Income distribution and the allocation of public agricultural investment in developing countries[*][†]

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Abstract

We compare the effects, on rural wages and farmer income, of public investments in either a traditional or a modern agricultural sector. The traditional sector has constant or decreasing returns to scale. Economies of scale that are external to farms in the modern sector create a possible engine for growth. We show that if there are moderate returns to scale in the modern sector, and labor has a moderate share of the wage bill in that sector, there is a "critical size" of the modern sector. If the modern sector is close to this critical level at the time of the public investment, then investment in the traditional sector can derail growth and worsen poverty, whereas investment in the modern sector is likely to promote growth and reduce poverty. As the magnitude of the increasing returns to scale parameter or the labor-share parameter increase, this "critical size" of the modern sector falls, increasing the likelihood that investment in the traditional sector is inimical to both growth and poverty reduction, while investment in the modern sector promotes both of these goals.

Keywords: agricultural labor, migration, growth, public investment, increasing returns to scale

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1 Introduction

We compare the distributional effects of public investment in either a traditional or a modern agricultural sector. Public support can take many forms, including extension services, the creation of infrastructure, and output subsidies. This support increases the value of output in the sector where it is provided. Supporting the traditional sector directly benefits the farmers in that sector. Supporting the modern sector might increase the wage and employment there, providing farmers in the traditional sector with a better alternative and an escape from poverty. Growth in the modern sector also increases the value of land. If traditional farmers rent land, they are harmed by the higher rents.

We assume that the traditional sector has constant or decreasing returns to scale; farms in the modern sector have decreasing returns to scale at the farm level, but they create positive externalities for that sector. For example, an increase in the size of the modern sector lowers average distribution costs, encouraging the growth of a distributional network. The creation of an export channel typically requires fixed costs, so a larger modern sector may help to create export opportunities by making it possible to spread the fixed costs over a greater volume. Since these kinds of benefits are external to farms in the modern sector, a competitive equilibrium is in general not efficient. Traditional farmers can move to the modern sector, where they become workers. There may be migration costs, and reverse migration might also be possible.[1]

The assumption that there are industry-wide increasing returns to scale in the modern sector implies that if the policy objective is to increase aggregate national income, it is (probably) efficient to use scarce public funds to support the modern rather than the traditional sector. However, when the objective is to alleviate poverty, the recommendation is less clear. The growth resulting from support of the modern sector is shared amongst the factors of production, which include entrepreneurs in the modern sector. The economic gain from the public support to the traditional sector may also be shared by various owners of factors, but poor farmers who own land are likely to receive a substantial share of this gain.[2] Here we consider the effect on agricultural poverty of the choice of sectors to target. In our model, the agricultural poor are

[1] There have been many papers written using two-sector migration models with increasing returns in one sector. The QJE papers by Matsuyama and Krugman in 1991 are important examples. This paper includes a second mobile factor of production, and it emphasizes poverty reduction rather than national income.

[2] If that support increases marketed output, consumers could benefit by a fall in price. We ignore these kinds of secondary (price) effects.

traditional farmers and workers in the modern agricultural sector.

We distinguish between the static and the dynamic effects (on poverty) of public investment. The static effect incorporates a change in income prior to factor adjustment, and the dynamic effect incorporates the induced factor adjustment. Consider two investments, one in either sector, that both increase national income by the same amount at a given allocation of factors. The static effect on reducing poverty is greater if the investment is made in the traditional sector, simply because of our assumption that farmers in that sector are the residual claimants. The comparison of dynamic effects is more interesting. We find that investment in the modern sector is likely to do more to decrease poverty in the long run, provided that the increasing returns to scale and the rural workers' share of revenue are sufficiently large, and that the initial size of the modern sector is sufficiently large.

The next section presents the model. The following section presents the results. In the interests of simplicity, we emphasize a static model, and our analysis uses comparative statics. However, the issues that we discuss are inherently dynamic, so we include an informal discussion of a dynamic version of the model. A conclusion follows.

2 Model

We first discuss the equilibrium in the modern sector, taking as given the supply of factors in that sector. We then consider the traditional sector.

2.1 The modern sector

At the farm level there are decreasing returns to scale. However, at the sectoral level there are increasing returns, for the reasons discussed in the introduction. These increasing returns are external to the individual farm.

The parameter m is a measure of the entrepreneurial talent in the modern sector. We speak of m as the number of entrepreneurs, each of whom runs a modern farm. (This convention allows us to use sums rather than integrals to represent the size of the modern sector, simplifying the presentation.) Farm i in this sector employs L_i units of labor and N_i units of another factor (possibly a composite of more than one factor), which we hereafter refer to simply as "land".

