price-cost margins for the manufacturers are given by (3) with the only change that the ownership matrix in the expression is now T_w and not T_r.

Third, for the Hybrid model (Scenario 3), retailer 1 is vertically integrated with respect to its own private label a_1, therefore $p^w_{a1} = c^w_{a1}$. Compared to scenario 1, this has no impact in the price-cost margins of the competing retailer 2 and of the other brands sold by retailer 1. This is due to the fact that the private label brand a_1 is not sold at the other retailer. Therefore the retail price-cost margins are given again by (3). On the contrary, by vertically integrating into the upstream industry, the retailer affects the price-cost margins of the other manufacturers since p^w_{a1} is not optimized over. Price-cost margins for the manufacturers are obtained from (6) by ignoring all rows and columns that correspond to the first product a_1. This yields

$$\begin{bmatrix} p^w_{b1} - c^w_{b1} \\ p^w_{b2} - c^w_{b2} \end{bmatrix} = -\left[\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} \frac{\partial s_{b1}}{\partial p^w_{b1}} & \frac{\partial s_{b1}}{\partial p^w_{b2}} \\ \frac{\partial s_{b1}}{\partial p^w_{b2}} & \frac{\partial s_{b2}}{\partial p^w_{b2}} \end{bmatrix}\right]^{-1} \begin{bmatrix} s_{b1} \\ s_{b2} \end{bmatrix}.$$

In terms of the economic intuition, by vertically integrating into the upstream market the retailer eliminated the double margin in its own product. So manufacturers of other products see the final retail price of product $a1$ fall. Accordingly, demand for the products that they produce changes and they need to adjust their own wholesale prices, since the ones that they were charging before are no longer optimal.

Fourth, in the manufacturer collusion model (Scenario 4), if manufacturers are colluding, the relevant ownership matrix for the manufacturers is T_1, an ownership matrix full of ones. It is as if a single manufacturer was deciding the wholesale prices for all the products. For the retailers, the relevant ownership matrix is still T_r. The price-cost margins for retailers are the same as in scenario 1, given by (3). The price-cost margins for the manufacturers are given by (6) but substituting T_w by T_1, that is,

$$\begin{bmatrix} p^w_{a1} - c^w_{a1} \\ p^w_{b1} - c^w_{b1} \\ p^w_{b2} - c^w_{b2} \end{bmatrix} = -\left[\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} \frac{\partial s_{a1}}{\partial p^w_{a1}} & \frac{\partial s_{b1}}{\partial p^w_{a1}} & \frac{\partial s_{b2}}{\partial p^w_{a1}} \\ \frac{\partial s_{a1}}{\partial p^w_{b1}} & \frac{\partial s_{b1}}{\partial p^w_{b1}} & \frac{\partial s_{b2}}{\partial p^w_{b1}} \\ \frac{\partial s_{a1}}{\partial p^w_{b2}} & \frac{\partial s_{b1}}{\partial p^w_{b2}} & \frac{\partial s_{b2}}{\partial p^w_{b2}} \end{bmatrix}\right]^{-1} \begin{bmatrix} s_{a1} \\ s_{b1} \\ s_{b2} \end{bmatrix}.$$

Fifth, in the retail collusion model (Scenario 5), the price-cost margins for the retailers are given by (3) changing the ownership matrix to T_1 since they are the ones colluding now. The price-cost margins for manufacturers are the same as in scenario 1, given by equation (6).

Sixth and finally, in the monopolist model (Scenario 6), the relevant ownership matrix is T_1, a matrix full of ones, since joint profit maximization is done. The sum of retailers' and wholesalers' price-cost margins are given by (3) with the unitary ownership matrix instead of T_r.

1.4. The Response Matrix Δ_r

Δ_r is a matrix with derivatives of all shares with respect to all retail prices, with general element $\Delta_r(j,k) = \frac{\delta s_k}{\delta p_j}$. For the Logit model, its expression is straightforward:

$$\Delta_r = \begin{bmatrix} \alpha s_{a1}(1-s_{a1}) & -\alpha s_{a1}s_{b1} & -\alpha s_{a1}s_{b2} \\ -\alpha s_{b1}s_{a1} & \alpha s_{b1}(1-s_{b1}) & -\alpha s_{b1}s_{b2} \\ -\alpha s_{b2}s_{a1} & -\alpha s_{b2}s_{b1} & \alpha(1-s_{b2})s_{b2} \end{bmatrix}.$$

For the random coefficients model, Δ_r is computed by simulation. Given N_s random draws of unobserved and observed consumer characteristics,

$$\Delta_r = \begin{bmatrix} \sum_{n=1}^{N_s} \alpha_n s_{a1,n}(1-s_{a1,n}) & -\sum_{n=1}^{N_s} \alpha_n s_{a1,n}s_{b1,n} & -\sum_{n=1}^{N_s} \alpha_n s_{a1,n}s_{b2,n} \\ -\sum_{n=1}^{N_s} \alpha_n s_{b1,n}s_{a1,n} & \sum_{n=1}^{N_s} \alpha_n s_{b1,n}(1-s_{b1,n}) & -\sum_{n=1}^{N_s} \alpha_n s_{b1,n}s_{b2,n} \\ -\sum_{n=1}^{N_s} \alpha_n s_{b2,n}s_{a1,n} & -\sum_{n=1}^{N_s} \alpha_n s_{b2,n}s_{b1,n} & \sum_{n=1}^{N_s} \alpha_n (1-s_{b2,n})s_{b2,n} \end{bmatrix},$$

