An Overlapping Generations Model of a Renewable Resource with Imperfect Property Rights

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Abstract
We use a two-sector OLG model to study the intergenerational effects of a tax designed to conserve a natural resource such as the global atmosphere. The traded asset capitalizes future environmental benefits created by the tax, benefiting the current asset owners, the old generation. Absent a transfer, the tax harms the young generation by decreasing their real wage. Future generations benefit from the higher environmental stock created by the tax. The intergenerational conflict arising from public policy is between generations alive at the time society imposes the policy, not between generations alive at different times. A simple transfer from the current old to the current young leads to a Pareto-improving policy that can be implemented and sustained in a setting where any policy change requires a supermajority.

Keywords: Open-access resource, two-sector overlapping generations, Ricardo-Viner model, resource tax, distributional effects, generational conflict, climate policy

JEL classification numbers: E24, H23, Q20, Q52, Q54

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1 Introduction

Conventional wisdom holds that environmental policy requires people alive today to make sacrifices in order to protect the environment, preserving consumption opportunities for those alive in the future. We propose a challenge to this view and show that in certain situations all generations can benefit from such policy.

In our overlapping generations model (OLG) environmental policy benefits asset holders (the old generation) by increasing the value of their assets, and tends to harm wage earners (the young generation) by decreasing their real wage. Future generations benefit from the improved environment. The only possible conflict to arise is between generations alive at the time society imposes the policy, not between those alive today and those alive in the distant future. The old however can compensate the young currently alive, making both groups better off. Future generations do not need to share their welfare improvement with the present through an intergenerational transfer scheme.

Specifically, we study a two-sector OLG model with a single endogenous stock, the environment. Lack of property rights to this resource causes producers to exploit the environmental stock too heavily, reducing future productivity. The absence of property rights to the environment also means that there is no market for the environmental stock, so it cannot be used as a store of value. There is a single store of value, a constant (or exogenously changing) stock of man-made capital. The old generation uses this capital for production and then sells it to the young generation.

A policy such as an environmental tax reduces over-production of the environment-intensive commodity and increases the value of the traded asset, the fixed (or exogenously changing) capital. This change in asset value is a mechanism for transferring welfare from future generations, who benefit from the higher future environmental stock caused by the tax, to the current owners of the asset, the old in the period when society imposes the tax. Those asset owners always benefit from the environmental tax. The current young generation, which buys the higher priced asset, can be made better off by appropriate allocation of the revenue from the environmental tax. In particular, it is not necessary to give the current young a share of the increase in asset value. In future periods, the allocation of tax revenues is constant across generations. The policy we propose emerges and is stable in a political equilibrium that requires a supermajority to change the environmental tax.
The literature that examines environmental policy in OLG models has neglected the role of asset prices so far. Kemp and van Long (1979) and Mourmouras (1991) are among the first to use the OLG framework of Samuelson (1958) and Diamond (1965) to assess the economics of renewable resources. Mourmouras (1993) demonstrates that a social planner can implement welfare-improving conservation measures in a model with environmental externalities and capital accumulation. John et al. (1995) discuss the steady state inefficiencies due to intergenerational disconnectedness in the presence of private goods with negative externalities; John and Pecchenino (1994) consider the transitional dynamics in this setting. These contributions recognize that environmental policy affects different generations unevenly because costs are immediate but benefits arise in the future; they do not take into account the possibility of Pareto improvements through intergenerational transfers in the issuance public debt. Bovenberg and Heijdra (1998, 2002) and Heijdra et al. (2006) use a continuous time OLG model to explore this possibility and to show how different policy instruments (e.g. profit, wage, or lump-sum taxes) affect the distributional impacts of environmental policy across present and future generations. Our contribution extends their insights by emphasizing the role of asset price effects and by finding that Pareto-improving tax policy can be implemented and sustained through an endogenous political process. In particular, an environmental tax can improve current generations’ welfare even in the absence of a government that uses bonds to distribute income across generations. Our model framework is close to that of Koskela et al. (2002); it differs by separating conventional capital and the renewable resource into different sectors and by allowing for open-access in the latter.

In addition to examining the welfare effects of environmental policy in OLG models with a renewable resources with imperfect property rights, we also contribute to the debate on climate policy. By calibrating our model

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1 The emphasis of these policies is to implement sustainable consumption paths, i.e. paths on which consumption is not falling. Howarth (1991, 1996), Howarth and Norgaard (1990, 1992), and Krautkraemer and Batina (1999) also use OLG models to analyze the social welfare of sustainability.

2 Marini and Scaramozzino (1995) analyze the intertemporal effects of environmental externalities and optimal, time-consistent fiscal policy in continuous time.

to the case of a degenerating, overused renewable resource such as the atmosphere, we use climate policy as an exemplary application of the type of environmental policy we are discussing.\(^4\) As with environmental policy in general, most models in this tradition include assumptions that imply that meaningful climate policy requires a reduction in current consumption and current utility. Climate policy changes the future trajectory of consumption, and eventually leads to higher utility flows than the levels under Business as Usual (BAU). Comparison of the consumption trajectories under BAU and under a climate policy depends on the social discount rate, a parameter (or function) about which there is considerable disagreement (Stern, 2007; Nordhaus, 2007).\(^5\) Of course, there is also disagreement about the level of abatement costs and climate-related damages, but we do not deal with that debate.

In the context of climate policy, there already exist two challenges to what we call the conventional wisdom. First, if other market failures are correlated with the climate externality, there may be “win-win” opportunities, so that correcting the externalities jointly makes it possible to protect the environment without reducing current consumption. Second, Foley (2009) notes that when saving is endogenous, there may be opportunities to reallocate savings, reducing saving of man-made capital, e.g. industrial infrastructure, and increasing saving of environmental capital in such a way that leaves all generations better off than under BAU. In this situation, where each generation is better off under the environmental policy, the welfare comparison does not depend on the social discount rate. Rezai et al. (2010), using a model that resembles DICE, find that Foley’s conjecture is plausible. Even proponents of what we refer to as the conventional view recognize the possibility that Foley emphasizes\(^6\) but this recognition seldom plays much role in policy analysis.

The fact that our model contains a single endogenously changing stock,

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\(^4\)We partly use numerical methods to do so, as in Howarth (1998) and Rasmussen (2003).

\(^5\)Schneider, Traeger and Winkler (2010) use an OLG model to explain the problems with using parameters that describe individual preferences to calibrate a social discount rate, in the context of climate change policy.

\(^6\)For example, Nordhaus (2007) discusses an option that “... keeps consumption the same for the present but rearranges societal investments away from conventional capital (structure, equipment, education and the like) to investments in abatement of greenhouse gas emissions (in ‘climate capital’, so to speak).”
the environment, means that we rule out the possibility of win-win situations and of a reallocation of the savings portfolio, as Foley describes. Agents alive in the current period have only one way to influence the future, by changing their current use of the environmental stock. In addition, agents in our OLG model are not altruistic; they care only about their own lifetime welfare. Thus, questions about the social discount rate are essentially irrelevant, although each generation’s discounting of its own future consumption still matters. Our assumptions that there is a single endogenous stock and that agents are indifferent about the welfare of their successors make the model tractable, and equally important, the assumptions bias the model against finding that environmental policy improves welfare for each generation. However, we find that an environmental tax, with appropriate allocation of tax revenues, creates a Pareto improvement and can be implemented and sustained in a political equilibrium that requires a supermajority to change the policy.

Our model directs attention away from the conflict between those alive today and those alive in the distant future, a conflict that has been the focus of much academic attention, to the conflict between different type of actors alive today. Much of the political (as distinct from academic) dispute about climate change policy centers on a conflict between developed and developing countries. Populations of the developing countries are younger and poorer than those of developed countries. The correspondence between levels of development in the world and generations in an OLG model is inexact but useful. Our results show that environmental policy benefits the current rich (the old) and harms the current poor (the young). The current rich should compensate the current poor not because the former are able and morally obliged to do so, but because climate policy benefits the former and, absent the transfer, harms the latter.