The value of the farm's output is

$$pF_T^{\mu-\theta}\left(F_i\left(N_i,L_i\right)\right)^{\theta},\text{ with }F_i\left(N_i,L_i\right)=N_i^{1-\rho}L_i^{\rho}.$$

The average revenue in the modern sector is p. We can interpret p as a function of agricultural policy; a larger value of p might reflect a price subsidy or an increase in productivity due to a public investment in infrastructure or extension services. We can also think of p as a measure of total factor productivity. The parameter $\mu > 1$ is a measure of sector-wide increasing returns to scale, and $0 < \theta < 1$ is a measure of the farm-specific decreasing returns to scale. As $\mu \to 1$ the positive externality in the sector vanishes, and as $\theta \to 1$ the farm has constant returns to scale. The share of labor in the function $F_i\left(\right)$ is ρ. (Table 1 collects all definitions.)

Each entrepreneur takes F_T as given. This function captures the positive externalities in the sector. It is defined as

$$F_T\equiv\sum_i F_i\left(N_i,L_i\right)=mF_i\left(N_i,L_i\right), \tag{1}$$

where the equality follows from the assumption of a symmetric equilibrium. Let L and N be the aggregate stocks of labor and land in the modern sector.

2.2 Factor prices conditional on the sectoral allocation of factors

Here we find the relation between the factor prices and the model parameters, given a particular amount of labor and land in the modern sector. The entrepreneur takes the price of labor and of land, w and r, as given, and chooses inputs to maximize the value of output, leading to the equilibrium conditions

$$\begin{array}{c} p\theta F_T^{\mu-\theta}\left(F_i\left(N_i,L_i\right)\right)^{\theta-1}\frac{\partial F_i(N_i,L_i)}{\partial L}=w \\ p\theta F_T^{\mu-\theta}\left(F_i\left(N_i,L_i\right)\right)^{\theta-1}\frac{\partial F_i(N_i,L_i)}{\partial N}=r. \end{array} \tag{2}$$

Cost minimization requires that expenditures on labor and land, as a share of F_i, equal ρ and $1-\rho$ respectively, leading to the equilibrium condition $\frac{wL_i}{rN_i}=\frac{\rho}{1-\rho}$. This relation implies

$$\frac{(1-\rho)\,L}{\rho N}=\frac{r}{w}. \tag{3}$$

The assumption of symmetry means that the labor/land ratio in each of the modern farms equals the sectoral ratio.

Further manipulation of the optimality conditions (2) (reported in the appendix) produces the second equilibrium condition for the modern sector

$$\lambda N^{\mu-1}\left(\frac{r}{w}\right)^{\rho\mu-1}=w \tag{4}$$

with

$$\lambda\equiv p\theta\rho^{\rho\mu}\left(1-\rho\right)^{1-\rho\mu}m^{1-\theta}>0. \tag{5}$$

Using equations (3) and (4) we can solve for the equilibrium factor prices, taking as given the supply of factors in the modern sector:

$$w=\rho\theta pm^{1-\theta}L^{\rho\mu-1}N^{\mu(1-\rho)} \tag{6}$$

$$r=\left(1-\rho\right)\theta pm^{1-\theta}L^{\rho\mu}N^{-1+\mu-\rho\mu}. \tag{7}$$

The appendix derives the return per entrepreneur (= total revenue in the modern sector minus payments to land and labor, divided by the number of entrepreneurs):

$$\pi=pN^{\mu(1-\rho)}L^{\rho\mu}m^{-\theta}\left(1-\theta\right). \tag{8}$$

A simple calculation shows that the shares of the value of output in the modern sector that accrue to labor, land, and entrepreneurs are $\rho\theta$, $(1-\rho)\,\theta$, and $1-\theta$, respectively. Thus, labor's and land's shares of the payment to factors are equal to ρ and $1-\rho$, just as under constant returns to scale.

An increase in the returns to scale parameter increases output in the sector, and increases the returns to labor, land, and entrepreneurs[3]:

$$\frac{dw}{d\mu}=\rho\theta pm^{1-\theta}L^{\rho\mu-1}N^{-\mu(-1+\rho)}\left(\rho\ln L+(1-\rho)\ln N\right)>0$$
$$\frac{dr}{d\mu}=\theta pm^{1-\theta}L^{\rho\mu}N^{-1+\mu-\rho\mu}\left(1-\rho\right)\left(\rho\ln L+(1-\rho)\ln N\right)>0$$
$$\frac{d\pi}{d\mu}=pN^{-\mu(-1+\rho)}L^{\rho\mu}m^{-\theta}\left(1-\theta\right)\left(\rho\ln L+(1-\rho)\ln N\right)>0.$$

[3]Total output is

$$F_T^{\mu}m^{1-\theta}=\left(mN_i\left(\frac{L}{N}\right)^{\rho}\right)^{\mu}m^{1-\theta}=\left(N\left(\frac{L}{N}\right)^{\rho}\right)^{\mu}m^{1-\theta}.$$

We assume that a larger value of μ corresponds to a higher level of output – as must be the case if μ is a valid measure of increasing returns to scale. Total output is an increasing function of μ if and only if

$$N\left(\frac{L}{N}\right)^{\rho}>1,$$

which is equivalent to $L>N^{\frac{\rho-1}{\rho}}$. We assume that this inequality holds; it requires that N is sufficiently large.

