

# **Intersectoral Adjustment and Policy Intervention: the Importance of General-Equilibrium Effects**

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## **Abstract**

We model adjustment costs in a general-equilibrium setting using a “transport sector.” This sector provides services needed to reallocate a factor of production across two other sectors. A market imperfection in the transport sector causes adjustment to occur too slowly in the absence of government intervention. The government has a restricted menu of second-best policies to remedy this imperfection. Given this restricted menu, the optimal policy choice depends on the government’s ability to make commitments. The key to these results is our replacement of the black box of adjustment costs with an explicit model of these costs.

## **1. Introduction**

Many economies suffer from massive misallocation of resources, particularly labor. For example, transitional economies in Central and Eastern Europe began the reform process with bloated state sectors and anemic private sectors. Other developing countries pursuing market reforms face adjustment costs as labor and other factors of production leave formerly protected sectors. To a lesser extent, the process of multi-lateral trade liberalization and economic unification in developed countries (e.g. NAFTA and the EU) requires the reallocation of resources. We study the conditions that affect a government’s ability to manage sectoral reallocation of factors of production when adjustment is costly and the government has a limited policy menu.

The costs associated with factor adjustment may be large and take varied forms. Most of the academic policy debate treats adjustment costs as a black box (Hamer-mesh and Pfann, 1996).<sup>1</sup> These models typically assume that the costs of changing a quasi-fixed factor (such as the stock of labor in a particular sector) depend only on the amount of intersectoral migration. A common assumption, used by Krugman (1991), Karp and Paul (1994), and Terra (1999) is that the *supply function* for “migration services” is fixed, and that the equilibrium level of migration is endogenous.

Following Mussa (1978), we include a “transport sector” in a model of intersectoral labor migration. The transport sector provides services, such as education and retraining. Workers need these services in order to move from one sector to another. The equilibrium supply function for these services depends on factor prices, and is therefore endogenous.

Taxing or subsidizing a traded commodity influences the migration decision directly, because it affects the equilibrium wage differential across sectors. If the tax/subsidy affects the equilibrium wages of factors that are used in the transport sector, the policy also changes the equilibrium supply function for migration services. The change in this supply function alters the migration decision. We refer to the effect of the tax/subsidy

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on the migration decision that operates via the endogenous supply function for migration services as the “indirect link.” This indirect link is absent in models of adjustment costs that exclude the transport sector.

It appears to be widely accepted that adjustment costs are important in developing and transitional economies, and that policies designed to manage adjustment are also important. However, we might guess that the indirect link described above would have only second-order welfare effects. Instead, we find that this indirect link qualitatively alters the nature of government policies. The main contribution of this paper is to improve our intuition for how these kinds of indirect (general-equilibrium) links affect policy in dynamic models. The introduction of the transport sector means that the *current* policy level affects current migration, via the indirect link. In the absence of the transport sector—as in Karp and Paul (1994) and Terra (1999)—only *future* policy levels affect the current migration decision. These future policies operate via the change in the stream of future wage differentials.

The inclusion of the transport sector is important not only because it affects the policy analysis, but also because the internal consistency of the model requires an explanation of the source of adjustment costs. Presumably these costs arise because adjustment uses scarce factors. These factors receive a payment, so the costs to one sector of the economy represent returns to another sector. Excluding the transport sector implies that adjustment costs simply disappear from the economy.

A model with adjustment costs but without a transport sector is consistent with a situation where foreigners supply adjustment services; i.e. these services are a traded good. (For example, OECD countries provide higher education for many workers from developing countries.) However, if we view transport services as a traded good, and also maintain the small-country assumption, the marginal cost of these services is constant. In that case, the adjustment to a new equilibrium occurs in a single period: “adjustment costs” provide no rationale for dynamics, and the policy questions that we study become moot. In contrast, when the transport services are domestically produced, the supply function for these services is upward sloping. In this case, adjustment occurs over many periods, giving rise to the policy questions that we study.

In our model, nonstrategic workers with rational expectations decide whether to migrate. Their equilibrium decision rules depend on their beliefs about future wage differentials and migration costs, which depend on government actions. In order to model general-equilibrium effects in this setting, in a manner simple enough to improve our intuition, we have to make a number of assumptions. We now outline these assumptions and explain why they are reasonable and how they contribute to the goal of creating a useful model. We then describe the major insight derived from this model.

We use a three-factor, three-sector model of a small open economy. The aggregate supplies of the three factors are fixed. There are two traded goods sectors, referred to as the growing sector and the shrinking sector, and a “transport sector” that produces nontraded educational services required for intersectoral migration. One of the three factors, skilled labor, is used in all three sectors. Skilled labor is a mobile factor, i.e. it can be reallocated across sectors costlessly. A second factor, semi-skilled labor, is used in only the two traded goods sectors. Semi-skilled labor is quasi-fixed—the allocation is fixed in the short run, but semi-skilled labor can move across sectors over time by incurring a convex adjustment cost (faster adjustment entails a higher cost). The third factor is fixed and is used only in the transport sector. The presence of this factor is used only as a justification for decreasing returns to scale in the transport sector; decreasing returns to scale imply that the marginal cost of migration is positively

sloped; i.e. adjustment costs are convex. The identity of the two factors used in the traded goods sectors is not important to our results, but it is important that there are costs of moving one of these factors across sectors. The supply of educational services, and thus the amount of intersectoral migration of semi-skilled workers at a point in time, depends on the amount of skilled labor in the transport sector.

In the first period, the wages of semi-skilled workers are different in the two traded goods sectors: the economy therefore is not initially at a steady state. This disequilibrium may be the result of a recent terms-of-trade shock, a political upheaval that has changed the rules of the economy (as in Central and Eastern Europe in the early 1990s) or a trade agreement (as with Mexico following NAFTA). In the absence of government intervention, semi-skilled workers want to leave the low-wage sector, so we refer to that as the shrinking sector. Government intervention can change the incentive for migration, thereby changing the equilibrium amount of migration. However, in our model, the identity of the shrinking sector does not change, regardless of whether the government intervenes and regardless of the type of policy that it uses.

In each period semi-skilled workers decide whether to move from the shrinking to the growing sector. Workers migrate if their current cost of migration (the price of education) is less than the present discounted value of the stream of future wage differentials. In the absence of uncertainty, workers with rational expectations correctly forecast future wage differentials and migration costs. These depend on future government policies.

A market imperfection causes the equilibrium level of migration to be slower than the social optimum. If the government could use a first-best policy, e.g. a policy that targets the transport sector, it would achieve the social optimum. That government has no need to commit to future policies. In the real world, most feasible policies are second-best, i.e. they give rise to a secondary distortion. In order to model this aspect of reality, we assume that the government is restricted to using an output tax/subsidy on only one of the traded goods, and we treat the targeted sector as exogenous.<sup>2</sup> These policies are less distortionary than a tariff, which also creates a consumption distortion. A key assumption in our model is that any available policy creates a secondary distortion; we do not require that the distortion is severe. The use of second-best policies to influence forward-looking agents typically leads to a time-consistency problem, raising the issue of commitment.

Our analysis illuminates a simple point, applicable in situations where a government is restricted to using a particular class of second-best policies (a commodity tax/subsidy for a traded good, in our setting). If this government is able to make commitments, the outcome may be rather insensitive to the precise choice of the second-best policy (the sector that receives the tax/subsidy). Commitment ability enables the government to achieve nearly the primary objective (the optimal level of migration), while distributing the secondary distortions optimally over time. A government that is not able to make commitments is in a weaker position, and incurs larger secondary distortions. The indirect policy links described above—the influence of the policy on the supply function for migration services—can either reinforce or weaken the direct effects operating through the change in the wage differential. These indirect links can therefore ameliorate the lack of commitment ability, or they can give rise to perverse incentives that cause well-intentioned policy to reduce welfare.

Section 2 presents the model. Section 3 describes the two types of equilibria that we study. In the first equilibrium, the government is able to make binding commitments regarding future policy levels, at the initial time. In the second (Markov-perfect) equilibrium, the government cannot make commitments about future policies. We use the

equilibrium conditions to obtain intuition for the relation between the degree of commitment ability and the welfare effects of policy; this relation depends on the sector that is targeted—a decision that is exogenous in our model. Section 4 presents numerical experiments that test this intuition. These experiments show that when the market failure in the transport sector results in migration being too slow in the competitive equilibrium, then: (i) If the government is able to make credible commitments about future policy, it should target the growing sector. If it makes a mistake and instead targets the shrinking sector, the loss in welfare is small. (ii) If the government is not able to make credible commitments about future policy, it should target the shrinking sector. If it makes a mistake and instead targets the growing sector, social welfare is lower than under non-intervention. In this case, well-intentioned intervention is disadvantageous.

## 2. The Model

The important assumptions of our model are: (i) Agents have rational expectations and they incur a cost of reallocating factors of production. This reallocation is like any other investment decision; it requires a comparison of current costs and future benefits. (ii) The reallocation cost depends on the quantity of factors being reallocated at a point in time, and also on the prices of factors used in the sector that provides the reallocation services. These factor prices are endogenous (unlike in most models of adjustment cost). (iii) A market failure causes the adjustment trajectory to differ from the first-best trajectory. (iv) The government has a restricted policy menu. Any instrument it can use leads to a secondary distortion.

In order to develop a concrete model, we need to specialize each of these assumptions. The specializations that we adopt are: (i) The factor being “reallocated” is semi-skilled labor. (ii) The cost of moving from one sector to another arises from the need for retraining, which is provided by skilled labor. Skilled labor is mobile—at least in relation to semi-skilled labor. (iii) An externality in the sector that provides retraining leads to an inefficient provision of services. (iv) The government is able to use only sector-specific output taxes or subsidies.