2 Model

We use a two-sector Ricardo-Viner discrete time overlapping generations model. In each period \( t \) a cohort of constant size \( N \equiv 1 \) is born. Agents live two periods; they are risk-neutral and maximize their intertemporal additive, homothetic utility. They have no bequest motive.

One sector, “manufacturing”, produces a good \( M \) using labor and a sector-specific input, capital. The stock of capital is fixed, \( K \equiv 1 \); later
we relax this assumption. The other sector produces a good $F$ using labor and an endogenously changing resource stock, $x$. There are perfect property rights for the stock of manufacturing capital, and no property rights for the resource stock. Labor is perfectly mobile, and in the absence of an environmental policy competes away all rent in the resource-intensive sector.

Young agents receive a wage, income from the resource sector, and, possibly, a share of tax revenues. They divide their budget between consumption of the two goods and saving in the form of buying manufacturing capital. The old generation earns the profits of its manufacturing firm, the proceeds from selling the firm, and its share of the tax revenue. Because agents are non-altruistic, the old generation consumes all of its assets.

The labor and commodity markets are competitive and clear in each period. Employment in the resource sector equals $L$ and free movement of labor between the sectors ensures that the return to labor there equals the manufacturing wage. Manufacturing is the numeraire good and the relative price of the resource-intensive good is $p$. Output in the resource-intensive sector (“harvest”) is $F = Lf(x)$ with $x$ the stock of the resource and $f(x)$ the productivity per unit of labor. The function $f(.)$ is increasing and concave. Manufacturing output is $M = M(1 - L)$ where $M$ is increasing and concave. There are decreasing returns to scale and therefore profits in this sector. Manufacturing firms are the only asset of the economy. They are privately owned by the old generation and sold to the young generation.

The open access of the resource sector means that excess labor (relative to first best) moves to this sector. This misallocation can be reduced by imposing an ad-valorem tax, $T$, on the resource harvest. The revenue accruing to workers in the resource sector, under the tax, equals $p(1 - T)Lf(x)$. Society returns the tax revenue, $R = pTLf(x)$, in a lump sum, but possibly different shares, to the young and old generations.

We examine the distributional effect of a tax that protects the renewable resource. We explore how changes in the value of the manufacturing firm, the wage, and the relative price of the resource-intensive good, caused by the tax, affect generations alive at the time the tax is imposed. The tax also affects welfare of future generations because it promotes the conservation of the resource.

The lifetime decision problem of the representative agent, assuming in-
tertemporal additive utility, is
\[
\max_{c_F,t,c_M,t,c_{F,t+1},c_{M,t+1}} \ U(c_{F,t},c_{M,t}) + \frac{1}{1+\rho} U(c_{F,t+1},c_{M,t+1})
\]
subject to
\[
\begin{align*}
    w_t + \chi R_t &\geq p_t c_{F,t} + c_{M,t} + \sigma_t \\
    \tilde{\sigma}_{t+1} + (1 - \chi) \tilde{R}_{t+1} + \tilde{\pi}_{t+1} &\geq \tilde{p}_{t+1} c_{F,t+1} + c_{M,t+1}
\end{align*}
\]
given
\[
    w_t, p_t, \tilde{p}_{t+1}, \sigma_t, \tilde{\sigma}_{t+1}, \chi, R_t, \tilde{R}_{t+1}, \rho, \tilde{\pi}_{t+1}
\]
with \( p \) the price of \( F \) in terms of \( M \), \( c_{i,j} \) the consumption level of good \( i \) at time \( j \), \( \rho \) the pure rate of time preference, and \( \chi \) the tax revenue share allocated to the young; the endogenous variables \( \sigma_t \) and \( \tilde{\sigma}_{t+1} \) are the current and the expected next-period value of the firm and \( \tilde{\pi}_{t+1} \) is the expected next-period manufacturing profit. The young agent dedicates all of her time to working and the old agent manages the manufacturing firm.

The assumption of identical homothetic preferences implies that the share of income devoted to each good is independent of both the level and distribution of income, so prices do not depend on the distribution of income. Two of the first order conditions of the representative agent’s decision problem can be rearranged to yield the consumption portfolio in each period as a function of the relative price, \( p \):
\[
    \frac{U_F}{U_M} = p \Rightarrow \frac{M}{F} = s(p).
\]
The share of commodities in the consumption bundle, \( s(p) \), depends only on the relative price, \( p \), with \( s' > 0 \).

The requirements that workers are indifferent between working in one sector or the other, and that manufacturing firms maximize profits, determine the wage and the allocation of labor. The manufacturing wage, \( w \), equals the value of labor’s marginal product. This equality and the full employment condition give manufacturing output as a function of \( w \), and resource output as a function of \( w \) and the resource stock, \( x \):
\[
    M = M(1 - L(w)) \equiv g(w) \quad \text{and} \quad F = L(w)f(x) \equiv h(w,x)
\]
\[
    \Rightarrow \frac{M}{F} = \frac{g(w)}{h(w,x)}.
\]
The equality of the wage rate with the return to labor in the resource-intensive sector implies:

\[ p(1 - T)f(x) = w \iff p = \frac{w}{(1 - T)f(x)}. \]  

(1)

The condition that the ratio of demand equals the ratio of supply closes the system and yields the static equilibrium condition

\[ s \left( \frac{w}{(1 - T)f(x)} \right) = \frac{g(w)}{h(w, x)}. \]

Solving this condition for \( w \) gives the wage as a function, \( q(\cdot) \), of \( x \) and \( T \); then using equation (1) gives the relative price as a function of \( x \) and \( T \):

\[ w = q(x, T) \quad \text{and} \quad p = \frac{q(x, T)}{(1 - T)f(x)}, \]

with the equilibrium price \( p \) decreasing in \( x \) and increasing in \( T \) and the equilibrium wage \( w \) increasing in \( x \) and decreasing in \( T \).

Having solved the static equilibrium, we consider the effects of the introduction of a tax. The imposition of the tax reduces the returns to labor in the resource sector and, hence, reduces the misallocation of labor. This reallocation of labor increases the resource stock in future periods. As labor flows into the manufacturing firm, wages fall and manufacturing output and nominal profits rise. A no-arbitrage condition introduced below implies that the value of the firm is equal to the discounted sum of future profits. The environmental tax increases these future profits, thereby increasing the asset price and raising the transfer of resources from the young to the old generation in the period the tax is imposed. The old generation pockets the asset price jump. Because the tax increases the relative price \( p \), decreases the wage, and increases the young generation’s savings, it would, in the absence of transfers of tax revenue, necessarily decrease the young generation’s utility from consumption in the current period. Appendix A shows that even if the young generation obtains all of the tax revenue in the first period, their first period utility of consumption might fall, due to the lower wage, higher price of the resource-intensive good, and higher price of the asset.

Figure 1 shows how the tax alters consumption and the sum of the young and old agents’ utility (aggregate utility) in the current period. Under BAU, current consumption is at point A, a level that maximizes current aggregate
Figure 1: Consumption expansion paths under BAU and under an environmental policy

utility, ignoring the environmental externality. The tax moves consumption to point $B$, where aggregate utility is lower: at least one of the two agents has lower current utility at $B$ than at $A$.

