Indeterminacy with Environmental and Labor Dynamics*

Larry Karp† and Thierry Paul‡

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Abstract

We study the joint dynamics of labor reallocation and environmental change when workers have rational expectations and incur migration costs. We emphasize the relation between parameter values and the area of state space in which indeterminacy of equilibria can occur. Unlike the one-dimensional model in which the wage differential adjusts instantaneously, here the measure of the region of indeterminacy is not monotonic in the cost of adjustment.

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†207 Giannini Hall,Department of Agricultural and Resource Economics, University of California, Berkeley CA 94720. Email: karp@are.berkeley.edu

‡IDEP/ GREQAM, 2 rue de la Charité, Centre de la Vieille Charité, 13002 Marseille. Email : tpaul@univ-aix.fr

1 Introduction

Rapid economic development and large-scale industrialization are associated with inter-sectoral labor migration and environmental damage. In some cases this damage affects the productivity of resource-based industries, increasing the incentive for labor migration. We study the interaction between environmental change and the sectoral reallocation of factors of production. In particular, we want to understand how the rates of change in the environment and labor allocation – the two stock variables – affect the existence and the severity of coordination problems in a competitive equilibrium.

The city of Ansan in Korea illustrates the effects of pollution on environmentally sensitive industries. Located 30 kilometers south of Seoul, Ansan was originally designed to relieve the pressures associated with pollution in Seoul. Employment and development opportunities led to rapid growth, with the population increasing from 300,000 to 580,000 between 1990 and 2000 (Bai 2002). High levels of industrial wastewater discharge caused local farmers and fishermen to lose their traditional means of livelihood. Meanwhile, in Seoul, the relocation of firms and workers to Ansan provided more land for urban development and more parks in central districts, improving the environmental quality there and favoring tourism. Similarly, in the Aral Sea region the massive schemes used to irrigate cotton crops destroyed the local ecosystem, causing traditional industries to collapse. In Northern California and Oregon, agricultural use of water threatens local fisheries and tourism. The allocation of water affects the ecosystem and workers' incentives to move to a different sector.

These examples illustrate the interactions between economic development, the environment, and the sustainability of some industries. Production in one sector causes pollution or otherwise harms an environmentally sensitive sector. This impact can accelerate industrial restructuring and create further environmental changes. Such self-reinforcing processes might result in large changes.

In some cases the rational expectations competitive equilibrium is unique, and in other cases the equilibrium outcome depends on what agents think is going to happen. For example, if agents believe that the environmentally sensitive sector is doomed, workers may move to the pollution-creating sector, causing further damage to the environmental sector. If instead agents think that prospects are good in the environmentally sensitive sector, they may remain in (or move to) that sector, reducing pollution and benefiting the environmental sector. When either prophecy can be self-fulfilling, the equilibrium is indeterminate.

Regardless of whether the equilibrium is unique or indeterminate, the presence of the externality means that the equilibrium is not socially optimal. However, the policy problem in the two situations is different. When the equilibrium is unique, standard policies can induce agents to internalize the externality. When the equilibrium is indeterminate, the effect of a policy is harder to predict because it depends on agents' (indeterminate) beliefs. Part of the policy problem in this case is to help agents coordinate on a good equilibrium. We examine the relation between indeterminacy and the relative and absolute speeds of adjustment of labor allocation and the environment.

We use Krugman (1991)'s model as a starting point. In that model, inter-sectoral migration involves adjustment costs, the magnitude of which depends on the amount of migration and on an exogenous parameter. Agents base their migration decision on a comparison of current migration costs and the present value of the future wage differential. An externality causes the wage in one sector to increase with the number of workers in that sector. The competitive equilibrium is indeterminate when the externality is strong or the adjustment costs are low, or when agents are very patient.

This type of one-dimensional model is ideal for illustrating the possibility of indeterminacy and for understanding the economic fundamentals associated with it. However, a one-dimensional model does not adequately describe the interactions between an environmental stock and the growth of a sector. The importance of these interactions in the development process justifies formal modeling. It is obvious that the same kinds of coordination problems that can occur in a one-dimensional state variable model can also arise with a higher dimensional state space. We briefly describe our model and introduce our research questions.

In our model, output in the manufacturing sector creates pollution, which harms a renewable environmental stock. A change in the environmental stock affects labor productivity in agriculture and therefore affects the wage differential between sectors. Manufacturing output and the flow of pollution depend on the stock of labor in manufacturing. Labor migration between sectors is costly, so the sectoral reallocation of labor is not instantaneous; the magnitude of adjustment costs affects the speed of adjustment. Workers have rational expectations. As either the speed of adjustment of the environment or the speed of adjustment of labor become infinite (adjustment costs go to zero), we obtain a one-dimensional model. In the general model the adjustment speed is finite and positive for both stocks.

We use this model to address two questions: (1) What is the role of relative versus absolute speeds of adjustment (in the environmental stock and the sectoral allocation of labor) in determining the qualitative dynamics? (2) How does a change in one parameter affect the "likelihood" of indeterminacy, with respect to both (a) other parameters and (b) the initial value of the state variables? We now motivate these questions.

For an arbitrary interval of time, suppose that there is the potential for a large adjustment of one stock variable relative to the potential adjustment of the other variable. For example, labor might be able to adjust quickly relative to the environment. It might seem that the qualitative dynamics would not depend on the absolute magnitude of adjustment, since that varies with the arbitrary magnitude of the interval of time. This statement suggests that the qualitative dynamics depend on relative rather than absolute speeds of adjustment. There is some truth to this conjecture, but it is wrong concerning the existence of indeterminacy. The model helps to clarify the importance of relative and absolute adjustment speeds in determining qualitative dynamics.

There are two ways of thinking about the influence of a parameter (such as an adjustment speed) on the "likelihood" of indeterminacy. As a particular parameter changes, we can examine the change in the measure of the set of remaining parameters for which indeterminacy is a possibility (Question 2a). Alternatively, as a particular parameter changes, we can examine the change in the measure of the set of initial conditions of the predetermined states (labor allocation and the environmental stock) for which the equilibrium is indeterminate (Question 2b).

In either case, there is an obvious sense in which indeterminacy becomes "more likely" – and the policy question more interesting – as the measure of the set increases. Most of the previous literature emphasizes Question 2a; by construction of the models, Question 2b does not arise. Krugman's model is an exception, since it addresses both issues. However, in that one-dimensional model, both questions have the same answer, and it might appear that they are essentially the same question. Our two-dimensional model shows that the two questions can have opposite answers.

If both stocks adjust instantaneously, i.e. if labor adjustment costs are 0 and the wage differential is proportional to the labor allocation, the presence of the externality means that agents play a static coordination game. There are two Nash equilibria in this static game, specialization in either sector. Adding (only) adjustment costs, thereby slowing labor adjustment, tends to reduce the set of initial allocations at which the equilibrium is indeterminate; thus, slower adjustment of labor makes indeterminacy "less likely". An increase in adjustment costs also decreases the set of other parameters at which indeterminacy can occur. In a more general setting, where there are adjustment costs for labor and where the wage differential depends on the environment, both stock variables change smoothly. We find that in this case, a slower speed of adjustment for labor does *not* necessarily decrease the set of initial conditions at which indeterminacy occurs, although (as in the one-dimensional model) it does reduce the set of other parameters at which indeterminacy occurs.