### *Production*

The small open economy consists of two traded goods sectors, 1 and 2, and the transport sector. Each traded goods sector uses skilled and semi-skilled labor, denoted  $N$  and  $L$ . Skilled labor is perfectly mobile between sectors, but semi-skilled labor requires retraining in order to be used in a different sector, and is therefore a quasi-fixed factor. The fixed supplies of skilled and semi-skilled labor in the economy are  $\bar{N}$  and  $\bar{L}$ . We use the following restricted profit function:

$$W^i(p_i, v, L^i) = \max_{N^i, Y^i} \{p_i Y^i - v N^i \mid Y^i \leq F^i(N^i, L^i)\},$$

where  $v$  is the skilled labor wage, and  $p_i$  and  $Y^i$  are sector  $i$  price and output;  $F^i(N^i, L^i)$  is sector  $i$ 's constant-returns-to-scale production function. Since  $W^i$  is linearly homogeneous, we have:

$$W^i(p_i, v, L^i) = L^i w^i(p_i, v).$$

We assume free entry into the traded goods sectors, leading to zero profits. This assumption means that the equilibrium wage of semi-skilled labor in sector  $i$  is the

function  $w^i(p_i, v)$ .<sup>3</sup> Partial derivatives of this function with respect to  $p_i$  and  $v$  equal sector  $i$  output per unit of semi-skilled labor ( $w_p^i$ ) and minus sector  $i$  input demand of skilled labor per unit of semi-skilled labor ( $w_v^i$ ). The equilibrium wage function has the following properties:

$$\begin{aligned} w_{pp}^i &\geq 0, \quad w_{vv}^i \geq 0, \quad w_{vp}^i = w_{pv}^i \leq 0, \\ w^i &= p^i w_p^i + v w_v^i, \quad p^i w_{pp}^i + v w_{vp}^i = 0 \\ p^i w_{pv}^i + v w_{vv}^i &= 0, \quad w_{pp}^i w_{vv}^i - (w_{vp}^i)^2 = 0. \end{aligned} \quad (1)$$

Hereafter we set  $N = N_2$  and  $L = L_2$ , the sector 2 level of skilled and semi-skilled labor.

We assume that in the absence of adjustment costs, both traded goods would be produced in a competitive equilibrium. This assumption means that it is unnecessary to consider corner solutions—an irrelevant complication.

### *Migration*

All consumers face constant world prices and have the same linearly homogeneous utility function, so they have constant marginal utility of income. In this case, the benefit of migration is proportional to the present discounted value of the stream of future wage differentials. If a worker migrates to sector  $i$ , he pays the migration costs in this period and begins receiving the sector  $i$  wage in the next period. In our continuous-time model, the “next period” occurs immediately.

We assume that at time 0, *in the absence of government intervention* the semi-skilled wage is lower in sector 1:

$$w^2(p_{2,0}, v_0) > w^1(p_{1,0}, v_0). \quad (2)$$

This inequality means that the initial allocation of semi-skilled workers is not a steady state (i.e. a long-run equilibrium), as we discuss in section 3. Workers move from sector 1 to sector 2 to take advantage of the higher wage: sector 1 is the declining sector.

For a discount rate  $r$ , the present value of migrating in period  $t$  is  $q_t$ :

$$q_t = \int_{\tau=0}^{\infty} e^{-r\tau} [w^2(p_{2,t+\tau}, v_{t+\tau}) - w^1(p_{1,t+\tau}, v_{t+\tau})] d\tau,$$

which implies:

$$\dot{q}_t \equiv \frac{dq}{dt} = r q_t - [w^2(p_{2,t}, v_t) - w^1(p_{1,t}, v_t)]. \quad (3)$$

Hereafter we suppress the time subscripts when there is no ambiguity.

To move from one sector to another, a semi-skilled worker must pay for retraining. He buys these “transport services” from the transport sector. There are  $n$  (an exogenous constant) identical, competitive firms in the transport sector. These firms employ skilled labor and a sector-specific factor, and they operate under decreasing returns to scale (e.g. because of the sector-specific input). Each firm creates an externality. If a representative firm  $j$  hires  $N_j$  units of skilled labor, and the total amount of skilled labor in the transport sector is  $N_T$ , then firm  $j$  retrains (transports)  $u_j$  workers per unit of time. The transport sector production function for firm  $j$  is

$$u_j = \sqrt{2\tilde{\gamma}} N_T^{0.5-\theta} N_j^\theta, \quad j = 3, 4, \dots, n+2, \quad (4)$$

where  $\sqrt{2\tilde{\gamma}}$  is a scaling coefficient. (We suppress the sector-specific input in writing the production function.) The parameter  $\theta$ ,  $0 < \theta < 1$ , determines the extent and direction of the externality in the transport sector. If  $\theta < \frac{1}{2}$ , as we hereafter assume, other firms' costs decrease when firm  $j$  hires an extra skilled worker. In this case, there is a positive externality in the transport sector, and the competitive equilibrium results in migration occurring too slowly. We can think of each of the transport firms as a training institution. When  $\theta < \frac{1}{2}$ , these institutions create positive spillovers.

Migrants are willing to pay  $q$  to move into sector 2, and firm  $j$  must pay  $v$  for every skilled worker it hires, so firm  $j$ 's profits are

$$q[\sqrt{2\tilde{\gamma}}N_T^{0.5-\theta}N_j^\theta] - vN_j. \tag{5}$$

The firm's optimal purchase of skilled labor is given by the first-order condition

$$\theta q[\sqrt{2\tilde{\gamma}}N_T^{0.5-\theta}N_j^{\theta-1}] = v. \tag{6}$$

The externality is evident in this equation. When choosing its production level, firm  $j$  does not take into account how its decision affects the sector's productivity, via  $N_T^{0.5-\theta}$ . (In contrast, in Krugman (1991) there is increasing returns in the production of one of the traded goods; when deciding whether to migrate, workers do not take into account the effect of their decision on marginal productivity in that sector.) Using the assumption of identical firms,  $N_T = nN_j$ , we can solve equation (6) to obtain the equilibrium value of  $N_T$ :

$$\sqrt{N_T} = \frac{\sqrt{2\tilde{\gamma}\theta q}}{v} n^{1-\theta}. \tag{7}$$

The price  $q$  affects the equilibrium number of skilled workers in the transport sector. It therefore affects the equilibrium supply of migration services and the equilibrium flow of migrants. The flow of workers into the growing sector is

$$\dot{L}_t \equiv \frac{dL_t}{dt} = u_t = nu_{j,t}.$$

Using equations (4) and (7) we can write the change in the stock of labor in sector 2 as a function of the current prices,  $q$  and  $v$ :

$$\dot{L}_t = u_t = nu_{j,t} = \frac{2\theta\tilde{\gamma}q_t}{v_t} n^{2-2\theta} = \frac{2\theta\gamma q_t}{v_t}, \tag{8}$$

where the last equality uses the definition  $\gamma \equiv \tilde{\gamma}n^{2-2\theta}$ .

The market-clearing condition for skilled labor is:

$$\bar{N} = -(\bar{L} - L)w_v^1(p_1, v) - Lw_v^2(p_2, v) + N_T.$$

The first two terms on the right-hand side are the demand for skilled labor in sectors 1 and 2, and the third term is the amount of labor used in the transport sector. Using (7), the market-clearing condition becomes:

$$\bar{N} = -(\bar{L} - L)w_v^1(p_1, v) - Lw_v^2(p_2, v) + \frac{2\theta^2\gamma}{v^2} q^2. \tag{9}$$

We refer to the equilibrium trajectory of  $L$  in the absence of government intervention as the private adjustment equilibrium (PAE). The initial condition is  $L_0$ , the predeter-

mined amount of semi-skilled labor in sector 2 at time 0. In our model, the marginal cost of adjustment (migration) approaches 0 as the level of migration approaches 0. In the PAE, the unique steady state is  $L_\infty$ ; this value is obtained by setting the intersectoral wage differential (for both skilled and semi-skilled labor) equal to 0. (Here we use the assumption of an interior equilibrium.) At the steady state,  $q_\infty = 0$ . In addition to these three boundary conditions, the PAE satisfies equations (3), (8), and (9).

In the absence of a distortion ( $\theta = \frac{1}{2}$ ) the competitive equilibrium trajectory of  $L$  is efficient. If there is a distortion and the government can target the migration sector, the optimal policy supports the efficient outcome.<sup>5</sup> We refer to this outcome—whether it occurs because there is no distortion, or because the government has a first-best policy to correct the distortion—as the first-best equilibrium (FBE). The FBE satisfies the same boundary conditions and equations as the PAE—only the parameter  $\theta$  is different.

For future use, we note (using equation (9)) the following relations between the equilibrium skilled wage,  $v$ , and the output prices in the tradable sectors,  $p^i$ , for fixed values of  $L$  and  $q$ :

$$\frac{\partial v}{\partial p_1} = -\frac{(\bar{L} - L)w_{vp}^1}{(\bar{L} - L)w_{vv}^1 - Lw_{vv}^2 + (4\theta^2\gamma/v^3)q^2} \geq 0 \quad (10)$$

$$\frac{\partial v}{\partial p_2} = -\frac{Lw_{vp}^2}{(\bar{L} - L)w_{vv}^1 - Lw_{vv}^2 + (4\theta^2\gamma/v^3)q^2} \geq 0. \quad (11)$$

### *National Welfare*

At this point we need to refine the notation in order to distinguish between the world price and the domestic producer price of a tradable good. Denote the world price of good  $i$  as  $p_i$ , a constant. If the government targets sector  $i$ , the domestic producer price is  $\tilde{p}_{i,t} = p_i + s_{i,t}$ , where  $s_{i,t}$  is the sector  $i$  subsidy. ( $s_{i,t} < 0$  means that sector  $i$  is taxed.) If the government does not intervene in sector  $i$ , then  $\tilde{p}_{i,t} = p_i$ .