2.3 The traditional sector

For our "basic model" we assume that each traditional farmer produces the value of output a using σ units of land. Farmers rent land at the price r, so their net return is $a - \sigma r$: farmers are the residual claimant in the traditional sector. (There is no substitutability between land and labor in the traditional sector, and production in that sector has constant returns to scale.) Each traditional farmer that migrates to the modern agricultural sector releases σ units of land.

In a variation of this model, which we discuss below, the traditional sector has decreasing returns to scale. We mention two of the many possible reasons for decreasing returns to scale. First, there may be substantial underemployment in the traditional sector, and for cultural reasons jobs in that sector are shared. In that case, all traditional farmers have the same average income, but that average would increase if there were fewer farmers. Second, traditional farmers may have different skill levels. If the least efficient farmers are the first to leave the sector, the average productivity in the sector increases as the sector shrinks. In this case, the infra-marginal farmers have higher income than the marginal farmer.

In a still more complicated model (which we do not consider further), land and labor would be substitutes in the traditional sector (as they are in the modern sector). This complication requires the use of an additional equilibrium condition, needed to determine land allocation between the modern and traditional sector. By assuming that land and labor are used in fixed proportions in the traditional sector, we obtain a tractable model.

3 Effect of public investment on poverty

Rural workers and traditional farmers are the poor. Since we take commodity prices as given, the nominal income of these two groups is a valid measure of their real income, i.e. of poverty. Migration is possible between the modern and traditional sectors, so in a steady state either all labor is in one sector or the wage equals the return to traditional farmers. Outside a steady state, while there is migration, the income levels might differ. We want to compare the effects, on poverty, of public investment in either the traditional or the modern sector.

Public investment in the traditional sector causes an increase in average productivity in that sector, a. Public investment in the modern sector causes an increase in the parameter p. The assumption that traditional farmers are the residual claimant means that they obtain all of the benefit from the public investment. In contrast, rural workers obtain only the share $\rho\theta$ from the increased value of investment in the modern sector. In addition, that investment raises the rent, harming traditional farmers.

Public investment can have both a "static" and a "dynamic" effect on poverty. The static effect takes into account the effect of the investment on poverty, *given the allocation of factors*. The dynamic effect takes into account the reallocation of factors induced by the investment.

3.1 The static effect of public investment

The *static* effect depends on the extent to which the investment increases productivity, and how this productivity gain is shared by factors, at a given allocation. A simple example illustrates an argument for investing in the traditional rather than the modern sector, when the objective is to alleviate poverty. Normalize the aggregate amount of labor to 1, so $1 - L$ is the amount of labor in the traditional sector. Suppose that both types of investment lead to a one unit increase in national income. By the assumptions of our model, if investment is made in the traditional sector, the aggregate income of traditional farmers increases by one unit and rural workers are not affected. If the investment is made in the modern sector, the rural wage bill, wL, increases by $\rho\theta$; the rental bill, rN, increases by $(1-\rho)\,\theta$, so r increases by $\frac{(1-\rho)\theta}{N}$. Total income of urban workers and traditional farmers increases by

$$\rho\theta - \sigma\frac{(1-\rho)\,\theta}{N} < 1.$$

Thus, given the assumptions of our model, if the two investments are equally profitable, investment in the traditional sector does more to reduce poverty when we take into account only the static effect of the investment. Of course, the point of this model is to consider the dynamic effect of public investment, taking into account the policy-induced reallocation of factors.

3.2 The dynamic effect of public investment

For the purpose of studying the dynamic effects, we adopt the following assumption on parameters

$$\mu < \max\left\{\frac{1}{\rho}, \frac{1}{1-\rho}\right\}. \tag{9}$$

This inequality together with $\mu > 1$ means that the magnitude of increasing returns to scale is moderate rather than extremely large.[4]

We begin by finding the effect of migration on the wage, w, and the return to traditional farmers, $a - r$. Labor can move between the agricultural sectors, so public investment in a sector attracts labor and land into that sector. An increase in labor and land in the modern sector has the following effects on the equilibrium wage:

$$\begin{aligned} \tfrac{\partial w}{\partial L} &= \rho\theta p m^{1-\theta} L^{\rho\mu-2} \left(\rho\mu - 1\right) N^{-\mu(-1+\rho)} \\ \tfrac{\partial w}{\partial N} &= \rho\theta p m^{1-\theta} L^{\rho\mu-1} N^{-1+\mu-\rho\mu} \mu \left(1-\rho\right). \end{aligned} \tag{10}$$

An increase in land in the modern sector increases the marginal productivity of labor and always increases the equilibrium wage. By inequality (9), an increase in labor (holding the land allocation fixed) decreases the equilibrium wage. The magnitude of increasing returns to scale is not large enough for additional workers in the sector to lead to a higher wage, unless additional factors also enter.

