The Income and Production Effects of Leakage*

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Abstract

In general equilibrium, Home's stricter environmental policy can either increase or decrease trading partners' pollution levels, resulting in positive or negative leakage, respectively. This experiment holds fixed partners' policies. Home's stricter policy leads to a reallocation of its factors of production, creating an income and a production effect. These combined effects determine the sign of leakage. The production effect is related to a technical feature of production, "separability". A partial equilibrium analog to a general equilibrium model does not take into account endogenous income and factor prices. Under plausible conditions this partial equilibrium model overstates the magnitude of leakage.

Keywords: carbon leakage, trade and the environment, environmental policy, general equilibrium, separable production function.

JEL classification numbers C72, H4, Q54

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1 Introduction

Stricter climate policies in one region may cause countries with weaker environmental regulation to increase production of carbon-intensive goods. The possibility of "carbon leakage" makes it harder to reach sub-global climate agreements. Non-regulating countries' ability to undermine the regulatory actions of other countries has been a prominent theme in environmental economics for decades (Hoel 1992). Carbon leakage is either a special case of, or closely related to, the pollution haven effect (Copeland and Taylor, 2003, 2004; Karp, 2011).

Much of our intuition about these effects is based on partial equilibrium models, where the sign of leakage is unambiguous, and the only issue is its magnitude. In a partial equilibrium setting, Home's stricter environmental policies increase production costs in its regulated ("dirty") sector, shifting up (= in) its excess supply for that product. Regardless of whether the country is a net importer or exporter of the good, or whether it is large or small, market forces cause the country's trading partners to increase their production of the dirty good. Holding fixed partners' policies, Home's stricter environmental regulation increases pollution in the partner countries. Leakage is positive in this partial equilibrium setting.

With this intuition, most of the research effort in this area has been directed to attempts to assess the likely magnitude of leakage. Limited data, due to the dearth of meaningful climate policies, and econometric problems endemic to this setting, make it difficult to econometrically estimate the magnitude of leakage. Most of the literature on leakage uses simulation, often with computable general equilibrium (CGE) models. These models yield clear predictions, but their complexity makes them hard for outsiders to evaluate.

As an alternative, this paper steps back, using an analytic general equilibrium model to assess leakage. The primary insight is that, contrary to general perception based on partial equilibruim intuition, leakage can be either positive or negative. In the later case, Home's stricter environmental policy causes partners' to reduce their equilibrium emissions, even in the absence of any policy response in those countries.

The intuition for this possibility is based on the fact that the stricter environmental policy leads to reallocation of factors across Home's clean and dirty sectors. This reallocation leads to an income effect and a production effect; the production effect contains two components. Stricter environmental regulation always shrinks at least one sector, and possibly both, and it always reduces national income. With lower income and normal goods, Home's demand for the regulated good falls, shifting out its excess supply for that good, creating the income effect. Holding factor prices constant, the stricter regulation increases dirty-sector production costs, shifting in the domestic supply (and thus the excess supply) of dirty goods, exactly as in the partial equilibrium setting; this is the first component of the production effect. If the reallocation of factors changes factor prices, there is a second component to the production effect. The factor price changes could further increase dirty-sector production costs, leading to an additional inward shift in Home's excess supply of that good. Alternatively, the factor price changes might more than offset the direct cost increase caused by a higher emissions tax, so that on balance the regulation lowers the domestic production cost of the regulated good. In that case, the aggregate production effect shifts out the excess supply function; here, both the income and the production effect promote negative leakage. Regardless of the sign of the aggregate production effect, the income effect can be strong enough to generate negative leakage.

The production effect of stricter environmental policy is related to a technical characteristic of the dirty-sector production function, "separability". Suppose that stricter environmental policy leaves factor prices unchanged; here, the stricter policy certainly increases production costs in the dirty sector. The dirty-sector production function is separable in emissions and a composite of other factors (capital, labor, etc) if and only if firms in the sector do not want to change their factor mix (e.g., their capital/labor ratio) in response to the stricter policy. Previous multi-factor models implicitly assume that production is separable. This strong and not-widely-recognized assumption plays an important role in determining the sign and magnitude of the aggregate production effect, and therefore in determining whether leakage is positive or negative. Although separability greatly simplifies analysis, it is not especially plausible: faced with a higher emissions tax or stricter emissions standards, firms are likely to change their production methods, and thus their factor mix, regardless of whether factor prices change.

I also compare leakage estimates in a general equilibrium model, and in a partial equilibrium analog that "turns off" the factor price and income effects. I find that in a Heckscher-Ohlin-Samuelson version of the model, the partial equilibrium model always overstates the magnitude of leakage. A Ricardian version of this model can overturn that conclusion.

Related literature The 1999 Kyoto Protocol special issue of *Energy Journal* reviews the pre-2000 carbon leakage literature. Five papers in that symposium use CGE models and four use partial equilibrium models. More recently, Burniaux and Martins (2012), using a small CGE model, numerically examine the reasons for the 2% – 21% range of leakage estimates produced by earlier models. They conclude that, where fuel markets are integrated and fuel supply quite inelastic, most leakage occurs through the "energy markets channel":

environmental policies reduce the demand for fossil fuels in abating countries, reducing fuel prices and increasing fuel consumption and non-abating countries' emissions. They find that the pollution haven effect (the "non-energy markets channel") is a smaller reason for leakage.

Mattoo, Subramanian, van der Mensbrugghe, and He (2009) estimate leakage of less than 4% when high income countries' reduce emissions; their second, simpler model, implies 11% leakage. They also examine a Border Tax Adjustment's (BTA) effects on leakage and welfare. Fischer and Fox (2009b) and Fischer and Fox (2009a) estimate leakage using both a CGE and a partial equilibrium model. Their CGE estimates of leakage are 28% for energy intensive manufacturing and 14% overall. Like most partial equilibrium models, their's assumes that marginal production costs increase with abatement but are constant with respect to output, implying infinite supply elasticities. Home and foreign firms produce differentiated products. Most CGE models also assume constant returns to scale (CRTS) and differentiated products (the Armington assumption). Their partial equilibrium leakage estimates span 10% – 60%.

Babiker (2005) builds a model that includes an imperfectly competitive energy intensive tradable goods sectors with increasing returns to scale (IRTS). His leakage estimates range from 25% with IRTS and differentiated products, to 60% with CRTS and homogeneous products, to 130% with IRTS and homogenous products. He concludes that if energy intensive sectors produce homogenous products under IRTS, then unilateral climate policies may increase global emissions.

McKibbin and Wilcoxen (2009) examine BTAs between countries with different carbon prices. The EU's effective tariff ranges from 1% – 4%, depending on whether they base the BTA on US or China's carbon intensity. They conclude that small estimated leakage and small effective tariffs at moderate carbon prices create modest environmental benefits; these do not justify the BTA's efficiency cost and administrative complexity.