The flow of utility at a point in time depends only on the consumption of traded goods. (Migration services are valuable to the economy, but they do not contribute to the flow of current utility.) Under the assumption that the economy is unable to borrow or lend, the balance-of-payments constraint requires equality between the value of consumption of and production of traded goods, evaluated at world prices. This constraint implies that the level of national income in a period ( $Y_t$ ) equals the value of production in the two traded goods sectors:

$$Y_t \equiv [p_1(\bar{L} - L)w_p^1(\tilde{p}_{1,t}, v) + p_2Lw_p^2(\tilde{p}_{2,t}, v)]. \quad (12)$$

By assumption, the country is small and it does not use trade policies, so domestic consumer prices equal world prices. An intervention in sector  $i$  changes the argument of the wage function for that sector but it does not change the social shadow value (= the world price) of that sector's output.

Because of our assumption that agents have identical homothetic preferences, national welfare at a point in time is proportional to national income,  $Y_t$ . The factor of proportionality depends on the constant world prices. Since the value of this function is constant, it does not affect the government's optimization problem. If the government chooses a trajectory of interventions  $\{\tilde{p}_{i,t}\}_{t=0}^\infty$ , the present discounted value of future national income (social welfare) at time  $t$  is



$$J(L_t; \{\tilde{p}_{1,s}\}_{s=t}^{\infty} \{\tilde{p}_{2,s}\}_{s=t}^{\infty}) \equiv \int_0^{\infty} e^{-r\tau} Y(t+\tau) d\tau.$$

Social welfare depends on the current allocation of the quasi-fixed factor,  $L_t$ , and on the government’s policy sequence.

### 3. Equilibrium Policies

We state the government’s maximization problem and describe the two equilibrium concepts that we use to model different levels of commitment ability. We then state and explain two conjectures. The next subsections use these conjectures and the necessary conditions to the government’s optimization problems (under the two polar assumptions about its commitment ability) to provide intuition for the nature of the equilibria.

#### *The Government’s Problem and the Two Equilibria*

The government’s problem is

$$\max \int_0^{\infty} e^{-r\tau} Y(t+\tau) d\tau \quad \text{s.t. (3), (8), (12), and (9), } L_t \text{ given.} \tag{13}$$

The government is able to use an output subsidy/tax in either but not in both of the traded goods sectors. If the government intervenes in sector  $i$ , then  $\tilde{p}_{j,t} = p_j$ , for  $j \neq i$ . Given the assumption that there is a positive externality in the transport sector, migration occurs too slowly in the absence of government intervention. By targeting one sector, the government is able to alter the wage differential for semi-skilled workers, and the wage for skilled workers, but it cannot choose these two variables independently.

In order to complete the description of the government’s problem, we need to specify the extent to which it can commit to future policies. We consider two polar assumptions about its ability to make commitments. These two assumptions lead to two different control problems, and two different equilibrium trajectories. The extent of the government’s ability to make commitments about the policy level—infinite in the first case and zero in the second—distinguishes the two types of equilibria. The government’s objectives and agents’ decision problems (migration and allocation of skilled labor) are the same under both equilibria. In every case we assume that the government takes the policy instrument (as opposed to the policy level) and the sector that it targets as given.

*The open-loop equilibrium* In the open-loop equilibrium (OLE) the government can credibly commit to all future policies at time 0; here we need only solve the problem stated in (13) for  $t = 0$ . Given that the government has decided to target sector  $i$ , the equilibrium policy in the OLE is the function of time and the initial value of  $L$ ,  $\{\tilde{p}_{i,s}(L_0)\}_{s=0}^{\infty}$ , that solves (13) for  $t = 0$ .

We noted in section 2 that the steady state is  $L_{\infty}$  in both the PAE and the FBE. More generally, the steady state is equal to  $L_{\infty}$  in any open-loop program, even when the government uses a second-best policy. It is feasible to maintain a steady state not equal to  $L_{\infty}$ , but such a policy cannot be optimal. At a steady state, migration is 0, so the marginal adjustment cost is 0. At a steady state not equal to  $L_{\infty}$  there is a first-order welfare cost due to the production inefficiency. A small amount of migration leads to a first-order improvement in this inefficiency, and entails only second-order adjustment costs. Con-



sequently, the steady state is  $L_\infty$  in the PAE, the FBE, and in any open-loop equilibrium that uses a second-best policy. In view of inequality (2), the initial stock of semi-skilled labor in sector 2 is less than  $L_\infty$ . Sector 2 grows along the equilibrium trajectory.

*The no-commitment equilibrium* In “no-commitment equilibria” the government is free to choose the current policy level in every period, but cannot commit to future policy levels. There are two approaches in modeling this kind of equilibria. One approach allows agents to condition their expectations and their decisions on some part of the history of the game. In this case, there typically exist many subgame-perfect equilibria. Outcomes that give the leader a payoff approximately equal to the full-commitment level can sometimes be sustained using trigger strategies. In this situation the commitment problem becomes unimportant.<sup>6</sup>

The other alternative to modeling “no-commitment” equilibria, which we adopt, assumes that agents base their expectations and their decisions only on the “payoff-relevant” state variable. For workers, this state is the current level of prices and the current level of  $L$ ; for the government, the state is the current level of  $L$ . (In the current period, the government chooses prices by means of the tax/subsidy.) A subgame-perfect equilibrium in which strategies depend only on the payoff-relevant state variable is “Markov-perfect.” In order to obtain a Markov-perfect equilibrium (MPE) we need to find a decision rule for the government,  $s_t = s_t^{MPE}(L)$ , that maps the current level of  $L$  into the current subsidy. The requirement of subgame-perfection implies that this decision rule solves the problem given by (13) for every feasible value of  $L$  (and thus, for every  $t$ ). This decision rule must be optimal given the government’s beliefs about how agents will behave in the future. In equilibrium, agents’ beliefs about future prices and the government’s beliefs about future migration decisions are correct.

In the MPE, the government’s optimization problem at time  $t$  does not involve the flow of utility over the time interval  $(0, t)$ . At time  $t$  the government cannot influence previous events; these events are therefore irrelevant to the government’s decision problem at time  $t$ , except in so far as they affect the current state,  $L_t$ . The government at time  $t$  cares about current and future levels of  $Y$ . It cannot commit to future subsidy levels, but it may be able to influence these levels by influencing the change in  $L$ . In the OLE, the government takes into account the flow of  $Y$  over  $(0, t)$  when choosing the time  $t$  subsidy  $s_t^{OLE}(t)$ . (Recall that the government chooses  $s_t^{OLE}(t)$  at time 0.) In both the OLE and the MPE, the subsidy that agents expect the government to use at time  $t$  affects their migration decisions before time  $t$ ; the migration decision depends on agents’ beliefs about future prices, and thus about future subsidies.

Government intervention in the OLE cannot possibly be disadvantageous (i.e. it cannot possibly lead to a loss in social welfare), because in the OLE it is feasible for the government to commit to using no subsidy. In the MPE, on the other hand, the government cannot make this—or any other—kind of commitment. The Markov assumption also excludes the possibility of using reputation (or some kind of trigger strategy) to sustain a good outcome. Therefore, government intervention can be disadvantageous in an MPE.<sup>7</sup>

It is a matter of judgment whether the MPE or a history-dependent equilibrium provides a more reasonable description of situations where a government cannot commit to future behavior. We already noted that with history-dependent strategies/beliefs, there typically exists a great variety of subgame-perfect equilibria. This non-uniqueness makes it difficult to compare outcomes under complete commitment (the

OLE) and under no commitment, if the latter involves history-dependent strategies. (We would have to decide which of the history-dependent equilibria to use as a basis for comparison.)

However, even when agents use Markov-perfect strategies, the equilibrium may not be unique. There are two general reasons for non-uniqueness. First, if agents use strategies that are discontinuous in the state variable (here,  $L$ ) a great variety of equilibrium outcomes can be supported, as in Dutta and Sundaram (1993). These kinds of strategies are similar to trigger strategies, in so far as a small change in the state can precipitate large changes in beliefs and in equilibrium actions. We exclude this reason for non-uniqueness by assuming that strategies are not only continuous, but also differentiable in the state variable. This assumption means that a small change in the level of the state variable causes only a small change in agents' beliefs and equilibrium actions.

Even under the assumption of differentiable strategies, the MPE is typically not unique because the model has an "incomplete transversality condition" (alternatively, a "missing boundary condition") as in Tsutsui and Mino (1990) and Karp and Paul (1998). In order to select a unique equilibrium (within the class of differentiable strategies), we assume that workers believe that the government will maintain a zero tax/subsidy whenever the labor allocation is at the first-best steady state, defined above as  $L_{\infty}$ . This assumption provides the missing boundary condition and enables us to determine the equilibrium using the necessary conditions to the agents' and the government's control problems. The assumption is reasonable, because if agents have these beliefs, the optimal government policy confirms them, and the outcome is efficient in the steady state. That is, the steady state in the MPE is  $L_{\infty}$ .

### *Two Conjectures*

We are not able to determine analytically the properties of this model. However, the necessary conditions for optimality provide a great deal of information about the equilibrium, provided that two plausible conjectures hold. We now explain these conjectures. The next subsections use them to interpret the necessary conditions. Section 4 provides a numerical example for which the conjectures hold.