The next section discusses related literature. Section 3 presents the model. Section 4 describes the dynamics with rational expectations and considers extreme cases in which the speeds of adjustment of the two stocks differ by orders of magnitude, but both remain positive and finite. Section 5 presents numerical simulations.

2 Literature review

There are two types of models of indeterminacy; they have many features in common, but they are used to describe quite different economic issues. Most of the literature deals with the situation where there are multiple equilibrium paths leading to a particular stable steady state. We refer to this as "local indeterminacy". Either this steady state is globally asymptotically stable, or by assumption the initial condition is in the basin of attraction of this steady state. Models of local indeterminacy provide a possible explanation for economic fluctuations.

We define "global indeterminacy" to mean the situation where (as in Krugman (1991) and Matsuyama (1991)), starting from the same initial condition, there exist different equilibrium paths that approach different steady states. (There may also be different paths to the same steady state, as with local indeterminacy.) With these models there are at least two stable steady states, each with a basin of attraction. "Global indeterminacy" means that the basins of attraction intersect. This kind of model provides a possible explanation for different development paths.

Benhabib and Farmer (1999)'s survey emphasizes local indeterminacy, reflecting the amount of attention in the literature given to this aspect of the topic. Local indeterminacy exists if the number of stable eigenvalues of the dynamic system that describes the economy is greater than the number of predetermined variables. Much of the literature on this topic investigates the characteristics of the model – e.g., increasing returns to scale or some other type of externality

– where this occurs. For example, Benhabib, Meng, and Nishimura (2000) show that indeterminacy can occur with socially constant but privately decreasing returns to scale; Nishimura and Shimonura (2002) shows that with constant returns to scale, indeterminacy can arise if the rate of time preference is an increasing function of the level of consumption.

A growing body of literature assesses the empirical importance of local indeterminacy. Wen (1998b) finds that adding empirically plausible adjustment costs to a one-sector Real Business Cycle model eliminates the possibility of indeterminacy. Adjustment costs "convexify" the technology, and counteract the forces (such as increasing returns to scale or another kind of externality) that promote indeterminacy. Herrendorf and Valentinyi (2000) address the same question using a two-sector neoclassical growth model with adjustment costs and sector-specific externalities in the capital-producing sector. These and many other empirically based papers emphasize the role of adjustment costs. Wen (1998a) shows that indeterminacy is empirically plausible in a model that takes into account the relation between capacity utilization and capital depreciation.

The existing empirically based literature takes as its starting point a model in which the initial condition of the state variable(s) is in the basin of attraction of the steady state. This literature therefore has no interest in assessing the "size" of the basin – i.e., the likelihood that the initial condition is indeed in this basin. This literature addresses our Question 2a but by construction of the models it ignores Question 2b. In contrast, the assessment of the empirical importance of models of global indeterminacy depends on how likely it is that an initial condition lies in more than one basin of attraction.

As the previous discussion of Krugman's model indicates, two types of comparative statics questions may have the same qualitative answer. For example, lower adjustment costs increase the set of parameter values at which indeterminacy can occur (as is also the case with models of local indeterminacy), and lower adjustment costs increase the measure of the intersection of the basins of attraction. This conclusion might suggest that, in the presence of certain externalities, a low adjustment cost increases the empirical importance of global indeterminacy. Our model shows why this conjecture can be incorrect.

Existing literature shows that the presence of a non-convexity in technology or preferences can cause indeterminacy, and this insight obviously applies in an environmental economics setting. However, few papers study indeterminacy in environmental economics. Koskela, Ollikainen, and Puhakka (2000), an exception, use an overlapping generations model with a renewable resource that is both a factor a production and a store of value. In this setting there may be cycles and indeterminacy even in the absence of externalities or imperfect competition.

In a different context, Rubinstein (1989) and Carlsson and Van Damme (1993) show that a change in agents' information can eliminate indeterminacy Morris and Shin (1998) develop this idea to study speculative attacks on currencies. Karp (1999) adapts their argument to show that (under the assumption that strategies are "almost monotonic") lack of common knowledge eliminates indeterminacy in a variation of Krugman's model. Frankel and Pauzner (2000) show that adding exogenous uncertainty to Matsuyama (1991)'s model eliminates indeterminacy. In the same setting, Herrendorf, Valentinyi, and Waldman (2000) show that heterogeneity, rather than uncertainty, can eliminate indeterminacy. Chamley (1999) analyzes a model in which indeterminacy occurs even with exogenous uncertainty.

We noted above that the magnitude of adjustment costs can affect the existence and importance of indeterminacy. There is a growing body of empirical literature that attempts to quantify adjustment costs in different sectors and for different factors. Contributions to this literature include Anderson (1993), Buhr and Kim (1997), Hall (2002), Epstein and Denny (1983), Fernandez-Cornejo, Gempesaw, Elterich, and Stefanou (1992), Hayashi and Inoue (1991), Luh and Stefanou (1991), and Pindyck and Rotemberg (1983).

Milik, Prskawetz, Feitchinger, and Sanderson (1996) study a model in which different state variables evolve at speeds that differ by orders of magnitude. They show that the resulting dynamics can involve transitions from one regime to another that are virtually unpredictable. That model does not involve forward-looking agents, and there is no indeterminacy. However, the model does emphasize relative and absolute speeds of adjustment, and therefore is related to our Question 1.

3 The Model

The model describes a small open economy with two sectors; Manufacturing produces commodity M and Agriculture produces commodity A. There are two primary factors of production: labor (L) and environmental capital (E). The total endowment of labor in the economy is 1. The environmental stock and the allocation of labor change endogenously but are predetermined at a point in time.

3.1 Technology and dynamics

The output of M is a linear function of labor in manufacturing, L_M: $M = 0.5L_M$. Manufacturing also creates pollution, at a rate proportional to output: pollution $= \lambda L_M,\ \lambda > 0$. The agricultural sector uses both labor, L_A and the stock of environment. Agricultural output is $A = EL_A$. The fixed relative price of the manufactured good is 1 and we define $L = L_M$, $L_A = 1 - L$. Given that E and L are bounded, the relevant state space is only a part of the (L, E) plane. Here we consider the dynamics in this part of the plane.

Environmental capital evolves as follows:

$$\dot{E} = g(Z - E) - \lambda L, \tag{1}$$

where the dot denotes a derivative with respect to time, and we omit time subscripts. The long-run level of the environment in the absence of pollution is Z; this is also the maximum level of the environment, assuming that the initial level is less than Z. The constant $g > 0$ is the recovery rate of the environment. At a point in time, the environment is degraded by pollution, λL.

