Fundamentals versus Beliefs under Almost Common Knowledge[1]

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Abstract

Modern economic growth models show that the equilibrium outcome may depend on agents' beliefs (expectations) rather than on economic fundamentals (history). In this situation, the equilibrium is indeterminate. However, if agents have "almost common knowledge" rather than common knowledge about the economic fundamentals, this indeterminacy vanishes in one of these models, under certain restrictions. In this situation, the unique competitive equilibrium can be influenced by government policy, just as in standard models.

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1 Introduction

The idea that important economic phenomena may depend as much on crowd psychology as on economic fundamentals has become widespread over the past decade. This view implies that some outcomes are unpredictable. Even if the economist had "the right model" and knew everything there was to know about the kinds of things we regard as "fundamental" (e.g. tastes and technology), we might be unable to say which of two widely different outcomes is more likely. This inability would be due to the fact that the outcome depends on agents' beliefs, and these beliefs are not pinned down by fundamentals. In this circumstance, the set of equilibrium outcomes may remain unchanged following a non-negligible change in the economic fundamental. Models of economic indeterminacy therefore do more than merely illustrate that an outcome might be extremely sensitive to fundamentals. These models point to the inherent limits of even the most precise modeling.

The possibility of indeterminacy is particularly important in the study of economic growth and development. Several papers, including Krugman [9] and Matsuyama [10] show that when agents solve dynamic investment problems and technology is non-convex, identical economies might follow completely different development paths. The outcome depends on agents' beliefs ("expectations") rather than simply on economic fundamentals ("history").[1] These models have a ring of plausibility, and their apparent robustness has led to the widespread acceptance that indeterminacy may be an important feature of economic development.

The models assume that agents have common knowledge about economic fundamentals. In a variation of Krugman's model, replacing the assumption of common knowledge with almost common knowledge removes the indeterminacy. The distinction between fundamentals and beliefs (history and expectations) vanishes. Without common knowledge of economic fundamentals, the model becomes standard: the unique equilibrium is determined by economic fundamentals, and it can be modified by standard forms of government intervention.

Common knowledge of an event (e.g. the event that economic fundamen-

[1] Farmer [4] surveys a related literature on multiplicity of equilibria. He suggests that the indeterminacy can be resolved by augmenting the state vector to include variables that define beliefs. Blanchard and Fischer [1] also discuss a number of earlier models that have multiple equilibria. It appears that the importance of the assumption of common knowledge was not recognized in these models.

tals lie in a particular set) means that everyone knows the event, everyone knows that everyone knows, and so on *ad infinitum*. A frequently used definition of "almost common knowledge" is that everyone believes that the event is extremely likely, everyone believes that everyone believes the event is extremely likely, and so on up to a large but finite order of beliefs.

Game theorists are well aware of the importance of the assumption of common knowledge.[2] Several recent papers, including Rubinstein [13], Carlsson and Van Damme [2] and Morris and Shin [12], show that under almost common knowledge there may be a unique equilibrium to a coordination game. This paper is closely related to [12], as explained in subsequent footnotes.

The next section provides a simplified version of Krugman's model, reproducing his conclusion that in some circumstances the equilibrium is unique, in which case it depends on economic fundamentals (history); in other circumstances there are two equilibria, in which case the outcome depends on agents' beliefs (expectations). The following two sections describe and analyze the model with "almost common knowledge". The concluding section assesses the plausibility of this information structure in the context of a dynamic model, an issue raised by Chamley [3]. That section also discusses the relation between the results here and in two earlier papers, Herrendorf *et al*. [7] and Frankel and Pauzner [5], which investigate the robustness of indeterminacy using Matsuyama's model.

2 A Simple Migration Model

The small, open, competitive economy[3] consists of two sectors. The stock of (domestically mobile) labor is normalized to 1. The amount of labor in the manufacturing sector is L, and the amount of labor in the agricultural sector is $1 - L$. The manufacturing sector has increasing returns to scale. The wage in manufacturing is $a + bL$. The agricultural sector has constant

[2] For example, Fudenberg and Tirole [6] page 555, note that "apparently small perturbations in the information structure starting from common knowledge of the payoffs may change the equilibrium set considerably. That is, some equilibria of the game in which payoffs are common knowledge are not near any equilibrium of the perturbed game, even if with high probability all players know that the payoffs are as in the original game."

[3] For the small open economy commodity prices are fixed, so workers' real income equals their nominal income, which equals their wage. Therefore, their migration decision depends on the comparison of wages in the two sectors. Labor is not internationally mobile.

returns to scale, and the wage there is c, a constant. In this simple model with certainty, the parameters satisfy $c > a > c - b$. Thus, the wage in agriculture exceeds the wage in manufacturing if and only if $L < \theta \equiv \frac{c-a}{b}$. Other things equal, *a larger value of* θ *makes it more attractive to be in the agricultural sector.*

The *net* number of migrants into manufacturing is u. With common knowledge, there is never two-way migration in equilibrium, so $\mid u \mid$ equals the total (the gross) number of migrants. Given the initial allocation L, the number of workers in manufacturing in the next (and final) period is $L + u$. In order to change sectors, a worker needs to pay for "migrations services", such as education. The price he pays depends on the aggregate amount of migration, and equals $\delta \mid u \mid$, $\delta > 0$.[4] The total amount of migration is constrained by $-L \leq u \leq 1 - L$.

The discount factor is β. Each worker decides whether to migrate (i.e., he makes a 0-1 decision). An agent who migrates pays the price of migration in the current period, and benefits from the wage differential in the next period. If an interior equilibrium involves migration into manufacturing ($1-L > u \geq 0$) the equilibrium satisfies $\beta\left[a + b(L+u) - c\right] - \delta u = 0$. In this case, the private cost of migration equals the present value of the wage differential. If an interior equilibrium involves migration into agriculture ($-L < u \leq 0$), the equilibrium satisfies $\beta\left[c - a - b(L+u)\right] + \delta u = 0$. If neither of these conditions hold, the equilibrium is on the boundary ($u = -L$ or $u = 1 - L$).

To describe the equilibrium, define

$$f(L;\theta) \equiv \frac{\beta(a - c + bL)}{\delta - \beta b} = \frac{\beta b}{\delta - \beta b}\left(L - \theta\right),$$

the value of u at an interior equilibrium. Figure 1 graphs the two constraints $u \geq -L$ and $u \leq 1 - L$ and the function $f(L)$ for the two cases where $\delta > \beta b$ (Figure 1a) and $\delta < \beta b$ (Figure 1b). L_1 denotes the intersection of $f(L)$ and the constraint $u = 1 - L$, and L_2 denotes the intersection of $f(L)$ and the constraint $u = -L$. The values of L_i are

$$L_1 = \frac{1}{\delta}\left(\beta b\theta + \delta - \beta b\right); \qquad L_2 = \frac{\beta b}{\delta}\theta. \tag{1}$$

[4] Section 4 discusses the manufacturing production function and the cost function for migration services, needed for welfare analysis. In order to determine the equilibrium we need only the wage function, $a + bL$, and inverse supply function for migration services, $\delta \mid u \mid$.