CONJECTURE 1. *The Hamiltonian to the government's control problem is globally concave in the control.*

If the necessary conditions to the government's control problem are satisfied, the Hamiltonian must be locally concave, so Conjecture 1 is a technical assumption. It enables us to make inferences about the sign of the optimal policy by evaluating the derivative of the Hamiltonian at a level of the policy equal to 0.

CONJECTURE 2. *In the OLE and the MPE, the marginal cost of migration is less than the social marginal benefit of migration.*

The economic basis for this conjecture is straightforward. A second-best policy causes a secondary distortion, so using the policy to increase the amount of migration is costly. Suppose, contrary to Conjecture 2, that the equilibrium second-best policy supports the first-best level of migration; in that case, a small decrease in the policy would cause a first-order increase in welfare (due to the decreased secondary distortion) but only a second-order loss in welfare due to the lower migration. The net

result would be a gain in welfare, contradicting the hypothesis that a second-best policy supports the first-best level of migration.<sup>8</sup>

We can state Conjecture 2 more precisely with a bit of additional notation. Using equations (4) and (8) we can write  $L^2 = 2\gamma N_T$ , which implies

$$\frac{dN_T}{dL} = \frac{\dot{L}}{\gamma}.$$

Since the price of skilled labor is  $v$ , the dollar cost of the additional labor needed to produce the marginal unit of migration (the marginal cost of migration) equals  $v\dot{L}/\gamma$ . The private value of migration is simply  $q$ . When the government does not intervene (i.e. in the private adjustment equilibrium—PAE) the gap between the marginal cost and the private marginal value of migration is

$$\frac{v\dot{L}}{\gamma} - q = \frac{v\dot{L}}{\gamma} - \frac{v\dot{L}}{2\gamma\theta} < 0, \quad (14)$$

where we use equation (8) to eliminate  $q$ , and the assumption  $\theta < 0.5$ . With the distortion, the social marginal cost of migration is less than the private marginal benefit of migration.

Note that in the absence of a distortion (i.e. if  $\theta = 0.5$ ) the marginal cost of migration equals the private marginal benefit. In this case, the competitive equilibrium is efficient: the marginal social cost of migration equals the private marginal benefit of migration. (Here, the inequality in equation (14) becomes an equality.)

If the government can use a first-best policy, it can support the distortion-free equilibrium. Denote  $\beta^{FB}$  as the marginal social value of having one more unit of labor in the growing sector rather than in the shrinking sector (i.e. the social marginal value of  $L$ ) in the case where the government is able to use a first-best policy.<sup>9</sup> In this case, it is straightforward to show that

$$\frac{v\dot{L}}{\gamma} - \beta^{FB} \equiv 0. \quad (15)$$

This equation says that, in a first-best equilibrium, the marginal cost of migration always equals the social shadow value of labor in the growing sector—as is true in the private adjustment equilibrium when there is no distortion ( $\theta = 0.5$ ). If there is no externality, or if the government can use a first-best policy to cause firms to internalize the externality, the social shadow value of labor in the growing sector ( $\beta^{FB}$ ) equals a worker's private value of being in the growing sector ( $q$ ).

Denote  $\beta^{SB}$  as the marginal social value of  $L$  when the government uses a second-best policy.<sup>10</sup> The value of  $\beta^{SB}$  depends on which second-best policy the government uses (OLE, MPE, targeting sector 1 or 2); here we use the term to denote a generic second-best policy. We can now restate Conjecture 2 as

$$\frac{v\dot{L}}{\gamma} - \beta^{FB} = 2\theta q - \beta^{SB} < 0, \quad (16)$$

which uses equation (8). Conjecture 2 simply states that the second-best policy does not eliminate the gap between the marginal cost of migration and the social marginal benefit of migration.

*The Open-Loop Equilibrium*

The two cases, where the government targets sector 1 (so that  $\tilde{p}_2 = p_2$ ) and where it targets sector 2 (so that  $\tilde{p}_1 = p_1$ ) are analytically symmetric. We concentrate on the first case, and then briefly discuss the second.

*Targeting sector 1* When the government is able to make binding commitments and seeks to maximize the discounted flow of national income, its current value Hamiltonian is

$$H^O \equiv [p_1(\bar{L} - L_t)w_p^1(\tilde{p}_{1,t}, v_t) + p_2L_tw_p^2(p_2, v_t)] + \beta_{1,t} \left[ \frac{2\theta\gamma q_t}{v_t} \right] + \beta_{2,t} [rq + w^1(\tilde{p}_{1,t}, v_t) - w^2(p_2, v_t)]. \tag{17}$$

(The superscript of  $H^O$  emphasizes that this is the open-loop equilibrium.) The costate variables associated with the states  $L$  and  $q$  are  $\beta_1$  and  $\beta_2$  and the function  $v_t = v(\tilde{p}_{1,t}, L_t, q_t)$  is implicitly given by equation (9).  $\beta_1$  is the social marginal value of  $L$  under a particular second-best policy; i.e. it is a particular value of the generic variable  $\beta^{SB}$  used in equation (16). Recall the definition  $\tilde{p}_{i,t} = p^i + s_{i,t}$ , where  $s_{i,t}$  is the sector  $i$  at time  $t$ . The necessary condition for maximization of the Hamiltonian (derived in the Appendix) is

$$\frac{\partial H^O}{\partial \tilde{p}_1} = 0 = P + S + C; \tag{18}$$

where:

$$P \equiv -s_1(\bar{L} - L) \left( w_{pp}^1 + w_{pv}^1 \frac{\partial v}{\partial \tilde{p}_1} \right);$$

$$S \equiv \frac{\partial v}{\partial \tilde{p}_1} \frac{2\theta\gamma q}{v^2} (2\theta q - \beta_1) = \frac{\partial v}{\partial \tilde{p}_1} \frac{2\theta\gamma q}{v^2} \left( \frac{vL}{\gamma} - \beta_1 \right);$$

$$C \equiv \beta_2 \left[ w_p^1 + \frac{\partial v}{\partial \tilde{p}_1} (w_v^1 - w_v^2) \right].$$

Equation (18) decomposes the effect of a sector 1 subsidy into three terms.

- The first term on the right-hand side of (18), denoted  $P$ , is the *production distortion effect*. The production distortion arises because the policy causes a misallocation of skilled labor between the two traded goods sectors. This distortion arises from a direct effect via a change in the producer price and an indirect effect via the price of skilled labor,  $v$ .
- The second term of (18), denoted  $S$ , is the *skilled wage effect*.  $S$  is an indirect, or general-equilibrium effect. An increase in the subsidy increases the wage of skilled workers (see expression (10)) thereby increasing the cost of retraining semi-skilled labor. The increase in  $v$  reduces migration by the amount  $-(2\theta\gamma q/v^2)$  (which equals  $\partial L/\partial v$  from equation (8)). The gap between the marginal cost of migration and the social value of an additional migrant is  $2\theta q - \beta_1 = (vL/\gamma) - \beta_1$ , which by Conjecture 2 (equation (16)) is negative. Thus, using equation (10), the skilled wage effect is non-positive. It is strictly negative whenever sector 1 uses a positive amount of skilled labor (so that  $\partial v/\partial \tilde{p}_1 > 0$ ).

- The last term of (18), denoted  $C$ , is the *commitment effect* of the subsidy. The subsidy affects the current wage differential and thus affects *previous* values of  $q$ . The effect of the subsidy on the evolution of  $q$  is

$$\frac{d\dot{q}}{ds_1} = w_p^1 + \frac{\partial v}{\partial \tilde{p}_1} (w_v^1 - w_v^2)$$

from equation (3). The variable  $\beta_2$  is the current value costate variable associated with the jump state  $q$ ; that is,  $\beta_2$  is the current shadow value of  $q$ .

The optimal policy requires balancing these three effects.

*The initial sector 1 open-loop production subsidy* To determine the sign of the open-loop sector 1 subsidy at time 0 we evaluate condition (18) at  $t = 0$  and  $s_{1,0} = 0$ , using the boundary condition  $\beta_2(0) = 0$ .<sup>11</sup> The result of this calculation is:

$$\left. \frac{\partial H^o}{\partial \tilde{p}_1} \right|_{t=0, s_1=0} = \left( \frac{\partial v}{\partial \tilde{p}_1} \frac{2\theta\gamma q}{v^2} \right) (2\theta q - \beta_1). \tag{19}$$

Since sector 2 is the growing sector, equation (8) implies  $q > 0$ ; this inequality and equation (10) imply that the term in the first parenthesis on the right-hand side of equation (19) is positive. If Conjecture 2 is true, the term in the second parenthesis is negative, so the right-hand side of equation (19) is negative.

If, in addition, the Hamiltonian is globally concave in the policy (Conjecture 1), equation (19) implies that the initial sector 1 subsidy is negative (a tax). The steady-state policy is always 0, since there is no migration in the steady state and therefore no distortion.

*Targeting sector 2* The analysis when the government targets sector 2 proceeds along the same lines. We can define the government’s Hamiltonian and obtain a first-order condition, and its decomposition, as in equation (18); we omit the details.

In order to accelerate migration, the government needs to increase the wage differential (relative to its level in the absence of intervention) over at least part of the trajectory. To achieve this increase in  $w^2 - w^1$  using a sector 2 output policy, the government needs to subsidize sector 2 during at least part of the trajectory. The somewhat surprising result is that the initial sector 2 policy involves *a tax rather than a subsidy*. The government’s first-order condition (for maximization of the Hamiltonian) when it targets sector 2, evaluated at  $t = 0 = s_2$ , is

$$\left. \frac{\partial H^o}{\partial \tilde{p}_2} \right|_{t=0, s_2=0} = \frac{\partial v}{\partial \tilde{p}_2} \frac{2\theta\gamma q}{v^2} (2\theta q - \beta_1). \tag{20}$$

(We abuse notation by again using  $\beta_1$  to denote the social shadow value of  $L$ , given that the government uses an open-loop policy and targets sector 2.)