Figure 1 also illustrates the conventional view that environmental policy creates a conflict between those alive today and those alive in the future. The consumption expansion path under BAU moves along the curve from $A$ to $A'$, a trajectory that incorporates changes in both environmental and man-made capital stocks, including technological change. An environmental policy causes current consumption to move to point $B$, leading to a fall in current aggregate utility. The consumption trajectory under the environmental policy moves along the curve from point $B$ to $B'$. Agents alive at the initial time have higher utility under trajectory $AA'$, and those alive later have higher utility under trajectory $BB'$, so a welfare comparison depends on the social discount rate. The two previous challenges to the conventional view, the existence of win-win situations or the possibility of reallocating the investment portfolio, imply that environmental policy moves society from trajectory $AA'$ to trajectory $B''B'$. With this move, agents in every period have higher utility under environmental policy.

Our model rules out both of the previous challenges: there are no win-win opportunities, and the assumption that the environment is the only
endogenously changing stock excludes the possibility of reallocating investment across stocks. Environmental policy in our model does lower aggregate utility of consumption in the first period. The current old live for a single period, so the policy increases their lifetime welfare if and only if it increases their utility in the current period. The current young will also be alive in the next period. Even if the policy lowers their current utility, their lifetime welfare can increase if their utility in the next period increases sufficiently. We specialize the model in the next section to explore this possibility.

3 The Cobb-Douglas Specialization

In order to assess the welfare effects of a resource tax, we need to specify the functional forms of the utility function, \( U \), the production function in manufacturing, \( M \), and the production function and growth function in the resource sector. We choose Cobb-Douglas functions:

\[
U(.) = c^\alpha F_t M_t^{1-\alpha} + \frac{1}{1+\rho} c^\alpha F_{t+1} M_{t+1}^{1-\alpha}
\]

and

\[
M(.) = (1-L)^\beta.
\]

With \( \alpha \) the budget share for the resource-intensive good, the ratio of commodities is

\[
s(p) \equiv \frac{M}{pF} = \frac{1-\alpha}{\alpha} \Rightarrow \frac{M}{F} = \frac{1-\alpha}{\alpha} p
\]

and indirect utility with expenditure level \( e \) is

\[
\hat{v}(e,p) = \left( \frac{\alpha e}{p} \right)^\alpha \left( \frac{(1-\alpha)e}{1} \right)^{1-\alpha} = \alpha^\alpha (1-\alpha)^{1-\alpha} p^{-\alpha} e = \mu p^{-\alpha} e,
\]

with \( \mu \equiv \alpha^\alpha (1-\alpha)^{1-\alpha} \). The young and old generation’s expenditure level is

\[
e^y = w + \chi R - \sigma(x) \text{ and } e^o = \pi + (1-\chi) R + \sigma(x),
\]

where \( \sigma(x) \) is the amount that the young pay the old to purchase the manufacturing firms.
Given the expressions for $s(\cdot)$ and $M(\cdot)$, we can solve for the static equilibrium values of $w$, $L$, and $p$ as functions of $T$ and $x$:

$$L = \frac{1 - T}{\frac{1}{\alpha} - \beta + 1 - T}$$

$$w = \beta \left(1 + \frac{1 - T}{\frac{1}{\alpha} - \beta}\right)^{1-\beta}$$

$$p = \frac{w}{(1 - T)f(x)} = \beta \left(1 + \frac{1 - T}{\frac{1}{\alpha} - \beta}\right)^{1-\beta} \frac{1}{(1 - T)f(x)} \equiv \psi(T)^{f(x)}.$$

Under Cobb-Douglas technology and preferences, the equilibrium allocation of labor and the wage do not depend on the resource stock, $x$, only on the tax $T$ and the parameters $\alpha$ and $\beta$. The price level is a multiplicative function of $f(x)$. This fact is important in computing the firm’s value. The tax revenue, $R$, firms’ profits, $\pi$, and the sectoral values of output, $pF$ and $M$, depend also only on $T$ and model parameters:

$$\pi = \frac{1 - \beta w(1 - L)}{\beta}$$

$$R = \frac{T}{1 - T} L w$$

$$M = (1 - L)^{\beta}$$

$$pF = \frac{\alpha}{1 - \alpha} (1 - L)^{\beta}.$$

The levels of expenditure reduce to

$e^y = w + \chi R - \sigma(x, T) = w \left(1 + \chi \frac{T}{1 - T} L\right) - \sigma(x, T)$ and

$e^a = \pi + \sigma(x, T) + (1 - \chi) R = w \left(\frac{1 - \beta}{\beta} (1 - L) + (1 - \chi) \frac{T}{1 - T} L\right) + \sigma(x, T).$

For the resource sector, we assume a logistic growth function and constant returns-to-scale harvest function. The resource transition equation is

$$x_{t+1} = x_t + r x_t \left(1 - \frac{x_t}{C}\right) - L(T)\gamma x_t = \left(1 + r \left(1 - \frac{x_t}{C}\right) - L(T)\gamma\right) x_t$$

$$= (1 + \bar{r}(T, x)) x_t; \text{ with } \bar{r} \equiv \left(r \left(1 - \frac{x_t}{C}\right) - L(T)\gamma\right),$$
with \( r \) the intrinsic growth rate, \( C \) the carrying capacity of the resource, 
\( f(x) = \gamma x \) resource labor productivity, and \( \bar{r} \) the actual growth rate of the resource. A higher tax conserves the resource because \( \frac{dL}{dT} < 0 \Rightarrow \frac{dx}{dT} > 0 \Rightarrow \frac{dx_{t+1}}{dT} > 0 \). The steady state stock of the resource is

\[
x^* = C \left( 1 - \frac{\gamma L(T)}{r} \right).
\]  

(2)

A higher tax increases the steady state stock.

We now derive the price of manufacturing capital using a simple no-arbitrage condition.

**Proposition 1** The value of a firm is equal to the sum of the discounted utility from its future profits evaluated at current prices.

**Proof.** The price of a firm this period is \( \sigma_t \) and the expectation of the next-period price is \( \tilde{\sigma}_{t+1} \). In equilibrium the young generation buys one firm today and sells it in the next period. With intertemporally additive, homothetic lifetime utility, the present value of total utility of the young agent is:

\[
U_y = \mu p_t^{-\alpha} e_t^{y} + \frac{1}{1+\rho} \mu \tilde{p}_{t+1}^{-\alpha} e_{t+1}^{y} = \mu \left( \frac{w}{(1-T)\gamma} \right)^{-\alpha} \times
\left( x_t^\alpha (w + \chi R - \sigma(x_t, T)) + \frac{1}{1+\rho} \tilde{x}_{t+1}^\alpha ((1 - \chi)R + \pi + \tilde{\sigma}(x_{t+1}, T)) \right).
\]  

(3)

If a young person buys a unit of the factory today, costing \( \sigma_t \), the loss in utility is \( \mu p_t^{-\alpha} \sigma_t \). Purchase of one factory today increases expenditures next period by \( \pi + \tilde{\sigma}_{t+1} \); the increase in the present value of utility next period due to the purchase of the factory is \( \frac{1}{1+\rho} \mu \tilde{p}_{t+1}^{-\alpha} (\pi + \tilde{\sigma}_{t+1}) \). The equilibrium price-of-factory sequence requires that excess demand for the asset is 0, which requires satisfaction of the no-arbitrage condition

\[
\mu p_t^{-\alpha} \sigma_t = \frac{1}{1+\rho} \mu \tilde{p}_{t+1}^{-\alpha} (\pi + \tilde{\sigma}_{t+1}).
\]  

(4)

Assuming that expectations are formed in a model-consistent manner, appendix [B] shows using repeated substitution that equation (4) implies that

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\(^7\)With logistic growth and CRS harvest functions this non-trivial steady state is unique and stable.
value of the firm is

$$\sigma_t = p_t^\alpha \sum_{i=1}^{\infty} \pi (1 + \rho)^{-i} p_{t+i}^{-\alpha} = x_t^{-\alpha} \pi \sum_{i=1}^{\infty} (1 + \rho)^{-i} x_{t+i}^{\alpha},$$

(5)

where the first expression equals the sum of discounted utility from future profits evaluated at current prices and the last expression uses the fact that profits do not depend on the resource stock and that the price level is multiplicative in the resource stock. ■

The no-arbitrage condition on the price of the firm also has implications for the lifetime utility of present and future young generations.