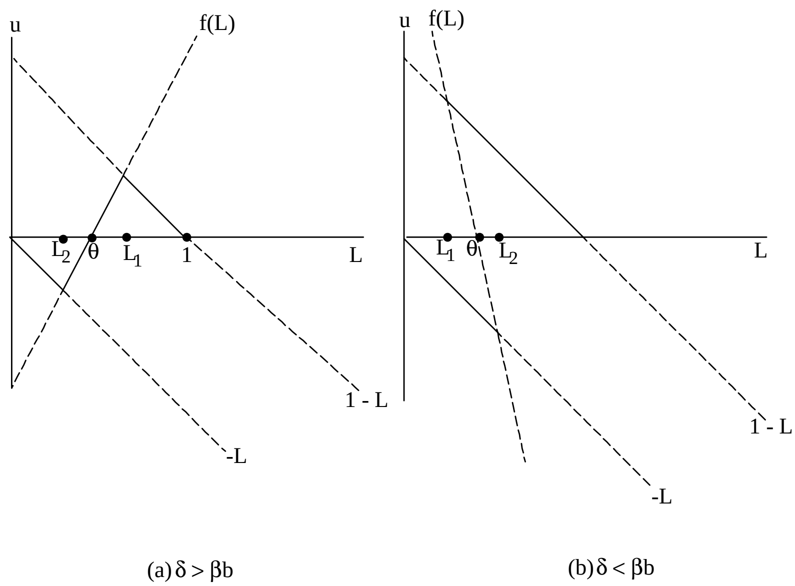


Figure 1: The Equilibrium with Perfect Information

The graph of $f(L)$ shows levels of u at which the present value of the wage differential equals the price of adjustment. If the constraint $-L \leq u \leq 1-L$ is not binding, $u = f(L)$ is an equilibrium, but not necessarily a *stable* equilibrium. The solid lines in the two panels show the set of stable equilibria as a function of the initial allocation, L.

The equilibrium is unique if $\delta > \beta b$ (Figure 1a). In this case all labor moves to manufacturing if $L \geq L_1$ and all labor moves to agriculture if $L \leq L_2$. For $L \in (L_2, L_1)$ the equilibrium is interior.

If $\delta < \beta b$ (Figure 1b) the equilibrium is either unique or indeterminate, depending on the value of L. All labor moves to manufacturing if $L \geq L_2$ and all labor moves to agriculture if $L \leq L_1$. For $L \in (L_1, L_2)$ there are two stable equilibria and an unstable equilibrium $f(L)$.[5] In the stable equilibria all labor ends up in either manufacturing or in agriculture.

The interesting case is $\delta < \beta b$, where the equilibrium is indeterminate for $\{L : L_1(\theta) < L < L_2(\theta)\}$. The length of the set of L at which the equilibrium

[5]The outcome $u = f(L)$ is "unstable" in the usual sense: if the actual amount of migration differs from $f(L)$ by a small positive measure, all other agents would follow that deviation, thus moving the outcome away from $f(L)$.

is indeterminate is $L_2 - L_1 = \frac{\beta b}{\delta} - 1$. This set of indeterminacy increases with $\frac{\beta b}{\delta}$ and collapses to the null set as $\frac{\beta b}{\delta} \rightarrow 1$. A larger value of b implies a greater degree of increasing returns to scale ("more non-convexity"), and a larger value of β makes this non-convexity more important to agents' decisions. A larger value of δ increases the slope of the inverse supply function for migration services and dampens workers' incentive to imitate other migrants. The measure of the set of indeterminacy depends on the relative strengths of the two forces that increase or moderate the model's non-convexity.

It is useful to rewrite the region of indeterminacy as a function of L rather than θ, since the next section treats L as common knowledge, and θ as imperfectly observed. For the case where $\delta < \beta b$, define the "set of indeterminacy" $I(L)$ as

$$I(L) = \left\{ \theta \mid \frac{\delta L}{\beta b} < \theta < \frac{\delta L + \beta b - \delta}{\beta b} \right\} \tag{2}$$

using equation (1). The equilibrium is indeterminate for $\theta \epsilon I(L)$.[6]

3 Migration with Almost Common Knowledge

This section relaxes the assumption of common knowledge about economic fundamentals. Suppose that the parameters δ, β, and b are common knowl-

[6] The one period migration model (with $\delta < \beta b$) and Morris and Shin's [12] model of speculative attack have striking similarities and two important differences. In both models, the economic fundamental falls into one of three intervals. If the currency is very strong or very weak in their model, and if θ is very large or very small here, each agent (speculators in their model, workers here) have a dominant strategy. In these cases, the equilibrium is unique. In an intermediate interval, corresponding to the "ripe for attack" range in their model, and to the set $I(L)$ here, each agent's payoff depends on what other agents do. For economic fundamentals in this intermediate interval, the equilibrium is indeterminate under common knowledge.

There are two differences between the models. The most important of these is that there are two types of agents in the migration model, those who are initially in agriculture and those in manufacturing. In the speculative attack model there is a single type of agent, speculators. This difference leads to a substantive difference in the analysis, discussed in the next section.

The second difference is that for economic fundamentals in the intermediate range in Morris and Shin's model, the payoff of agents who speculate depends only on whether the number of other speculators exceeds or falls short of a critical level. In the migration model, the payoff depends continuously on the actions of other agents. This difference makes the calculations more complicated here, but is not qualitatively important.

edge and that $c - a$ is the unknown "economic fundamental". In order that the economic fundamental has the same units as L, define $\theta \equiv \frac{c-a}{b}$ as the unknown parameter. Workers begin with diffuse priors about θ.

Each worker receives a private signal, x, about this economic fundamental, and knows that this signal is drawn from a uniform distribution with support $[\theta - \varepsilon, \theta + \varepsilon]$. The parameter ε (which measures the heterogeneity of information) and the form of the distribution are common knowledge. If $\varepsilon = 0$ the economic fundamental is common knowledge once agents receive their signal. For any value of $\varepsilon > 0$, the economic fundamental is never common knowledge, but for small positive values of ε, the economic fundamental is "almost common knowledge". The signals received by workers in the different sectors are independently distributed; for example, half the workers in each sector receive a signal less than θ.

3.1 The Role of Almost Common Knowledge

Even an arbitrarily small value of ε means that it is never common knowledge that θ lies in a strict subset of its prior support.[7] Almost common knowledge (heterogeneity of information) causes uncertainty to "expand", leaving agents unsure what other agents will do, even for ε arbitrarily small.

Under common knowledge ($\varepsilon = 0$), the equilibrium is indeterminate if $\theta \epsilon I(L)$. The "expansion of uncertainty" caused by heterogenous information eliminates the possibility of multiple equilibria. Suppose to the contrary, that there did exist a set of θ for which the equilibrium is indeterminate under almost common knowledge. Since a worker's migration decision depends on his signal x, this hypothesis means that there is a set of signals for which the equilibrium decision is indeterminate. For example, we might conjecture that the equilibrium decision is indeterminate for signals that leave the representative worker, Agent k, certain that $\theta \epsilon I(L)$. This conjecture implies

[7] For a given value of ε, Agent k who observes the signal x can place bounds on the information that other agents have. Agent k knows that the true value of θ is in the interval $[x - \varepsilon, x + \varepsilon]$, and therefore knows that other agents have received a signal within the interval $[x - 2\varepsilon, x + 2\varepsilon]$. Agent k therefore knows that all other agents know that every agent has received a signal within the interval $[x - 4\varepsilon, x + 4\varepsilon]$. The bounds that an agent can place on "higher order beliefs", i.e. beliefs about what other agents know that other agents know that other agents know.... and so forth ... grow arbitrarily wide, regardless of the value of ε. The support of Agent k's higher order beliefs becomes arbitrarily large for a sufficiently "high order" of beliefs. If the prior support of θ is bounded, there is a finite value of n at which the support of the agent's nth order beliefs equals the prior support.

that the set of signals for which the equilibrium decision is indeterminate is $\left\{x \mid \frac{\delta L}{\beta b}+\varepsilon<x<\frac{\delta L+\beta b-\delta}{\beta b}-\varepsilon\right\}$. [See equation (2).]

This conjecture is unsatisfactory. It does not take into account what Agent k thinks that other workers (who observe different signals) know. We might try to improve upon the original conjecture by supposing that the equilibrium decision is non-unique only for signals that leave Agent k certain that *all workers* know that $\theta \epsilon I(L)$. This "improvement" implies that the set of signals for which the equilibrium is non-unique is $\left\{x \mid \frac{\delta L}{\beta b}+3\varepsilon<x<\frac{\delta L+\beta b-\delta}{\beta b}-3\varepsilon\right\}$. Still, the conjecture is unsatisfactory, because now it does not take into account what Agent k knows about what the other workers know about what other workers know. As we continue to refine the conjecture by requiring (in order for the equilibrium decision to be indeterminate) a higher order of certainty, we decrease the set of signals that satisfy the refinement. Regardless of the value of ε, by requiring a sufficiently high order of certainty, we reduce the candidate set of signals to the null set.