Hereafter we set $\lambda = gZ$. This restriction reduces the number of parameters in the model and it implies that the recovery rate of the environment and the flow of pollution are of the same order of magnitude. The restriction allows us to focus on the relative speeds of adjustment of the two state variables, ignoring the location of the unstable steady state. As an additional simplification, we set $Z = 1$. These parameter restrictions imply that the state space of the model is the unit square. Substituting $\lambda = gZ = g$ into equation (1) gives:

$$\dot{E} = g\left(1 - L - E\right). \tag{2}$$

The aggregate adjustment costs associated with migration is convex in aggregate migration. Each (price-taking) worker pays the marginal migration cost, which is positive whenever migration is positive. The aggregate cost is quadratic in the rate of migration: $c(\dot{L}) = \left(\dot{L}\right)^2/2\gamma$ where γ affects the speed of labor adjustment. A worker's migration decision is based on a comparison of the price of migration (equal to the absolute value of the marginal cost of migration, $c'(\dot{L}) = \dot{L}/\gamma$) and the value of being in the manufacturing sector, defined as q, an endogenous variable; q is the present discounted value of the stream of future wage differentials (Manufacturing wage minus Agriculture wage). Equilibrium *while migration occurs* (i.e., when $\dot{L} \neq 0$) requires that the price an agent has to pay to migrate is equal to the value of changing sectors:

$$\dot{L}/\gamma = q \Rightarrow \dot{L} = \gamma q. \tag{3}$$

When $q > 0$ (and $L < 1$) workers migrate into Manufacturing, and when $q < 0$ (and $L > 0$) they move into Agriculture.

Equation (3) need not hold if all of the labor is in one sector. If, for example, $L = 0$ (all labor is in Agriculture), it may be the case that $q < 0$. (This inequality means that the present discounted value of the agricultural wage exceeds that of the manufacturing wage.) In that case, the constraint $L \geq 0$ prevents further migration into Agriculture, so equation (3) is *not* satisfied. When $L = 0$ and $q < 0$, the equilibrium requires that $\dot{L} = 0$. Including the constraint $0 \leq L \leq 1$, the evolution of L satisfies the complementary slackness conditions

$$\begin{aligned} L = 0 &\Longrightarrow \dot{L} = \max\{0, \gamma q\} \\ L = 1 &\Longrightarrow \dot{L} = \min\{0, \gamma q\} \\ 0 < L < 1 &\Longrightarrow \dot{L} = \gamma q. \end{aligned} \tag{4}$$

Labor markets are competitive, so the wages in the two sectors are $w_A = E$, and $w_M = 0.5$. The wage differential at time t is $\omega(t) = 0.5 - E(t)$, and agents' point expectation at time 0 of the wage differential at time t is $\omega^e(t)$. The point expectation at time t of the present value of being in the manufacturing sector is therefore

$$q = \int_t^T \omega^e(s) e^{-r(s-t)} ds \tag{5}$$

where r is the real interest rate. The upper limit T of the integral is the first time that the equilibrium level of migration is 0. At this time, a worker could migrate at 0 cost.

To simplify the exposition, we use a deviation-form of the model obtained by defining $e = E - 0.5 = -\omega, \ \ l = L - 0.5$. Setting $\bar{l} = \overline{L} - 0.5 = 0.5$, equations (2) and (3) simplify to:

$$\dot{e} = -g\left(l + e\right) \tag{6}$$

$$\dot{l} = \gamma q. \tag{7}$$

We also use

Definition 1 *The state space* Ω *is defined by:* $-0.5 \leq l \leq 0.5$ *and* $-0.5 \leq e \leq 0.5$.

3.2 Some remarks about the model

This model contains only three parameters, the discount rate and the speeds of adjustment of labor and the environment (r, γ, g). We will emphasize the two adjustment speed parameters. At a given value of one of these parameters, we can imagine a probability distribution over the remaining two parameters, and over the initial values of the state variable. With this interpretation, if the prior distributions are uniform, the likelihood of indeterminacy is simply the size of the area in parameter space – or the size of the area of state space – (relative to the entire feasible parameter space or state space) for which indeterminacy occurs. Hereafter, when we speak of the measure of indeterminacy, we mean this relative area.[1]

As we discussed in Section 2, in models of local indeterminacy, and in Krugman's model of global indeterminacy, a lower adjustment cost (a higher value of γ in our setting) increases the empirical relevance of indeterminacy. Our model shows that this result need not hold in a more general model of global indeterminacy.

When the environmental dynamics are instantaneous ($g = \infty$), we obtain Krugman's model; there the existence of indeterminacy depends on the costs of adjustment, the size of the externality (which is fixed given our parameter restrictions) and the interest rate. In that model, the wage adjusts instantaneously to the reallocation of labor. In contrast, when g is finite, the externality has a delayed and cumulative effect via the stock of environment.

When the costs of adjustment are zero so that the labor market dynamics are instantaneous, we obtain Copeland and Taylor (1999)'s model. In that case the equilibrium is indeterminate if and only if the initial level of the environment is such that wages are equal in the two sectors; for all other initial conditions, the competitive equilibrium is unique.

The general model in which γ and g are both finite can be interpreted as a learning-by-doing model, in which production in each sector depends on the stock of knowledge accumulated in that sector, as in Matsuyama (1992). These stocks depend on cumulative production, which depends on the history of the distribution of labor across sectors. As workers move towards a particular sector, the stock of knowledge there increases, raising future productivity.

[1]The choice of a uniform distribution is innocuous insofar as the we are interested in the measure of parameter space for which indeterminacy occurs, but it is not innocuous concerning the measure of state space, as a later footnote explains.

4 The rational expectations dynamics

This section discusses the dynamics of the economy assuming that workers have rational expectations. We explain the boundary conditions for the model, and then discuss basins of attraction for the different steady states. We close by considering extreme forms of the model.

4.1 The differential system

When workers have rational point expectations, $\omega = \omega^e$. Using $e = -\omega$ and differentiating equation (5) we obtain

$$\dot{q} = rq + e. \tag{8}$$

The two state variables e and l are predetermined; q is a jump variable, for which the initial condition is endogenously determined.

The three differential equations (6), (7) and (8) comprise the three-dimensional dynamic system of the economy between the two boundaries $l = -0.5$ and $l = 0.5$ (in the interior of state space Ω). The unique *interior* steady state is $U \equiv (0, 0, 0)'$, which we show below is unstable. For example, suppose that the system is initially at point U, and a positive measure of workers "deviates from equilibrium" by leaving Agriculture and moving to Manufacturing. The increased flow of pollution caused by this deviation reduces the future agricultural wage, increasing the incentive to leave the agricultural sector. That is, the equilibrium response moves the system away from U; the deviation is not self-correcting, so U is an unstable steady state.

The dynamics on the boundaries, $l = -0.5$ and $l = 0.5$, are slightly more complicated, because no equilibrium trajectory can cross these boundaries. We noted this constraint in writing the complementary slackness conditions, equation (4). An analogous condition holds for the model written in deviation form. Each of the two boundaries, $l = -0.5$ and $l = 0.5$, consists of two intervals. For each boundary, on one of these intervals all three differential equations (6), (7) and (8) hold; for the other interval $\dot{l} = 0$, so equation (7) *does not* hold.

We first describe the dynamics on the boundaries, and then provide the economic intuition. We discuss the boundary $l = -0.5$; the dynamics on the boundary $l = 0.5$ are symmetric. The point $(l, e, q) = (-0.5, 0.5, -0.5/r)$ is a steady state of the dynamic system: at $l = -0.5$ and $q = -0.5/r$, the complementary slackness conditions in equation (4) imply that $\dot{l} = 0$; equation (6) implies that $\dot{e} = 0$, and equation (8) implies that $\dot{q} = 0$. At this steady state all labor is in Agriculture and the environment is at its highest level, so that the agricultural wage exceeds the manufacturing wage. The wage differential is -0.5, so the present discounted value of the wage differential is $q = -0.5/r$. No more labor can enter Agriculture, and there is no incentive for labor to leave, so all labor remains in Agriculture. By symmetry, the point $(l, e, q) = (0.5, -0.5, 0.5/r)$ is also a steady state.