Again, Conjectures 1 and 2 imply that the initial sector 2 policy is a tax, just as was the case with sector 1. (Of course, the level of the policies would not, in general, be the same.) Intervention in either sector causes a misallocation of the mobile resource, so the production effect is always negative. A small tax or subsidy creates only a second-order production loss. At the initial time, the commitment effect is 0, in view of the boundary condition  $\beta_2(0) = 0$ . Therefore the only incentive to intervene in the initial instant is due to the skilled wage effect. This indirect general-equilibrium effect works in the same direction, regardless of the sector being targeted. *A decrease in the initial price of either of the traded goods lowers the wage of skilled workers and makes migration cheaper, leading to faster migration.*

We have three pieces of information about the OLE policy that targets sector 2: (i) it begins as a tax, (ii) it must be a subsidy over some interval, and (iii) it eventually approaches 0. Therefore, the policy trajectory (as distinct from the trajectory of the state variable  $L$ ) is necessarily nonmonotonic. There is “phasing in and phasing out” of intervention.

*Comparing intervention in the two sectors* Although we cannot make definitive welfare comparisons between the two types of intervention, the forces that favor one form or the other are worth describing. Whatever sector the government targets, it has two channels for promoting migration. An increase in the future stream of the wage differential,  $w^2 - w^1$ , makes it more attractive to be in sector 2 and increases current migration via the commitment effect. This channel of influence encourages the government to tax sector 1 or to subsidize sector 2. The second channel of influence is the indirect effect, via the wage of skilled workers. Promoting current migration of semi-skilled workers requires reducing the wage of skilled workers. This reduction is achieved by taxing either of the traded goods sectors.

Thus, the two channels of influence work in the same direction when the government targets sector 1. Both channels call for an output tax: the indirect general-equilibrium effect reinforces the direct effect that occurs via the wage differential.

When the government targets sector 2, the indirect general-equilibrium effect works against the direct effect. We noted that the government needs to subsidize sector 2 over some interval to promote migration. However, the beneficial effect of this subsidy is muted because it is partly offset by the indirect general-equilibrium effect. This contrast favors targeting sector 1.

An opposing force makes the comparison ambiguous. Define  $s_i^*$  as the *ad valorem* equivalent of the unit subsidy:  $s_i^* \equiv s_i/p_i$ , so  $w^i = (1 + s_i^*)p_i F_L^i$ . Since sector 2 is the growing sector, its value of marginal productivity of semi-skilled labor (evaluated at world prices) is higher than in sector 1. Consequently, the direct effect on the wage of a small *ad valorem* subsidy is greater when the government targets sector 2:

$$\frac{\partial w^2(L, N_2)}{\partial s_2^*} = p_2 F_L^2 > p_1 F_L^1 = \frac{\partial w^1(1-L, N_1)}{\partial s_1^*}, \quad (21)$$

where  $N_i$  is the equilibrium level at world prices. The misallocation of skilled labor between the two traded goods sectors (the production distortion effect) causes a welfare loss regardless of which sector is targeted. The extent of this loss depends on the magnitude of the policy level. Equation (21) implies that a small level of intervention (expressed in *ad valorem* terms) has a greater effect on the wage differential when sector 2 is targeted.

In summary, targeting sector 2 gives the government more direct leverage, in the sense that it can achieve a larger change in the wage differential for a given level of intervention. However, the indirect general-equilibrium (skilled wage) effect reinforces the direct effect when the government targets sector 1, whereas it mutes the direct effect when the government targets sector 2. Thus, it remains an open question which of the two forms of intervention lead to higher welfare.

### *The Markov-Perfect Equilibrium*

In an MPE the government is unable to commit to future actions. At every point in time agents base their current decision on the current state variable,  $L_t$ , and on their

expectations of future events. In the absence of reputational effects, these expectations depend on the current state. In particular, agents' expectations about the future trajectory of the wage differential depend on  $L_t$ , so their expectations about the present discounted value of the stream of the wage differentials is a function of  $L_t$ . We denote agents' point expectation at time  $t$  of the present value of the wage differential as  $Q(L_t)$ . We assume that expectations are sufficiently regular that the derivative  $dQ/dL$  exists and is continuous.

The function  $Q(L)$  summarizes all of the information concerning agents' expectations that the government needs in order to solve its control problem. Since the government cannot commit to future policies, it is not able to choose the function  $Q(L)$ . That is, in solving its control problem, the government takes the function  $Q(L)$  as given.

Of course, the function  $Q(L_t)$  is not arbitrary. By virtue of its definition, it must satisfy the "consistency condition"

$$\frac{dQ}{dt} \equiv \frac{dQ}{dL} \frac{dL}{dt} = \frac{dq}{dt},$$

where the last derivative is given by equation (3). This consistency condition and the other necessary conditions for the government's control problem result in a system of equations with one degree of freedom. This indeterminacy is a consequence of the incomplete transversality condition mentioned in the "no-commitment equilibrium" subsection, above. The assumption that workers believe that the government will maintain a zero tax/subsidy whenever the labor allocation is at the first-best steady state provides the missing boundary condition, and enables us to solve for a unique equilibrium.

As above, we first consider the case where the government targets sector 1. The Hamiltonian for the government's problem is:

$$H^M = p_1(\bar{L} - L)w_p^1(\tilde{p}_1, v) + p_2Lw_p^2(p_2, v) + \beta \frac{2\theta\gamma}{v}Q(L), \tag{22}$$

where the superscript  $M$  emphasizes that this is the Hamiltonian for the MPE. In the MPE there is only one state variable, with costate variable  $\beta$  (a particular case of the shadow value under a second-best policy,  $\beta^{SB}$ ). The first-order condition for maximization of the Hamiltonian is (see the Appendix):<sup>12</sup>

$$\frac{\partial H^M}{\partial \tilde{p}_1} = 0 = P + S^M; \tag{23}$$

$$P \equiv -s_1(\bar{L} - L) \left( w_{pp}^1 + w_{pv}^1 \frac{\partial v}{\partial \tilde{p}_1} \right); \quad S^M \equiv \frac{\partial v}{\partial \tilde{p}_1} \frac{2\theta\gamma Q(L)}{v^2} (2\theta Q(L) - \beta).$$

Here we decompose the right-hand side of the first-order condition into two terms. For convenience, we repeat the definition of  $P$ , the production distortion effect, which is the same as in the OLE. The second term, the skilled wage effect  $S^M$ , has the same form as the skilled wage effect in the OLE ( $S$ ) except that the function  $Q(L)$  replaces the state variable  $q$ , and the costate variable  $\beta$  replaces  $\beta_1$ .

The comparison of equations (18) and (23) emphasizes the difference in the OLE and the MPE. In the MPE, the government is not able to use announcements of future policies to influence current migration, so there is no commitment effect. If we evaluate  $\partial H^M / \partial \tilde{p}_1$  at  $s_1(t) = 0$  (so that  $P = 0$ ) we obtain



$$\text{sign}\left(\frac{\partial H}{\partial \tilde{p}_1}\right)_{s_1=0} = \text{sign}(2\theta Q(L) - \beta) = \text{sign}\left(\frac{v}{\gamma} \dot{L} - \beta\right). \tag{24}$$

Once again, Conjectures 1 and 2 imply (using equation (24)) that the MPE sector 1 subsidy is negative, i.e. a tax.

We showed above that the OLE policy (when either sector is targeted) is a tax at  $t = 0$  (see equation (20)), but in the MPE the conclusion holds at all times. In the OLE the commitment effect is 0 at  $t = 0$  in view of optimality, but in the MPE the commitment effect is identically 0, because the government cannot make commitments.

We can repeat the above analysis when the government targets sector 2. We obtain a first-order condition that has the same form as equation (23), replacing the subscript 1 with 2. If we evaluate this condition at  $s_2(t) = 0$  we obtain a relation analogous to equation (24). Again, this relation leads to the conclusion (under Conjectures 1 and 2) that the MPE sector 2 policy is a tax at every point.

Thus, under Conjectures 1 and 2, the MPE involves an output tax *at every point in time, regardless of which sector is targeted*. In contrast, in the OLE (under the same hypotheses) both sectors are taxed at the initial time, but not necessarily at other times. In an MPE the only source of leverage for the government is the skilled wage effect. The government can encourage migration in the current period only by reducing the price of skilled labor, which requires taxing either sector 1 or 2.

Even though the government in the MPE is unable to use future policies to influence current migration, workers correctly anticipate the value of future policies. Their expectation of these policies affects their current decisions. When the government targets sector 1 and agents recognize that sector 1 output will be taxed in the future, the present value of the stream of the wage differential increases (relative to its non-intervention trajectory). The sector 1 tax encourages migration via agents' expectations.

However, when the government targets sector 2 and agents anticipate that the sector will be taxed in the future, the expectation of the present discounted value of the stream of the wage differential is lower (relative to non-intervention). The *current* sector 2 tax encourages current migration via the skilled wage effect, but the *anticipation of future* taxes discourages current migration, because it reduces the value of moving to sector 2. Since the MPE sector 2 policy involves a tax at every point in time, whereas the optimal (open-loop) sector 2 policy must involve a subsidy over at least part of the trajectory, government intervention can be disadvantageous.