3.2 The Migration Decisions

There are two types of agents in the migration model. Type M is currently in manufacturing and Type A is currently in agriculture. A worker makes his decision conditional on the signal he receives. Since workers in the different sectors have different decision problems, their decision rules are different.

Under common knowledge, where two-way migration never occurs in equilibrium, a single variable (u) denotes net migration into manufacturing; $\mid u \mid$ equals gross migration. (If $u<0$, workers migrate into agriculture.) Under almost common knowledge, we cannot exclude the possibility of two-way migration, and therefore need to extend the notation. Henceforth, u_i denotes the number of workers who leave Sector i for Sector j, $i, j \epsilon \{A, M\}$, so $u_i \geq 0$. If initially there are L workers in manufacturing, the number of workers in manufacturing after migration is $L-u_M+u_A$. The *net* migration into manufacturing is u_A-u_M, and the *gross* migration is u_A+u_M.

There are a number of ways to model adjustment costs and the corresponding price of migration, when net and gross migration differ. If the main cost of intersectoral migration consists of the costs of retraining a worker who moves to a different sector, then the total costs depends on gross migration, u_M+u_A. This assumption, together with the specification in the previous section, implies that every migrant pays the price $\delta\left(u_M+u_A\right)$ in order to move to a different sector.

The equilibrium value of u_i depends on the equilibrium decision rules and the value of θ. Denote the fraction of Type i workers who migrate when they receive signal x as $\pi_i(x)$, an integrable function. Denote $h_i = h(\theta;\pi_i)$ as the fraction of Type i who migrate when the economic fundamental is θ, given that Type i's behavior is described by the function π_i.

$$h_i = h(\theta;\pi_i) = \frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta+\varepsilon}\pi_i(x)dx. \tag{3}$$

The number (measure) of workers who leave manufacturing is $u_M \equiv h(\theta,\pi_M)L \geq 0$ and the number who leave agriculture is $u_A \equiv h(\theta;\pi_A)(1-L) \geq 0$.

The expected benefit of migration for a manufacturing worker who observes signal x when aggregate behavior is described by the pair of strategies $\Pi \equiv (\pi_M,\pi_A)$ is $G_M(x;\Pi)$.

$$\begin{aligned} G_M(x;\Pi) &= E_{\theta|x}\left\{\beta\left[c-(a+b\left[L-u_M+u_A\right])\right]-\delta\left(u_M+u_A\right)\right\} \\ &= E_{\theta|x}\left\{\beta b\left[\theta-L\right]+u_M\left(\beta b-\delta\right)-u_A(\beta b+\delta\right\}. \end{aligned} \tag{4}$$

The expected benefit of migration for an agricultural worker, G_A is

$$G_A(x;\Pi) = E_{\theta|x}\left\{\beta b\left[L-\theta\right]-u_M\left(\beta b+\delta\right)+u_A(\beta b-\delta\right\}. \tag{5}$$

3.3 Dominance regions

The different approaches to proving uniqueness – including the one that I follow – rely on the existence of "dominance regions", i.e. sets of values of the economic fundamental at which a particular action is optimal for an agent, regardless of other agents' actions. For example, a manufacturing worker finds it less attractive to migrate if he thinks that few other manufacturing workers will leave the sector, or if he thinks that many agricultural workers will enter the sector. If a manufacturing worker knows the value of θ and believes that $u_M = 0$ and $u_A = 1-L$. (the beliefs that make migration the least attractive for him) he wants to migrate if and only if $\beta b\theta - \beta bL - (1-L)(\beta b+\delta) \geq 0$ (using equation (4)). Migration is a dominant strategy for manufacturing workers if $\theta \geq \frac{\beta b+\delta(1-L)}{\beta b}$.

Table 1 lists the circumstances under which a particular action is dominant for a particular type of worker. The values of θ at which it is dominant for the two types of workers not to migrate (shown in the last row of the table) equal the bounds of the set of indeterminacy, $I(L)$, defined in equation

	M-worker	A-worker
dominant to migrate	$\theta \geq \frac{\beta b+\delta(1-L)}{\beta b}$	$\theta \leq \frac{-\delta L}{\beta b}$
dominant not to migrate	$\theta \leq \frac{\delta L}{\beta b}$	$\theta \geq \frac{\delta L+\beta b-\delta}{\beta b}$

Table 1: Conditions for the existence of dominant strategies

(2). The values at which it is dominant to migrate (the elements of the first row) are on either side of this set. We can replace the assumption that the prior on θ is diffuse with:

Assumption 0. Before observing their private signal, each agent's subjective distribution of θ is uniform over a set that includes the interval $\left[\frac{-\delta L}{\beta b}, \frac{\beta b+\delta(1-L)}{\beta b}\right]$.

For simplicity, I retain the assumption of a diffuse prior, so that Assumption 0 is obviously satisfied. The Appendix explains why the proof of uniqueness fails if Assumption 0 is not satisfied. Readers familiar with the earlier papers on this topic will not be surprised by this observation. Assumption 0 is important in evaluating the plausibility of this model, an issue addressed in the concluding section.

4 The Equilibrium

This section shows (subject to qualifications described below) that there is a unique equilibrium. This equilibrium depends on economic fundamentals, and government intervention has predictable effects.

A worker in Sector i does not migrate if the benefits are negative, i.e. if $G_i(x;\Pi) < 0$. Define the "migration set" $\Psi_i(\pi_i) = \{x \mid \pi_i(x) > 0\}$, i.e. the set of signals for which some Type i migrates. Equations (4) and (5) imply

$$G_M(x;\Pi) = -G_A(x;\Pi) - 2\delta E_{\theta|x}(u_M + u_A). \tag{6}$$

Since $E_{\theta|x}(u_M + u_A) \geq 0$ equation (6) implies that the two types would never migrate given the same signal. Lemma 1 restates this conclusion.

Lemma 1 $\Psi_M(\pi_M) \cap \Psi_A(\pi_A) = \emptyset$.

4.1 A qualification: monotonicity

A large value of x is a signal that θ is likely to be large. Recall that an increase in θ increases the advantage of being in the agricultural sector, for a given allocation of labor. Therefore, *for a given amount of migration, the expected advantage of being in the agricultural sector is an increasing function of the signal x*. Thus, it is reasonable to expect that migration becomes more attractive for a manufacturing worker if he receives a higher signal (larger x). Similarly, migration becomes more attractive for an agricultural worker if he receives a lower signal. These assertions imply that $\pi_M(x)$ is nondecreasing in x, and $\pi_A(x)$ is nonincreasing in x. A pair of functions with this property is *Monotonic:*[8]

Definition 1 *The pair of strategies $(\pi_A(x), \pi_M(x))$ is Monotonic iff for all $x' > x''$, $\pi_M(x') \geq \pi_M(x'')$ and $\pi_A(x') \leq \pi_A(x'')$.*

In order to evaluate the "reasonableness" of monotonicity, consider the following analogy. Suppose that a gregarious gourmet's utility from visiting a restaurant increases with the quality of the restaurant and with the size of the crowd; this person likes to see and be seen while eating. The gourmet receives a signal of the restaurant's quality and knows that this signal is positively related to the signals that other gourmets receive, because all signals are positively correlated with the true quality. The gourmet in this example corresponds to an agricultural worker.[9] A higher quality of the restaurant corresponds to a lower value of θ – something that makes it more attractive to go the restaurant, other things equal. The gourmet's fondness for crowds corresponds to increasing returns to scale in the manufacturing sector.