The term α_n is the marginal utility of price for a certain consumer n and is given by

$$\alpha_n = \alpha + \Gamma_\alpha D_n + \Upsilon_\alpha v_n$$

where α is the mean across consumers of the marginal utility of price, D_n are the observed characteristics of consumer n, v_n are the unobserved characteristics of consumer n, Γ_α is the first column of Γ and Υ_α is the first element of Υ.

1.5. The Response Matrix Δ_w

This matrix Δ_w contains the derivatives of all shares with respect to all wholesale prices and has the general element $\Delta_w(j,k) = \frac{\delta s_k}{\delta p_j^w}$. An expression for this matrix is very complicated even in the Logit case. After noting that $\Delta_w = \Delta_p' \Delta_r$, where Δ_p is a matrix of derivatives of all the retail prices with respect to all the wholesale prices, one needs only to find an expression for Δ_p to get Δ_w. Let us start by totally differentiating, for a given j, equation (2) with respect to all prices $(dp_{a1}, dp_{b1}, dp_{b2})$ and a wholesale price p_f^w, with variation dp_f^w:

$$\sum_{k=a_1,b_1,b_2} \underbrace{[\frac{\partial s_j}{\partial p_k} + \sum_{i=a_1,b_1,b_2} (T_r(i,j)\frac{\partial^2 s_i}{\partial p_j \partial p_k}(p_i - p_i^w - c_i^r)) + T_r(k,j)\frac{\partial s_k}{\partial p_j}]}_{g(j,k)} \; dp_k \; - \; \underbrace{T_r(f,j)\frac{\partial s_f}{\partial p_j}}_{h(j,f)} dp_f^w = 0. \tag{8}$$

Putting all j products together, let G be the matrix with general element $g(j,k)$ and let H_f be the J dimensional vector with general element $h(j,f)$. Then

$$G \; dp - H_f \; dp_f^w = 0. \tag{9}$$

Solving for the derivatives of all prices with respect to the wholesale price f the f-th column of Δ_p is obtained:

$$\frac{dp}{dp_f^w} = G^{-1}H_f. \tag{10}$$

Stacking all N columns together, $\Delta_p = G^{-1}H$, which has the derivatives of all prices with respect to all wholesale prices, with general element $(i,j) = \frac{\partial p_j}{\partial p_i^w}$.

The only expression in (8) that needs to be obtained is the (3 by 3 by 3) Hessian, a matrix of second derivatives of the shares with respect to all retail prices. For the Logit case the Hessian is given by

$$\frac{\partial^2 s_{it}}{\partial p_{jt}\partial p_{kt}} = \begin{cases} \alpha^2(1-2s_{it})s_{it}(1-s_{it}) & \text{if } i=j=k, \\ -2\alpha^2 s_{it}s_{jt}s_{kt} & \text{if } i\neq j \text{ and } j\neq k \text{ and } i\neq k, \\ \alpha^2 s_{it}s_{jt}(2s_{jt}-1) & \text{if } i\neq j \text{ and } j=k, \\ \alpha^2 s_{it}s_{kt}(2s_{it}-1) & \text{if } i\neq k \text{ and } i=j, \\ \alpha^2 s_{it}s_{jt}(2s_{it}-1) & \text{otherwise.} \end{cases} \tag{11}$$

The Hessian for the random coefficients models is obtained by simulation. Given N_s random draws of unobserved and observed consumer characteristics, the Hessian is computed as

$$\frac{\partial^2 s_{it}}{\partial p_{jt}\partial p_{kt}} = \begin{cases} \sum_{n=1}^{N_s}\alpha_n^2(1-2s_{it,n})s_{it,n}(1-s_{it,n}) & \text{if } i=j=k, \\ -2\sum_{n=1}^{N_s}\alpha_n^2 s_{it,n}s_{jt,n}s_{kt,n} & \text{if } i\neq j \text{ and } j\neq k \text{ and } i\neq k, \\ \sum_{n=1}^{N_s}\alpha_n^2 s_{it,n}s_{jt,n}(2s_{jt,n}-1) & \text{if } i\neq j \text{ and } j=k, \\ \sum_{n=1}^{N_s}\alpha_n^2 s_{it,n}s_{kt,n}(2s_{it,n}-1) & \text{if } i\neq k \text{ and } i=j, \\ \sum_{n=1}^{N_s}\alpha_n^2 s_{it,n}s_{jt,n}(2s_{it,n}-1) & \text{otherwise.} \end{cases}$$