Since each unit of labor that enters the modern sector releases to that sector σ units of land, the aggregate wage effect of migration into the sector is

$$\begin{gathered} \tfrac{dw}{dL} = \tfrac{\partial w}{\partial L} + \sigma \tfrac{\partial w}{\partial N} = \\ \rho\theta p m^{1-\theta} L^{\rho\mu-2} N^{\mu(1-\rho)-1} \left(N\left(\rho\mu - 1\right) + \sigma L \mu \left(1-\rho\right)\right). \end{gathered} \tag{11}$$

Thus, migration increases the equilibrium wage if and only if

$$\frac{L}{N} > \frac{(1-\rho\mu)}{\sigma\mu\left(1-\rho\right)} \equiv \left(\frac{L}{N}\right)^{*}. \tag{12}$$

By inequality (9), $\left(\frac{L}{N}\right)^{*} > 0$.

Recall that N is actually a composite of inputs, including land, but excluding labor and entrepreneurial talent. Therefore, N does not consist solely of land that has left the traditional sector. In other words, we do not assume that $N = \sigma L$. If the amount of migration is Δ, then the subsequent "labor/land" ratio in the modern sector is

$$\frac{L+\Delta}{N+\sigma\Delta} \Longrightarrow \frac{d\left(\frac{L+\Delta}{N+\sigma\Delta}\right)}{d\Delta} = \frac{N-\sigma L}{\left(N+\sigma\Delta\right)^2}.$$

[4] If inequality (9) is reversed, the minimum points of the curves in Figures 1 and 2 occurs at $L = 0$ and the interior stable steady state denoted as s in those figures vanishes.

Thus, migration increases the labor/land ratio in the modern sector providing that[5]

$$\frac{1}{\sigma} > \frac{L}{N}. \tag{13}$$

We assume that this inequality holds over the domain of our analysis.

The rental rate varies with the amount of land and labor in the modern sector:

$$\frac{\partial r}{\partial L} = (1-\rho)\,\theta p m^{1-\theta} L^{\rho\mu-1} \rho\mu N^{-1+\mu-\rho\mu}$$
$$\frac{\partial r}{\partial N} = -(1-\rho)\,\theta p m^{1-\theta} L^{\rho\mu} N^{-2+\mu-\rho\mu} \left(1-\mu\,(1-\rho)\right).$$

As labor leaves the traditional sector, the change in the rental rate is

$$\frac{dr}{dL} = \frac{\partial r}{\partial L} + \sigma\frac{\partial r}{\partial N} =$$
$$\theta p m^{1-\theta}\,(1-\rho)\,L^{\rho\mu-1} N^{\mu-\rho\mu}\left(N\rho\mu + \sigma\,(\mu-\rho\mu-1)\,L\right).$$

Using inequality (9), migration into the modern sector increases the rental rate if and only if

$$\frac{L}{N} > \frac{\rho\mu}{\sigma\,(1-\mu\,(1-\rho)-1)} \equiv \left(\frac{L}{N}\right)^{**}.$$

A calculation confirms that

$$\left(\frac{L}{N}\right)^{**} > \left(\frac{L}{N}\right)^{*}.$$

Figures 1 and 2 show the heavy curves labelled w and $a-\sigma r$, the rural wage and the return to traditional farmers, prior to investment, as functions of L. As demonstrated above, both of these curves are decreasing for small values of L and increasing for large L, and the minimum of the graph of $a-\sigma r$ occurs at a larger L, compared to the the minimum of the graph of w. Note that an increase in labor's share in the modern sector ($\rho\theta$), or an increase in the returns-to-scale parameter (μ), both cause a decrease in the turning point of the graph of w. That is, larger values of μ or ρ increase the range of labor allocations for which increasing returns to scale are significant to the economy.

There are two points, shown as points s and u, at which there is no incentive for labor to migrate, because at these points $w = a - \sigma r$. Point s is a stable equilibrium, because any

[5]Note that

$$\mu > 1 \Leftrightarrow \frac{1}{\sigma} > \frac{(1-\rho\mu)}{\sigma\mu\,(1-\rho)}.$$

Since the first inequality is assumed, there always exists a range of $\frac{L}{N}$ for which both equations (12) and (13) are satisfied.

perturbation leads to a new equilibrium in the neighborhood of point s. Point u is an unstable equilibrium, because any perturbation leads to a new equilibrium far from point u.

In both figures there is at least one stable equilibrium (not shown) to the right of point u. This equilibrium might involve shutting down the traditional sector or it might involve an economy with both sectors. If the economy is sufficiently large (enough labor and land), then the largest stable equilibrium necessarily involves higher rural income than the point s.[6] At a stable equilibrium to the right of point u the economy is sufficiently developed that it is taking advantage of the increasing returns to scale in the modern sector. We are interested in situations where the economy has not yet reached such a point.