Fowlie (2009), Ponssard and Walker (2008) (for the cement industry), and Ritz (2009) (for steel) estimate leakage using partial equilibrium models with Nash-Cournot competition and homogenous products. The technology is CRTS and abatement increases production costs. Estimated leakage ranges from a few percent to over 70%. Demailly and Quirion (2008) use a partial equilibrium model to study the effect of EU policy on the EU iron and steel sector.

A large econometric literature tries to measure the pollution haven effect for pollutants other than carbon; as noted above, carbon leakage is an example of the pollution haven effect. A fundamental problem runs through this literature. This problem is not specific to, but is evident in Aichele and Felbermayr (2012). This paper estimates leakage using

Kyoto ratification as a "treatment"; the goal is to determine to what extent ratification altered the amount of carbon emissions associated with a country's consumption. The difficulty is that the very nature of international trade almost certainly violates the "stable unit treatment value assumption" (SUTVA). This assumption is needed in order to attribute to the treatment, differences between the treated and the untreated groups (countries that signed and those that did not sign the Protocol, respectively). In this context, SUTVA states that one country's carbon emissions do not depend on another country's Kyoto status. How can trade connect actions in one country and outcomes in another country (e.g., signing Kyoto and increased emissions) if SUTVA holds?

Analytic general equilibrium models complement CGE modeling and econometric estimation. Fullerton, Karney, and Baylis (2013) also use a small analytic model.[1] They construct a two-sector closed economy model, in which both sectors use a single inelastically supplied factor and both create emissions. Only one sector is regulated. (They note that this model can be interpreted as a global model in which commodities and the single factor are freely traded.) Their abstract explains that "the taxed sector substitutes away from carbon into clean inputs, so it may absorb resources, shrink the other sector, and reduce their emissions". This intersectoral factor reallocation is also key in my setting.

Despite this overlap in the core message, the settings in the two papers are quite different. *Some* price must change in response to stricter regulation. In their model, the equilibrium relative commodity price (of the regulated and unregulated sectors) changes with the reallocation of factors; the sign of leakage therefore depends on the elasticity of subsitution in demand, between the regulated and the unregulated commodities. With a single factor in their model, there can be no changes in relative fator prices, thus eliminating the second component of the production effect described above. In my setting, the possibility of negative leakage holds for either fixed or endogenous commodity prices and/or factor prices. In the simplest case, it depends on changes in national income, arising from changes in factor allocation, together with a balance of payments constraint. Here, commodity trade is central. Our two papers also differ because mine considers both the relation between a general equilibrium model and its partial equilibrium analog, and the role of separability between emissions and a composite of other factors. (No such composite exists in their one-factor model, so the issue of separability does not arise there.)

[1] Our two papers were both first distributed in late 2010, and our research was conducted independently.

2 Preliminaries

A country has a clean sector, y, and a dirty sector, x, both using capital and labor. The same symbol denotes a sector and the good it produces. The relative price of the dirty good is p. The dirty sector produces joint outputs: the dirty good, x, and emissions, z. Two ways of modeling pollution are commonly used. The usual approach inverts the joint production function to write the output of sector x as a function of the amount of capital and labor in the sector and the amount of emissions produced by the sector; the result is a standard production function with three inputs, capital, labor and emissions, and one output, x. A different approach introduces a third sector that produces abatement services, A; the dirty sector produces x and z and can purchase abatement services to reduce z. Firms in the dirty sector can be thought of as providing their own abatement services, making the two approaches similar.[2] Except where noted otherwise, I adopt the first approach.

One country, Home, imposes an emissions tax, t, exogenous to the model, making emissions endogenous. A larger tax corresponds to a stricter environmental policy. The Rest of World (ROW) trading partner has fixed (possibly non-existent) policies. I describe ROW using an excess supply or demand function for the dirty good. The two factors, capital and labor, are not traded, have fixed supply, and endogenous prices, r (capital) and w (labor).

Positive leakage means that stricter domestic environmental policy increases pollution abroad. Unless needed for clarity, I refer to this merely as "leakage", and use "negative leakage" to mean that stricter domestic policy reduces pollution abroad. Leakage requires that the higher pollution tax shifts out a country's excess demand for the dirty good in the neighborhood of the equilibrium commodity price. This shift leads to increased dirty-good production, and therefore increased pollution, abroad. Thus, a necessary and sufficient condition for positive leakage, arising from a small increase in the pollution tax, is that the higher tax shift out the country's excess demand for the dirty good *at the equilibrium price.*

To help fix ideas, and to emphasize the relation between partial and general equilibrium models, it helps to consider the case where preferences are homothetic and production is Constant Returns to Scale (CRTS). In this case, the relative demand $(\frac{x}{y})$ depends only on the relative price, p, and the relative supply depends on p and t, as shown in the left panel of Figure 1. The pollution tax (possibly zero) induces autarchic equilibrium prices p^a, w^a, r^a. A higher tax induces new autarchic prices, p', w', r'. The higher tax therefore shifts out

[2] The two approaches are not exactly equivalent. With the first approach, equilibrium requires zero profits in the clean and in the dirty sector. With the second approach, equilibrium also requires zero profits in the third sector, producing abatement services. Section 3.4 clarifies this difference.

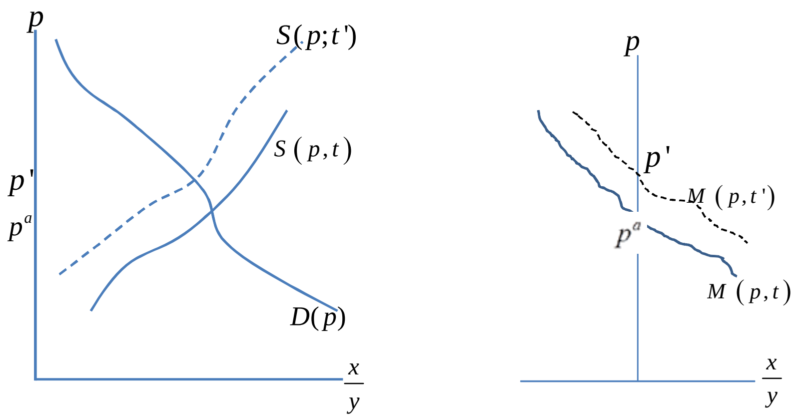

Figure 1: Left panel: the relative supply and demand curves under two levels of tax, $t' > t$. The higher tax increases the autarkic relative price to p'. Right panel: the corresponding excess relative demands.

the country's excess *relative* demand, $M(p, t)$, as shown in the right panel.

In a partial equilibrium setting, a tax increase raises production costs, shifting in (or up) the domestic supply curve, without altering the demand curve. The higher tax therefore shifts up the partial equilibrium excess demand curve, causing leakage. Leakage is inherent to partial equilibrium models, where the question is its magnitude, not its sign. In the partial equilibrium model, the statement that a higher tax increases the autarchic price implies that leakage is (weakly) positive.