Both of these boundary steady states are asymptotically stable. That is, for initial conditions in Ω that are sufficiently close to the respective boundary steady state, there is an equilibrium trajectory that takes the system to that steady state. Consider again the point $(l, e, q) = (-0.5, 0.5, -0.5/r)$. A trajectory might approach this point from within Ω (i.e., with $l > -0.5$) before the trajectory reaches the steady state, or the trajectory might first hit the boundary $l = -0.5$ and then approach the steady state along this boundary. In the latter case, the trajectory must hit the boundary at a value of e greater than a critical level. This critical level divides the boundary $l = -0.5$ into the two intervals mentioned above. Points on the boundary above this critical value of e are on trajectories that approach the steady state ($e = 0.5$), with $l = -0.5$, a constant. Points on the boundary below the critical level of e are on trajectories that take the system into the interior of Ω. The symmetry that results from our assumptions about parameter values implies that this critical value is $e = 0$. (See Proposition 3 below.)

In summary, there are three steady states to this model. The interior steady state is unstable, and the two boundary steady states are asymptotically stable. The equilibrium is indeterminate if the intersection of the basins of attraction of the two stable steady states has positive measure. In that case, there are initial conditions from which either steady state might be approached in equilibrium.

We now consider the dynamics in the interior of Ω. The qualitative dynamics in the neighborhood of U determine the properties of the model. The characteristic polynomial associated with the dynamic system is[2]

$$F(X) = X^3 + (g - r)X^2 - Xgr + g\gamma.$$

The roots of $F(X)$ are the eigenvalues of the system. The signs of the real parts and the

[2] This model is a reversal of the linearized version of the resource model in Berck and Perloff (1984). In their setting, increased production creates a negative externality in the *same* sector, so the stable manifold for the interior equilibrium is two-dimensional; the interior equilibrium is saddle-point stable.

possible existence of complex roots determine the nature of the dynamics. In order to analyze this polynomial we define the following variables,

$$u = \frac{1}{6}\left(-gr(g-r) - 3g\gamma\right) - \frac{1}{27}(g-r)^3$$
$$v = -\frac{gr}{3} - \frac{1}{9}(g-r)^2$$

$$\phi \equiv u^2 + v^3 = -\tfrac{1}{108}g^4r^2 - \tfrac{1}{54}g^3r^3 - \tfrac{1}{108}g^2r^4 + \tfrac{1}{4}g^2\gamma^2 + \tfrac{1}{27}\gamma g^4 - \tfrac{1}{27}g\gamma r^3 + \tfrac{1}{18}g^3r\gamma - \tfrac{1}{18}g^2r^2\gamma. \quad (9)$$

We have the following

Proposition 1 *(i)* $F(X) = 0$ *has two roots with positive real parts and one negative root. (ii)* $F(X) = 0$ *has complex roots if and only if* $\phi > 0$*.*

Proof. Part (i) follows from applying Descartes' rules of signs, and part (ii) follows from the formula for the solution of a cubic equation. ■

If $\phi > 0$, any trajectory in the neighborhood of the unstable steady state U oscillates; trajectories reaching one or the other specialized equilibria can oscillate, depending on the initial condition. In the one-dimensional model (where $g = \infty$) a simple proof establishes that indeterminacy can occur if and only if the roots of $F(X)$ are complex. Section 4.2 discusses the relation between the existence of indeterminacy and the complexity of the roots in the two-dimensional case. Here we provide a generalization of a comparative statics result from the one-dimensional model.

Define $\gamma^*(g,r)$ as the value of γ that satisfies $\phi = 0$. Straightforward calculations (contained in an appendix which is available upon request) establish the following

Proposition 2 *(i) For all finite* g, r *there exists a unique positive value of* $\gamma^*(g,r)$*. (ii)* $\phi > 0 \Longleftrightarrow \gamma > \gamma^*(g,r)$*, (iii)* $\gamma^*_g(g,r) < 0$ *and* $\gamma^*_r(g,r) > 0$*. (iv)* $\phi > 0 \Rightarrow \gamma > \frac{r^2}{4}$*.*

Parts (i) and (ii) imply that for sufficiently low adjustment costs (large γ) the roots of $F(X)$ are complex (so indeterminacy is a possibility). Part (iv) states that a necessary condition for complex roots is that the speed of adjustment is large relative to the discount rate. Parts (i) - (iii) imply that if we increase either g or γ or decrease r, the set of the two remaining parameters under which the roots are complex, strictly increases. Proposition 2 therefore generalizes the comparative statics results from the one-dimensional model by allowing a finite value of g.

When agents are more patient or when it is cheaper to change sectors, complex roots can occur for a wider range of other parameter values. As g increases, the externality becomes more important, also increasing the range of other parameter values for which the roots are complex.

These results provide a partial answer to Question 2a posed in the Introduction, since they show how a change in one parameter affects the range of other parameters where indeterminacy can occur. They also provide a partial answer to Question 1: If we fix the relative speeds at a constant $\alpha = \frac{\gamma}{g}$ and let γ increase from 0, we pass from the region where $\gamma < \gamma^* (g, r)$ to the region where $\gamma > \gamma^* (g, r)$. Thus, the qualitative dynamics clearly depend on absolute as well as relative speeds of adjustment. Section 4.4 provides another perspective on these issues.

We now consider the two stable equilibria, where the economy is specialized in Agriculture or in Manufacturing. When all labor is in Agriculture ($l = -0.5$) the steady state for the environment is $e = 0.5$; when all labor is in Manufacturing ($l = 0.5$) the environmental steady state is $e = -0.5$.

The economy reaches the stable steady states only asymptotically. However, it can reach a boundary $l(T) = k, k \in \{-0.5, 0.5\}$ in finite time. Consider a trajectory beginning at time 0 at a point in the interior of state space, Ω. Define T as the first time at which the economy becomes specialized in one sector. One boundary condition for this trajectory is $l(T) = k, k \in \{-0.5, 0.5\}$.

The boundary condition for q is $q(T) = 0$, as in Fukao and Benabou (1993). To establish this claim, suppose that the economy becomes specialized in Manufacturing at T and that (contrary to our claim) $q(T) \neq 0$. Clearly $q(T) < 0$ cannot hold, or workers would be moving into Agriculture near time T. Suppose then that $q(T) > 0$. In that case, at $T - \epsilon$, q is bounded away from 0, so adjustment costs are strictly positive. In this situation, a worker who is "supposed to move" at $T - \epsilon$ would like to wait until all other workers have moved. This delay causes a small loss in the present discounted value of wages because the worker postpones entry into the high-wage sector. It results in a non-negligible reduction in adjustment costs, since at $T + \epsilon$ migration is free. The delay is therefore profitable, so $q(T) > 0$ cannot be an equilibrium. Consequently, $q(T) = 0$. A similar argument holds if the economy becomes specialized in Agriculture.

The following proposition bounds the feasible value of the environmental state at the time when the economy becomes specialized.