We noted above that the possibility of disadvantageous intervention when the government cannot make commitments is not a new result. Our example is different, because it depends on an apparently innocuous choice of which sector the government targets.

#### 4. A Numerical Example

This section provides a numerical example which satisfies Conjectures 1 and 2. We use the example to illustrate the relative advantage of targeting different sectors in the OLE and the MPE, and to show that intervention can be disadvantageous when the government targets sector 2 in the MPE.

*Model specification and policy scenarios* We use the following Cobb–Douglas production functions:

$$F^1(N, L) = \left(\frac{N}{0.3}\right)^{0.3} \left(\frac{L}{0.7}\right)^{0.7} \quad \text{and} \quad F^2(N, L) = \left(\frac{N}{0.6}\right)^{0.6} \left(\frac{L}{0.4}\right)^{0.4}.$$

The associated equilibrium wage functions are:

$$w^1 = \tilde{p}_1^{\frac{1}{0.7}} v^{(1-\frac{1}{0.7})} \quad \text{and} \quad w^2 = \tilde{p}_2^{\frac{1}{0.4}} v^{(1-\frac{1}{0.4})}.$$

The parameters of the economy are set to the following values:  $\bar{L} = \bar{N} = 1$ ,  $p_1 = p_2 = 1$ ,  $\gamma = 1$ , and  $L(1) = 0.1$ . The distortion parameter  $\theta$  is set to  $\frac{1}{4}$  ( $< \frac{1}{2}$ ) so that migration is too slow under private adjustment. To implement the numerical experiments, we use a finite horizon, discrete time version of the model presented in section 2 and compute the equilibria using the algorithm introduced by Fair and Taylor (1983). The time horizon is 20 years and the discount rate  $r = 0.05$ , so  $\delta = (1/1.05)$ . The boundary value  $L(20)$  is free and there is no scrap value.

We study the following policy scenarios: the private adjustment equilibrium (PAE), the first-best equilibrium (FBE), the open-loop equilibrium when (OLE) the government targets sector  $i$  (OLE $i$ ), and the Markov-perfect equilibrium when the government targets sector  $i$  (MPE $i$ ). In the OLE, the government commits to a sequence of tax/subsidies at the beginning of the initial period. In the MPE, the government announces a level of tax/subsidy at the beginning of each period, taking as given the current amount of labor in the growing sector. In choosing its policy level in the MPE, the government maximizes the present discounted value of current and future welfare, ignoring the past.

In both the OLE and the MPE, agents correctly forecast the current (post-migration) wage and all future wages; the value of  $q$  at the beginning of a period is calculated using these forecasts. An agent makes the migration decision at the beginning of the period; an agent who migrates pays for education in that period and begins to receive the wage in the growing sector in the next period. In all scenarios, the steady state is reached after about 15–17 years. Since the externality vanishes at steady state, the steady-state government's subsidy is zero in all cases.

*Policy trajectories* Figure 1 shows the equilibrium policy trajectories OLE $i$  and MPE $i$  ( $i = 1, 2$ ). The OLE1 and MPE1 subsidies are both negative. When sector 1 is targeted, a tax increases the speed of labor reallocation because it lowers the price of skilled labor and because it increases the relative benefit of moving to the second sector. The ability to announce future policies in the OLE allows the government to use a smaller first-period tax (relative to the MPE), thus decreasing the initial welfare cost associated with the production distortion. In other words, with open-loop policies the government is able to use announcements to obtain benefits (faster migration) in the current period, and defer costs to later periods.

The OLE is nonmonotonic—the tax is phased in and then phased out. If the only means of influencing migration is via the commitment effect, the policy necessarily involves this kind of hump-shaped trajectory. The nonmonotonicity of policy in the current circumstance suggests that the commitment effect is important. The MPE policy trajectory, on the other hand, is monotonic, because the commitment effect is absent.

When the government targets sector 2, the open-loop policy begins with a tax in order to promote migration by reducing the initial cost of skilled labor. Subsequent open-loop policies involve sector 2 subsidies in order to promote migration by increasing the relative advantage of working in sector 2 (i.e. to exploit the commitment effect).

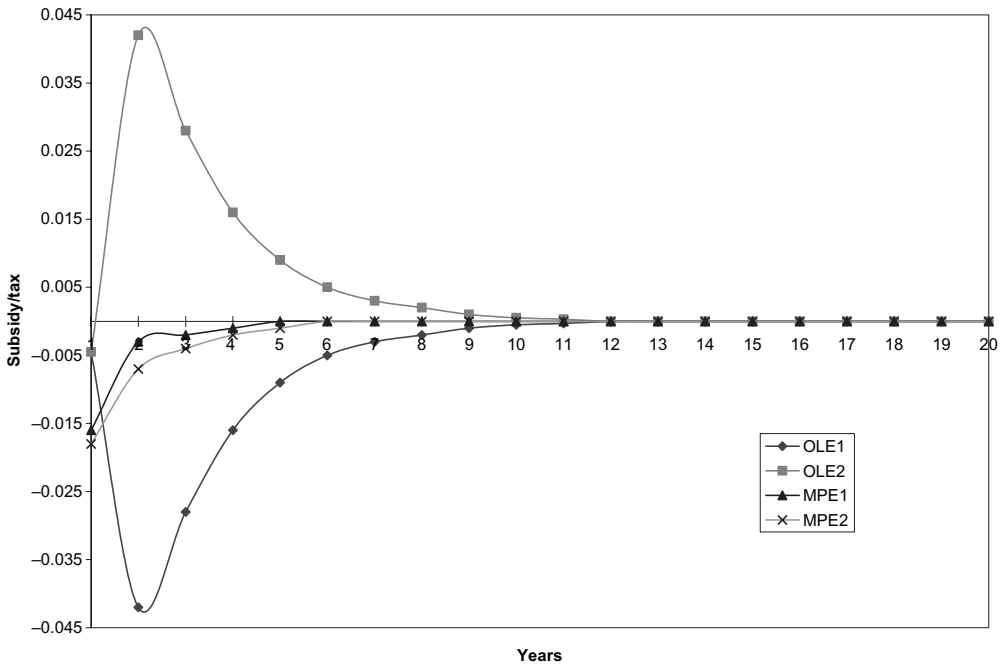


Figure 1. Equilibrium Policies:  $L(1) = 0.1$ ;  $\alpha_1 = 0.3$ ;  $\alpha_2 = 0.6$

Again, the policy trajectory is nonmonotonic. When the government targets sector 2 in the MPE, the equilibrium policy is always a tax. In every period the government tries to increase migration by lowering the cost of skilled labor. Being unable to promote migration by promising to subsidize sector 2 in the future, the government’s equilibrium behavior involves using an apparently perverse current policy.

*Welfare comparisons* In order to compare welfare under different policies, we calculate  $R$ , defined as the percent of the “potential increase” in the present discounted value of the stream of social welfare that a particular policy actually achieves. Let  $J_{FBE}$  be the payoff (the present discounted value of the stream of welfare) under the first-best policy (or in the absence of a distortion), and  $J_{PAE}$  be the welfare in the absence of government intervention (in the private adjustment equilibrium). The “potential increase” in welfare is  $J_{FBE} - J_{PAE}$ . Let  $J_i$  be the equilibrium value of welfare, given that the government targets a particular sector and is constrained to a particular equilibrium (open loop or Markov). The percent of the potential welfare increase that is actually achieved in a given equilibrium is

$$R = \left( \frac{J_i - J_{PAE}}{J_{FBE} - J_{PAE}} \right) 100.$$

Table 1 reports the values of  $R$  for the four cases.<sup>13</sup> As expected, OLE1 does much better than MPE1: the ability to commit to future policies is important. However, even if the government cannot make commitments, its ability to affect the current price of skilled labor gives it some leverage over migration, enabling it to achieve 6% of potential gains.

If the government is able to make commitments, it does slightly better subsidizing the growing sector rather than taxing the shrinking sector (achieving 84% rather than

*Table 1. Relative Efficiency of OLE and MPE (R)*

	<i>OLE</i>	<i>MPE</i>
Sector 1	80.0	6.1
Sector 2	84.0	-22.1

80% of potential gains). In the previous section we explained that this comparison is ambiguous. When the government targets the shrinking sector, the indirect general-equilibrium (skilled wage) effect reinforces the commitment effect. However, when the government targets the growing sector, it is able to induce a larger wage differential for a given level of distortion, making the policy less costly in terms of the secondary distortion. For this model, the second effect dominates.

If the government cannot make commitments, and targets sector 2, intervention is disadvantageous. The cost of intervention is 22% of the maximum potential gain. This conclusion is consistent with our explanation in the previous section, and the previous simulation result that the MPE2 policy has the opposite sign as the OLE2 policy.

*Sensitivity results* We conducted sensitivity results by changing the initial value  $L(0)$ <sup>14</sup> and changing the factor intensities and  $\gamma$ . In every case, intervention is disadvantageous in the MPE when the government targets the “wrong” sector (when the MPE policy has the opposite sign as the OLE over most of its trajectory). The one surprising result is that for all simulations, welfare is higher when the government targets the growing sector in the OLE.<sup>15</sup>

If the cost share for skilled labor (the exponent of  $N$  in the production function) is small in either sector, then the production distortion (the misallocation of mobile factors between the traded goods sectors) is small. In this case, the OLE does nearly as well as the first-best policy, and the advantage of targeting the growing sector is negligible. In the limiting case where skilled labor is not needed in one sector, the government has only one target: the allocation of skilled labor between the two sectors where it is used. In this case, an open-loop tax/subsidy on either traded goods sector is a first-best policy. If the cost share for skilled labor (in the sector that the government targets) is small but positive, then the MPE has negligible effects—it cannot be very disadvantageous or very beneficial.