**Corollary 1** (i) The asset price does not affect the lifetime utility of current and future young generations. (ii) The current old generation benefits from a higher asset price.

**Proof.** (i) The imposition of the no-arbitrage condition simplifies the lifetime welfare expression of the young, equation (3), to:

$$U_y = \mu p_t(T, x)^{-\alpha} \left[ w(T) + \chi R(T) + \frac{(1 + \bar{r}(T, x))^\alpha}{1 + \rho} (1 - \chi) R(T) \right].$$

(6)

The presence of the growth rate $\bar{r}$ in the second term in the parentheses in equation (6) ensures that second period’s tax revenue is valued at that period’s price. The no-arbitrage condition implies that the young generation’s utility is independent of the asset price. A loss in utility from the higher asset price in the first period equals the discounted utility gain from increased profits and asset price in the second period. As a consequence, the young generation’s expenditure equals wage income in the first period and their share of the tax revenue in the first and second period. Their welfare considerations are limited to these expenditure components and the price effects.

(ii) The same holds for all future generations. Asset prices only enter the welfare expression of the current old generation. Only they, the current owners of the asset, capture all of possible future benefits reflected in a changed asset price. ■
3.1 Welfare effects of a tax

We turn to the welfare effects of a tax. We noted above that the tax cannot increase both generations’ first-period utility of consumption, simply because the pre-tax equilibrium is Pareto optimal with respect to first period utility (ignoring the resource externality). Our specific functional forms enable us to aggregate first-period utility for the two generations and to show:

**Proposition 2** Environmental policy such as tax to conserve the renewable resource leads to a fall in aggregate utility at the time of its introduction.

**Proof.** Aggregate utility is

\[ U_1 = \mu p^{-\alpha} (w + R + \pi). \]  

\[ (7) \]

Differentiating this expression with respect to \( T \) and simplifying gives

\[ \frac{dU_1}{dT} = -\mu p^{-\alpha} \frac{(1 - \alpha)B}{(1 - T)^2} (w + R + \pi) < 0. \]  

\[ (8) \]

A tax reduces the value of the aggregate utility of first period consumption because the tax creates a deadweight loss. This result provides a utility measure of the movement from A to B in Figure 1 and reflects the sacrifice made by agents in the current period for agents alive in the future. The OLG model enables us to take into account the fact that some agents alive in the current period also obtain benefits from the higher resource stock in the future, and it helps to disentangle the distributional effects for agents alive at the time society imposes the tax.

**Proposition 3** For all \( \chi \in [0, 1] \), the current old generation benefits from the introduction of small tax. Its welfare increases in its tax share, \( 1 - \chi \).

**Proof.** The old generation’s remaining lifetime welfare consists of the utility it obtains from current consumption,

\[ U^o = \mu p (x_t, T)^{-\alpha} \left( (1 - \chi)R + \pi \sum_{i=0}^{\infty} (1 + \rho)^{-i} \left( \frac{x_{t+i}}{x_t} \right)^\alpha \right). \]  

\[ (9) \]

The condition for a small resource tax to benefit the old generation is

\[ \frac{dU^o}{dT} \big|_{T=0} > 0 \iff \frac{\alpha}{1 - \alpha} (1 - \chi) + (1 - \beta) \sum_{i=0}^{\infty} (1 + \rho)^{-i} \left( \frac{dx_{t+i}}{dT} \frac{1}{x_t} \right)^\alpha > 0. \]  

\[ (10) \]
With \( \frac{dx_{t+1}}{dt} > 0 \) and all other terms on the left side of the second inequality positive, this condition holds for all \( \chi \in [0, 1] \) and is decreasing in \( \chi \). Note that equation (10) is evaluated at \( T = 0 \). At sufficiently large values of \( T \) the negative price effect outweighs positive income effect of the tax. ■

**Corollary 2** The introduction of a tax always leads to a fall in first-period welfare of the current young generation.

**Proof.** Proposition 2 and 3 state that aggregate current welfare falls while welfare of the old generation rises. Therefore, first-period welfare of the current young must fall. ■

In general, a price change creates winners and losers. The OLG framework shows that a policy that discourages over-extraction of a resource benefits asset holders and in the first period harms the young agents, who extract the resource. Of course, the policy also changes the consumption of the current-young in the next period. This creates the possibility of improvements in the lifetime welfare.

**Proposition 4** (i) For \( \chi \in [0, 1] \), the introduction of a small tax increases lifetime welfare of the current young generation if and only if: (a) it receives less than the entire tax revenue \( (\chi < 1) \), and (b) the pure rate of time preference is less than the positive effect of lower prices due to the higher resource stock. (ii) If the renewable resource is degrading, the young generation suffers a welfare loss due to a conservationist tax policy.

**Proof.** To examine the effect of a small tax on the young agent’s lifetime welfare, given by equation (6), we use the following relation:

\[
\frac{dU_y}{dT}\big|_{T=0} > 0 \iff (1 - \chi) \left( \left( 1 + \bar{r}(0, x) \right)^{\alpha} - 1 \right) > 0. \tag{11}
\]

(11)

(Appendix D) With \( \chi \in [0, 1] \), a small tax increases the lifetime welfare of the young generation if and only if \( \chi < 1 \) and \( (1 + \rho)^{-1} < (1 + \bar{r}(0, x))^{-\alpha} \). A small tax creates a zero first order welfare effect for the young generation that receives all tax revenue \( (\chi = 1) \). Condition (b) in the Proposition is equivalent to \( \bar{r}(0, x) > (1 + \rho)^{\frac{1}{\alpha}} - 1 \). For \( \rho > 0 \), the expression on the right side of the previous inequality is positive. Thus, a necessary condition for the young to benefit from a tax is that the resource is below its 0-tax steady state, and is in the process of recovery. ■
The case relevant for most problems involving environmental stocks, and climate policy in particular, is where the resource is being degraded, i.e. where $\bar{r}(0,x) < 0$. In this circumstance, condition (b) in the Proposition fails, and the tax necessarily reduces the lifetime welfare of the young.

Even when a small tax harms the young, it makes sense to ask whether the young would prefer to receive a larger share of tax revenue when young or old, holding the tax fixed.

**Proposition 5** The young generation prefers $\chi = 0$ if and only if it benefits from a tax introduction.

**Proof.** Because the young generation’s welfare is linear in $\chi$, equation (6) shows that
\[
\frac{dU_y}{d\chi} < 0 \iff T \left( \frac{(1 + \bar{r}(T,x))^{\alpha}}{1 + \rho} - 1 \right) > 0. \tag{12}
\]
We also have $\frac{d\bar{r}}{dT} > 0$. Using this inequality and inequalities (11) and (12), we see that if the young benefit from a small tax, then for any tax level they prefer to receive all of the tax revenue when they are old, i.e. they prefer $\chi = 0$. In contrast, if the young are harmed by a small tax, then provided that the tax is small they prefer to receive all of the tax revenue when young ($\chi = 1$).

In summary, if the environmental problem is that the resource is below its 0-tax steady state and therefore recovering, but just not recovering sufficiently quickly, then the young potentially would support a tax that speeds recovery. In that circumstance, both the young and the old generations want all of the tax revenue to go to the old. In the more relevant circumstance where the environmental objective is to keep the resource from degrading excessively, the young would oppose a tax that helps to solve the problem. If such a tax were forced upon them, they would prefer to receive all of the tax revenue while young. Thus, in the case that is relevant to most problems involving environmental stocks, this OLG model shows that there is a conflict between generations alive at the time society imposes the tax. The old generation favors the environmental policy because some of the future benefits of that policy are capitalized into the asset value. The current young obtain none of those capitalized benefits, and they do not live long enough to reap significant benefits from the improved environment.