The analysis of point s is non-controversial. Unstable points like u are sometimes considered uninteresting, on the grounds that an economy is not likely to remain for long in the neighborhood of such a point, so a policy change is not likely to occur when the economy is close to such a point. In our view, the stable and unstable points are equally "plausible". An economy might be near a steady state (e.g., stagnating), or it might be in the process of change. Stable points are useful for studying the former case. Unstable points are useful for studying an economy in that is the process of development, but which has not yet reached a path of sustained growth. During this period of "fragile development" a policy change might enable the economy to reach sustained development, or it might cause the economy to regress. The following section discusses a genuinely dynamic version of this model, and explains in greater detail why unstable points like u are interesting for policy analysis.

Before discussing the effects of public investment in the two sectors, a word of caution about Figures 1 and 2 is in order. The figure shows the point u with a smaller income level (for traditional farmers and agricultural workers – the poor) than the point s. The assumption (in our basic model) of constant returns to scale in the traditional sector is consistent with this ranking, but does not imply it. If the point u is to the right of the minimum of the graph of $a - \sigma r$, point u could lie above the point s. In addition, if there are decreasing returns to scale in the traditional sector because of underemployment there, the average product is an increasing function of L: $a = a(L)$, with $a'(L) > 0$, because more labor in the modern sector means that

[6] In general, we cannot rule out the possibility that point s is the stable equilibrium with the highest rural income. That occurs if, for example, point u is below point s, and the economy is so small that all of the available labor is in the modern sector in the neighborhood of point u. In this case, the economy is too small to take advantage of the increasing returns to scale in the modern sector. We do not consider this case further.

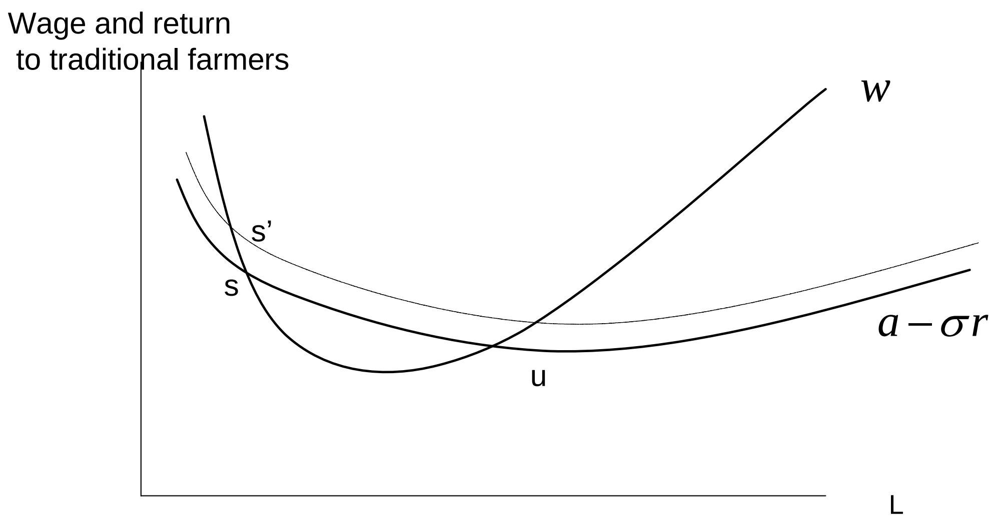

Figure 1: Effect of investing in traditional sector

there is less labor in the traditional sector.[7] If the slope of $a(L) - \sigma r$ becomes positive "close enough" to point s, then point u lies strictly above point s. At this level of generality, we can only say that constant returns to scale in the traditional sector make it "more likely" that income for the poor is higher at point s than at point u, and decreasing returns to scale in the traditional sector make the reverse more likely.

With this caveat, Figure 1 illustrates the effect of public investment in the traditional sector. This investment increases a, causing the curve $a - \sigma r$ to shift up from the heavy to the light curve. If the initial equilibrium is at point s, investment causes labor to flow into the traditional sector. At point s there are (local) decreasing returns to scale in the modern sector, so as labor leaves the sector the wage increases. This migration also causes the rental rate on land to decrease, benefiting traditional farmers. In this situation, the dynamic effect reinforces the static effect: the new equilibrium s' is higher than the point on the light line directly above point s. That is, the dynamic effect magnifies the increase in income of the poor.

[7]If there are decreasing returns to scale in the traditional sector because of heterogenous skill levels, it is still the case that $a(L) > 0$. However, in that case the income of the marginal traditional farmer is lower than the income of the average farmer. Migration stops when the marginal (not average) farmer income equals w. A graphical analysis along the lines of figures 1 and 2 is still possible (replacing $a(L)$ by marginal product), but we would then need a more complicated measure of real income for traditional farmers. That added complexity would obscure the purpose of this model, so we do not pursue this variation.

