

# Regulation of Stock Externalities with Correlated Abatement Costs<sup>★</sup>

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**Abstract.** We study a dynamic regulation model where firms' actions contribute to a stock externality. The regulator and firms have asymmetric information about serially correlated abatement costs. With price-based policies such as taxes, or if firms trade quotas efficiently, the regulator learns about the evolution of both the stock and costs. This ability to learn about costs is important in determining the ranking of taxes and quotas, and in determining the value of a feedback rather than an open-loop policy. For a range of parameter values commonly used in global warming studies, taxes dominate quotas, regardless of whether the regulator uses an open-loop or a feedback policy, and regardless of the extent of cost correlation.

**Key words:** asymmetric information, choice of instruments, correlated costs, learning, pollution control

**JEL classifications:** C61, D8, H21, Q28

## 1. Introduction

The possibility of global warming has revived interest in comparing taxes and quotas when the regulator and firms have asymmetric information about abatement costs. Weitzman (1974) showed that there is a simple criterion for ranking the policies when abatement costs and environmental damages are quadratic functions of the *flow* of pollution, uncertainty enters additively (i.e., it affects the level but not the slope of the firm's marginal costs), and the optimal quantity restriction is binding with probability one.<sup>1</sup> When the externality is caused by a *stock* rather than a *flow* the regulatory problem is dynamic, and the

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comparison of policies is more complicated. When stocks decay slowly, as do greenhouse gases, current emissions may cause future environmental damages. Regulatory policies should balance current abatement costs and the stream of future environmental damages. In formulating these policies, the regulator should also consider the possibility of learning about the firms' abatement costs, thereby reducing the future asymmetry of information.

We show how the regulator's ability to update information about serially correlated abatement costs affects the regulation of a stock pollutant, and also alters the comparison of taxes and quotas. In a dynamic model with serially correlated private information about abatement costs, past observations can provide information about current costs. In order to take advantage of this information, the regulator needs to use a feed-back policy.

Uncertainty and learning about abatement costs is an important component of the global warming problem. Arguably, uncertainty and learning about the relation between greenhouse gas stocks and environmental damages is at least as important; this topic is addressed in Karp and Zhang (2002b). These two aspects of the problem are distinct, and a model that incorporates both would be quite complicated. Here, we consider only uncertainty and learning about abatement costs.

Staring (1995) considers the simplest dynamic model where the regulator uses an open-loop policy, i.e., he announces the entire policy trajectory at the initial time. Weitzman's basic result, and the intuition for it, still holds: a steeper marginal damage function or a flatter marginal abatement cost function favor the use of quotas. A lower discount rate or a lower stock-decay rate – both features that increase the importance of future damages resulting from current pollution – favor the use of quotas.

Hoel and Karp (2002) show that both the ability to change the policy frequently and the use of a feedback rather than an open-loop policy favors the use of taxes.<sup>2</sup> They assume that costs are uncorrelated, so past observations provide no information about the current cost shock. The effect of all parameters on the policy ranking is qualitatively the same in the open-loop and feedback settings. Newell and Pizer (2003) extend the open-loop model by allowing costs to be serially correlated. They show that a more positive degree of autocorrelation favors the use of quotas. They study only the open-loop case, where the regulator learns nothing about either the evolution of stocks or abatement costs.

We consider the stock-regulation problem with correlated costs when the regulator uses a feedback policy. Correlated costs increase the value of feedback rather than the open-loop decision rules, because the regulator has the opportunity to learn about both the evolution of stocks and costs. Not surprisingly, most of the intuition developed in the earlier papers survives in this more general setting. With a feedback policy, the distinction between tradeable and non-tradeable quotas is important, because the two types of policies provide the regulator with different amounts of information. This

paper thus extends our intuition for the stock regulation problem and confirms that previous results hold in a more general setting.

Although these theoretical insights are valuable, the implications for empirical work are probably more important. For some pollution problems (such as global warming) where we would like to compare taxes and quotas, we have only rough estimates (or guesses) of the slopes of marginal damages and abatement costs, and estimates of other parameters such as decay and discount rates and cost correlations. An empirical challenge is to use the existing data to rank policies. This challenge cannot be met merely by knowing the *qualitative* characteristics of a problem. For example, Hoel and Karp (2002) show that despite the lack of a qualitative difference, there is a large quantitative difference in the criterion for ranking taxes and quotas, depending on whether the regulator uses open-loop or feedback policies; however, for all plausible parameter estimates, taxes dominate quotas for the control of greenhouse gases under either open-loop or feedback policies. Newell and Pizer (2003) reach a similar conclusion with respect to changes in the cost correlation parameter, given that the regulator uses an open-loop policy. The formulae that we derive enable us to rank taxes and quotas in a more realistic and general setting (e.g., with or without cost correlation, under open-loop or feedback policies, with or without trade in quotas).