The similarity between partial equilibrium supply and demand curves, and the general equilibrium relative supply and demand curves may lead to the mistaken view that an increase in the autarchic price implies positive leakage. Copeland and Taylor (2003) and Krishna (2010) find conditions under which a higher tax increases a country's autarchic price, $p^a(t)$. The price intercept of a country's general equilibrium import demand function equals the autarchic price. The import demand function shifts in the same direction as the change of the autarchic price, in the neighborhood of the autarchic price. If the country trades at a price in this neighborhood, then indeed leakage occurs if and only if the higher tax increases the autarchic price. If, however, the equilibrium occurs where p^a and the world price are not similar (e.g., if the volume of trade is non-negligible) then knowing the direction of change in the autarchic price, due to a change in the tax, provides no information about the sign of leakage. An increase in p^a due to a higher pollution tax is neither necessary nor sufficient for leakage in general.

I assume throughout that agents in Home have identical homothetic preferences. This assumption implies that at a fixed commodity price, expenditure shares are constant, both goods are normal, and the economy consumes both goods if it has positive income. I also assume that pollution has no effect on the ratio of marginal utility of the two goods. Therefore, pollution affects commodity demand only via its effect on income and/or price. Negative imports of a good means that exports of that good are positive.

I ignore pathological cases, and assume that, for example, a reduction in Home demand either reduces or leaves unchanged the equilibrium world price. For most of the paper, merely to simplify exposition, I assume that Home faces an infinitely elastic ROW excess supply curve, i.e. Home is a small country. This assumption can affect the magnitude, but not the sign of leakage. For example, suppose that Home imports the dirty good and that a higher domestic emissions tax lowers its excess demand for that good. If the world price is constant, so is ROW consumption. In this case, reduced Home imports is achieved by reducing ROW production of the dirty good. If the world price falls, then ROW production of the good also falls. In both cases, leakage is negative. As this example illustrates, the sign of leakage does not depend on whether Home is a large or a small country. It is simpler to discuss a small country, where one less price is endogenous.

3 Results

This section presents four results. The most important of these shows how production and income effects interact to determine whether leakage is positive or negative in a general equilibrium (GE) setting. I then explain the relation between leakage and "separability", a feature of the dirty sector production function. I then provide conditions under which a leakage estimate from a partial equilibrium (PE) model is likely to overstate the leakage estimate from a GE model, when the two models are calibrated to the same initial equilibrium. The final subsection shows that when one of these conditions does not hold, the PE estimate might either under- or over-state the actual level of leakage. The Appendix contains all proofs.

3.1 Production and income effects

Here I provide the conditions under which leakage is positive or negative in a GE setting where a small country faces an arbitrary world relative commodity price, p. The higher tax changes the country's equilibrium production point and reduces its real income, changing

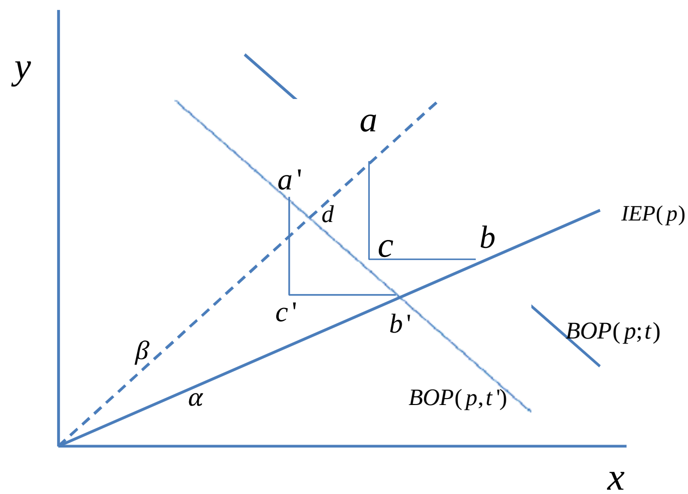

Figure 2: The balance of payments constraint before, $BOP\left(p;t\right)$, and after, $BOP\left(p;t'\right)$, the increase in emissions tax. Corresponding consumption points are b and b'. The initial production point is a and the post-tax increase production point lies somewhwhere on $BOP\left(p;t'\right)$.

domestic demand for the two goods. The tax-induced change in imports therefore involves a production effect and an income effect. These two effects jointly determine the sign of leakage. The production set is convex.

The country remains in balance of trade equilibrium: the value of its imports equal the value of its exports. In Figure 2, a country facing relative price p and an emissions tax t produces at a. Point a lies on the country's production possibility frontier (not shown) at the initial tax. The dashed line, with slope β, shows the set of points where relative production of the dirty and clean good equals the ratio at point a. Denote $\varepsilon^{x,t}\left(t;p\right)$ and $\varepsilon^{y,t}\left(t;p\right)$ as, respectively, the (total) tax elasticity of dirty and clean good domestic production. The modifier "total" means that the elasticities take into account any change in factor prices due to the higher tax; the commodity price is constant because this is a small country. A higher tax reduces equilibrium emissions, causing the production possibility frontier to move towards the origin. Therefore, a higher tax must reduce the supply of at least one of the two goods: at least one elasticity is negative. Denote $\psi\left(t;p\right)=\frac{x}{y}\left(t;p\right)$ as the equilibrium relative domestic production of the dirty good, for a given emissions tax and price, so

$$\frac{d\psi}{dt}\frac{t}{\psi}=\varepsilon^{x,t}\left(t;p\right)-\varepsilon^{y,t}\left(t;p\right). \tag{1}$$

If $\varepsilon^{y,t}=\varepsilon^{x,t}$, then a higher tax causes the production point to move southwest down the dashed line in Figure 2. If $\varepsilon^{y,t} > \varepsilon^{x,t}$ then the higher tax causes a smaller percent reduction in the clean sector than in the dirty sector, and the new production point lies above the dashed line. If $\varepsilon^{y,t} < \varepsilon^{x,t}$, the new production point lies below the dashed line.

The line $BOP\,(p;t)$ shows the country's Balance of Payments (BOP) constraint at the initial production point, a, corresponding to the initial tax, t. The line $IEP(p)$ shows the country's Income Expansion Path, a straight line due to homothetic preferences; the slope of this line is α, a constant because the commodity price is constant for this experiment. The ratio $\frac{\alpha(p)}{\alpha(p)+p}$ is the share of the clean good in expenditures. The figure shows the case $\beta > \alpha$, where the country imports the dirty good. At a fixed commodity price, a higher tax changes the production point, lowering the value of national income, causing the balance of payments constraint to shift in, and changing the consumption point and the level of dirty-good imports.