## 5. Conclusion

We studied a rational-expectations equilibrium in which semi-skilled workers choose whether to move to a different sector. To change sectors, these workers must be retrained, a process which requires a mobile factor, skilled labor. A market imperfection causes adjustment to occur too slowly. At each point in time, the equilibrium depends on the price of three goods (the two traded goods and “migration services”). The government cares about two relative prices, and has a single second-best instrument. We investigated the welfare effects of targeting different sectors under polar assumptions concerning the government’s ability to make commitments about future policy.

The critical feature of this model is that retraining uses a mobile factor. The government’s policy affects migration via its effect on (future) wage differentials, thus affecting the benefit of migration. The policy also affects the price of skilled labor, and

thus affects the cost of migration. The presence of this indirect channel of government influence is key to our results.

In the open-loop equilibrium, where the government can make binding commitments, the advantage of targeting one sector rather than another is small. Our simulation results suggest that when adjustment is too slow, there is a small advantage of subsidizing the growing sector rather than taxing the shrinking sector. This result appears to be robust for Cobb–Douglas functional forms. Under plausible assumptions, we showed analytically that the open-loop trajectory begins with a tax and is nonmonotonic, regardless of which sector is targeted.

In a Markov-perfect equilibrium, where the government is not able to make commitments, it relies exclusively on the indirect general-equilibrium channel to affect current migration. (In the absence of this indirect effect, the government does not intervene in the MPE.) When it targets the shrinking sector, its equilibrium policy at least has the right sign, and the government achieves some increase in welfare. If, however, it targets the growing sector, its inability to make commitments creates perverse incentives, and intervention can lower welfare.

Several of our specific assumptions are ad hoc, including the exact nature of the market imperfection and the exact policy menu. We do not view our model as an approximation to any particular economy. Rather, it is intended to illustrate a situation where adjustment costs involve general-equilibrium linkages, there is *some* market failure, and the policy menu is restricted in a nontrivial manner. The intuition for our results does not rely on specific assumptions.

The possibility that the inability to make commitments leads to disadvantageous government intervention in a rational-expectations equilibrium has been shown in previous settings. A novel feature in our model is that disadvantageous intervention is associated with acting in a particular sector, rather than with using a particular policy instrument, and it arises only because of indirect general-equilibrium effects.

The importance of our results depends on the significance of the question that we have asked, and on the plausibility of our assumptions. We close with a discussion of these issues.

It appears to be widely accepted that adjustment costs are important. The economics profession has devoted enormous effort to the analysis of models based on adjustment costs. Casual observation and formal econometric work support the existence of these costs. Whether general-equilibrium linkages are an important component of these costs is less clear. There has been very little attention given to these linkages, and many economists might consider them of second-order importance. If the government can make commitments, our examples show that indirect general-equilibrium effects are indeed of second-order importance. However, if the government cannot make commitments, these indirect general-equilibrium linkages are important. In static models the issue of commitment does not arise, but it is central to dynamic models. The intuition developed from static models may be a poor guide to many things—including the assessment of the importance of general-equilibrium linkages in a dynamic setting.

## Appendix

### *Open-Loop Production Subsidy in Sector 1 (OLE1)*

First, we present the Hamiltonian and necessary conditions to the control problem. Secondly, we explain how to get the three terms of condition (A2) below (or equivalently condition (18) of the text):

**1. Hamiltonian and necessary conditions**

The *Hamiltonian* of the open-loop control problem is:

$$H^0 = p_1(\bar{L} - L)w_p^1(\tilde{p}_1, v) + p_2Lw_p^2(\tilde{p}_2, v) + \beta_1\frac{2\theta\gamma}{v}q + \beta_2[rq + w^1(\tilde{p}_1, v) - w^2(\tilde{p}_2, v)]. \tag{A1}$$

where  $v \equiv v(\tilde{p}_1, L, q)$  is implicitly given by equation (9) of the text and where  $\beta_1$  and  $\beta_2$  are the costate variables associated with the states  $L$  and  $q$  respectively.

The *necessary condition for maximization* of the Hamiltonian is:

$$\begin{aligned} \frac{\partial H^0}{\partial \tilde{p}_1} = 0 = & -s_1(\bar{L} - L)\left(w_{pp}^1 + w_{pv}^1\frac{\partial v}{\partial \tilde{p}_1}\right) + \frac{\partial v}{\partial \tilde{p}_1}\frac{2\theta\gamma q}{v^2}(2\theta q - \beta_1) \\ & + \beta_2\left[w_p^1 + \frac{\partial v}{\partial \tilde{p}_1}(w_v^1 - w_v^2)\right] \end{aligned} \tag{A2}$$

where:

$$\frac{\partial v}{\partial \tilde{p}_1} = -\frac{(\bar{L} - L)w_{vp}^1}{(\bar{L} - L)w_{vv}^1 + Lw_{vv}^2 + (4\theta^2\gamma/v^3)q^2} \geq 0.$$

The *adjoint conditions* of the problem are:

$$\begin{aligned} \dot{\beta}_1 = & r\beta_1 + p_1w_p^1 - p_2w_p^2 \\ & + \frac{\partial v}{\partial L}\left[-p_1(\bar{L} - L)w_{pv}^1 - p_2Lw_{pv}^2 + \beta_1\frac{2\theta\gamma q}{v^2} - \beta_2(w_v^1 - w_v^2)\right] \end{aligned} \tag{A3}$$

$$\begin{aligned} \dot{\beta}_2 = & r\beta_1 - \left[\frac{\partial v}{\partial q}[p_1(\bar{L} - L)w_{pv}^1 + p_2Lw_{pv}^2] \right. \\ & \left. + \frac{\beta_1 2\theta\gamma}{v^2}\left[v - \frac{\partial v}{\partial q}q\right] + \beta_2\left[r + \frac{\partial v}{\partial q}(w_v^1 - w_v^2)\right]\right]. \end{aligned} \tag{A4}$$

The *boundary values and transversality conditions* are:

$$\begin{aligned} L(0) = L_0, \quad \lim_{t \rightarrow \infty} q(t)e^{-rt} = 0; \\ \lim_{t \rightarrow \infty} \beta_1(t)e^{-rt} = 0; \quad \beta_2(0) = 0. \end{aligned} \tag{A5}$$

**2. Intermediate calculations to get maximization condition (A2)**

The derivative of  $H^0$  with respect to  $\tilde{p}_1$  gives:

$$\begin{aligned} \frac{\partial H^0}{\partial \tilde{p}_1} = 0 = & p_1(\bar{L} - L)\left(w_{pp}^1 + w_{pv}^1\frac{\partial v}{\partial \tilde{p}_1}\right) + p_2Lw_{pv}^2\frac{\partial v}{\partial \tilde{p}_1} \\ & - \beta_1\frac{2\theta\gamma q}{v^2}\frac{\partial v}{\partial \tilde{p}_1} + \beta_2\left[w_p^1 + \frac{\partial v}{\partial \tilde{p}_1}(w_v^1 - w_v^2)\right]. \end{aligned}$$

We rewrite the above expression by adding and subtracting the term

$$s_1(\bar{L} - L) \left[ w_{pp}^1 + w_{pv}^1 \frac{\partial v}{\partial \tilde{p}_1} \right].$$

The result is

$$\frac{\partial H^0}{\partial \tilde{p}_1} = 0 = \Theta - s_1(\bar{L} - L) \left( w_{pp}^1 + w_{pv}^1 \frac{\partial v}{\partial \tilde{p}_1} \right) + \beta_2 \left[ w_p^1 + \frac{\partial v}{\partial \tilde{p}_1} (w_v^1 - w_v^2) \right], \tag{A6}$$

where

$$\Theta = \tilde{p}_1(\bar{L} - L)w_{pp}^1 + \frac{\partial v}{\partial \tilde{p}_1} \left[ \tilde{p}_1(\bar{L} - L)w_{pv}^1 + p_2Lw_{pv}^2 - \beta_1 \frac{2\theta\gamma q}{v^2} \right]$$

and  $\tilde{p}_1 = p_1 + s_1$ . Using properties (1), we want to simplify  $\Theta$ . Substituting  $\partial v/\partial \tilde{p}_1$  by (10) into  $\Theta$  leads to:

$$\begin{aligned} \Theta = \frac{1}{D} & \left[ \tilde{p}_1(\bar{L} - L)^2 w_{pp}^1 w_{vv}^1 + \tilde{p}_1(\bar{L} - L)Lw_{pp}^1 w_{vv}^2 + \tilde{p}_1(\bar{L} - L)w_{pp}^1 \frac{4\theta^2\gamma}{v^3} q^2 \right. \\ & \left. - \tilde{p}_1(\bar{L} - L)^2 (w_{pv}^1)^2 - p_2(\bar{L} - L)Lw_{vp}^1 w_{pv}^2 + \beta_1 \frac{2\theta\gamma q}{v^2} (\bar{L} - L)w_{vp}^1 \right], \tag{A7} \end{aligned}$$

where

$$D = (\bar{L} - L)w_{vv}^1 + Lw_{vv}^2 + \frac{4\theta^2\gamma}{v^3} q^2.$$

Using the fact that  $w_{pp}^1 w_{vv}^1 - (w_{pv}^1)^2 = 0$  (see properties (1)), the first and fourth terms of the above sum cancel each other out. Next using the fact that  $p_2 w_{pv}^2 = -v w_{vv}^2$  (see (1)), the fifth term can be rewritten as  $+v(\bar{L} - L)Lw_{vp}^1 w_{vv}^2$ . From  $\tilde{p}_1 w_{pp}^1 = -v w_{pv}^1$ , this fifth term is also equal to  $-\tilde{p}_1(\bar{L} - L)Lw_{pp}^1 w_{vv}^2$ . This expression together with the second term of (A7) cancel each other out. Finally, expression (A7) reduces to:

$$\Theta = \frac{1}{D} \left[ \tilde{p}_1(\bar{L} - L)w_{pp}^1 \frac{4\theta^2\gamma}{v^3} q^2 + \beta_1 \frac{2\theta\gamma q}{v^2} (\bar{L} - L)w_{vp}^1 \right].$$

Since  $\tilde{p}_1 w_{pp}^1 = -v w_{pv}^1$ , this simplifies to:

$$\Theta = \frac{-(\bar{L} - L)w_{pv}^1}{D} \frac{2\theta\gamma q}{v^2} [2\theta q - \beta_1] = \frac{\partial v}{\partial \tilde{p}_1} \frac{2\theta\gamma q}{v^2} [2\theta q - \beta_1].$$

Substituting this new expression for  $\Theta$  into (A6) gives condition (A2) (or condition (18) of the text).