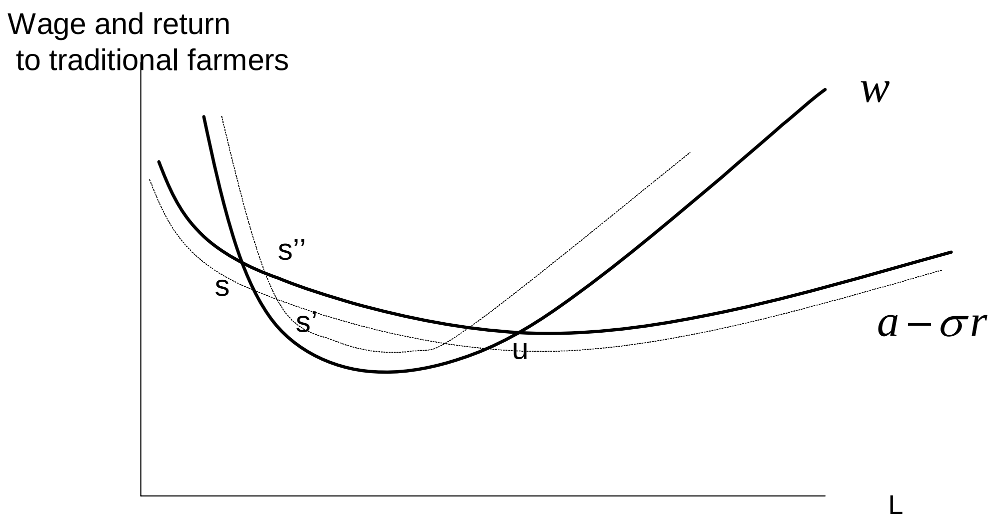

Figure 2: Effect of investment in the modern sector

If the initial equilibrium is at point u, the investment makes it more attractive to work in the traditional sector, drawing in workers. Along the transition path traditional farmers have higher income, but the rural wage initially decreases until reaching the minimum level. The eventual equilibrium is at point s.

This description understates the disadvantage of investing in the traditional sector, because of the ambiguous relation between income at points s and u. It is useful to think of an economy starting near, but not at point u. If the economy is to the left of point u, the modern sector would have shrunk even in the absence of the investment. However, if the economy is near but to the right of point u, it was in the process of "fragile development". In that case, investment in the traditional sector overturns this process, causing the economy to regress.

The effect of public investment in the modern sector is slightly more complicated. As noted in Section 3.1, this investment causes an increase in the wage and the rental rate. The higher wage benefits rural workers but the higher rental rate harms traditional farmers. Thus, investment in the modern sector causes the heavy curves in Figure 2 to shift to the light curves. If the initial allocation is at point s, the new equilibrium is at point s'. We can decompose the total movement into two parts. The migration into the modern sector causes the wage to fall, to point s''. The higher rental price causes more labor to leave the traditional sector; this increased migration further reduces the wage, until the point s' is reached. In this situation, the dynamic effect (due to induced factor adjustment) tends to undermine the static effect on poverty reduction. Figure 2 illustrates a case in which investment lowers rural income (point s'' is below point s). This outcome is possible, but it need not necessarily occur, if for example there are decreasing returns to scale in the traditional sector.

If the initial equilibrium is at (or near) point u, investment in the modern sector attracts labor into that sector. The economy eventually reaches a steady state to the right of point u, with a significantly higher level of rural income.

3.3 Other dynamic processes

The previous section assumes that the dynamic process is myopic, so that labor moves into the sector with the higher return. If agents incur an adjustment cost from moving to a different sector, they are likely to regard migration as an investment decision. As such, they may base this decision on their beliefs about the future rather than the current income differential. If in addition they have rational expectations, a number of interesting possibilities arise.

One possibility is that economic fundamentals – the current income levels – determine the eventual outcome. This possibility tends to be more likely when agents have high discount rates, so that their decision is guided chiefly by near-term income differentials. In this case, an unstable equilibrium like point u in the figures divides the L axis ("state space") into two regions. For initial conditions (a labor allocation) to the left of point u, the economy moves toward equilibrium s. For initial conditions to the right of point u the modern sector grows, leading to an equilibrium with a large modern sector and high rural income.

Public investment shifts the location of the unstable equilibrium u, by moving one or both of the graphs of w and $a - \sigma r$. Investment in the traditional sector causes the critical point u to shift right, and investment in the modern sector causes it to shift left. For example, the current value of L might be close to but to the right of point u, so that the modern sector is growing. Investment in the traditional sector might cause the new intersection (the "new" point u) to be larger than the current L. In that case, the investment causes the economy to switch trajectories; L begins to decrease rather than increase. Even though the investment has short run benefits for farmers, it derails a growth process, causing a rebirth of the traditional sector and shrinking the modern sector. In contrast, investment in the modern sector promotes the growth process.