Proposition 1 *An increase in the tax shifts out the import demand function, causing positive leakage, if and only if*

$$\frac{\beta}{\alpha}\varepsilon^{y,t} > \varepsilon^{x,t}. \tag{2}$$

If a higher tax reduces production of the dirty good ($\varepsilon^{x,t} < 0$) and increases domestic production of the clean good ($\varepsilon^{y,t} > 0$), then inequality (2) is satisfied for all $\frac{\beta}{\alpha}$, so the higher tax causes positive leakage. If a higher tax increases production of the dirty good ($\varepsilon^{x,t} > 0$) and decreases domestic production of the clean good ($\varepsilon^{y,t} < 0$), then inequality (2) is never satisfied, so the higher tax causes negative leakage. However, if the higher tax reduces supply of both goods, there is no presumption that leakage is positive. The parameter β and the elasticities $\varepsilon^{x,t}$ and $\varepsilon^{y,t}$ depend on the tax, the production side of the economy (factor supplies, production functions), and the world price. The parameter α depends only on preferences and the world price; a smaller α increases the share of the dirty good in consumption.

In order to understand the role of α, take the case where both $\varepsilon^{x,t}$ and $\varepsilon^{y,t}$ are negative. In this case, a change in preferences that reduces α makes it "more likely" that leakage is negative. The explanation for this result relies on the relation between α and the income elasticity of demand for the two commodities. Denoting national income as I, and D^x and D^y as consumption of goods x and y, the income accounting identity requires $pD^X + D^y = I$. Differentiating with respect to I and using $D^y = \alpha D^x$ gives a relation between the elasticities of demand with respect to income:

$$p\frac{\partial D^x}{\partial I}\frac{I}{D^x} + \alpha\frac{\partial D^y}{\partial I}\frac{I}{D^y} = \frac{I}{D^x} = \alpha\frac{I}{D^y}.$$

As $\alpha \to 0$, the economy spends all income on the dirty good, and the income elasticity of demand of the dirty good equals 1. In this case, a reduction in income due to the higher tax causes a proportional reduction in demand for the dirty good, and (using the assumption $\varepsilon^{x,t}$ and $\varepsilon^{y,t}$ are negative) necessarily leads to negative leakage. At the other extreme, as $\alpha \to \infty$, the income reduction resulting from the higher tax has no effect on domestic consumption of the dirty good (equal to zero), but (by assumption $\varepsilon^{x,t} < 0$) reduces domestic production of the dirty good. ROW production of the dirty good therefore increases, leading to positive leakage.

Proposition 1 holds for arbitrary p. If the world price equals the autarchic price, then $\beta = \alpha$. In this case, positive leakage occurs if and only if the percent fall in supply of the dirty good exceeds the percent fall in the supply of the clean good, i.e. if and only if the higher tax reduces the relative supply $\frac{x}{y}$. A leftward shift of the relative supply function corresponds to a higher autarchic relative price (Figure 1). Thus, Proposition 1 generalizes previous results that use the change in autarchic price to infer the sign of leakage. However, the proposition shows that information about the change in the autarchic price may not be informative about leakage, if the world and autarchic prices are dissimilar.

Figure 2 provides intuition for the proposition. At the initial equilibrium the consumption point is b and the trade triangle, Δabc, shows imports, the length $\| cb \|$. The point $(\| cb \|, p)$ lies on the country's import demand function, at the initial tax. A higher tax, and resulting lower emissions, decreases dirty sector factor productivity, lowering the value of national income. The BOP constraint shifts in, e.g. to $BOP(p;t')$, and the consumption point occurs at point b'. By construction, triangles $\Delta a'b'c'$ and Δabc are identical. From the property of congruent triangles, point a' lies above the dashed line, northwest of d.

If the new production point lies northwest of a' (on $BOP(p,t')$) then the dirty sector has contracted more than the clean sector; moreover, this difference in contraction rates is sufficient to offset the reduction in demand for the dirty good caused by the tax-induced fall in income. The net effect is to increase imports of the dirty good at the original price p: imports exceed $\| cb \|$. The higher tax shifts out the import demand curve for the dirty good, just as in a PE model, resulting in positive leakage. However, if the new production point lies southeast of a' (on $BOP(p,t')$) then the higher tax shifts in the import demand function for the dirty good, leading to negative leakage. For this interval, the income effect dominates the production effect. Point d on the dashed curve corresponds to a situation where the tax causes an equal proportional contraction in both sectors. A sufficient condition for negative leakage is therefore that the production point lie at or southeast of d, and that the country is initially importing the dirty good.

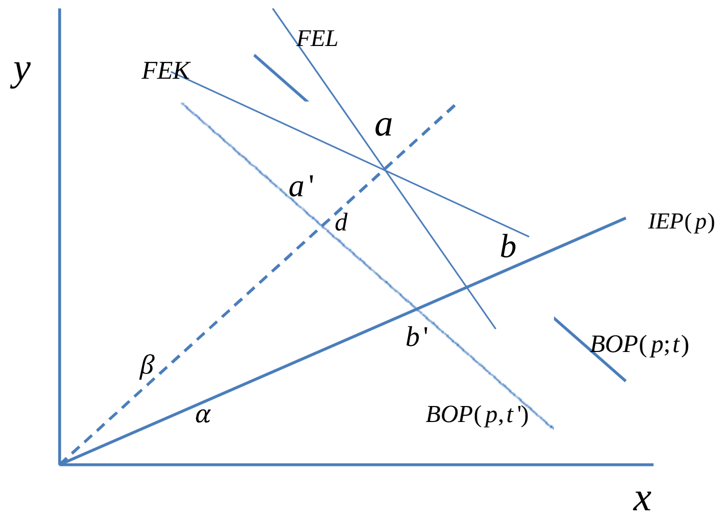

Figure 3: Lines have the same interpretation as in Figure 2. The new lines, FEL and FEK, show the set of points at which there is full employment of capital and labor, given the supplies of these factors.

Example 1 Suppose that both sectors use capital and labor in fixed proportions, and emissions are proportional to output of the dirty good: $z \propto x = \min\left\{\frac{K_x}{a_{x,k}}, \frac{L_x}{a_{x,L}}\right\}$, where K_x and L_x are the amount of capital and labor in sector x and the numbers $a_{x,K}$ and $a_{x,L}$ are the unit factor requirements. Firms in the dirty good can buy abatement services at price p^A; each unit of abatement reduces emissions by one unit. A unit of abatement uses some combination of capital and labor. I assume that all factors are fully employed, both sectors operate, and abatement is positive. Figure 3 reproduces Figure 2 except that it eliminates the trade triangles and adds two new "full employment" lines, FEK and FEL. These lines show the set of points at which both capital and labor are fully employed, given the quantities of those factors remaining in the initial equilibrium after subtracting (from the exogenous aggregate factor supply) the factors used in the abatement sector. An increase in the pollution tax increases the demand and supply of abatement services. If, for example, the abatement sector uses only capital, then the higher tax moves the production point southeast along FEL; if the abatement sector uses only labor, the higher tax moves the production point northwest, along FEK. In the first case there is leakage, and in the second there is negative leakage, by Proposition 1. If the abatement sector uses both capital and labor, the production point moves to a point in the cone spanned by FEL and FEK, containing the dashed line with slope β. In that case, leakage can be positive or negative.