The meaning of "non-monotonic" in this analogy is that there exists a set of signals for which the gourmet is less likely to go the restaurant when he receives a signal that the quality is high. The only rationale for this "perverse" response is that the gourmet believes that other gourmets are less likely to go to the restaurant when they think the quality is high. Moreover, the disadvantage arising from a smaller expected crowd more than offsets the higher expected quality of the dinner. Therefore, in order for the gourmet to

[8]This definition requires that the graphs of the functions have the "right slope" as well as being monotonic. However, if they are monotonic, they must have the right slope. For sufficiently small x'', manufacturing workers' dominant strategy is not to migrate, so $\pi_M(x'') = 0$; for sufficiently large x' manufacturing workers' dominant strategy is to migrate, so $\pi_M(x') = 1$. Thus, $\pi_M(x)$ could not be monotonic and also have the "wrong slope".

[9]To make the analogy more exact, we can think of some customers who are waiting at the bar of the restaurant (i.e. the manufacturing workers), who are trying to decide whether to eat there or go home.

respond "perversely" to the signal, he must believe that a substantial number of other gourmets respond perversely to their signals. That is, the degree to which monotonicity fails must be "substantial" in order for an equilibrium to be non-monotonic. An equilibrium in which the inequalities in Definition 1 fail by an arbitrarily small amount could not exist.

To show that there is a unique equilibrium to the migration game, I adopt the assumption that strategies are *Nearly Monotonic*, which means that the inequalities in Definition 1 are allowed to fail by a small amount by not a large amount. More precisely:

Definition 2 *The pair of strategies* $(\pi_A(x), \pi_M(x))$ *is Nearly Monotonic iff it satisfies*

$$\frac{4\varepsilon^2\beta b}{(\beta b+\delta)(1-L)} > \int_x^{x+2\varepsilon} \pi_A(y)dy - \int_{x-2\varepsilon}^{x} \pi_A(y)dy \tag{7}$$

$$\frac{4\varepsilon^2\beta b}{(\beta b+\delta)L} > \int_{x-2\varepsilon}^{x} \pi_M(y)dy - \int_x^{x+2\varepsilon} \pi_M(y)dy. \tag{8}$$

Any strategy that is monotonic is also *Nearly Monotonic,* but the converse is not true.

4.2 Uniqueness

If all workers in a sector have the same response to a signal, then $\pi_i(x)\epsilon\{0,1\}$. If, in addition, the strategies are monotonic (in the sense of Definition 1), then they are step functions. Suppose also that a worker who is indifferent does not migrate. (We need some tie-breaking assumption, but the particular assumption is unimportant.) Under these assumptions, a manufacturing worker migrates if and only if he receives a signal greater than a threshold x_M, and an agricultural worker migrates if and only if he receives a signal less than a threshold x_A. For these strategies, the functions $\pi_i(x)$ are step functions: $\pi_i(x) = I_i(x; x_i)$:

$$I_M(x;x_M) = \left\{ \begin{array}{c} 1 if \ x > x_M \\ 0 if \ x \leq x_M \end{array} \right\} \tag{9}$$

$$I_A(x;x_A) = \left\{ \begin{array}{c} 1 if \ x < x_A \\ 0 if \ x \geq x_A \end{array} \right\}. \tag{10}$$

Obviously, these strategies are monotonic, and therefore *Nearly Monotonic*.

The following Proposition states that there exists a unique equilibrium to the game within the class of *Nearly Monotonic* strategies; this equilibrium consists of the step functions in equations (9) and (10).[10]

Proposition 1 *Assume that $\beta b - \delta > 0$ and agents have diffuse priors over θ. Within the class of Nearly Monotonic strategies, there exists a unique equilibrium to the migration game with almost common knowledge. This equilibrium consists of the pair of threshold signals $X = (x_M, x_A)$, with $x_A < x_M$. A manufacturing worker migrates if and only if he receives a signal $x > x_M$ and an agricultural worker migrates if and only if he receives a signal $x < x_A$.*

4.3 Qualitative properties

For $\varepsilon > 0$, two-way migration may occur, and it becomes more likely as the amount of uncertainty, ε, increases. Define $z \equiv x_M - x_A$. In order for two-way migration to occur, it must be the case that there exists a value of θ such that $x_A > \theta - \varepsilon$ and $x_M < \theta + \varepsilon$. These inequalities require $z \equiv x_M - x_A < 2\varepsilon$.

Proposition 2 *Assume $\beta b - \delta > 0$. In the unique equilibrium (within the class of Nearly Monotonic functions): (i) The difference between the threshold signals, $z \equiv x_M - x_A$, is independent of the initial allocation L. In addition $z < 2\varepsilon$, so for all parameters β, b, and δ there exists realizations of θ such that two-way migration occurs. (ii) An increase in the heterogeneity of information (ε) increases the set of θ for which two-way migration occurs. There exists θ such that two-way migration occurs and more than 50% of the workers leave one sector if and only if $\frac{5\delta - 3\beta b}{8\beta b} < \varepsilon$. (iii) An increase in the adjustment cost parameter δ decreases the set of θ for which two-way migration occurs, and decreases the expected amount of two-way migration. (iv) The migration threshold for workers in both sectors are linear increasing functions of L.*

[10] The proof modifies Morris and Shin's [12] argument in order to take into account the two differences described in footnote 6. Type i must believe that Type $j \neq i$ is using a Nearly Monotonic strategy to ensure that Type i's equilibrium threshold strategy (the step function) is unique. Lemma 3 in the Appendix shows the importance of the monotonicity assumption. In Morris and Shin's model of speculative attack, there is only one type of agent, so the question of what one type believes that another type is doing simply does not arise.

The appendix contains the proof and further discussion of Proposition 2. Part (i) says that whenever $\beta b - \delta > 0$ there is the possibility of two-way migration under almost common knowledge. Part (ii) implies that an increase in heterogeneity of information increases the likelihood of two-way migration. This result is reasonable. With common knowledge, we expect two-way migration to occur if workers in the two sectors are different in some respect. Individuals who find themselves in Sector i when they are better suited to work in Sector j would migrate. The greater the difference between individuals, the greater is the likelihood of two-way migration.[11] Without common knowledge, workers have heterogeneous information. If the amount of heterogeneity exceeds a critical level, $\varepsilon = \frac{5\delta - 3\beta b}{8\beta b}$, then some workers may be entering a sector even though most workers in that sector are leaving it. This critical level of heterogeneity is an increasing function of δ and a decreasing function of βb. An increase in the adjustment cost parameter δ makes two-way migration less likely.

The equilibrium in the limiting case where the amount of uncertainty is arbitrarily small, is particularly simple.

Proposition 3 *Assume that $\beta b - \delta > 0$. In the limit as $\varepsilon \to 0$, $z \to 0$, and*

$$x_M = x_A \equiv x^* = \frac{1}{2} - \frac{\delta}{\beta b}\left(\frac{1}{2} - L\right). \tag{11}$$

The midpoint of the set of indeterminacy, $\hat{L} \equiv \frac{L_1 + L_2}{2}$ [where L_i is given in equation (1)] defines a critical stock. If the initial allocation is $L > \hat{L}$ all agricultural workers move to manufacturing, and for $L < \hat{L}$ all manufacturing workers move to agriculture.

4.4 Policy Intervention

The fact that the equilibrium is determinate under almost common knowledge ($\varepsilon > 0$) means that government policies have predictable effects. The optimal policy intervention depends on the government's information and the instruments it has available, just as in "standard" models. By announcing

[11] Empirically, the amount of two-way migration is substantial. Jovanovic and Moffitt [8] present US data for which the ratio of net to gross flows among sectors ranges from approximately 0.04 to 0.26. over the 1960s and 1970s. Cameron et al.'s [14] summary of US census data shows that the ratio of net to gross flows, either among sectors or geographical areas, is approximately 0.1.