A second possibility is that the equilibrium outcome depends on agents' beliefs about the future (rather than on economic fundamentals). This possibility is more likely to arise when agents have very low discount rates, so that their current migration decision depends largely on distant income differentials. In this case, there are typically multiple rational expectations equilibria when the initial condition is in the neighborhood of an unstable equilibrium like point u. Some of these equilibria may cause the modern sector to grow, and others cause it to shrink. (This situation is formally a "coordination game" with non-atomic agents.) Public investment can have a particularly important role in cases where there are multiple rational expectations equilibria. By targeting the modern sector, the government can help agents coordinate on the equilibrium that produces growth (and higher incomes) – here, the "good" equilibrium.

4 Conclusion

Public investment in infrastructure or education can increase agricultural productivity, leading to higher rural wages and higher returns to farmers. These investments also induce the reallocation of factors of production. We studied a model with a traditional and a modern agricultural sector. Farmers in the traditional sector use land and their labor; production has constant or decreasing returns to scale. Entrepreneurs in the modern sector hire land and labor. Individual farms in the modern sector produce under decreasing returns to scale, but a positive externality causes the average productivity in the sector to increase with the size of the sector. Labor can move between the traditional and the modern sector. In the long run, migration between sectors either causes the returns to labor to be equal in the two sectors (at an interior steady state), or it causes the sector with the lower return to close down (at a boundary steady state).

The rural wage (in the modern agricultural sector) and the returns to traditional farmers – which must be equal in an equilibrium where both sectors operate – is our index of poverty. We used the model to compare the poverty effects of public investment in the two sectors. The investment increases productivity in the sector where it is made, and it can also affect the other sector via adjustments of factors of production. We emphasized the distinction between the short run poverty effects of the investment (those that arise before adjustment of factors of production) and the dynamic effects (those that take into account the induced factor adjustment).

We assumed that traditional farmers rent their land, and that the price of land is determined by equilibrium in the modern sector. Traditional farmers are the residual claimants to output in their sector. Rural workers share any increase in the value of output. These assumptions mean that if we compare two investments that yield the same increase in national income, the investment in the traditional sector always has a larger *static* effect on poverty reduction, compared to investment in the modern sector. Not only do rural workers share (with entrepreneurs and land-owners) the increased value resulting from investment in the modern sector, but the resulting higher rental rate harms the traditional farmers.

The main point of the model, however, is to study the dynamic effects of the two types of investment. These effects depend on three considerations: the extent of increasing returns to scale in the modern sector, the wage bill as a share of revenue in that sector, and the size of the modern sector.

We emphasized the case in which the extent of increasing returns to scale and labor's share of revenue are both moderate. We studied the case in which there are (at least) two interior steady states, corresponding to either a small or a large modern sector. The small steady state is stable: if the economy begins at this steady state, investment in either sector causes the new equilibrium to be close to the original equilibrium. The large steady state is unstable: beginning here, investment in either sector causes the economy to move toward a distant equilibrium.

The small steady state describes a relatively stagnant economy, in which small public investments can have only marginal effects. The large steady state describes an economy in "fragile" development. This development is fragile because the economy might either continue to develop, or it might regress.

If the economy is at the stable steady state with a small modern sector, the dynamic effect of the investment reinforces the poverty reduction when the investment occurs in the traditional sector, and it undermines the poverty reduction when the investment occurs in the modern sector. Investment in the traditional sector attracts labor, increasing the wage in the modern sector because of (local) decreasing returns to scale in that sector. The outflow of labor from the modern sector reduces the marginal value of land, causing the rent to fall and further benefiting traditional farmers. In contrast, investment in the modern sector attracts labor, causing the rural wage to fall and increasing the rental rate.

The picture is much different if the economy is at (or near) the unstable steady state. In that case, a small investment in the traditional sector causes a large fall in the size of the modern sector, destroying the opportunity to take advantage of economies of scale. Investment in the modern sector promotes growth of that sector. Providing that the economy is large enough to take advantage of the economies of scale in the modern sector, there is a stable equilibrium with a large modern sector at which rural income is higher than at the stable equilibrium where the modern sector is small.

The simplest policy implication of our model is that if there are moderate returns to scale in the modern sector, and labor has a moderate share of the wage bill, there is a “critical size” of the modern sector. If the modern sector is close to this critical level at the time of the public investment, then investment in the traditional sector can derail growth and worsen poverty, whereas public investment in the modern sector is likely to promote growth and reduce poverty. As the magnitude of the increasing returns to scale parameter or the labor-share parameter increase, this “critical size” of the modern sector falls, increasing the likelihood that investment in the traditional sector harms both growth and poverty reduction, while investment in the modern sector promotes both of these goals. If the modern sector smaller than the critical size at the time of investment, a marginal investment has only a small effect on income and poverty.

We emphasized a myopic adjustment process, in which people move to the sector with the highest current return. We also considered adjustment processes in which workers base their migration decision on their expectation of future wage differentials. This extension is important because it justifies our attention to the unstable steady state for our comparative statics analysis. It also highlights the value of public investment in the modern sector as a means of inspiring confidence and promoting entry to that sector (i.e. as a “coordination device”).