This result is based on the assumption that the parameter $\chi$ is constant, i.e. that the old in each period receive the same share of tax revenue, $1 - \chi$. 

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That assumption is not reasonable, because the old in the period when the tax is imposed – unlike the old in any other period – capture the future benefits that are capitalized in the asset price. In addition, the young in future periods benefit from a higher resource stock (relative to BAU) in both periods of their life; the young in the current period benefit from environmental protection in only the second period of their life. Therefore, it is reasonable to treat the old and the young in the period when the tax is first imposed differently than their counterparts in future periods. In particular, the current young should receive a larger share of tax revenues, compared to the young in future periods.

4 Transfers

Here we consider the role of transfers when under BAU the resource is degrading, $\bar{r}(0, x) < 0$. Inequality (11) shows that a small tax has only a second order welfare effect on the young if they receive all of the tax revenue while young ($\chi = 1$). We noted above that the old obtain a first order welfare gain even if they receive none of the tax revenue. These two results imply that for a small tax, it is always possible for the old generation to make a transfer to the young, in addition to giving them all of the tax revenue, so that both generations are better off.\footnote{With homothetic preferences, the indirect utility function is linear in income, so a transfer of $\$1$ of income corresponds to a $\mu p(T, x)^{-\alpha}$ transfer of utils. This util measure depends on the tax and the resource stock, but not on the level of the transfer.}

We find that using only a share of their tax income, $\xi$, the old can always finance a transfer to the young such that both generations are better off under the combined (small) tax and transfer. It is not necessary for the old to give the young a share of the increase in the asset value.

Proposition 6 There exists a tax transfer rate $0 \leq \xi < 1$ from the current old to the current young such that a small tax creates a Pareto improvement.

Proof. With $\xi$ the share of the old generation’s tax revenue transferred to the young in the period when the tax is first imposed, the first period’s tax receipts are now $(\chi + (1 - \chi)\xi)\bar{R}$ for the young and $(1 - \chi)(1 - \xi)\bar{R}$ for the old. The condition for the young to benefit from the combined transfer and tax is $^9$

\footnote{This derivation follows essentially appendix D}
\[
\frac{dU^y}{dT}\bigg|_{T=0} > 0 \Leftrightarrow (1 - \chi) \left(\frac{(1 + \bar{r}(0, x))^\alpha}{1 + \rho} - (1 - \xi)\right) > 0.
\] (13)

Setting \(\xi = 0\), equation (13) reproduces equation (11). For

\[\xi > \xi^\text{crit} \equiv 1 - \frac{(1 + \bar{r}(0, x))^\alpha}{1 + \rho}\]

the young strictly prefer the combined tax and transfer compared to the status quo. Even if the resource is degrading on the BAU trajectory, \(\xi^\text{crit} < 1\). Therefore, by transferring less than their entire share of the tax revenue to the young, the old can always make the young better off under a small tax. Because Proposition 3 states that the tax improves the old generation’s welfare even if they receive none of the tax revenue, the old are obviously better off under the combined tax and transfer, compared to the status quo. Since the young gain under such a scheme, both generations prefer \(\chi = 0\).

The fact that a tax and transfer combination creates a Pareto improvement for the generations alive at the time society imposes the policy is our chief result. This result is noteworthy because it arises in a model that appears biased in favor of finding that an environmental policy harms some generation. Agents alive at the time the policy is imposed do not care about the welfare of future generations. In addition, they have only one means of accumulation: protecting the environment. That protection always requires that aggregate first period utility of consumption falls.

The key to the result that the tax+transfer increases the lifetime welfare of both generations alive at the time society imposes the policy, is that some of the future benefit of the future increase in the resource stock is capitalized in the asset value, even though there is no change in the asset stock. This increased asset value benefits only the asset owners, the old generation. Their benefit is large enough that they do not need to be compensated by tax revenue. If the young generation gets some tax revenue in the future (\(\chi < 1\)) and in addition obtains a transfer financed by the old generation’s tax revenue (but not financed by the increase in the asset value), then the young are also made better off.

Generations sufficiently far in the future are also better off due to a small tax. A small tax has only a second order effect on “static efficiency”, the efficiency calculation that holds the trajectory of the resource stock fixed. However, the tax has a first order effect on the steady state resource stock,
and that increased stock creates a first order welfare gain in the steady state. Absent transfers, the tax is more likely to benefit future generations compared to the current young generation: the tax-induced higher stock benefits each of the future generations in two periods, whereas it benefits the current young generation in only one period.¹⁰

A qualified caveat Increased asset values are the means by which some benefits of environmental policy are transferred from the future to the present. This transfer is the basis for our result that such policy is likely to involve a conflict between generations alive when the policy is first imposed, rather than between generations who live at different points in time.

In practice, environmental policy is likely to reduce the value of some assets, e.g. coal-fired power plants in the case of climate policy. In the interest of transparency, our model contains a single asset. A richer but less tractable model would include two or more assets, and environmental policy might reduce the value of some of these. The exact implications of an environmental tax for asset prices depend on the environment’s future effect on asset returns. For example, climate policy might increase the asset price of even a coal-fired power plant if this policy averts major climate crises that severely reduce future returns to the plant.

At an extreme, it is possible that environmental policy reduces the value of most assets, and that moreover the stock of these assets is really fixed. In that case, environmental policy causes society to write down the value of the stock of its assets, possibly reducing the welfare of all generations. Another possibility is that environmental policy reduces the value of most assets, but it is feasible to invest in new, low-carbon assets. In that case, policy would be likely to harm all generations currently alive, and benefit only some.

¹⁰The condition for the next generation to benefit from the tax is

\[
\frac{dU_{t+1}^y}{dT}\big|_{T=0} > 0 \iff (1 - \chi) \left( \frac{(1 + (1 - r_{x_0})\bar{t}(0, x_0))^{\alpha}}{1 + \rho} - 1 \right) > -\frac{1}{1 + \bar{r}(0, x_0)(1 - \alpha)} \beta L(0)
\]

Comparing this condition to inequality [11], we see that when the stock is degrading (i.e. \(\bar{r}(0, x) < 0\)), a small tax is more likely to benefit the next young generation compared to this generation, which always loses in the absence of transfers: the \((1 - r_{x_0})\bar{t}(0, x_0)\) term makes the left side less negative and with \(\bar{r}(0, x) > -1\) (because the stock in the next period is positive) the right side of the inequality above is negative. Therefore, the condition here is weaker than the condition in inequality [11] and it is satisfied in the numerical example we give below.

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future generations. That scenario would produce results analogous to those of integrated assessment models based on an infinitely lived agent.

More plausibly, only a small fraction of society’s non-environmental assets would lose value from conservationist policy, while the valuations of the majority of assets would increase due to the policy-induced outward shift of the production possibility frontier. In that case, our model with a single asset provides a useful basis for understanding the intergenerational effects of environmental policy.

5 Exogenous productivity Growth

In the context of most environmental problems, the natural resource is degrading on the 0-tax trajectory. In our model of constant productivity and capital, the world becomes poorer and future generations have lower welfare on that trajectory. This section introduces exogenous productivity growth in both sectors. Let \( a \geq 0 \) be the growth rate of total factor productivity in manufacturing and \( b \geq 0 \) the growth rate of efficiency in output per unit harvested. Sectoral output is

\[
M = e^{at}(1 - L)\beta \quad \text{and} \quad F = e^{bt}L(x_t)\gamma.
\]

This model of resource productivity growth implies that each extracted unit of the resource increases the supply of the resource-intensive commodity. If we think of the resource as being energy, the assumption means that the economy becomes less energy intensive, as indeed is the case for developed economies.\footnote{Given the Cobb-Douglas specialization, this form of productivity growth leaves the allocation of labor between the sectors, and, hence, the trajectory of the renewable resource unchanged. A different model of increased productivity would allow each unit of the resource to be harvested with less labor. That type of productivity growth, unlike the model we use, would worsen resource degradation.}

With productivity growth, utility grows at \( e^{(1-\alpha)a + \alpha b} \), the price level at \( e^{(a - b)(1 + \bar{r}(T, x_t))} \) and all other variables \( (w, R, \) and \( \pi) \) at \( e^{a} \).