Proposition 3 *A trajectory that satisfies equations (6), (7), (8) and the boundary condition*

$q(T) = 0$ *reaches* $l(T) = -0.5$ *(respectively* $l(T) = 0.5$*) from inside* Ω *with* $0 \leq e(T) \leq 0.5$ *(respectively* $-0.5 \leq e(T) \leq 0$*).*

Proof. Consider a trajectory that satisfies $l(T) = -0.5$, $q(T) = 0$, and $l(T-\epsilon) > -0.5$, $-0.5 < e(T-\epsilon) < 0.5$ for small $\varepsilon > 0$. That is, the trajectory approaches the boundary from the interior of Ω. (The argument for a trajectory that reaches $l = 0.5$ is symmetric.) Since l is decreasing before T it must be the case that $q(T-\epsilon) < 0$. This fact and $q(T) = 0$ imply that $\dot{q}\,(T-\epsilon) > 0$. This inequality and the continuity (with respect to time) of all variables implies that $\frac{dq^-}{dt} \geq 0$, where

$$\frac{dq^-(T)}{dt} \equiv \lim_{t \to T^-} \left(rq(t) + e(t)\right) = e(T).$$

Therefore, $e(T) \geq 0$. The assumption that the trajectory approaches the boundary from inside Ω implies $e(T-\epsilon) < 0.5$. This inequality and continuity imply that $e(T) \leq 0.5$. ■

Hereafter, when we refer to a "boundary condition" of the dynamic system we mean a point $q = 0$, and either $l = -0.5$, $0 \leq e \leq 0.5$ or $l = 0.5$, $-0.5 \leq e(T) \leq 0$.

4.2 The Basins of attraction

An equilibrium trajectory from a point in Ω converges either to the unstable equilibrium U or to one of the boundaries. Define $\bar{P}$ as the projection on to the (l, e) plane of the eigenvector associated with the negative root of F. By definition, only points on $\bar{P}$ have trajectories that converge to U. Equilibrium trajectories emanating from all other points in Ω converge to a boundary $l \in \{-0.5, 0.5\}$.[3]

We use the following

Definition 2 *The basin of attraction* $B(-0.5)$ *(respectively* $(B(0.5))$ *is the set of points in* Ω *from which there exists at least one trajectory that satisfies equations* (6), (7) *and* (8)*, and reaches* $l = -0.5$ *(respectively,* $l = 0.5$*) and the associated boundary conditions, and which remains in* Ω *for* $t \leq T$.

[3]The iso-planes of our system divide (e, l, q) space into eight iso-sectors. For each iso-sector we can identify the direction of the trajectory. Therefore, we can identify the iso-sector that a trajectory must pass through before reaching a boundary $l = k$, $k \in \{-0.5, 0.5\}$, $q = 0$. Denote this as the "prior iso-sector for k".

The three-dimensional phase portrait does not help in identifying the basins of attraction. From any point in Ω there are trajectories that enter both "prior iso-sectors".

Definition 3 *The Region of Indeterminacy (hereafter, "ROI") equals the intersection of the basins of attraction:* $ROI = B(-0.5) \cap B(0.5)$.

We need to consider the basins of attraction for two sets of points (involving specialization in either Agriculture or Manufacturing) rather than for two steady states. This complication occurs because the two stable steady states are on the boundaries of state space, rather than in the interior; this feature arises because of the linearity of the model. The dynamics change on the boundary of state space, where the inequality constraint $-0.5 \leq l \leq 0.5$ is binding. This complication is avoided in a non-linear model for which the stable steady states are in the interior of state space. A model that does not involve specialization in the long run is also more plausible. However, the linear model has two advantages. First, it enables us to use graphical methods to see how the addition of a state variable affects the measure of the ROI, as a in Section 4.3. Second, the fact that we have only three parameters makes it easy to gain insights using numerical methods, as in Section 5.

The following Remark is a simple consequence of the model's symmetry.[4]

Remark 1 *The sets $B(-0.5)$ and $B(0.5)$ are symmetric with respect to $U = (0, 0, 0)$. That is, if two points in Ω, x and x', are symmetric with respect to U, then $x \in B(-0.5)$ if and only if $x' \in B(0.5)$. Therefore, the ROI is symmetric with respect to U.*

We will use the following

Definition 4 *An iso-T curve is the set of points in (l, e) space such that a trajectory that satisfies equations (6), (7) and (8) can reach $q = 0$ and $l = k \in \{-0.5, 0.5\}$ in a given amount of time, T.*

The linearity of the model leads directly to the following

Remark 2 *The iso-T curves are lines.*

[4] Note that even in this symmetric model, if a social planner were to supervise labour reallocation, the basins of attractions would not be symmetric. The reason is as follows: the steady state flow of welfare under complete specialization in agriculture (equal to 1) is higher than the steady state flow of welfare in manufacturing (equal to 0.5). Therefore the social planner would accept larger adjustment costs during the transition path leading to specialization in agriculture (relative to specialization in manufacturing). Therefore, the basins of attraction for the social planner's problem are not symmetric.

The proof of this Remark is in an appendix, available upon request.

We have not been able to exclude the possibility that a trajectory (more precisely: the projection onto the (l, e) plane of this trajectory) that begins in Ω leaves Ω before satisfying the boundary condition. Such a trajectory satisfies the differential system and the boundary conditions but is not an equilibrium, since equilibria must remain in Ω in transit to the boundary. The inability to exclude such a possibility means that qualitative analysis of the differential system and the boundary conditions does not enable us to make conclusions about the ROI.[5] (The appendix, available on request, explains this issue in greater detail.)

Extensive numerical experiments, described in the next section, fail to find any trajectories that begin in Ω, subsequently leave Ω, and then reach a boundary. We therefore provide a description of the ROI under the unproven assumption that no such trajectories exist. That is, we ignore the possibility that trajectories that begin in Ω leave Ω before hitting a boundary.

Subject to this qualification, Remark 2 implies that through every point in $B(k)$ there is an iso-T line; all points on this line in Ω are also in $B(k)$. If in addition, $B(k)$ is convex[6], it follows that the boundary of $B(k)$ inside Ω is a particular iso-T line. By Remark 1, the boundaries of $B(-0.5)$ and $B(0.5)$ are parallel and symmetric with respect to U. Thus, the ROI is bounded by parallel lines. The ROI includes $\bar{P}$, the projection of the eigenvector associated with the stable eigenvalue. If the ROI is of measure 0, then $\bar{P}$ is the boundary of $B(k)$.

Define the intersection of the (l, e) plane and the stable manifold of the system in *reversed* time as P^*. (This manifold is the plane spanned by the eigenvectors associated with the positive roots of F.) If the positive roots of F are complex, projections of trajectories in the reversed time system that begin on the boundary, but not on P^*, eventually spiral around $\bar{P}$. The possibility of indeterminacy exists when the roots are complex, as Figure 1 illustrates. The point e^* (specialization in Agriculture) lies on P^*, so it is on the stable manifold of the reversed time system. The trajectory emanating from e^* (in reversed time) asymptotically approaches U with spirals. The point $-e^* - \varepsilon$ (specialization in Manufacturing) is close to but not on P^*,

[5]This limitation of our analysis prompted us to investigate a two-period version of this continuous time model, reported in Karp and Paul (2004). We obtain a complete characterization of the equilibrium set in the two-period model; all of the qualitative features of that model are consistent with the results of the continuous time model studied here. An appendix to our two-period paper discusses a non-linear version of the continuous time model studied here. This general model provides intuition for indeterminacy, but does not lead to analytic results.