3.2 Separability and the production effect

The previous section shows how the income and production effects jointly determine the sign of leakage. Here I abstract from the income effect to consider the production effect more carefully. The goal is to understand the determinants of the sign of $\frac{d\psi}{dt}\frac{t}{\psi}$, the tax elasticity of relative domestic production of the dirty good. As above, I hold the commodity price fixed. I invert the dirty sector joint production function, writing dirty good output as a function of capital, labor and emissions, and I assume CRTS production.

A production function for good x that is separable in emissions and a composite of capital and labor (hereafter, "separable") can be written as $x = F(z, h(K_x, L_x))$, with the function h positive and increasing in both arguments, and F increasing in z and the composite, h. Previous papers implicitly assume separability. This functional assumption is central in determining the production effect, and thus the sign of leakage, so it is important to understand its economic meaning. With separable production, we can think of the dirty firm as using capital and labor to produce the joint products, "potential output" and emissions, and then using a fraction of potential output to reduce emissions (Copeland and Taylor, 2004). Here, production and abatement use the same capital-labor ratio. With non-separable production in the dirty sector, this kind of two-stage process does not occur. Reducing emissions does not merely require the use of some "potential output" to abate; it instead may lead to a different capital/labor ratio in the dirty sector, even without changes in factor prices. To assess the plausibility of the separability assumption, we can ask whether, *in the absence of factor price changes*, we would expect firms to change their capital/labor ratio, following a policy-induced emissions reduction. If firms change their input mix in this circumstance, then their production function is not separable.

Denote the unit factor requirements in the two sectors as $a_{x,K}$, $a_{x,L}$, $a_{y,K}$ and $a_{y,L}$. These unit requirements are functions of the endogenous factor prices (unlike in Example 1, where they were exogenous parameters). I define η_{ij} as the tax elasticity of the j'th unit input requirement in sector i. For example, $\eta_{xK} = \frac{da_{xK}}{dt}\frac{t}{a_{xK}}$; this is a total elasticity. The higher tax may change $a_{x,K}$ even in the absence of factor price changes, and the tax induced change in $\frac{w}{r}$ may lead to additional changes in $a_{x,K}$. The economy's exogenous capital/labor ratio is k and the endogenous capital/labor ratios in sectors x and y are k_x and k_y. Using these definitions, I have:

Proposition 2 *Suppose that preferences are homothetic, sector-y production is CRTS in* *capital and labor, and sector-x production is CRTS in capital, labor, and emissions. Then*

$$\frac{d\psi}{dt}\frac{t}{\psi} = \frac{\eta_{yK}k_y - \eta_{yL}k}{k_y - k} + \frac{k_x\eta_{xK} - k\eta_{xL}}{k - k_x}. \tag{3}$$

(i) If sector-x production is separable, then $\frac{d\psi}{dt}\frac{t}{\psi} < 0$. (ii) If sector-$x$ production is non-separable, then the sign of $\frac{d\psi}{dt}\frac{t}{\psi}$ is ambiguous. Sufficient conditions for $\frac{d\psi}{dt}\frac{t}{\psi} > 0$ are: (a) $\frac{\eta_{yK}k_y - \eta_{yL}k}{k_y - k} \approx 0$ and (b) $\frac{k_x\eta_{xK} - k\eta_{xL}}{k - k_x} > 0$.

If sector-x production is non-separable and both inequalities (a) and (b) hold, then the tax increase raises $\frac{x}{y}$. If in addition, the country imports the dirty good ($\frac{\beta}{\alpha} > 1$), leakage is negative.

Example 2 Suppose sector-x production is non-separable and there is little substitutability between capital and labor in the clean sector, and in addition $k_y - k > 0$ and non-negligible. In this case, condition (a) in Proposition 2 is satisfied. Condition (b) can be rewritten as

$$\frac{\left(k_x\frac{da_{xK}}{a_{xK}} - k\frac{da_{xL}}{a_{xL}}\right)100}{(k - k_x)\left(\frac{dt}{t}\right)100} > 0, \tag{4}$$

where $\frac{da_{xK}}{a_{xK}}100$ and $\frac{da_{xL}}{a_{xL}}100$ are the tax-induced percentage changes in sector-x unit factor requirements. If, for example, the clean sector is relatively capital intensive (and $dt > 0$), then the denominator of the left side of inequality (4) is positive, so the inequality holds if the percentage change in a_{xK}, weighted by k_x, exceeds the percentage change in a_{xL}, weighted by k. The dirty sector faces offsetting pressures. The higher tax leads to a fall in the relative factor price $\frac{w}{r}$, which promotes a decrease in the equilibrium dirty-sector capital/labor ratio, $\frac{a_{xK}}{a_{xL}}$. If, however, capital provides a better substitute than labor for the reduced emissions, the sector's capital/labor ratio might increase.

3.3 Comparing GE and PE leakage estimates

The PE model holds factor prices and income fixed, whereas the GE model treats these as endogenous. To compare leakage estimates under PE and GE models, it is necessary to first specify the relation between the two. I *define* the PE model as a specialization of the GE model, obtained by turning off the factor price and income adjustments. Given this definition, I compare leakage estimates under the two models, by comparing the effect of an increase in the emissions tax on the position of Home's excess supply function for the dirty good. A tax-induced upward shift in the home country's excess supply function leads to

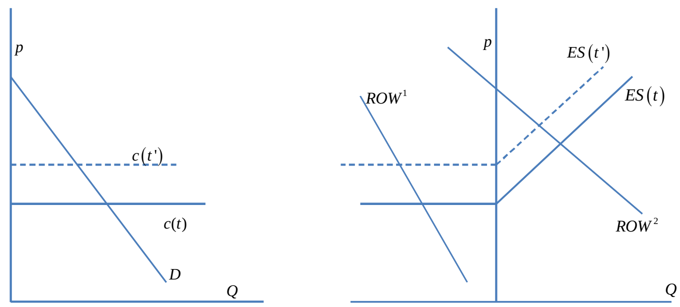

Figure 4: Left panel: domestic market; right panel: world market with two possible ROW excess demand functions. Home imports the dirty good under ROW^1 and exports the dirty good under ROW^2. A higher tax, $t' > t$, shifts a country's export supply from $ES(t)$ to $ES(t')$ in a partial equilibrium setting, increasing production of the dirty good in ROW.

higher ROW dirty good production, and leakage.

I assume that production in both sectors is CRTS. The partial equilibrium analog of the GE model therefore has constant costs for a given emissions tax, leading to a horizontal domestic supply function, as in the left panel of Figure 4. (The linearity of the demand function is irrelevant to this discussion.) An increase in the tax, increases costs in the dirty sector, shifts up the domestic supply function and (in this PE setting) does not affect the domestic demand function; the higher tax thus shifts up the excess supply function. Here, the higher tax creates leakage, regardless of whether the country is a net importer or exporter of the dirty good.