**Proposition 7** An improvement in the welfare of the current young due to a (small) tax introduction is more less likely if the growth rate of productivity in the resource sector is greater (lower) than in manufacturing.
Proof. The condition for a welfare improvement for the young generation, in the absence of a transfer from the old, reflects the growth of the price level
\[
\frac{dU_y}{dT} \Big|_{T=0} > 0 \iff (1 - \chi) \left( \frac{e^{-(a-b)x} (1 + \bar{r}(0, x))^\alpha}{1 + \rho} - 1 \right) > 0. \quad (14)
\]
Under proportional growth \((a = b)\), the condition for the young to benefit from the tax is unchanged from equation (11). If productivity growth is greater in manufacturing than in the resource sector \((a > b)\), the resource price increases each period due to the productivity growth differential in addition to the dynamics arising from the resource stock. For the young generation to benefit from a tax, the conservation effect of the tax has to offset this additional adverse impact of rising prices. Thus, for \(a > b\), the current young are less likely to support a resource tax, and they would need a larger transfer from the old to induce them to support a tax. If, however, the productivity in the resource sector grows much faster than in the manufacturing sector \((b >> a)\), the young might support a tax even when the resource is shrinking on the 0-tax trajectory, and in the absence of a transfer. ■

6 Numerical Results

Both current generations can gain from a tax given proper allocation of tax revenues. To find the appropriate tax and transfer levels and to explore the political economy details, we calibrate the model and solve it numerically.

The parameter \(\alpha\) is the share of the resource-intensive commodity in the consumption basket. The US NIPA suggest that non-durable goods are roughly 20% of total consumption; we use this value to set \(\alpha = 0.2\).[13] The wage share in manufacturing, \(\beta\), in the US is around 0.6. The annual pure rate of time preference equals 1%/year which gives \(\rho = 0.01\) assuming one period lasts 35 years.

This derivation follows essentially appendix D.

[12] NIPA Table 2.3.5. states personal consumption expenditures. Non-durable goods which comprises (i) food and beverages purchased for off-premises consumption (8%), (ii) clothes/footwear (4%), (iii) energy (3%), and (iv) other non-durable (8%) accounts for roughly 24% of consumption. While energy and parts of non-durable goods probably should not be counted as stemming from a renewable resource, parts of food services and accommodations (6%) should.
In light of the climate policy motivation, we model the renewable resource as easily exhaustible, slowly regenerating, and half-depleted; these choices reflect the view that climate change is a serious environmental problem. We choose units of the resource stock, $x$, such that its carrying capacity is one, $C \equiv 1$. This choice implies that $x$ is equal to capacity rate. The productivity parameter $\gamma$ equals the inverse of the amount of labor that would exhaust the resource in a single period, starting from the carrying capacity $x_0 = 1$. We set $x_0 = 0.5$, $\gamma = 3.33$, and $r = 1.37$ which is equivalent to a 2.5%/year growth rate. On a 0-tax trajectory the resource continues to degrade to a steady state of $x^* = 0.285$. Table 1 summarizes the parameter assumptions. We begin the numerical study by finding the optimal tax levels and their welfare implications for these parameter values. Throughout this section we set $\chi = 0$, a value that gives the old the ability to finance substantial transfers from tax revenue, and gives the current young all of the tax revenue in the next period.\footnote{In addition, in the political process introduced below current generations always choose $\chi = 0$ in their preferred tax+transfer scheme.}

<table>
<thead>
<tr>
<th>Baseline Value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.20</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.60</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.41</td>
</tr>
<tr>
<td>$x_0$</td>
<td>0.50</td>
</tr>
<tr>
<td>$r$</td>
<td>1.37</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.33</td>
</tr>
</tbody>
</table>

Table 1: Parameter Values

Figure 2 plots the lifetime welfare of both generations as a function of $T$ relative to the 0-tax level ($W^*_T - W^*_0$). The left figure shows that in the absence of additional transfers the tax harms the young generation. This result follows from equation (13) and the assumption that $\bar{r}(0, x) < 0$. The exact computation of the welfare of the old generation requires evaluating an infinite sum involving the future resource stock. Since the stock trajectory approaches the steady state value quickly, we can approximate this sum by calculating the first five elements and replacing the stock with its steady state level for the remainder of the sequence.\footnote{We can find bounds on the sum by replacing the sequence of $x_{t+i}$ with its initial}
tax of up to 0.92 and their preferred rate is 0.61, giving them a welfare gain of 16%.

The young generation would not agree to a tax unless they receive transfers from the old. Under a 100% transfer (i.e., $\xi = 1$), a tax benefits both generations. The right panel of figure [2] shows this scenario. The young generation now gains from a tax; their preferred tax rate is 0.55, leading to a 7% increase in their welfare. The old generation’s welfare increases up to a tax of 0.77. Their maximal welfare gain occurs at $T = 0.48$, which gives them a 5% welfare increase.

![Figure 2: Young and Old Generation’s Welfare as a Function of $T$.](image)

An increase in the level of the transfer increases the welfare and the preferred tax level of the young and lowers the welfare and preferred tax level of the old generation. Under such opposing interests, what would be the equilibrium transfer and tax level? Suppose that instituting or changing the tax requires a supermajority and that in the first period society chooses the lower of the two preferred taxes, e.g. because of political inertia. Then we can conceive of the following decision-making process:

- The old generation proposes a transfer rate, $\xi$.
- Both generations name their welfare-maximizing tax rate given $\xi$.
- Society implements the lower of the two rates.

and the steady state stock. Although the asset price trajectory may not be monotonic as a function of time, the trajectory for the resource stock is monotonic. This fact, and the monotonicity of the asset price in the resource stock, imply that the current value of the firm lies between its value at the current level in perpetuity and at the steady state: $\sigma_t \in (\pi \frac{1+r}{p} \left( \frac{\pi}{\sigma_t} \right)^{\alpha}, \pi \frac{1+r}{p})$. 

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Under complete information, the old generation will propose the transfer level, $\xi^*$, that maximizes their welfare level over the constraint set

$$ (\xi, T(\xi)) = (\xi, \min\{\arg\max_T U^y(T, \xi, .), \arg\max_T U^o(T, \xi, .)\}). $$

Figure 3 plots the preferred tax rates for both generations as a function of $\xi$. The lower envelope of these curves is the implementable tax associated with the transfer rate. For our parameter values, the young generation chooses the lower tax for most transfer rates. The old generation evaluates welfare for these $(\xi, T(\xi))$ pairs and chooses the welfare maximizing transfer rate, $\xi^*$, for its initial proposal.

Figure 4 plots both generation’s attainable welfare. The old maximize their welfare by proposing a transfer rate of $\xi^* = .62$, and at that value the young generation’s preferred tax rate is $T = 0.4$\footnote{This tax level does not maximize the old generation’s welfare given a certain transfer rate, but it is the tax level that maximizes welfare over the set of feasible $(\xi, T(\xi))$-combinations.}. Because $\chi = 0$, this policy means that in the first period the young get 62% of the tax revenue and in each subsequent period the old get all of the tax revenue. This combination yields a welfare increase of 8% for the old and of 2% for the young relative to 0-tax levels. The tax almost doubles the steady state resource stock, from 0.28 to 0.51. Introducing policies that protect the environment from
Figure 4: Old generation’s (solid curve) and young generation’s (dashed curve) welfare as a function of the transfer rate, $\xi$.

degradation always leads to welfare improvements in our framework. The percentage welfare gains from such measures are in the single digits, similar to levels suggested in the climate change debate.