[6]We cannot rule out the possibility that the boundary of $B(k)$ is formed by the envelope of iso-T lines. Therefore, the convexity of $B(k)$ is only a conjecture, again supported by numerical analysis.

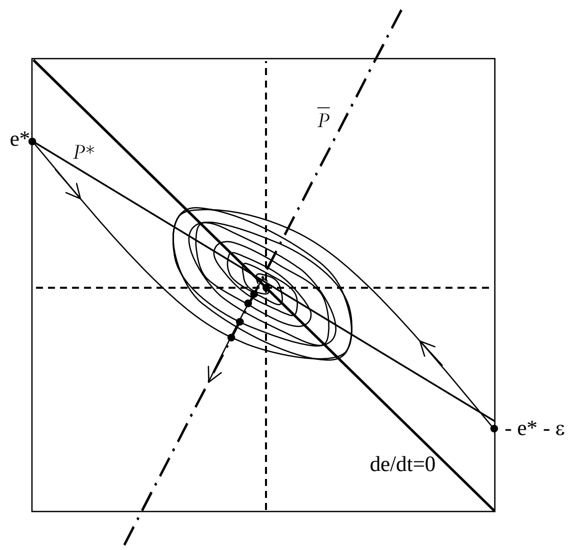


Figure 1: Illustration of indeterminacy with complex roots

with $l = 0.5$. Trajectories emanating from this point (in reversed time) spiral toward the neighborhood of U, but then diverge in the direction $\bar{P}$, the projection of the *unstable* eigenvector in reversed time. Consequently, the projections of the two trajectories must intersect. Those points of intersection are in the interior of the ROI. Thus, the ROI is not empty when the roots are complex.

4.3 Extreme cases

We consider "extreme" cases in which the speeds of adjustment are of different orders of magnitude, but both are positive and finite. We distinguish between an "extreme" case and a limiting case in which the parameter takes a limiting value, 0 or ∞. In the limiting case, the dynamics can be described using two-dimensional phase portraits, shown in Figure 2. This analysis helps in addressing the two questions posed in the Introduction. (We present the analysis using the "asymptotic to" symbol "$\sim$". For example, the statement $\phi \sim y$ as $g \longmapsto s$, for $s = 0$ or $s = \infty$ means that $\lim_{g \to s} \frac{y}{\phi} = 1$.)

• **Case (i):** $g \longmapsto 0, \gamma$ **fixed.** Using equation (9),

$$\phi \sim -\frac{1}{27} g \gamma r^3 < 0.$$

This relation implies that for given γ there exists $\bar{g} > 0$ such that for all $g < \bar{g}$, $\phi < 0$.

Thus, for sufficiently small g there is no indeterminacy. Workers move quickly relative to the adjustment of the environment. If e_0 is larger than 0, q will be negative for a long time because pollution persists a long time. In this situation, the dominant strategy for a worker is to go to the agricultural sector. Conversely, if e_0 is lower than 0, the dominant strategy is to go to the manufacturing sector. In Figure 2.i, as $g \longmapsto 0$ the frontier between the two basins of attraction approaches the line $e = 0$.

• **Case (ii):** $\gamma \longmapsto 0$, g **fixed.** In this case

$$\phi \sim -\frac{1}{108}g^4r^2 - \frac{1}{54}g^3r^3 - \frac{1}{108}g^2r^4 < 0,$$

so again there is no indeterminacy. The labor allocation changes slowly relative to the change in the environment. In Figure 2.ii, the line $e = -l$ is the steady state line of e (conditional on l). The economy approaches this line before a significant change in l occurs. If the economy starts at $l_0 > 0$, e becomes negative before the labor allocation has changed by much. Therefore, the manufacturing wage exceeds the agricultural wage for a long time. The dominant strategy is to go to the manufacturing sector. If $l_0 < 0$, the dominant strategy is to move to the agricultural sector.

• **Case (iii):** $\gamma \longmapsto \infty$, g **fixed.** Here

$$\phi \sim \frac{1}{4}g^2\gamma^2 > 0,$$

so there is indeterminacy for some initial conditions. Labor adjusts rapidly compared to the environment (Figure 2.iii). If e_0 is much larger (lower) than 0, the dominant strategy for a given worker is to go to Agriculture (Manufacturing), which is currently the high wage sector. That worker can later cheaply move to whatever sector has the high wage, an outcome that depends on the decisions of other workers. However, if $e_0 \approx 0$ the current wage differential is approximately 0, and the optimal decision of a worker depends on other agents' decisions, so there is indeterminacy. In the limit as $\gamma \rightarrow \infty$, the ROI collapses to the line $e = 0$. (The "??" on the line $e = 0$ in Figure 2.iii denotes indeterminacy.)

• **Case (iv):** $g \longmapsto \infty$, γ **fixed.** Here

$$\phi \sim -\frac{1}{108}\left(r^2 - 4\gamma\right).$$

Indeterminacy can occur if and only if $r^2 - 4\gamma < 0$ (Figure 2.iv). There can be indeterminacy if workers' decisions depend strongly on the future wage differential (r is small) or if adjustment costs are low (γ is large). This condition reproduces the criteria for indeterminacy in

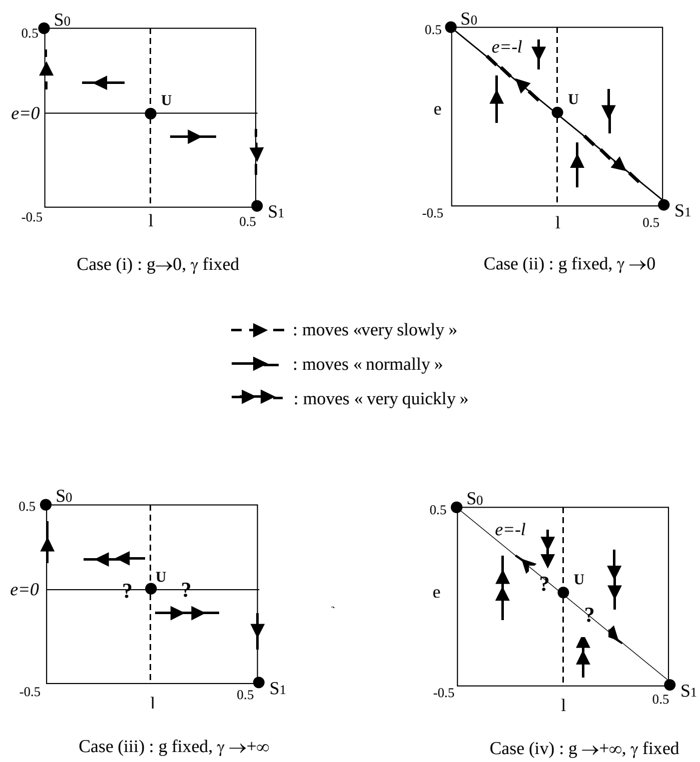


Figure 2: Dynamics with extreme parameter values

Krugman's model, which is the limiting case of our model, evaluated at $g = \infty$. In the limit, $\omega(t) = -e(t) = l(t)$; here the wage differential directly depends on l, a flow variable, rather than e, a stock variable. For large but finite g, the environment adjusts quickly, so the wage differential closely tracks the allocation of labor. Suppose, for example, that a point (e_0, l_0) lies in ROI and above the line $e = -l$. If g is large, then any trajectory from (e_0, l_0) moves quickly toward the line $e = -l$ in an almost vertical direction, and then can move to either boundary, $l = k \in \{-0.5, 0.5\}$. If we take a point directly North of (e_0, l_0), such as $(e_0 + \epsilon, l_0 + \epsilon)$, $\epsilon > 0$, the projection of the trajectory from that point passes close to (e_0, l_0). Thus, for large g we expect $(e_0 + \epsilon, l_0 + \epsilon)$ to also lie in ROI. A parallel argument applies for a point in ROI below the line $e = -l$. Consequently, for g sufficiently large, we expect the ROI to be approximately a rectangle.