Location of externality	Social Marginal values	Optimal Policy
1) both manufacturing and migration	$a + 2bL;\quad 2\delta\,(u_A + u_M)$	$\hat{\theta} = \frac{\theta}{2}$
2) manufacturing sector only	$a + 2bL;\quad \delta\,(u_A + u_M)$	$\hat{\theta} = \theta - 0.5$
3) no externality	$a + bL;\ \delta\,(u_A + u_M)$	$\hat{\theta} = \theta$

Table 2: Models with different social marginal values

a sector-specific tax or a migration tax/subsidy, the government is able to shift the threshold levels x_i and thereby change the amount of migration in a predictable manner.

The optimal policy also depends on the relation between the manufacturing wage ($a+bL$) and the social marginal value of labor in the manufacturing sector, and on the relation between the price of migration ($\delta\,(u_A + u_M)$) and the social marginal cost of migration. Table 2 presents three possibilities, together with the optimal policy in the limiting case as $\varepsilon \to 0$. The tax-inclusive value of θ is denoted $\hat{\theta}$. Thus, if $\hat{\theta} < \theta$, the social optimum is achieved by taxing the agricultural sector or subsidizing the manufacturing sector, in order to encourage migration into manufacturing.[12]

If there are Marshallian externalities in the manufacturing sector, the social marginal value of labor in that sector exceeds the wage. For example, if labor is the sole mobile input and there are constant returns to scale at the level of the firm, but increasing returns at the level of the sector, value added in manufacturing is $(a + bL)\,L$, so the social marginal value of labor in the sector is $a + 2bL$. The first two rows of Table 2 assume that this is the case.

Migration may cause congestion which workers do not internalize. For example, the social marginal cost of migration may be $2\delta\,(u_A + u_M)$, as in the first row of the table. Alternatively, migration services may be competitively supplied using a fixed factor, and there may be no externality in the sector. In that case, the social cost of migration is $\frac{\delta(u_A+u_M)^2}{2}$, and the social marginal cost equals $\delta\,(u_A + u_M)$, the price workers pay (as in the second row of Table 1).

The externality in the manufacturing sector implies a suboptimal amount of migration to that sector. An externality in the migration sector tends to

[12] A migration tax increases δ, which (from Proposition 2.iii) decreases migration. Clearly, different policies affect different parameters. In the interests of brevity, the text considers only policies that influence $c - a$.

cause an excessive amount of migration. However, under either alternative in the first two rows of Table 2, private decisions lead to circumstances where all labor moves into agriculture, when it is socially optimal for all labor to move into manufacturing. In both cases, the policy that insures that the socially optimal allocation is achieved regardless of the initial allocation, is to tax agriculture (or subsidize manufacturing). The last column of Table 2 shows the optimal policy for the two cases.[13]

The last row of the table shows the case where there is no externality. In order for the manufacturing wage to equal the social marginal value of labor in the sector, and for there to be increasing returns, there would have to be a fixed surplus in that sector, and an increase in the number of workers must enable them to capture a larger part of that surplus. Thus, this circumstance is rather artificial, and it is included only to emphasize the point that the need for government intervention arises because of the presence of one or more externality, just as in standard models. The lack of common knowledge solves the coordination problem.

5 Concluding comments

The modern theory of economic growth emphasizes that equilibrium growth paths may be indeterminate. This kind of result arises in situations where agents with rational expectations solve dynamic problems and technologies are non-convex. These assumptions seem plausible, and the indeterminacy result holds under a variety of model specifications. Two recent papers investigate the robustness of the indeterminacy result in Matsuyama's model.

Herrendorf *et al*. [7] assume that agents' have *heterogeneous ability* (while

[13]The assumption about social marginal values in each row of Table 2 implies an expression for social welfare – the present value of national product in the next period, minus the adjustment cost in the current period. For each set of assumptions, social welfare is a function of migration and the initial stock of labor. Under the maintained assumption that $\beta b - \delta > 0$ the social welfare function is convex and the optimal amount of migration is either L or $1 - L$, depending on the initial allocation. There is a critical value $\hat{L}^S(\theta)$ above which all labor should move to manufacturing. This value satisfies $\hat{L}^S(\theta) < \hat{L}(\theta)$, where the function $\hat{L}(\theta)$ is given by Proposition 3. Thus, in the absence of policy intervention, there are values of L such that all labor would move to agriculture even though it is socially optimal for all labor to move to manufacturing. To obtain the optimal policy, choose $\hat{\theta}$ to solve $\hat{L}^S(\theta) = \hat{L}(\hat{\theta})$. Of course, this policy is needed only if $\hat{L}^S < L < \hat{L}$; if these inequalities do not hold, private decision are socially optimal.

in my model agents have *heterogenous information).* They show that if the degree of heterogeneity of ability is sufficiently small, the equilibrium remains indeterminate, although if it is sufficiently large the equilibrium is determinate.

Frankel and Pauzner [5] assume that agents are uncertain about the future (while in my model, agents are uncertain about what other agents know in the current period). A parameter which affects the desirability of working in the increasing returns to scale sector evolves randomly and independently of agents' actions. In addition, there are sets of values of the random parameter (dominance regions) at which agents prefer a particular sector independently of the choices other workers make. Provided that the changes in the random variable occur frequently relative to the rate of change in the allocation of workers, an arbitrarily small variance leads to a determinate equilibrium.

If one regards the assumption of almost common knowledge as reasonable in the current context, the results here imply that the indeterminacy of equilibria is fragile. The critical question, then, is whether the assumption is reasonable. I have used a static model to represent a dynamic process, and it is natural to ask whether this simplification obscures anything important.

The advantage of the static formulation is its simplicity. It would be possible to introduce lack of common knowledge into a genuinely dynamic model, but this would be a difficult equilibrium problem. A simpler strategy begins with the observation that the indeterminacy in growth theory does not rely on complicated dynamics, and certainly does not rely on an infinite time horizon. Rather, it depends on a non-convex technology and agents with rational expectations. These two features can be imbedded in a static setting which (under common knowledge) reproduces the insights of Krugman's dynamic model. With this static model, it is straightforward to replace the assumption of common knowledge with the assumption of almost common knowledge.

With almost common knowledge, there is "very little uncertainty" after agents receive their signal. The uniqueness proofs require that there be "substantial" uncertainty before the signal, i.e., that there be dominance regions. An agent who observes a very high or a very low signal which leaves him uncertain that the economic fundamental is in the dominance region, must believe that a non-negligible fraction of agents have obtained even more extreme signals.

If the random parameter that describes the economic fundamental is serially correlated – as is reasonable – the information revealed in one period may cause the next period's priors to be so informative that either the dominance regions are eliminated, or that an agent who receives a extreme signal thinks that it is unlikely that other agents have received more extreme signals. In this case, public information obtained from history dominates private information, and there may be multiple equilibria.

Chamley [3] notes this possibility and concludes that the use of almost common knowledge to obtain a determinate equilibrium "is inappropriate when the game is played repeatedly, *by different agents in each period*, and the structural parameters change by small amounts between periods" (page 893, emphasis added). He provides an example to illustrate this claim. There are three interesting features of the claim and the supporting example.

First, the example shows that some specifications of uncertainty which are reasonable in a one-period problem may not lend themselves to a dynamic generalization. Adding serial correlation of the economic fundamental to the model with a uniform prior and uniform signal would require an unreasonably large innovation in each period, in order that publicly available information not dominate the private signal. For other specifications, the uniqueness result can survive in a dynamic setting, as shown by Morris and Shin's [11] extension of their earlier paper.

Second, Chamley's example involves a game that is played by different agents in each period, whereas the natural dynamic version of the model in this paper involves long-lived agents. We know from Frankel and Pauzner's paper that in this setting the lack of common knowledge is not needed to obtain a determinate equilibrium. However, in their paper, a long-lived agent (genuine dynamics) is needed for determinacy. Thus, with almost common knowledge we do not need dynamics, and with dynamics we do not need informational asymmetries, in order to obtain determinacy. In other words, there are at least two forces – three if we include the heterogeneity of ability in Herrendorf et al. – that promote a determinate equilibrium.