We focused on the poverty effects of investments. Even if the two investments have the same short run effects on poverty, they might have quite different effects on national income. If two investments have the same short run level of poverty reduction, the investment in the modern sector is more likely to lead to a larger increase in national income. The benefits of investment in the traditional sector accrue largely to farmers (at least in our model). In contrast, the benefits of investment in the modern sector are also shared by land owners and entrepreneurs, providing a broader political constituency to sustain the investment program.

Parameter name	meaning
σ	amount of land used per traditional farmer
m	a measure of entrepreneurial talent (the number of modern farms)
p	average revenue in the modern sector (a policy variable)
$\mu > 1$	a measure of sector-wide increasing returns to scale
$0 < \theta < 1$	a measure of farm-specific decreasing returns to scale
ρ	the share of labor in the function $F_i\left(\right)$
N	$= mN_i$, the aggregate land in the modern sector
L	$= mL_i$, the aggregate labor in the modern sector
λ	a function of parameters, given by $p\theta\rho^{\rho\mu}\left(1-\rho\right)^{1-\rho\mu} m^{1-\theta}$
w	wage of workers in the modern sector
r	rental rate of land
π	total profits of entrepreneurs
a	value of product of a traditional farmer using σ units of land

Table 1: Definition of parameters

5 Appendix:

Derivation of equation (4)

Using the Cobb-Douglas functional form, we have

$$\frac{\partial F_i\left(N_i, L_i\right)}{\partial L} = \rho\left(\frac{L_i}{N_i}\right)^{\rho-1} \text{ and } \frac{\partial F_i\left(N_i, L_i\right)}{\partial N} = \left(1-\rho\right)\left(\frac{L_i}{N_i}\right)^{\rho}. \tag{14}$$

Using equations (3) and (14), we can write the first optimality condition in equation (2) as

$$p\theta F_T^{\mu-\theta}\left(F_i\left(N_i, L_i\right)\right)^{\theta-1}\rho\left(\frac{\rho r}{\left(1-\rho\right)w}\right)^{\rho-1} = w. \tag{15}$$

(We only need to use one of these two equations, because equation (3) provides a second equilibrium condition.)

The definition of F_T and the symmetry assumption imply that

$$F_T^{\mu-\theta}\left(F_i\left(N_i, L_i\right)\right)^{\theta-1} = F_T^{\mu-\theta}\left(\frac{F_T}{m}\right)^{\theta-1} = F_T^{\mu-1}m^{1-\theta}. \tag{16}$$

Using equation (3), we have

$$\begin{aligned} F_i\left(N_i, L_i\right) = N_i \left(\tfrac{L_i}{N_i}\right)^{\rho} = N_i \left(\tfrac{\rho r}{(1-\rho) w}\right)^{\rho} \Longrightarrow \\ F_T = m N_i \left(\tfrac{\rho r}{(1-\rho) w}\right)^{\rho-1} = N \left(\tfrac{\rho r}{(1-\rho) w}\right)^{\rho}, \end{aligned} \tag{17}$$

where $N \equiv m N_i$ the aggregate labor in the modern agricultural sector. Substituting equations (16) and (17) into the optimality condition (15) , we obtain equation (4) and the definition (5).

Derivation of equation (8).

Denote the total return to the entrepreneurial class as Π, the value of output minus the payments to labor and land:

$$\begin{aligned} \Pi = p F_T^{\mu-\theta} m \left(F_i\left(N_i, L_i\right)\right)^{\theta} - wL - rN = \\ p F_T^{\mu-\theta} m \left(\tfrac{F_T}{m}\right)^{\theta} - wL - rN \\ p F_T^{\mu} m^{1-\theta} - wL - rN = \\ p \left(N \left(\tfrac{L_i}{N_i}\right)^{\rho}\right)^{\mu} m^{1-\theta} - wL - rN = \\ p \left(N \left(\tfrac{\rho r}{(1-\rho) w}\right)^{\rho}\right)^{\mu} m^{1-\theta} - wL - rN. \end{aligned} \tag{18}$$

Substituting in the expressions for the equilibrium wage and rental rate, equations (6) and (7), using the definition $\pi = \frac{\Pi}{m}$, and simplifying, leads to equation (8).

Factor shares

Labor's share of the value of output in the modern sector is

$$\frac{wL}{p \left(N \left(\frac{\rho r}{(1-\rho) w}\right)^{\rho}\right)^{\mu} m^{1-\theta}} = \rho\theta$$

and the share of land is

$$\frac{rN}{p \left(N \left(\frac{\rho r}{(1-\rho) w}\right)^{\rho}\right)^{\mu} m^{1-\theta}} = (1-\rho)\,\theta,$$

implying that the entrepreneurs' share is $1-\theta$.