Tax policy also affects future generations. Although current generations benefit from the tax/transfer policy, might future generations be worse off from a non-negligible tax? As the resource stock replenishes, the production possibility frontier in Figure 1 shifts outwards and potential welfare increases. Welfare might nevertheless fall due to the dead weight loss associated to the tax.

Figure 5 graphs the welfare gains of future young generations relative to the zero-tax policy scenario. All future generations benefit from the reduction of excess labor in the resource sector and the induced outward shift of the production possibility frontier. The sacrifice attributed to environmental policy is not present in this model.

The tax $T = 0.48$ maximizes steady state welfare of the young. Numerical simulations show that the dead weight loss exceeds the benefit only at tax levels much beyond this level. The political economy first period equilibrium tax, $T = 0.4$ is close to the level that maximizes steady state welfare.

Thus far, we assumed that agents expect the first period tax to persist forever, and they make their decisions accordingly. Having determined the tax preferred by the current generations, we relax this assumption and ask whether future generations would want to modify the tax regime. Given the supermajority rule, both generations would have to be made better off in
order to change the tax. In such a second round of negotiations, the position of
the old is much better than in the initial period, because now the default
option is the existing positive tax; the old do not need to make a transfer to
retain this tax. The young generation would increase lifetime welfare by the
abolition of the tax in the absence of transfers.

As a matter of fairness, it is reasonable that the future young should
not be allowed to demand a transfer in exchange for retaining a positive
tax. For example, if the young in the second period were to receive such a
transfer, then they would take from the young in the first period a transfer
that the latter received to compensate them for the loss in real income that
they would otherwise have suffered. The young in the second and subsequent
periods have greater potential to enjoy the higher resource stock, compared
to the young in the first period. Indeed, Figure 5 shows that each subsequent
generation obtains a higher percentage gain than their predecessor.

Our simulations show that once an equilibrium tax has been implemented,
the net benefit to future old generation of tax increases (a second asset price
jump minus compensating transfers) is never large enough for new legislation
to be passed. Likewise, the future young generations’ benefit from tax re-
ductions is never sufficient to compensate the old. While each future young
generation would increase their lifetime welfare by cutting the tax, all of
them benefit from the fact that no previous generation has done so.

In summary, open access to the renewable resource leads to an intertem-
poral coordination failure. The introduction of environmental policy by self-
ish present generations partially corrects the externality and implements a

Figure 5: Implications of Present Tax Policy for Future Generations’ Welfare.
Pareto improvement. The opposing interests of future young and old agents ensure that the protective tax remains in force.

7 Discussion

Many discussions about environmental policy start from the presumption that this policy requires current sacrifices in order to protect future generations. The two existing challenges to this presumption are that there may be win-win situations, and that it may be possible to reallocate current savings in order to make agents in each period better off. We provide a different perspective, using a model that excludes both of the existing challenges to the conventional view.

In a general equilibrium OLG model, current owners of the non-environmental asset benefit from the increased asset value caused by the imposition of an environmental tax, even though they do not survive long enough to enjoy the improved environment. Future generations benefit from the improved environment. The only possible loser is the young generation that is alive when society imposes the policy. This generation does not capture the increase in asset values, since it buys those assets. Moreover, it suffers current losses because the environmental tax increases the relative price and decreases the wage. The young generation is also not alive long enough to enjoy the benefit of the improved environment. Thus, there is a genuine intergenerational conflict, but it is not the conflict between those living today and those living in the distant future – the conflict that most of the literature emphasizes. Instead, it is the conflict between those who own assets and those who must purchase them: in our model, the old and the young currently alive.

The special status of the generations currently alive makes it easier to design Pareto improving environmental policy. They can “strike a bargain” between themselves without involving future generations, since the environmental improvements automatically leave the future generations better off. In this bargain, the current old keep all of the gains from the higher asset price, but they receive less of the environmental tax revenue, compared to old generations in the future. A simple political economy structure that requires a supermajority to institute or change a policy supports and sustains a Pareto improving tax+transfer.

Although the academic literature on climate policy emphasizes conflict
between current and distant future generations, the actual political dispute turns to a large degree on disagreements between developed and developing countries. Our model is too simple to accurately reflect the subtleties of that dispute, but the model does help to illuminate some important points. The developing nations are younger and poorer than developed nations. Our model shows that young and poor agents have a just claim on compensation from old and rich agents, if the former are to accept meaningful climate policy. It is not that the old rich can afford to and are morally obliged to make this compensation – a claim that may or may not be accepted. Rather, the old rich should make the compensation because the environmental policy benefits them and would, in the absence of the transfer, harm the young poor; in addition the old rich can finance the compensation using only a fraction of their increased benefits.

References


A Effect of Tax on First Period Utility of Young

Welfare effect of a change in tax in the current period, when $y$ is the value of consumption. Indirect utility is

$$v = v(p, y)$$
$$dv = v_p dp + v_y dy$$
$$\frac{dv}{dT} \frac{v_y}{v} = \left( \frac{v_p}{v_y} \frac{dp}{dT} \frac{T}{y} + \frac{dy}{dT} \frac{y}{T} \right)$$
$$= \left( -F \frac{dp}{dT} \frac{p}{y} + \frac{dy}{dT} \frac{y}{T} \right)$$
$$= (\eta_T - \alpha \eta_p T) \frac{y}{T}$$

where $\eta$'s are elasticity and $\alpha$ is consumption share spent on the renewable resource good. If workers get all of the tax revenue, their income, after spending $\sigma$ on the manufacturing firm, is

$$y = pf (1 - T + TL) - \sigma$$
$$\eta_T = \frac{dy}{dT} \frac{T}{y} = \left( \frac{dp}{dT} f (1 - T + TL) + pf \left( (L - 1) + T \frac{dL}{dT} \right) - \frac{d\sigma}{dT} \right) \frac{T}{y}$$
$$= \frac{dp}{dT} \frac{pf (1 - T + TL)}{y} + \left( pf \left( (L - 1) + T \frac{dL}{dT} \right) - \frac{d\sigma}{dT} \right) \frac{T}{y}$$

Here $y$ is expenditures on consumption, and $y + \sigma$ is total income. Define the savings rate

$$S = \frac{\sigma}{pf (1 - T + TL)}$$

which implies

$$pf (1 - T + TL) \frac{y}{y} = \frac{1}{1 - S}.$$

Define

$$\Delta = \left( pf \left( (L - 1) + T \frac{dL}{dT} \right) - \frac{d\sigma}{dT} \right) \frac{T}{y} < 0,$$

\footnote{We show that the equilibrium wage is $p(1-T)f$ and since there is unit mass of workers, the total wage to workers is also $p(1-T)f$. The tax revenue is $TpL$. So if workers get all of the tax revenue their income is $y$, as shown.}
where the inequality arises because $L < 1$, $\frac{dT}{dT} < 0$ and $\frac{dT}{dT} > 0$ (since a tax increases today’s and future profits, it also increases the value of the firm).