Third, in order to *overturn* the unique equilibrium, Chamley's example requires that the structural parameter change slowly, so that the publicly available knowledge swamps the private information. This requirement has the same flavor as Frankel and Pauzner's condition that the public information change relatively quickly, in order to *maintain* the unique equilibrium.

Regardless of whether agents are uncertain about what other agents know, or uncertain about what will happen in the future, these examples suggest that the economic environment must change relatively quickly in order to support a unique equilibrium. In this sense, one might regard the introduction of randomness or the introduction of asymmetric information as being major changes to the original deterministic model. Under this view, the implications of these models do not challenge the robustness of the indeterminacy in the original model. The alternative view is that since the models in Frankel and Pauzner and in this paper involve either an arbitrarily small variance in the fundamental or an arbitrarily small degree of heterogeneity of information about the fundamental, they can be considered perturbations of the original model, and their results interpreted as a challenge to the indeterminacy result.

A Appendix: Proofs and Discussion of Propositions

To conserve notation, I replace the second and third arguments of $G_i(x;\pi_M,\pi_2)$ with x_i in the case where $\pi_i = I_i(x;x_i)$. For example, if the manufacturing worker's behavior is described by the general strategy $\pi_M(x)$ and the agricultural worker's behavior is described by the step function in equation (10), the expected benefit of migration to Type i who observes the signal x is $G_i(x;\pi_M,x_A)$. If both types use the step functions, the expected gain of migration is $G_i(x;x_M,x_A) = G_i(x;X)$, where $X = (x_M, x_A)$.

Consider an arbitrary reference strategy pair $\Pi = (\pi_M, \pi_A)$ and an alternative $\Pi' = (\pi'_M, \pi'_A)$ such that $\pi_M(x) \geq \pi'_M(x)$ and $\pi_A(x) \leq \pi'_A(x)$. Under the alternative strategy a manufacturing worker is no more likely to migrate and an agricultural worker is no less likely to migrate, relative to the reference strategy. A simple calculation (details omitted) establishes:

Lemma 2 *Assume $\beta b - \delta > 0$. In this case, $G_M(x;\Pi) \geq G_M(x;\Pi')$ and $G_A(x;\Pi) \leq G_A(x;\Pi')$.*

For example, it is more attractive for a manufacturing worker to migrate if he believes that more workers will leave manufacturing and that fewer workers will leave agriculture. If we were to reverse the assumption in the lemma, so that $\beta b - \delta < 0$, the conclusion would not follow. In that case, if a manufacturing worker believed that more workers would leave manufacturing, migration would be less attractive.

Lemma 3 *Assume that agents have diffuse priors over θ. (i) For any Nearly Monotonic function π_A, $G_M(x_M;x_M,\pi_A)$ is a continuous increasing function of x_M, and there exists a unique $\hat{x}_M(\pi_A)$ that satisfies $G_M(x_M;x_M,\pi_A) = 0$. (ii) For any Nearly Monotonic function π_M, $G_A(x_A;\pi_M,x_A)$ is a continuous decreasing function of x_A and there exists a unique $\hat{x}_A(\pi_M)$ that satisfies $G_A(x_A;\pi_M,x_A) = 0$.*

Proof. (i) When π_M is given by equation (9), $E_{\theta|x_M} h(\theta;\pi_M) = \frac{1}{2}$. Substituting this value into equation (4) gives

$$G_M(x_M;x_M,\pi_A) = \beta b\,[x_M - L] + \frac{\beta b - \delta}{2} L - (\beta b + \delta)(1 - L) E_{\theta|x_M} h(\theta,\pi_A). \quad (12)$$

Since $0 \leq h(\theta, \pi_A) \leq 1$ there exists sufficiently large and small values of x_M such that $G_M(x_M; x_M, \pi_A)$ is either positive or negative. Continuity follows from the integrability of π_i. The derivative of G_M is

$$\frac{d}{dx_M} G_M(x_M; x_M, \pi_A) = \beta b - \frac{(\beta b + \delta)(1-L)}{2\varepsilon} \left[h\left(x_M + \varepsilon; \pi_A\right) - h\left(x_M - \varepsilon; \pi_A\right) \right] > 0.$$

The inequality follows from the definition of h [equation (3)] and the assumption of *Near Monotonicity*, equation (7). The monotonicity of G_M in x_M and the fact that G_M can be positive or negative implies that there exists a unique $\hat{x}_M(\pi_A)$ that solves $G_M(x_M; x_M, \pi_A) = 0$. (ii) The proof of second part is identical, so I omit it. ■

Discussion of Lemma 3 (dominance regions). The first line of the proof and the statement that x_M can be sufficiently large or small use the assumption of diffuse priors. Suppose, instead, that the upper bound on the prior support is $\bar{\theta}$. In this case, for $x_M > \bar{\theta} - \varepsilon$, the marginal manufacturing worker (i.e., the one who observes $x = x_M$) has an expected value of θ equal to $\frac{\bar{\theta} + x_M - \varepsilon}{2} < x_M$ and believes that the fraction $\frac{\bar{\theta} - x_M + \varepsilon}{4\varepsilon} < \frac{1}{2}$ of manufacturing workers will migrate. For this worker, the lower bound on the expected benefit of migration (which equals the expected benefit conditional on all agricultural workers migrating) is

$$\beta b \left(\frac{\bar{\theta} + x_M - \varepsilon}{2} - L \right) + \frac{\bar{\theta} - x_M + \varepsilon}{4\varepsilon} \left(\beta b - \delta\right) L - \left(\beta b + \delta\right)\left(1 - L\right)$$

For small ε this expression is decreasing in x_M, so a necessary and sufficient condition for it to be non-negative (and thus for the expected benefit of migration to be non-negative) is that it is non-negative at $x_M = \bar{\theta} + \varepsilon$, which implies $\bar{\theta} \geq \frac{\beta b + \delta(1-L)}{\beta b}$. (A different bound is obtained if ε is large.)

Thus, when manufacturing workers use threshold strategies, the expected benefit of migration is positive for high signals provided that $\bar{\theta}$ is sufficiently large. Using a similar argument it is easy to show that the expected benefit of migration is negative for low signals, provided that the lower support of θ is small. For intermediate signals, the expected benefit is monotonically increasing and has a unique 0 root.

Without the assumption of dominance regions, the function $G_M(x_M; x_M, \pi_A)$ may have two (or more) zero roots In that case, the equilibrium (conditional on π_A) might still be unique, but it would not be monotonic in the signal, and probably would not be *Nearly Monotonic.*

The next step shows that π_i is a step function whenever π_j, $j \neq i$, is *Nearly Monotonic.*

Lemma 4 *Assume that $\beta b - \delta > 0$. (i) Suppose that π_A is Nearly Monotonic. Then the unique equilibrium response π_M is the step function defined in equation* (9)*, with $x_M = \hat{x}_M(\pi_A)$, defined in Lemma 3. (ii) Suppose that π_M is Nearly Monotonic. Then the unique equilibrium response π_A is the step function define in equation* (10)*, with $x_A = \hat{x}_A$.*

Proof. (i) Define $\bar{x}_M = \sup\{x \mid \pi_M(x) < 1\}$ and $\underline{x}_M = \inf\{x \mid \pi_M(x) > 0\}$. Then

$$\bar{x}_M \geq \sup\{x \mid 0 < \pi_M(x) < 1\} \geq \inf\{x \mid 0 < \pi_M(x) < 1\} \geq \underline{x}_1. \quad (13)$$

If $\pi_M(x) < 1$ some manufacturing workers do not migrate when they receive signal x, so $G_M(x;\Pi) \leq 0$. By continuity, $G_M(\bar{x}_M;\Pi) \leq 0$. Clearly, $I_M(x;\bar{x}_M) \leq \pi_M(x)$, so by Lemma 2, $G_M(x;\Pi) \geq G_M(x;\bar{x}_M,\pi_A)$ Evaluating this inequality at $\bar{x}_M$ and using the previous result implies

$$0 \geq G_M(\bar{x}_M;\Pi) \geq G_M(\bar{x}_M;\bar{x}_M,\pi_A).$$

Lemma 3 then implies that $\bar{x}_M \leq \hat{x}_M(\pi_A)$. A similar argument establishes that $\underline{x}_M \geq \hat{x}_M(\pi_A)$. These two inequalities and equation (13) imply that $\bar{x}_M = \hat{x}_M(\pi_A) = \underline{x}_M$ which establishes part (i) of the lemma. (ii) The proof of the second part is virtually identical. ■

The previous lemmas lead almost immediately to the proof of Proposition 1.

