Strategic overinvestment and/or strategic underinvestment

Basic framework:

Simple two period model

Period 1: incumbent, firm 1, chooses “investment” $k_1$, for example, capacity.
Firm 2, observes $k_1$, and then decides whether to enter.

If 2 does not enter,

$$\pi_2 = 0,$$
$$\pi_1 = \pi_1^m(k_1, X_1^m(k_1))$$

that is, firm 1 enjoys monopoly in the second period, choosing $X_1^m(k_1)$, that is monopoly X given k.

If 2 enters, firms choose simultaneously $X_1$ and $X_2$ and the profits are

$$\pi_1(k_1, X_1, X_2)$$
$$\pi_2(k_1, X_1, X_2)$$

where the entry cost of 2 is in there.

Given $k_1$, $X_1$ and $X_2$ are determined by Nash equilibrium: \{ $X_1^*(k_1), X_2^*(k_1)$ \}.

We’ll look at the effects of changing $k_1$ on the Nash equilibrium, assuming that this NE is unique and stable.

If $k_1$ is chosen such that

$$\pi_2^2(k_1, X_1^*(k_1), X_2^*(k_1)) \leq 0,$$ then entry is blockaded

$$\pi_2^2(k_1, X_1^*(k_1), X_2^*(k_1)) > 0,$$ then entry is accommodated.

Whether incumbent wants to accommodate or deter depends on the profits’ comparison on the incumbent doing one or the other option.
Assumptions:
\[ \pi^1(k_i, X_i^*(k_1), X_2^*(k_1)) \text{ and } \pi^m_i(k_i, X_i^m(k_1)) \]
are strictly concave in \( k_i \) and \( X_i^*(k_1) \) are differentiable.

Note: we’ll ignore the case when entry is blockaded since no strategic issues there.

DETERRENCE OF ENTRY

Incumbent chooses \( k_1 \) such that \( \pi^2(k_1, X_i^*(k_1), X_2^*(k_1)) = 0 \).

Which strategy relative to choosing for non strategic reasons (assuming firm 2 does not see \( k_1 \) before she enters, open loop, call that level of investment \( k_1^{OL} \)), should firm 1 use to make firm 2’s entry unprofitable?

Lets take the total derivative of firm 2’s profits with respect to \( k_1 \):

\[
\frac{d\pi^2}{dk_1} = \frac{\partial \pi^2}{\partial k_1} + \frac{\partial \pi^2}{\partial X_2} \frac{\partial X_2^*}{\partial k_1} + \frac{\partial \pi^2}{\partial X_1} \frac{\partial X_1^*}{\partial k_1}
\]

If \( \frac{d\pi^2}{dk_1} < 0 \) then investment in \( k_1 \) makes firm 1 tough, if \( > 0 \) then investment in \( k_1 \) makes firm 1 soft.

If investing makes the firm look tough then to deter entry the firm should over-invest, \( k_1 > k_1^{OL} \), to look “top dog”, very big and ready to fight.

If investing makes the firm look soft, then the firm should under-invest to deter entry, \( k_1 < k_1^{OL} \) looking “lean and hungry”.

ACCOMMODATION

Suppose deterring entry is too costly.
In deterring entry, \( k_1 \) was determined by post entry 2’s profits.
In accommodating firm 1’s behavior in first period \( k_1 \) is dictated by firm 1’s profit.

\[ \pi^1(k_1, X_i^*(k_1), X_2^*(k_1)) \]

The incentive to invest is given by the total derivative of the above with respect to \( k_1 \).
\[
\frac{d\pi^1}{dk_1} = \frac{\partial \pi^1}{\partial k_1} + \frac{\partial \pi^1}{\partial X_1} \frac{\partial X_1^*}{\partial k_1} + \frac{\partial \pi^1}{\partial X_2} \frac{\partial X_2^*}{\partial k_1}
\]

The DE exists regardless of firm 2 seeing \(k_1\) or not. So, firm should over-invest if \(\text{SE}>0\) and under-invest if \(\text{SE}<0\).

The key thing is figuring out the sign of \(\text{SE}\). The \(\text{sign}(\text{SE})\)

\[
\text{sign}\left(\frac{\partial \pi^1}{\partial X_2^*} \frac{\partial X_2^*}{\partial k_1}\right)
\]

Given that \(\frac{\partial X_2^*}{\partial k_1} = \frac{\partial X_2^*}{\partial X_1} \frac{\partial X_1^*}{\partial k_1} = R'_2 (X_1^*) \left(\frac{\partial X_1^*}{\partial k_1}\right)\) and 
\(\text{sign}\left(\frac{\partial \pi^1}{\partial X_2}\right) = \text{sign}\left(\frac{\partial \pi^2}{\partial X_1}\right)\), then

\[
\text{sign}\left(\frac{\partial \pi^1}{\partial X_2} \frac{\partial X_2^*}{\partial k_1}\right) = \text{sign}\left(\frac{\partial \pi^1}{\partial X_1} \frac{\partial X_1^*}{\partial k_1}\right) \text{sign}\left(\frac{R'_2 (X_1^*)}{R'_1 (X_1^*)}\right).
\]

In the neighborhood of equilibrium, quantities are strategic substitutes, \(R'^*<0\) and prices are strategic complements, \(R'^*>0\).