In the Introduction we suggested a basis for conjecturing that the qualitative dynamics depend on relative rather than absolute speeds of adjustment. There is some truth to this conjecture, but (as we previously noted following Proposition 2) it is wrong concerning the issue of indeterminacy.

In both Cases (i) and (iii) above, $\frac{\gamma}{g}$ is large: the labor market adjusts quickly relative to the environment. In Case (i) this occurs because the environment adjusts slowly and in Case (iii) it occurs because labor adjusts rapidly. Inspection of Figures 2.i and 2.iii show that in both of these cases, the trajectory moves relatively quickly toward specialization of labor, and the environment changes relatively slowly. In this sense, the qualitative dynamics are indeed similar in the two cases.

However, in Case (i) there is no indeterminacy, whereas in Case (iii) indeterminacy occurs for some initial conditions. In this respect, the qualitative dynamics are quite different in the two cases. That is, the qualitative dynamics depend on absolute and not merely relative speeds of adjustment. The comparison between Cases (ii) and (iv) is similar. In these two cases the environment adjusts quickly relative to the labor adjustment. In Case (ii) there is no indeterminacy, and in Case (iv) indeterminacy arises for some parameter values and initial conditions.

The mathematical explanation for the importance of the absolute and not merely the relative speeds of adjustment is obvious. The dynamic system is not homogenous with respect to γ, g: we cannot write an equivalent system as a function of the ratio of these two parameters. The economic explanation is that the dynamics depend not only on the relative speeds of adjustment, but also on the speed of adjustment of one sector relative to other parameters (including r and the parameters that we restricted, Z and λ.)

We assume that the "likelihood" of the occurrence of indeterminacy is positively related to the measure of ROI. Cases (iii) and (iv) show that if either γ or g is large relative to the other parameters, there is/might be indeterminacy. In the limit as $\gamma \to \infty$, we noted that the ROI collapses to the line $e = 0$, so the measure of ROI goes to 0. This limiting case reproduces Copeland and Taylor's result, where indeterminacy occurs only on a set of measure 0 and is therefore not economically interesting. In the limit as $g \to \infty$ we explained why the ROI approaches the shape of a rectangle, which has positive measure if $r^2 - 4\gamma < 0$. This limiting case reproduces Krugman's result, where indeterminacy occurs on a set of positive measure and is therefore potentially important.

In the one-dimensional model, the set of initial labor allocations for which the equilibrium is indeterminate is non-decreasing in γ. (It is strictly increasing when the measure is positive and less than unity.) We might be tempted to view this one-dimensional model as an approximation of the case where the wage adjusts rapidly but not instantaneously, i.e. g is large but finite. Case (iii) shows that the intuition from the one-dimensional model does not survive the addition of a state variable. When γ is small but positive (for a fixed finite value of g), the ROI does not exist; for large but finite γ, the ROI exists, but its measure vanishes as $\gamma \to \infty$. Consequently, faster labor adjustment (a lower cost of adjustment) does not necessarily increase the measure of ROI (the likelihood of indeterminacy).

5 Numerical simulations

This section numerically identifies the sets $B(i)$ and relates the measure of ROI to parameter values. We determine $B(i)$ by putting a grid on Ω; for each point (e_0, l_0) in this grid we identify to which basin(s) the point belongs. (Details of the algorithm are in the appendix, available upon request.) Before reporting these experiments we describe the choice of parameter values.

We choose units of time equal to one year and set $r = 0.05$ (a discount rate of 5%). We choose base-line values of g, γ to correspond to "moderate" speeds of adjustment, at which $\phi > 0$, so the measure of ROI is positive. We also use an interval for each parameter to represent a range from rapid to slow adjustment. We use the following assumptions to select reasonable parameter values.

First, suppose that in the *absence of flow pollution*, and given an initial environmental stock that is completely degraded ($e = -0.5$), it would take 15 years for the environmental stock to reach half of its maximum (steady state) level, $e = 0.5$. This assumption implies $g = 0.05$. We let g range from 0.015 to 8, values which correspond to an adjustment period (i.e. a half-life) of 45 years and one month, respectively.

Second, suppose that if all workers begin in Agriculture, and in the presence of a constant 20% wage differential (i.e., $\omega_M = 0.5$, $\omega_A = 0.4$ are constants) it would take approximately 25 years for half of the labor force to move into Manufacturing. This assumption implies $\gamma = 0.01$. We allow γ to vary from 0.0025 to 3, changing the adjustment period (the half-life) from 100 years to 1 month, respectively.

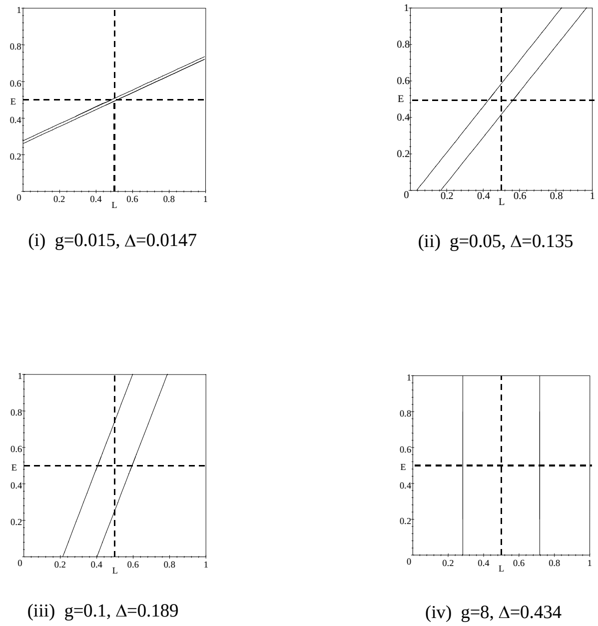
5.1 Region of Indeterminacy

We confirmed (numerically) that the speculations in Section 4.2 hold. All trajectories from inside Ω that satisfy the boundary conditions (specialization in one sector) remain in Ω before hitting the boundary. We also confirmed that the boundaries of $B(k)$ are parallel straight lines – iso-T lines. These lines are parallel to $\bar{P}$, the projection of the eigenvector associated with the stable eigenvalue. The interior of the ROI is non-empty if and only iff $\phi > 0$.