Proof. (Proposition 1) Lemma 4 means that we can restrict attention to step functions, so we need to show that there is a unique pair $X = (x_M, x_A)$ that satisfy $G_M(x_M;X) = 0 = G_A(x_A;X)$. First, we consider the graph of the curve that satisfies $G_M(x_M;X) = 0$. From Lemma 3 we know that $\frac{d}{dx_M}G_M(x_M;x_M,x_A) > 0$. From equations (4) and (3) we have

$$\frac{d}{dx_A}G_M(x_M;x_M,x_A) = -(1-L)(\beta b+\delta)\frac{d}{dx_A}E_{\theta|x_M}\max\left[0,\frac{x_A-\theta+\varepsilon}{2\varepsilon}\right] \leq 0. \quad (14)$$

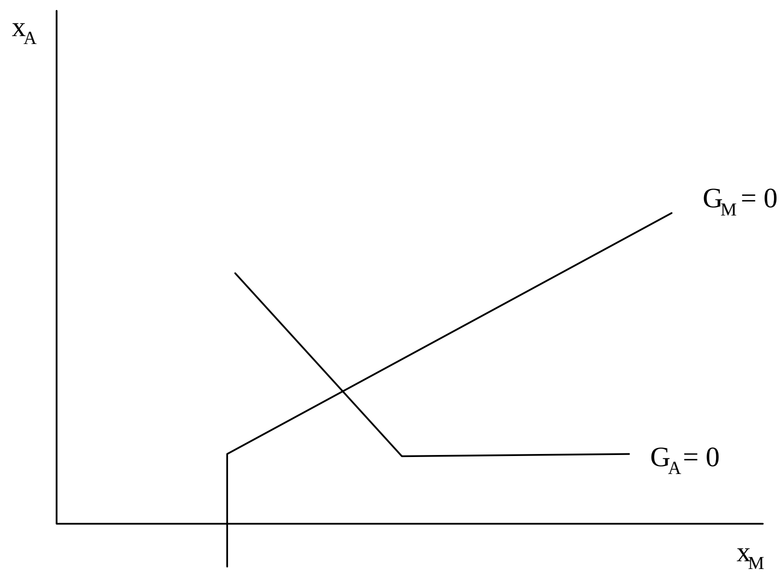


Figure 2: The Unique Threshold Signals

Treating x_M as an implicit function of x_A, defined by the equation $G_M(x_M; X) = 0$, we know from Lemma 3 that the domain of x_M is the entire real line. From Lemma 3 and equation (14) we have

$$\frac{dx_M}{dx_A}_{|G_M=0} \geq 0.$$

A similar argument implies that we can regard x_A as an implicit function of x_M defined by the equation $G_A(x_A; X) = 0$. The domain of this function $(x_A(x_M))$ is the entire real line, and

$$\frac{dx_A}{dx_M}_{|G_A=0} \leq 0.$$

(Figure 2 shows these graphs.) Thus, there is a unique solution to $G_M(x_M; X) = 0 = G_A(x_A; X)$. Lemma 1 implies that $x_A < x_M$. ■

The proof of Proposition 2 proceeds by calculating the roots of $G_M(x_M; X) = 0 = G_A(x_A; X)$ under the assumption that $z > 2\varepsilon$ and then under the assumption that $z < 2\varepsilon$.

Proof. (Proposition 2) (i) First, consider the possibility that $z > 2\varepsilon$. In this case, the marginal manufacturing worker (i.e. the worker who is indifferent) knows that no agricultural worker will migrate, and thus, $E[u_A \mid x_M] = 0$. Since $E[\theta \mid x_M] = x_M$, this marginal agent expects that half of the other manufacturing workers observe $x \geq x_M$. Consequently, $E[u_M \mid x_M] = \frac{L}{2}$. Substituting these expressions into equation (4) and evaluating G_M at $x = x_M$ gives the linear equation for x_M

$$G_M(x_M; X) = \beta b \left[x_M - L\right] + \frac{L}{2}\left(\beta b - \delta\right) = 0. \tag{15}$$

Repeating these steps for the agricultural worker yields

$$G_A(x_A; X) = \beta b \left[L - x_A\right] + \frac{1 - L}{2}(\beta b - \delta) = 0. \tag{16}$$

Adding these two equations gives

$$z = x_M - x_A = -\frac{\beta b - \delta}{2\beta b} \tag{17}$$

which implies that $z < 0$, in view of the assumption that $\beta b - \delta > 0$. Equation (17) violates Proposition 1. Consequently, $z > 2\varepsilon$ cannot occur in equilibrium.

When $z < 2\varepsilon$ and $\pi_i(x) = I_i(x; x_i)$, equation (3) implies

$$h(\theta; x_A) = \min\left(1, \max\left[\frac{x_A - \theta + \varepsilon}{2\varepsilon}, 0\right]\right). \tag{18}$$

Define the set $\Gamma = \{\theta \mid x_M - \varepsilon \leq \theta \leq x_A + \varepsilon\}$, the set of possible values of θ (given the signal x_M) where $h_A > 0$. Since $z < 2\varepsilon$, Γ is nonempty. For $\theta \epsilon \Gamma$ equation (18) implies

$$h(\theta; x_A) \mid \theta \epsilon \Gamma = \frac{x_A - \theta + \varepsilon}{2\varepsilon}. \tag{19}$$

We have

$$E\left(\theta \mid \theta \epsilon \Gamma\right) = \frac{x_M + x_A}{2}.$$

Using this result in equation (19) implies

$$E\left[h(\theta; x_A) \mid \theta \epsilon \Gamma\right] = \frac{x_A + \varepsilon}{2\varepsilon} - \frac{x_M + x_A}{4\varepsilon} = \frac{x_A - x_M + 2\varepsilon}{4\varepsilon} = \frac{2\varepsilon - z}{4\varepsilon}.$$

From the rules of conditional expectations, we have

$$E\left[h_A \mid x_M\right] = E\left[h_A \mid \theta\epsilon\Gamma\right] \cdot \Pr(\theta\epsilon\Gamma \mid x_M)$$

and from the definition of Γ,

$$\Pr(\theta\epsilon\Gamma \mid x_M) = \frac{x_A - x_M + 2\varepsilon}{2\varepsilon} = \frac{2\varepsilon - z}{2\varepsilon}.$$

The last three equations imply

$$E\left[h_A \mid x_M\right] = \frac{(2\varepsilon - z)^2}{8\varepsilon^2} \equiv H. \tag{20}$$

The definition of H, the fact that $E[\theta \mid x_M] = x_M$ and equation (4) enable us to write $G_M(x_M; X) = 0$ as

$$G_M(x_M; X) = \beta b x_M - \frac{\beta b + \delta}{2} L - H\left(1 - L\right)\left(\beta b + \delta\right) = 0. \tag{21}$$

Using parallel steps, we can write $G_A(x_A; X) = 0$ as

$$G_A(x_A; X) = -\beta b x_A + \frac{\beta b + \delta}{2} L - HL\left(\beta b + \delta\right) + \frac{\beta b - \delta}{2} = 0. \tag{22}$$