Comparability of the PE and GE models requires (at least) that they yield the same equilibrium net imports at an initial environmental tax. With this restriction, I say that the PE model "overstates the actual magnitude of leakage" if the upward (or inward) shift of the PE excess supply curve is greater than the upward shift in the corresponding GE model. The modifier "actual" means "taking into account the change in factor prices and income".

Proposition 3 *Suppose that (i) both sectors produce with CRTS and preferences are homothetic, and (ii) there are two sectors and two factors with fixed supply. Then the PE estimate of leakage overstates the actual magnitude of leakage.*

Section 3.4 illustrates the importance of assumption (ii). This assumption means that there are two endogenous factor prices and two zero profit conditions.

In the GE setting, the higher tax reduces national income, reducing the demand function for the dirty good. This effect is absent in the PE setting, which holds income fixed. The proposition then follows by showing that adjustment of factor prices acts to moderate the direct cost increase, in the dirty sector, arising from the higher tax. This moderating effect is absent in the PE setting, so the PE domestic supply function shifts up by more than the corresponding GE supply function. Consequently the higher tax creates a larger upward shift in the PE *excess* supply function compared to the GE excess supply function, and therefore leads to a higher estimate of leakage.

The conclusion that the PE model overstates the magnitude of leakage does not mean that it necessarily understates the effect of the pollution tax. In neglecting the income effect, the PE model ignores the tax-induced reduction in dirty-good demand, thereby understating the effect of the tax on equilibrium emissions. But in neglecting factor price changes, the PE model exaggerates the supply response to the tax. Both of these omissions contribute to a larger estimate of leakage, but only the first contributes to an underestimate of the effect of the tax on equilibrium emissions.

Most PE models assume CRTS in production. With such a model, in a homogenous product competitive equilibrium without capacity constraints or a similar source of non-linearity, all production occurs in the low cost country. In order to avoid this extreme outcome, the PE models cited above assume imperfect competition and/or product differentiation. Leakage estimates may be sensitive to these modeling assumptions. Because of its simplicity and familiarity, it is worth having a homogenous product, perfect competition PE model with an upward sloping supply curve. Such a model cannot be obtained by specializing a GE model with CRTS.[3]

3.4 A Ricardian model

Chau (2003) uses Cobb Douglas functions to study a three-sector model with dirty and clean tradable goods, and non-tradable abatement services. I use the same model with general functions in order to make the results more transparent and more easily compared with the results above. As in Example 1, there are three sectors, the clean and dirty sectors and the nontradable sector providing abatement services. Here, however, unit input requirements are endogenous. One unit of production of x creates one unit of emissions. Sector-x firms

[3] An earlier version of this paper provides this kind of PE model, and uses it to show the relation between leakage and supply and demand elasticities, emissions intensity, and the relative size of regulated and unregulated markets. I also used that model to discuss different types of Border Tax Adjustments. This material is available on request, but I have deleted it here in order to keep the paper short and focused.

face a unit emissions tax, t, and can buy a unit of abatement services, at price p^A, to eliminate a unit of pollution.

Production is CRTS, with unit cost functions for sectors x, y, and A (abatement) given by $c_y(w, r)$, $c_x(w, r, t)$ and $c_A(w, r)$. If $t > p^A$ then firms strictly prefer to abate all emissions. In order to avoid a taxonomy, I assume that this inequality does not hold, so $p^A \geq t$. The zero profit conditions imply

$$p^A \leq c_A(w,r) \quad 1 \leq c_y(w,r) \quad p \leq c_x(w,r,t). \tag{5}$$

If a sector operates, the corresponding inequality holds as an equality. In addition, if the abatement sector operates, firms must be indifferent between paying the tax and buying a unit of abatement services, so $P^A = t$.

With three sectors and two factors of production, the model generalizes the textbook Ricardian model of two sectors and one factor. The imbalance between sectors and factors violates assumption (ii) in Proposition 3, so the PE production effect might either over- or under-state the GE production effect. Therefore, the PE leakage estimate might overstate the actual leakage. However, as with the GE models considered above, leakage can be either positive or negative.

In a nontrivial trade equilibrium, where world price does not equal the autarchic price, at most two sectors operate: Home might produce both goods, but no abatement, or it might produce only the dirty good with abatement. Suppose, for example, that Home starts in a trade equilibrium producing both goods, and then faces an increase in the emissions tax. Three outcomes are possible; in each, the sign of leakage is ambiguous. First, if the tax change is sufficiently small that it remains an equilibrium not to abate, then the higher emissions tax is equivalent to an increase in a producer tax on the dirty good. This higher tax lowers the return to the factor used relatively intensively in the dirty sector and increases the price of the other factor, moderating the reduction in domestic dirty-good production (caused direct cost increase due to the higher tax). It also lower real income, as discussed in Section 3.1. Second, if the higher tax induces abatement, then the clean sector shuts down. The higher tax reduces national income and might either increase or decrease dirty good production. Third, if the tax is high enough to eliminate production of the dirty good, the economy specializes in production of the clean good. The higher tax again lowers national income.

Under autarchy, the dirty and clean good sectors both operate. If the abatement sector also operates, the three equilibrium conditions determine the three endogenous prices, p, w, and r (with $p^A = t$) independently of preferences. If the abatement sector shuts down, preferences and technology jointly determine equilibrium prices.

Proposition 4 *Suppose that in an autarchic equilibrium, firms are indifferent between paying the tax, t, and paying the price for abatement services ($t = p^A$). A change in the tax changes the equilibrium autarchic price by*

$$\frac{dp}{dt} = a_{xz} - \frac{a_{xL}(k_y - k_x)}{a_{AL}(k_A - k_y)}. \tag{6}$$

(i) The partial equilibrium price-effect of the higher tax (a_{xz}) exceeds the general equilibrium effect if and only if k_y lies between k_x and k_A. (ii) The higher tax lowers the autarchic dirty-good price if and only if

$$a_{AL}\frac{a_{xz}}{a_{xL}} < \frac{k_y - k_x}{k_A - k_y}$$

The intuition for part (i) of the proposition follows from the fact that at p, w, and r, the higher tax attracts factors to the abatement sector. Suppose, for example, that $k_A > k_y > k_x$, so the abatement sector and the clean sector are both more capital intensive than the dirty sector. The abatement sector competes with the clean sector for factors. As the abatement sector expands, the price of capital increases; to maintain zero profits in the clean sector, the wage must fall. The decline in $\frac{w}{r}$ benefits the dirty sector, partly offsetting the direct effect of the higher tax. Part (ii) of the proposition merely states that the dirty-good autarchic price falls if and only if the general equilibrium cost effect, mediated by the change in factor prices, is sufficiently large.

If the clean sector is either more capital intensive than both of the other two sectors, or less capital intensive than both of these two sectors, then the general equilibrium effects magnify the partial equilibrium effect. At constant factor prices, a marginal increase in the tax increases dirty-sector production costs by a_{xz}. If k_y does not lie between k_x and k_A, then $\frac{dp}{dt} > a_{xz}$. In this case, the general equilibrium factor price effects reinforce the partial equilibrium effect of the higher tax, and the partial equilibrium estimate of leakage might understate actual leakage.