Figure 3 shows the ROIs corresponding to four values of g, holding γ, r fixed; the figures also show the measure of ROI, denoted Δ. (For clarity, we show state space in the E, L plane, rather than using the model in deviation form.) The ROI consists of points inside the lines in the interior of Ω; those lines are the boundaries of $B(k)$. The NW and SE corners are, respectively, the steady states with specialization in Agriculture and Manufacturing.

The ROI is tilted along the NE-SW axis. Points to the NW of the upper line can lead only to specialization in Agriculture; points to the SE of the lower line can lead only to specialization in Manufacturing. The orientation of the ROI and the restriction on boundaries identified in Proposition 3 imply that the economy cannot oscillate between specialization in the two sectors. For example, any trajectory that approaches specialization in Agriculture satisfies $e \geq 0$ and is therefore outside of the $B(0.5)$; this trajectory cannot subsequently approach specialization in Manufacturing. Of course, en route to its final area of specialization, a trajectory can spiral around U, becoming more and then less concentrated in a sector.

When g is small, the ROI covers the entire L dimension; when g is large, the ROI covers the entire E dimension. For small g, the ROI is nearly flat, and has measure close to 0.



(i) g=0.015, Δ=0.0147

(ii) g=0.05, Δ=0.135

(iii) g=0.1, Δ=0.189

(iv) g=8, Δ=0.434

Figure 3: ROI for different values of g

This situation approximates Figure 2.i, except that here the measure of ROI is positive. The orientation of the ROI rotates counter-clockwise, becoming almost vertical as γ increases.[7] That is, for large g the ROI is approximately a rectangle, for the reasons explained in the discussion of Case (iv) in Section 4.3. The measure of ROI increases from 0.015 to 0.434 as g increases from 0.015 to 8.

We constructed similar figures (not included) that show how the ROI changes with γ. Increases in γ cause the ROI to rotate clockwise. This result is consistent with Figure 3: an increase in the environmental speed of adjustment relative to the labor speed of adjustment causes the same change in the orientation of the ROI. To see the effect of this change, choose a point in (for example) the SW region of Ω, where the environment is low and most labor is in Agriculture. Holding this point fixed, increase the *relative* speed of environmental adjustment, causing the $B(k)$ and ROI to change. The chosen point might initially be in the basin of

[7] The fact that a parameter change causes the ROI to rotate – and not merely to grow or shrink – explains why the choice of a uniform prior distribution for the initial condition is not an innocous assumption. The direction of change in the measure of the ROI following a change in a parameter could be different with different (non-uniform) probability distributions.

δ	0.0025	0.01	0.1	3.0
Δ	0.0154	0.135	0.105	0.0355

Table 1: Measures of ROI

attraction of Manufacturing outside the ROI; with higher $\frac{g}{\gamma}$ it is then inside the ROI; for still higher $\frac{g}{\gamma}$ the point is still in the basin of attraction for Agriculture, but no longer in the ROI.

The measure of ROI (given by Δ) is non-monotonic in γ. Table 1 shows four values of γ and corresponding values of Δ, with other parameters equal to the baseline values. For values of γ greater than approximately 0.05, Δ decreases with γ. This numerical result illustrates the point that we made at the end of Section 4.3: faster adjustment of labor does not necessarily increase the set of initial conditions at which indeterminacy occurs.

5.2 Myopic versus rational expectations

Figure 4 illustrates the difference between the dynamics under rational and under myopic expectations using the baseline parameter values. (We obtain the myopic expectations model by letting $r \to \infty$.) The ROI under rational expectations consists of regions II and V. The heavy line shows the separatrix for the myopic case, which is also the boundary of the basins of attraction in that setting.

If the slope of boundary of the basins were horizontal, the steady state would depend only on the initial value of the environment, not of labor. In this sense, a flatter boundary makes the environment "more decisive" in determining the steady state. By this measure, the environment is more decisive under myopic (relative to rational) expectations. In regions (III) and (VI) the nature of expectations (rational or myopic) is decisive. For example, starting in region (III), a rational expectation equilibrium trajectory leads to specialization in manufacturing, while with myopic expectations the economy becomes specialized in agriculture. Conversely, for initial conditions in regions (I) or (IV) the long run steady state is the same for either rational or myopic expectations.

Agents with rational expectations make more sophisticated use of information than do agents with myopic expectations. However, this sophistication aggravates a coordination problem. Since there is an externality under both types of equilibria, the comparison of the levels of social welfare is ambiguous.

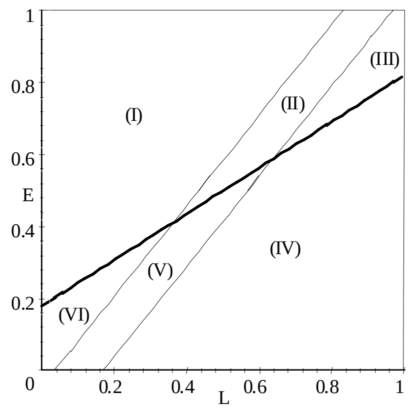


Figure 4: Basins of attraction under rational and myopic expectations

6 Concluding remarks

We studied a model in which the stock of labor in a sector causes an externality that affects the wage differential. Labor adjusts smoothly (rather than instantaneously) because of migration costs. The wage differential depends on a stock variable – such as an environmental stock or a stock of knowledge – which also adjusts smoothly. When both speeds of adjustment are positive and finite, the equilibrium depends on the interaction of the two stocks. When one speed of adjustment equals either zero or infinity we obtain familiar one-dimensional models.

The two-dimensional model, in which both speeds of adjustment are positive and finite, helps to clarify the role of relative and absolute speeds of adjustment. Some qualitative aspects of the dynamics depend on only the relative speeds. However, the possibility of indeterminacy depends on absolute speeds. We used this model to test the robustness of the intuition obtained from the one-dimensional models. In some respects, that intuition survives in a more general setting. An important exception is that faster adjustment of the labor stock may either increase or decrease the set of initial conditions for which the equilibrium is indeterminate.

The existence of indeterminacy complicates the evaluation of policies designed to remedy externalities. If the equilibrium is determinate, the policy objective is to decrease the externality. If the equilibrium is indeterminate, it is important to assist agents in coordinating on a "good" equilibrium. Assessing the empirical plausibility of indeterminacy is therefore an important question.

The progress in achieving this goal for models of local indeterminacy – designed to study economic fluctuations – raises the hope that similar progress might be achieved with models of global indeterminacy. The latter may be useful in explaining the adoption of different development paths – particularly where economic development creates environmental externalities.

The assessment requires a judgement of whether the set of parameter values for which indeterminacy occurs includes empirically plausible values. This assessment depends on the model (since different models create indeterminacy with different parameter values) and on the confidence intervals of parameter estimates. For models of global indeterminacy, the assessment also depends on the likelihood that the initial condition lies in the region of indeterminacy.

The ingredients for a rough estimate can be obtained – at least in principle. For example, with estimates of labor or capital adjustment costs, the magnitude of an environmental externality, and the speed of change of an environmental factor, it is possible to calibrate a more sophisticated version of the kind of model that we studied. This calibration can be used to assess whether indeterminacy ever occurs, and if so, whether the initial condition of the state is in the region of indeterminacy.

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