Adding equations (21) and (22) implies

$$-\beta b z - H(\beta b + \delta) + \frac{\beta b - \delta}{2} = 0. \tag{23}$$

Rearranging equation (23) using the definition of H, results in

$$z^2 - z\left(\frac{2\varepsilon\beta b + \beta b + \delta}{\beta b + \delta}\right) 4\varepsilon + \frac{8\delta\varepsilon^2}{\beta b + \delta} = 0. \tag{24}$$

To complete the proof of part (i) it is necessary to show that there exists a unique positive root of (24) which is always smaller than 2ε. Define

$$\sigma \equiv 1 + \frac{2\varepsilon\beta b}{\beta b + \delta} > 1; \qquad \Delta \equiv \sigma^2 - \frac{2\delta}{\beta b + \delta} > 0 \tag{25}$$

where the last inequality uses the fact that $\delta < \beta b \Longrightarrow \frac{2\delta}{\beta b+\delta} < 1$. With these definitions, we can write the roots of (24) as

$$2\varepsilon(\sigma \pm \sqrt{\Delta}).$$

Both roots are real; the larger exceeds 2ε, so the smaller root,

$$z^* \equiv 2\varepsilon(\sigma - \sqrt{\Delta}) \tag{26}$$

is only remaining candidate. Using the definitions in equation (25) we have

$$\sigma - \sqrt{\Delta} < 1 \Longleftrightarrow \varepsilon > \frac{\delta - \beta b}{4\beta b}. \tag{27}$$

For $\delta - \beta b < 0$ the second inequality in equation (27) is always satisfied, so $z^* < 2\varepsilon$ is the correct root.

(ii) The first sentence in this part of the proposition is equivalent to the claim that $\frac{d\left(\frac{z}{2\varepsilon}\right)}{d\varepsilon} < 0$. To establish this inequality, use equations (25) and (26) to obtain

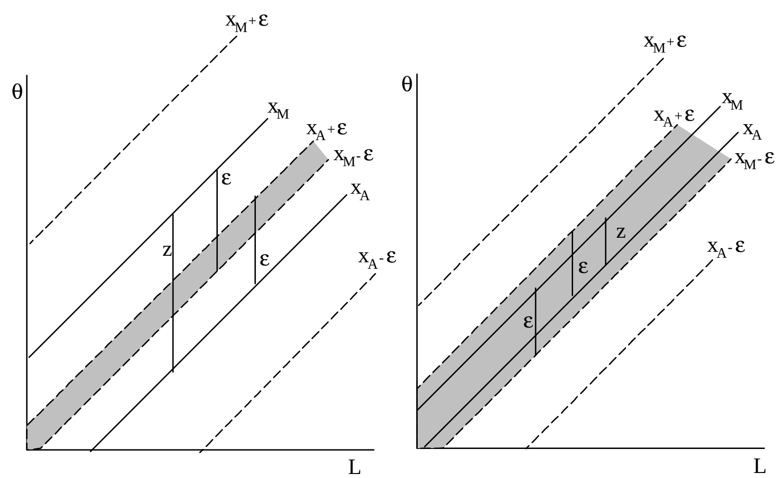
$$\frac{d\left(\frac{z}{2\varepsilon}\right)}{d\varepsilon} = \frac{d\sigma}{d\varepsilon}\frac{1}{\sqrt{\Delta}}\left(\sqrt{\Delta} - \sigma\right) < 0.$$

In order for there to exist θ such that two-way migration occurs when more than half of the workers leave one sector, it is necessary and sufficient that $z^* < \varepsilon$. This inequality holds if and only if

$$\begin{aligned} \sigma - \sqrt{\Delta} &< \frac{1}{2} \Longleftrightarrow \left(\sigma - \frac{1}{2}\right)^2 < \Delta \Longleftrightarrow \\ \frac{5\delta - 3\beta b}{8\beta b} &< \varepsilon. \end{aligned}$$

(iii) Part three of the proposition is equivalent to the claim that $\frac{dz}{d\delta} > 0$. When z increases, the distance between the two threshold signals increases, decreasing the set of θ that satisfy the two inequalities $\theta - \varepsilon < x_A$ and $\theta + \varepsilon > x_M$. The derivative $\frac{dz}{d\delta}$ equals

$$\frac{dz}{d\delta} = \frac{-4\varepsilon\beta b}{(\beta b + \delta)^2}\left[\varepsilon - \frac{2\varepsilon\sigma + \beta b}{2\sqrt{\Delta}}\right].$$



(a) $z > \varepsilon$ (b) $z < \varepsilon$

Figure 3: The Equilibrium with Almost Common Knowledge

A straightforward calculation shows that the term in square brackets is negative.

(iv) Solving equation (21) for x_M yields

$$x_M = \frac{1}{\beta b}\left[(\beta b + \delta)\left(\frac{1}{2} - H\right)L + H(\beta b + \delta)\right]. \tag{28}$$

The fact that $0 < z^* < 2\varepsilon$ (which was shown above) and the definition of H in equation (20) implies $H < \frac{1}{2}$ so x_M is increasing in L. Since x_M and x_A differ by the constant z^*, x_A is also increasing in L. ■

Figure 3 illustrates the proposition. The two panels illustrate the two cases where $\frac{5\delta - 3\beta b}{8\beta b} > \varepsilon$ and $\frac{5\delta - 3\beta b}{8\beta b} < \varepsilon$. In both panels the shaded area shows the set of θ such that two-way migration occurs. For example, in panel a, when $\theta > x_M + \varepsilon$ all workers in manufacturing migrate, and no agricultural workers migrate. For $\theta < x_A - \varepsilon$ all workers in agriculture, and no manufacturing workers migrate. For $\theta \epsilon (x_A + \varepsilon, x_M + \varepsilon)$ some (but not all) manufacturing workers, and no agricultural workers migrate. Similarly, for $\theta \epsilon (x_A - \varepsilon, x_M - \varepsilon)$ some (but not all) agricultural workers, and no manufacturing workers migrate. Panel b has a similar interpretation.

When $\frac{5\delta-3\beta b}{8\beta b} < \varepsilon$ (panel b) two-way migration occurs in some situations where more than half of the workers in one sector migrate. For example if $\theta \epsilon (x_M, x_A + \varepsilon)$ (panel b) then some agricultural workers move to manufacturing even though more than half of manufacturing workers move to agriculture. Note that the inequality $\frac{5\delta-3\beta b}{8\beta b} < \varepsilon$ holds for all ε whenever $\delta < \frac{3\beta b}{5}$. Thus, when δ is very small relative to βb – so that set of initial allocations that give rise to multiple equilibria under perfect information is large – the extent of two-way migration also tends to be large.

Proof. (Proposition 3) Taking the limit as $\varepsilon \to 0$ of equation (23) and using the fact that $\lim_{\varepsilon \to 0} z = 0$ implies

$$\lim_{\varepsilon \to 0} H = \frac{\beta b - \delta}{2(\beta b + \delta)}.$$

Using this result in the limiting form (as $\varepsilon \to 0$) of equation (28) implies equation (11). We can solve equation (11) for L and use the fact that in the limit as $\varepsilon \to 0$ workers receive perfect information about the state of the economy. Thus, we can replace the signal x by the parameter θ to write equation (11) as

$$\hat{L} = \frac{\beta b}{\delta}\theta + \frac{\delta - \beta b}{2\delta} = \frac{L_1 + L_2}{2}. \tag{29}$$

The last equality uses the definitions of L_1 and L_2 from equation (1). A signal x informs workers of the state of the economy, θ, and defines a critical allocation $\hat{L}$. If the current allocation, L, exceeds this critical level, all workers in agriculture move to manufacturing. Otherwise, all workers in manufacturing move to agriculture. The critical allocation is mid-way between the bounds of the set of allocations for which there exists multiple equilibria under perfect information. ■

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