4 Discussion

The intuition from partial equilibrium models has lead to the presumption that in a properly specified general equilibrium model, Home's stricter environmental policy leads to an upward (= inward) shift of its excess supply function for the dirty good. This upward shift increases trading partners' production of the dirty good, thereby increasing partners' pollution, regardless of whether Home is a large or a small country, or whether it imports or exports the dirty good. In this situation, there is (positive) leakage. This paper challenges that intuition, showing that in plausible circumstances, a stricter environmental policy can cause a downward (= outward) shift in Home's excess supply for the dirty good, resulting in negative leakage.

The intuition for this possibility is extremely simple. A stricter environmental policy leads to the reallocation of factors of production, resulting in an income and a production effect; the combination of these might cause a higher emissions tax to shift in or to shift out Home's excess supply for the dirty good.

The income effect is the most obvious: the higher tax reduces national income. If goods are normal (e.g., preferences are homothetic) and emissions do not directly effect the demand for goods, the reduction in national income shifts down the domestic demand for the dirty good, tending to shift out the country's excess supply of that good.

The direction of the production effect is ambiguous. The stricter environmental policy raises production costs in the dirty sector, at constant factor prices. In order to maintain zero profits in the dirty sector, the factor prices must change. The change in factor prices increase costs in the sector that uses intensively the factor whose relative price has increased; the change in factor prices decreases costs in the other sector. Without additional assumptions, we do not know which sector contracts the most, due to a higher environmental tax.

In addition to these basic points, the paper makes three additional contributions. First, it shows the role of separability in the dirty-sector production function. Most (or all) papers involving two or more factors implicitly assume separability. This functional restriction implies that at constant factor prices, a reduction in emissions does not alter the equilibrium dirty-sector capital/labor ratio. In reality, the use of a cleaner method of production is likely to change capital/labor ratios in the dirty sector, even in the absence of factor price changes. Separability is sufficient for a stricter environmental policy to cause a larger proportional contraction of the dirty than the clean sector. Thus, with separability, the production effect always promotes positive leakage; unless the income effect is sufficiently strong, equilibrium leakage is positive. Absent the separability assumption, the production effect might promote either positive or negative leakage.

Second, the paper provides conditions under which a partial equilibrium model that is calibrated to a general equilibrium model always overstates the magnitude of leakage. The partial equilibrium model turns off the income and factor price adjustments caused by stricter environmental policy, leaving only the policy's direct effect of increasing production costs in the dirty sector. Under the conditions given, the general equilibrium effects always moderate the direct effect, so the partial equilibrium estimate of leakage overstates the actual magnitude of leakage. Third, the paper provides an example where the factor price effects of the higher tax might either moderate or reinforce the partial equilibrium effect; in this case, a partial equilibrium model might under- or over-state actual leakage.

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A Proofs

Proof. (Proposition 1). Denote $S^x(t;p)$ and $S^y(t;p)$ as the equilibrium supply of the dirty and the clean good, respectively. Consumers' budget constraint requires $pD^x + D^y = I$, where $D^x(t)$, $D^y(t)$, $I(t)$ are demand for the dirty and the clean good and income, respectively; these are all functions of t. By homotheticity, $D^y = \alpha(p) D^x$, where α depends only on p, a constant. The budget constraint therefore implies $(p+\alpha) D^x(t) = I(t)$, or $\frac{dD^x(t)}{dt} = \frac{1}{p+\alpha}\frac{dI}{dt}$. National income is $I(t) = pS^x + S^y$ so $\frac{dI}{dt} = p\frac{dS^x}{dt} + \frac{dS^y}{dt}$. Putting these two results together gives

$$\frac{dD^x(t)}{dt} = \frac{1}{p+\alpha}\left(p\frac{dS^x}{dt} + \frac{dS^y}{dt}\right). \tag{7}$$

Imports of the dirty good equal $m(t;p) = S^x - D^x$. Differentiating this identity and using equation (7) gives

$$\begin{gathered}
\tfrac{dm}{dt} = \tfrac{dD^x}{dt} - \tfrac{dS^x}{dt} = \tfrac{1}{p+\alpha}\left(p\tfrac{dS^x}{dt} + \tfrac{dS^y}{dt}\right) - \tfrac{dS^x}{dt} = \\
\tfrac{1}{p+\alpha}\left((p-p-\alpha)\tfrac{dS^x}{dt} + \tfrac{dS^y}{dt}\right) = \tfrac{1}{t(p+\alpha)}\left(-\alpha\tfrac{dS^x}{dt}\tfrac{t}{S^x}S^x + \tfrac{dS^y}{dt}\tfrac{t}{S^y}S^y\right) = \\
\tfrac{1}{t(p+\alpha)}\left(-\alpha\varepsilon^{x,t}S^x + \varepsilon^y\beta S^x\right) = \tfrac{S^x}{\alpha t(p+\alpha)}\left(\varepsilon^y\tfrac{\beta}{\alpha} - \varepsilon^{x,t}\right) \Longrightarrow \\
\tfrac{dm}{dt} = \tfrac{S^x}{\alpha t(p+\alpha)}\left(\varepsilon^y\tfrac{\beta}{\alpha} - \varepsilon^{x,t}\right).
\end{gathered}$$

where the penultimate line uses the fact that at the initial equilibrium, $S^y = \beta S^x$. The last equality implies the proposition. ■

Proof. (Proposition 2) Using the full employment conditions, the relative supply of the dirty good, $\psi(t;p,w(p,t),r(p,t))$, equals

$$\psi = \frac{x}{y} = \frac{a_{yk} - ka_{yL}}{ka_{xL} - a_{xK}}.$$

Taking the derivative with respect to the tax, converting to an elasticity, and manipulating, gives the tax elasticity of the output ratio

$$\frac{d\psi}{dt}\frac{t}{\psi} = A + B \tag{8}$$

with

$$A = \frac{\eta_{yK}k_y - \eta_{yL}k}{k_y - k} \quad \text{and} \quad B = \frac{k_x\eta_{xK} - k\eta_{xL}}{k - k_x}.$$

For concreteness, I now assume that the clean sector is relatively capital intensive: $k_y >$

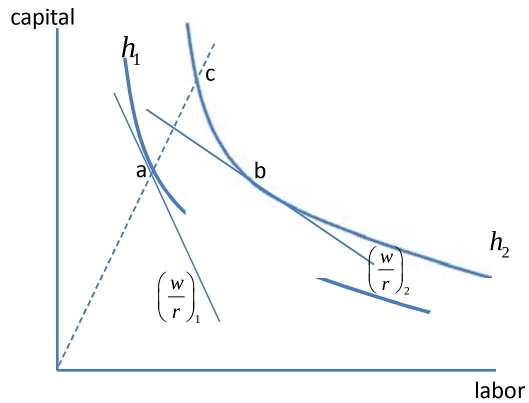

Figure 5: At constant commodity price, a higher tax and resulting changes in factor prices shifts unit capital and labor requirements form a to b. The percentage increase in labor per unit of the dirty good is greater than the percentage increase in capital per unit of the dirty good.