*Markov-Perfect Production Subsidy in Sector 1 (MPEI)*

The *Hamiltonian* for the government’s problem when the government is unable to commit to future actions is:

$$H^M = p_1(\bar{L} - L)w_p^1(\tilde{p}_1, v) + p_2Lw_p^2(p_2, v) + \beta \frac{2\theta\gamma}{v} Q(L) \tag{A8}$$



where  $v \equiv v(\tilde{p}_1, L)$  is implicitly given by equation (9) of the text in which the variable  $q$  is replaced by the function  $Q(L)$ ;  $\beta$  is the costate variable associated with the state  $L$ .

By using the same analytical manipulations than those used to obtain the maximization condition (A2), the *necessary condition for maximization* of  $H^M$  can be written as:

$$\frac{\partial H^M}{\partial \tilde{p}_1} = 0 = -s_1(\bar{L} - L) \left( w_{pp}^1 + w_{pv}^1 \frac{\partial v}{\partial \tilde{p}_1} \right) + \frac{\partial v}{\partial \tilde{p}_1} \frac{2\theta\gamma Q(L)}{v^2} (2\theta Q(L) - \beta) \quad (\text{A9})$$

where

$$\frac{\partial v}{\partial \tilde{p}_1} = - \frac{(\bar{L} - L)w_{vp}^1}{(\bar{L} - L)w_{vv}^1 + Lw_{vv}^2 + (4\theta^2\gamma/v^3)[Q(L)]^2} \geq 0.$$

The *adjoint condition* of the problem is

$$\begin{aligned} \dot{\beta} = & r\beta + p_1w_p^1 - p_2w_p^2 + \frac{\partial v}{\partial L} [-p_1(\bar{L} - L)w_{pv}^1 - p_2Lw_{pv}^2] \\ & - \beta \frac{2\theta\gamma}{v^2} \left[ \frac{dQ}{dL}v - \frac{\partial v}{\partial L}Q(L) \right]. \end{aligned} \quad (\text{A10})$$

The *boundary value and the transversality condition* are:

$$L(0) = L_0, \quad \lim_{t \rightarrow \infty} \beta(t)e^{-rt} = 0. \quad (\text{A11})$$

The *consistency condition* below is a restriction on the unknown function  $Q(L)$  to insure that when the government chooses its optimal policy rule  $s_1^M(L)$ , taking as given  $Q(L)$ , agents' expectations are confirmed in equilibrium:

$$\begin{aligned} \frac{dQ}{dL} \dot{L} = \dot{q} = & rQ(L) - w^1(\tilde{p}_1, v) + w^2(p_2, v) \\ \lim_{t \rightarrow \infty} Q(L_t)e^{-rt} = & 0. \end{aligned} \quad (\text{A12})$$

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## Notes

1. There is a growing body of empirical literature that attempts to quantify adjustment costs in different sectors and for different factors. Contributions to this literature include Anderson (1993), Buhr and Kim (1997), Epstein and Denny (1983), Fernandez-Cornejo et al. (1992), Hall (2002), Hayashi and Inoue (1991), Luh and Stefanou (1991), and Pindyck and Rotemberg (1983).
2. In a richer setting, the choice of the targeted sector would be imbedded in a political-economy model. In reality, any government has at best limited commitment ability, as a later government can overrule past commitments. In a dynamic, political-economy model, the decision to target one sector can create over time enough political opposition to the party in power to reverse government policies (Janeba, 2002). We exclude this, taking the choice of sectors as exogenous.

Nevertheless, it is reasonable to examine how this choice affects policy and welfare. There may be a variety of reasons, in addition to those identified by our analysis, why the government would choose to target a particular sector to change relative prices. For example, it may be politically easier to subsidize the growing sector rather than to tax the shrinking sector. On the other hand, if there is a deadweight cost of financing a subsidy, the government might want to tax the shrinking sector.

3. We assume that the factor intensities or the output prices are different in the two sectors. When both the factor intensities and output prices are equal, the wages in the two sectors are equal and there is no adjustment.

4. A larger value of  $n$  is equivalent to larger  $\gamma$ ; clearly, larger  $\gamma$  implies faster migration for a given level of  $q_t$  and  $v_t$ . However,  $q_t$  and  $v_t$  are also functions of  $n$ . (Numerical results show that equilibrium migration is faster when  $\gamma$  (or  $n$ ) increases.) The presence of an externality in production means that even if  $n$  were endogenous (e.g. if there is free entry together with sunk costs), its level would typically be socially inefficient—as would the amount of labor in the transport sector. Our results depend on the existence of a market imperfection that causes the competitive level of migration to differ from the first-best level. Since a distortion exists whether  $n$  is endogenous or fixed, and since a *general* model with endogenous  $n$  hugely increases the complexity of the model, we take  $n$  as exogenous. It would be easy to construct a *special* model in which  $n$  is endogenous, and its equilibrium level is approximately the same under all types of policies. For example, the entry cost is low up to a given level of  $n$  and then it rises very rapidly. Such a model would be virtually identical to the one we present. Finally, there are plausible circumstances under which  $n$  might literally be exogenous, as is the case if the government licenses training institutions.

5. The simplest way to obtain the FBE when  $\theta \neq \frac{1}{2}$  is to solve the PAE for  $\theta = \frac{1}{2}$  to find the first-best migration trajectory. We then find the migration tax that supports this as an equilibrium outcome in the PAE with the actual value of  $\theta$ .

6. Chari and Kehoe (1990) and Stokey (1991) explore this method of constructing subgame-perfect equilibria in a game between a government and non-atomic agents; Ausebel and Deneckere (1989) use this approach in the durable goods monopoly setting.

7. An increase in power to influence nonstrategic agents, and the associated loss of welfare, may be associated with greater cooperation amongst governments, as in Kehoe (1989) and Rogoff (1985), or from greater market power, as in Maskin and Newbery (1990) and Karp (1996).

8. This argument for the conjecture is straightforward, but it does not constitute a rigorous proof. The difficulty lies in excluding the possibility that at some states (levels of  $L$ ) the government causes migration to be faster than it would be in the first-best equilibrium, in order to compensate for excessively slow migration at other states.

9. The reader can think of  $\beta^{FB}$  as the social marginal value of migration. We prefer to define  $\beta^{FB}$  as the marginal social value of  $L$  because  $\beta^{FB}$  is simply the costate variable associated with the state variable  $L$ , in the control problem in which the government can use a first-best policy. The costate variable is often interpreted as a shadow price, or marginal value, of its corresponding state variable.

10. That is,  $\beta^{SB}$  is the costate variable associated with the state variable  $L$  in one of the control problems with second-best policies, described in the previous subsection.

11. This is the “standard” boundary condition for control problems with a “jump state.” It has been used in dynamic Stackelberg games at least since Simaan and Cruz (1973). The intuition is that the leader (here the government) chooses the current value of the jump state  $q$  by its choice of future policies. The optimal trajectory requires that  $q_0$  be chosen optimally. Since  $\beta_2$  is the shadow value of  $q$ , optimality requires  $\beta_2(0) = 0$ . This boundary condition is incorrect in certain Stackelberg differential games; see Xie (1997) and Karp and Lee (2003). This exception occurs when the  $q$  is not actually a jump state, i.e. when its value at every point is determined by the (ordinary) state (here,  $L$ ) and *not by future government policies*. This exception does not arise for our specification of the game.

12. In deriving equation (23) we use the necessary condition that the Hamiltonian is maximized with respect to the control. Equation (23) must hold for any function  $Q(L)$ . In order to obtain

the equilibrium (rather than merely a necessary condition) we also need to use the consistency condition and the boundary condition that the steady state is  $L_\infty$ .

13. The magnitudes  $J_i - J_{PAE}$  and  $J_{FBE} - J_{PAE}$  are both quite small, because the equilibrium policy intervention is quite small (a tax or subsidy of less than 5%) and it has a small effect on the equilibrium trajectories. Because a graph of small differences is not informative, we do not provide these graphs. We have encountered this kind of result—small absolute welfare changes resulting from policy changes—in previous work involving simulations. This kind of result is also common in simulation work by others (e.g. many models which simulate the welfare gains from trade liberalization). Despite the small absolute changes, the relative changes, measured by the variable  $R$ , are not small.

14. For  $L(0)$  greater than the steady state, sector 1 becomes the growing sector.

15. Our discussion in section 3 suggests that there are opposing forces which tend to favor different sectors, so we expected that we would be able to find parameter values for which it is optimal to target either the growing or declining sector in the OLE.