Dynamics of Information

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Sardines "1"

Squid "2"

e $(1-e)$

Per-period

$$U(e) = U_1(H_1) + U_2(H_2)$$

Deterministic / Full Knowledge Case

$H_1 = k_1 e$ $\quad$ $H_2 = k_2(1-e)$

$$U(e) = U_1(k_1 e) + U_2(k_2(1-e))$$

$$\boxed{k_1 U_1'(k_1 e) = k_2 U_2'(k_2(1-e))}$$

Stochasticity but no uncertainty

Patch 2: deterministic & known [k_2]

k_1 ~ known precisely

H_1 = $K_1^{(e)}$ = outcome of a visit to patch 1 when effort allocation is e.

[random variable]

$$Pr\{K_1 = k\} = e^{-\lambda e} \frac{(\lambda e)^k}{k!}$$

$$E\{K_1\} = \lambda e$$

Certainty Equivalence

$$U(e) = U_1(\lambda e) + U_2(k_2(1-e))$$

Boring.

Less Boring

$$U(e) = E_{K(e)}\{U_1(K)\} + U_2(k_2(1-e))$$

$$= \sum_{k=0}^{\infty} \frac{e^{-\lambda e}(\lambda e)^k}{k!} U_1(k) + U_2(k_2(1-e))$$

Not not boring: we still assume that λ and k_2 are known precisely

① λ has a distribution

② I can learn about ~~..~~ the parameters of that distribution

not boring

Learning About λ: Parametric, Bayesian Approach Based on a Conjugate Prior

$$Pr\{A|B\} = \frac{Pr\{A, B\}}{Pr\{B\}}$$

MCMC

Assume that λ has a gamma density with parameters ν and α

$$\lambda \sim f(\lambda) = \frac{e^{-\alpha\lambda} \lambda^{\nu-1} \alpha^{\nu}}{\Gamma(\nu)}$$

$0 \leq \lambda \leq \infty$

$$E\{\lambda\} = \int_0^{\infty} \lambda f(\lambda) d\lambda = \nu/\alpha$$

$$CV\{\lambda\} = 1/\sqrt{\nu} = SD/mean$$

Quiz

$$\int_0^{\infty} e^{-\alpha\lambda} \lambda^{\nu-1} d\lambda = \frac{\Gamma(\nu)}{\alpha^{\nu}}$$

$$\Gamma(\nu+1) = \nu\Gamma(\nu)$$

$$\Pr\{K=k\} = \int_0^\infty \Pr\{K=k,\ \lambda\}\, d\lambda$$

$$= \int_0^\infty \Pr\{K=k \mid \lambda\}\, f(\lambda)\, d\lambda$$

$$= \int_0^\infty e^{-\lambda e}\frac{(\lambda e)^k}{k!}\, e^{-\alpha\lambda}\frac{\lambda^{\nu-1}\alpha^{\nu}}{\Gamma(\nu)}\, d\lambda$$

$$= \frac{e^k}{k!}\frac{\alpha^\nu}{\Gamma(\nu)}\int_0^\infty e^{-\lambda(\alpha+e)}\,\lambda^{\nu+k-1}\, d\lambda$$

IIYH:

$$= \frac{e^k}{k!}\frac{\alpha^\nu}{\Gamma(\nu)}\frac{\Gamma(\nu+k)}{(\alpha+e)^{\nu+k}}$$

$$= \left(\frac{\alpha}{\alpha+e}\right)^{\nu}\left(\frac{e}{\alpha+e}\right)^{k}\frac{\Gamma(k+\nu)}{\Gamma(\nu)\,k!}$$

Negative Binomial Distribution

$$= p_k(\nu, \alpha, e)$$

$$\Pr\{K=0\} = \left(\frac{\alpha}{\alpha+e}\right)^{\nu}$$

$$E\{K\} = \frac{\nu}{\alpha} e$$

$$\mathrm{Var}\{K\} = \frac{\nu}{\alpha}e + \frac{1}{\nu}\left(\frac{\nu e}{\alpha}\right)^2$$

Bayesian Updating

$$\Pr\{A|B\} = \frac{\Pr\{A,B\}}{\Pr\{B\}} = \frac{\Pr\{B|A\}\Pr\{A\}}{\Pr\{B\}}$$

$$\Pr\{\lambda \mid K=k\}$$

prior

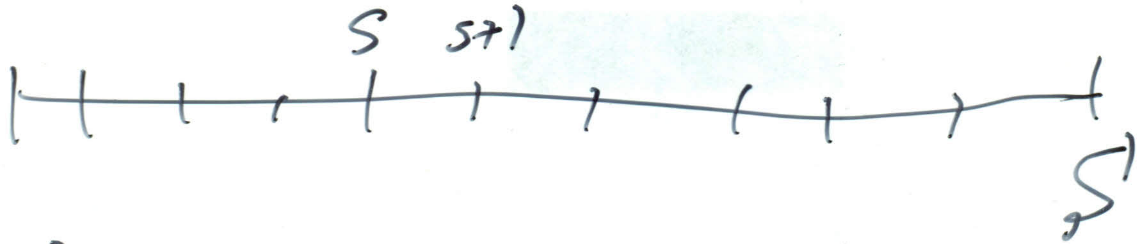
$$= \frac{\Pr\{K=k \mid \lambda\} f(\lambda)}{\int [\text{numerator}]\, d\lambda}$$