We have

$$\eta_{pT} = \eta_{pT} \frac{1}{1 - S} + \Delta.$$  

Substituting this into the expression for the change in utility, we have

$$\frac{dv}{dT} v_y = \left( \eta_{pT} \frac{1}{1 - S} + \Delta - \alpha \eta_{pT} \right) \frac{y}{T} = \left( \eta_{pT} \frac{1 - \alpha (1 - S)}{S} + \Delta \right) \frac{y}{T}.$$

The necessary and sufficient condition for welfare to fall is

$$\left( \eta_{pT} \frac{1 - \alpha (1 - S)}{S} + \Delta \right) < 0 \iff$$

$$\eta_{pT} < - \left( pf \left( (L - 1) + T \frac{dL}{dT} \right) - \frac{dσ}{dT} \right) \frac{1}{y} \frac{ST}{1 - \alpha (1 - S)}.$$  

Because

$$\eta_{pT} \frac{1 - \alpha (1 - S)}{S} > 0 > \Delta,$$

the sign of

$$\eta_{pT} \frac{1 - \alpha (1 - S)}{S} + \Delta$$

is ambiguous.

## B Value of the firm

Write the no-arbitrage condition, equation (4), as

$$\sigma_t = \frac{1}{1 + \rho} \left( \frac{p_t}{p_{t+1}} \right)^{\alpha} (\pi_{t+1} + \sigma_{t+1})$$

or

$$\sigma_{t+i} = \frac{1}{1 + \rho} \left( \frac{p_{t+i}}{p_{t+1+i}} \right)^{\alpha} (\pi_{t+i+1} + \sigma_{t+i+1}),$$

so

$$\sigma_t = \frac{1}{1 + \rho} \left( \frac{p_t}{p_{t+1}} \right)^{\alpha} \pi_{t+1} + \frac{1}{1 + \rho} \left( \frac{p_t}{p_{t+1}} \right)^{\alpha} \left[ \frac{1}{1 + \rho} \left( \frac{p_{t+1}}{p_{t+2}} \right)^{\alpha} (\pi_{t+2} + \sigma_{t+2}) \right].$$
By repeated substitution obtain

$$\sigma_t = \sum_{j=1}^{S} \left( \frac{1}{1+\rho} \right)^j \left[ \left\{ \Pi_{s=0}^{j-1} \left( \frac{p_{t+s}}{p_{t+s+1}} \right)^\alpha \right\} \pi_{t+j} \right] + \left( \frac{1}{1+\rho} \right)^S \left[ \left\{ \Pi_{s=0}^{S-1} \left( \frac{p_{t+s}}{p_{t+s+1}} \right)^\alpha \right\} \sigma_{t+S} \right]$$

If the system converges to a steady state, then the second term goes to 0 as $S \to \infty$ and

$$\sigma_t = \sum_{j=1}^{\infty} \left( \frac{1}{1+\rho} \right)^j \left[ \left\{ \Pi_{s=0}^{j-1} \left( \frac{p_{t+s}}{p_{t+s+1}} \right)^\alpha \right\} \pi_{t+j} \right].$$

Note that

$$\Pi_{s=0}^{j-1} \left( \frac{p_{t+s}}{p_{t+s+1}} \right)^\alpha = \left( \frac{p_t}{p_{t+j}} \right)^\alpha$$

Using this relation and the fact that $\pi$ is constant (independent of the stock) for fixed $T$ we have

$$\sigma_t = \pi \sum_{j=1}^{\infty} \left( \frac{1}{1+\rho} \right)^j \left[ \left\{ \Pi_{s=0}^{j-1} \left( \frac{p_{t+s}}{p_{t+s+1}} \right)^\alpha \right\} \right] = \pi p_t^0 \sum_{j=1}^{\infty} \left( \frac{1}{1+\rho} \right)^j p_{t+j}^{-\alpha}. \quad (17)$$

**C Condition for Welfare Improvement of Old Generation**

In this appendix we show that a tax introduction always increases the welfare of the old generation $U^{a}$. To do so, we restate lifetime welfare of the old, $U^{a}$, in terms of $T$ and $x$:

$$U^{a} = \mu p(x_t, T)^{-\alpha} \left( (1-\chi)R + \Pi \sum_{i=0}^{\infty} (1+\rho)^{-i} \left( \frac{x_{t+i}}{x_t} \right)^\alpha \right)$$

$$= (1-\alpha)^{-\alpha \alpha} \left( \frac{\beta}{(1-T)\gamma x} \right)^{-\alpha} \left( 1 + \frac{(1-T)\alpha}{(1-\alpha)\beta} \right)^{-\alpha(1-\beta)-\beta}$$

$$\left( \frac{\alpha T(1-\chi)}{1-\alpha} + (1-\beta) \sum_{i=0}^{\infty} (1+\rho)^{-i} \left( \frac{x_{t+i}(T)}{x_t} \right)^\alpha \right)$$
Differentiating this expression with respect to $T$ and evaluating the derivative at $T = 0$ gives
\[
\frac{dU^o}{dT}\bigg|_{T=0} = (1 - \alpha)^{1-\alpha} \alpha^\alpha \left( \frac{\beta}{\gamma x} \right)^{-\alpha} \left( 1 + \frac{\alpha}{\beta(1 - \alpha)} \right)^{-\alpha(1-\beta)-\beta} \left( \frac{\alpha}{1-\alpha} (1 - \chi) + (1 - \beta) \sum_{i=0}^{\infty} (1 + \rho)^{-i} \left( \frac{dT x_{t+i}(T)}{x_t} \right)^{\alpha} \right)
\]

Given that $0 < \alpha < 1$, $0 < \beta < 1$, $0 < \gamma$, $0 < \chi < 1$, $0 < x \leq K$, and $\frac{dT x_{t+i}(T)}{dT} > 0$, all terms are positive. A sufficiently small tax introduction always increases the welfare of the old generation.

D Condition for Welfare Improvement of Young Generation

In this appendix we derive the condition for a tax introduction which improves the lifetime welfare of the young generation (11). To do so, we restate lifetime welfare of the young, (6), in terms of $T$ and $x$:
\[
U^y = \mu p_t(T, x)^{-\alpha} \left( w(T) + \chi R(T) + \frac{(1 + \bar{r}(T, x))^{\alpha}}{1 + \rho} (1 - \chi) R(T) \right)
\]
\[
= (1 - \alpha)^{-\alpha} \alpha^\alpha \left( \frac{\beta}{(1-T)\gamma x} \right)^{-\alpha} \left( 1 + \frac{(1 - T)\alpha}{(1 - \alpha)\beta} \right)^{-\alpha(1-\beta)-\beta} \left( \beta(1 - \alpha) + \alpha(1 - T(1 - \chi)) + \frac{T\alpha}{1 + \rho} \left( 1 + r - \frac{rx}{K} - \frac{(1 - T)\alpha \gamma}{\beta(1 - \alpha) + \alpha(1 - T)} \right)^{\alpha} (1 - \chi) \right)
\]

Differentiating this expression with respect to $T$ and evaluating the derivative at $T = 0$ gives with $\bar{r}(0, x) \equiv r - \frac{rx}{K} - \frac{\gamma}{\alpha + \beta(1 - \alpha)}$
\[
\frac{dU^y}{dT}\bigg|_{T=0} = ((1-\alpha)^{-\alpha} \alpha^{1+\alpha} \left( 1 + \frac{\alpha}{\beta(1 - \alpha)} \right)^{-\alpha(1-\beta)-\beta} \left( \frac{\beta}{\gamma x} \right)^{-\alpha} \left( \frac{(1 + \bar{r}(0, x))^{\alpha}}{1 + \rho} \right) - 1) (1 - \chi)
\]

Given that $0 < \alpha < 1$, $0 < \beta < 1$, $0 < x \leq K$, $0 < \gamma$, and $0 \leq r$, the first four terms are positive. The sign of the derivative depends on the signs of the last two terms.

\[
\frac{dU^y}{dT}\bigg|_{T=0} > 0 \Leftrightarrow (1 - \chi) \left( \frac{(1 + \bar{r}(0, x))^{\alpha}}{1 + \rho} - 1 \right) > 0
\]

which is condition (11).