$k > k_x$. Results are not changed if the dirty sector is relatively capital intensive. With $k_y > k > k_x$, a tax increase lowers the equilibrium wage and increases the equilibrium rental rate. (This is a familiar result in GE models; the proof of Proposition 3 explains its rationale.) These changes imply $\eta_{yL} > 0 > \eta_{yK}$, which imply $A \leq 0$. The inequality is strict unless production in sector y is Leontieff. The sign of B is ambiguous. The higher emissions tax reduces emissions, making production of one unit of the dirty good infeasible at the initial level of capital and labor. One or both dirty-sector unit factor requirements increase.

Part i. With separable production, $x = F(z, h(K_x, L_x))$; the function h returns a composite of capital and labor that is positive and increasing in both arguments. With constant returns to scale in F, h also has constant returns to scale, and therefore is homothetic. Denote z_i as the optimal level of emissions per unit of output under the tax t_i. For two taxes with $t_2 > t_1$, $z_2 < z_1$. Denote h_i as the level of the composite of capital and labor needed to produce one unit of the dirty good when $z = z_i$; that is, z_i is the solution to $F(z_i, h_i) = 1$. Thus, $h_2 > h_1$.

Figure 5 shows: (i) two "h isoquants", $h_2 > h_1$; (ii) the equilibrium ratios, $\left(\frac{w}{r}\right)_1$ and $\left(\frac{w}{r}\right)_2$ corresponding to taxes $t_1 < t_2$; and (iii) the capital and labor per unit of production, the coordinates of a and b under the two taxes. From the comments above, $\left(\frac{w}{r}\right)_1 > \left(\frac{w}{r}\right)_2$ as the figure shows. The higher tax and lower wage/rental ratio increases the labor per unit of output, a_{xL}, and might increase or decrease a_{xK}. However, the proportional increase in

a_{xK} is less than the proportional increase in a_{xL}, implying $\eta_{xK} - \eta_{xL} \leq 0$; the inequality is strict unless sector x production is Leontieff. To confirm this claim, note that a straight line through the origin and point a gives the set of capital and labor requirements at different levels of h, holding fixed the factor price ratio $\left(\frac{w}{r}\right)_1$. This line passes through the curve h_2 at point c, northwest of b. At points a and c the capital/labor ratios are constant, so in moving from a to c capital and labor increase in equal proportions. Because b lies southeast of c, labor's proportional increase exceeds capital's in moving from point a to b. The inequality $\eta_{xK} - \eta_{xL} < 0$ (when sector x is not Leontieff) and $k > k_x$ imply $B < 0$.

Thus, in the case where the dirty sector production function is separable, a higher emissions tax reduces the relative supply of the dirty good.

Part ii. This result is immediate from equation (8) and the definitions of A and B. ■

Proof. (Proposition 3) The clean and dirty sector unit cost functions are $c_y(w, r)$ and $c_x(w, r, t)$. The envelope theorem implies:

$$\frac{\partial c_x}{\partial w} = a_{xL}, \quad \frac{\partial c_x}{\partial r} = a_{xK}, \quad \frac{\partial c_x}{\partial t} = a_{xz}, \quad \frac{\partial c_y}{\partial w} = a_{yL}, \text{ and } \frac{\partial c_y}{\partial r} = a_{yK},$$

where a_{ij} is the amount of factor $j \in \{\text{capital, labor, emissions}\}$ used to produce one unit of sector i output, $i \in \{x, y\}$. The zero profit conditions are

$$c_y(w, r) = 1 \quad \text{and} \quad c_x(w, r, t) = p.$$

Differentiating the zero profit conditions yields the system

$$\begin{bmatrix} a_{yL} & a_{yK} \\ a_{xL} & a_{xK} \end{bmatrix} \begin{pmatrix} \frac{dw}{dt} \\ \frac{dr}{dt} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{dp}{dt} - a_{xz} \end{pmatrix}. \tag{9}$$

In the partial equilibrium model, where $\frac{dw}{dt} = \frac{dr}{dt} = 0$ by definition, equilibrium requires $\frac{dp}{dt} = \frac{dc_x}{dt} = a_{xz}$, i.e. the commodity price must increase by the amount of emissions per unit of output, at the initial equilibrium. This price increase exactly offsets the cost increase. The horizontal partial equilibrium supply curve shifts up by this amount, and the partial equilibrium demand function does not respond to t.

In contrast, in the general equilibrium model, $\frac{dw}{dt}$ and $\frac{dr}{dt}$ are endogenous, changing to offset the cost increase resulting from the higher emissions tax. For example, if the commodity price is fixed by the world market, so that $\frac{dp}{dt} = 0$, then system (9) implies

$$\begin{pmatrix} \frac{dw}{dt} \\ \frac{dr}{dt} \end{pmatrix} = \begin{bmatrix} a_{yL} & a_{yK} \\ a_{xL} & a_{xK} \end{bmatrix}^{-1} \begin{pmatrix} 0 \\ -a_{xz} \end{pmatrix} = \frac{-a_{xz}}{D} \begin{pmatrix} -a_{yk} \\ a_{yL} \end{pmatrix},$$

with $D = a_{xL}a_{yL}(k_x - k_y)$, where k_x and k_y are the capital/labor ratios in sectors x and y. If, for example, the clean good is relatively capital intensive, then $D < 0$. Here, the higher tax lowers the nominal wage and increases the nominal rental rate, leading to a net fall in the combined cost of capital and labor exactly equal to the increase in the cost of emissions. By a similar argument, the factor price changes can adjust to maintain zero profits, for any commodity price change $\frac{dp}{dt} \in [0, a_{xz}]$. Thus, the required price change needed to support the original equilibrium in the general equilibrium setting can be as low as zero.

The higher tax shifts down the general equilibrium demand curve for the dirty good. Because the higher tax shifts up the supply curve by no more than (and typically, less than) a_{xz}, the excess supply curve shifts up by strictly less than a_{xz}. ■

Proof. (Proposition 4) Under the assumption that abatement and emissions are both positive in autarchy, the equilibrium conditions (using $p^A = t$) are $p = c_x(w, r, t)$, $1 = c_y(w, r)$, and $t = c_A(w, r)$. Differentiating these conditions and rearranging, gives the system

$$\begin{bmatrix} a_{xL} & a_{xK} & -1 \\ a_{yL} & a_{yK} & 0 \\ a_{AL} & a_{AK} & 0 \end{bmatrix} \begin{pmatrix} dw \\ dr \\ dp \end{pmatrix} = \begin{pmatrix} -a_{xz} \\ 0 \\ 1 \end{pmatrix} dt.$$

Solving for $\frac{dp}{dt}$ gives equation (6). The last two statements in the proposition follow directly from this equality. ■