$$= \frac{\dfrac{e^{-\lambda e}(\lambda e)^k}{k!} \; \dfrac{e^{-\alpha\lambda}\lambda^{\nu-1}\alpha^{\nu}}{\Gamma(\nu)}}{\int [\text{numerator}]\, d\lambda} \propto e^{-(\alpha+e)\lambda} \lambda^{\nu+k-1}$$

posterior

Before		After
α	Effort e	$\alpha+e$
	Harvest k	
ν		$\nu+k$
ν/α	Mean	$\dfrac{\nu+k}{\alpha+e}$
$1/\sqrt{\nu}$	CV	$1/\sqrt{\nu+k}$

SDP With Information Dynamics



~~F(v, q)~~

$$F(v, q, s) = \text{maximum } E\{\text{accumulation of utility from } s \text{ to } S' \mid \text{parameters at } s \text{ are } (v, q)\}$$

$$F(v, q, S')$$

NB

$$= \max_{e} \left[\sum_{k=0}^{\infty} \varphi_k(v, q, e)\, u_1(k) + u_2(k_2(1-e)) \right]$$

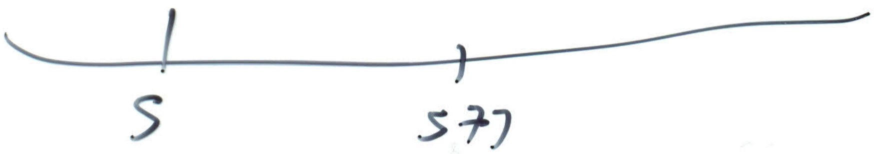
What about previous periods

$$s \longrightarrow s+1$$

$$(v, \alpha) \qquad (v+k, \alpha+e)$$

$$e \qquad \text{wp } p_k(v, \alpha, e)$$

$$F(v, \alpha, s) = \max_e \left\{ \sum_{k=0}^{\infty} p_k(v,\alpha,e) \Big[u_1(k) + u_2(k_2(1-e)) + F(v+k, \alpha+e, s+1) \Big] \right\}$$



Computational Considerations

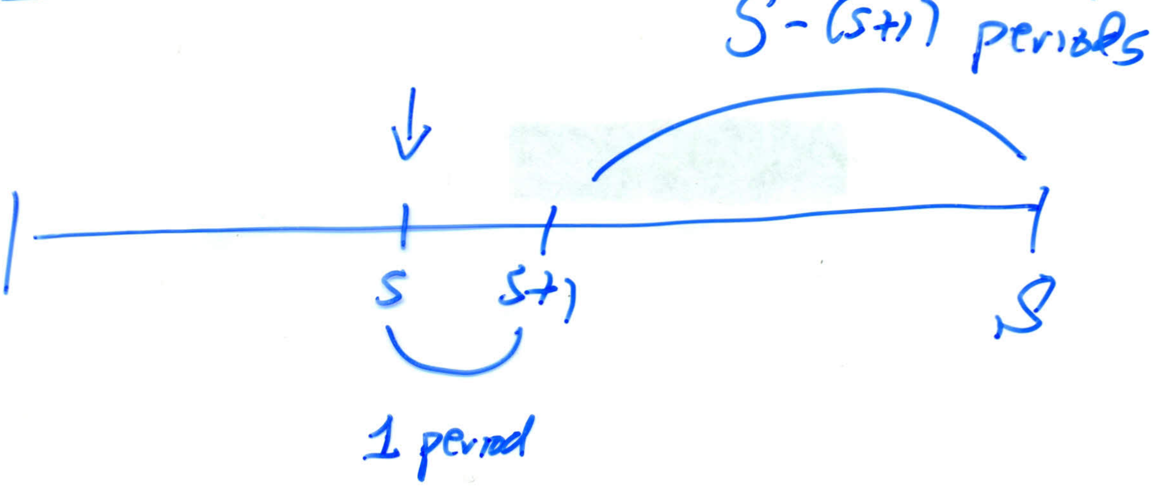
$$\sum_{k=0}^{\infty} \varphi_k(v, a, e)$$

$$\longrightarrow \sum_{k=0}^{k_{max}(\varepsilon)} \varphi_k(v, a, e) = 1 - \varepsilon$$

Renormalize

$$\sum_{k=0}^{k_{max}} \varphi_k'(v, a, e) = 1$$

"Myopic Approach"



$$F_M(v,\alpha,s) = \max_{e} \left\{ \sum_{k=0}^{k_{max}} \varphi_k^1(v,\alpha,e)\Big[U_1(k) + U_2(k,(1-e)) + F_M(v+k,\alpha+e,S^1)\,(S^1-(s+1))\Big] \right.$$