The literature to which this paper contributes compares efficient quotas and efficient taxes. To be consistent with this literature, we assume that when there is no trade in quotas, firms have the same marginal abatement costs. In this case, there would be no efficiency gain from trade. When firms have different cost shocks, we assume that there is trade in quotas, so the potential efficiency gain is realized. That is, in both of these cases the quota is efficient. The difference between the two cases is that with trade the equilibrium quota price contains the same information about the aggregate cost shock as does the equilibrium response to a tax; without trade, the regulator learns nothing about the cost shock when he uses a quota. Thus, we identify the role of trade in providing information (via the quota price). One way to interpret this model is that trade in quotas is always permitted. In equilibrium, trade actually occurs if and only if firms are heterogenous.

A third possibility, that we do not study (but mention in note 8), is that firms are heterogenous and there is no trade under quotas. In this case, the quota is inefficient. This model would illustrate the combined effects of the greater information and the greater allocative efficiency provided by taxes or by quotas with trade.

Section 2 describes the model and section 3 explains the intuition for our results. Section 4 generalizes Newell and Pizer's policy ranking under the open-loop assumption and shows that more positively correlated cost shocks always favors the use of quotas. Section 5 shows that under the feedback policy without quota trading, taxes tend to dominate quotas when the cost

shocks are highly positively or negatively correlated. Section 6 shows that efficient quota trading eliminates the informational advantage of taxes; as in the open-loop setting, higher autocorrelation of cost shocks then favors the use of quotas. Subsequent sections assess the likelihood that the policy ranking is different in the three scenarios (open-loop, feedback with and without quota trading), and provide an empirical illustration.

## 2. The Model

We begin the analysis with the assumption that all firms are identical, so there is no incentive for firms to trade quotas. Firms behave non-strategically towards the regulator. We fix units of time equal to years and assume that a period lasts for one year. All time-dependent variables are constant within a period. At time  $t$ , the stock of pollutant is  $S_t$  and the flow of new pollution is  $x_t$ . The fraction  $1 - \Delta$  of the stock decays within a period:

$$S_{t+1} = \Delta S_t + x_t. \quad (1)$$

In period  $t$ , the flow of environmental damages is  $D(S_t)$ :

$$D(S_t) = cS_t + \frac{g}{2}S_t^2, \quad g > 0.$$

The representative firm's business-as-usual (BAU) level of emissions in period  $t$  is  $x_t^b = \bar{x} + \tilde{\theta}_t$  where  $\tilde{\theta}_t$  is a random variable. With an actual emission level  $x_t < x_t^b$ , the firm's abatement cost is a quadratic function of abatement  $A(x_t) = \frac{b}{2}(x_t^b - x_t)^2$  with  $b > 0$ . The firm's benefit from higher emission equals the abatement costs that it avoids having to pay. Defining the cost shock  $\theta_t \equiv b\tilde{\theta}_t$ , we write the benefit as a linear-quadratic function, concave in the emission with an additive cost shock. The benefit function for a representative firm is defined as the flow of cost saving due to more pollution (less abatement):<sup>3</sup>

$$B(x_t, \theta_t) = f + (a + \theta_t)x_t - \frac{b}{2}x_t^2, \quad b > 0. \quad (2)$$

At time  $t$ , only the firm observes  $\theta_t$ ; there is persistent asymmetric information. The regulator knows the parameters of the  $AR(1)$  process that determines the evolution of  $\theta$ :

$$\theta_t = \rho\theta_{t-1} + \mu_t; \quad \mu_t \sim iid(0, \gamma^2) \quad (3)$$

for  $t \geq 1$ . The regulator's subjective distribution for  $\theta_0$  is

$$\theta_0 \sim (\bar{\theta}_0, \sigma_0^2).$$

The random variable  $\theta_0$  has (subjective) mean  $\bar{\theta}_0$  and variance  $\sigma_0^2$ .  $\theta_0$  and  $\mu_t$  are independent and the correlation coefficient  $\rho$  satisfies  $-1 < \rho < 1$ .

If the regulator uses quotas, we assume that these are always binding.<sup>4</sup> Depending on the choice of parameter values and the initial value of  $S$ , the probability that the quota is binding for an arbitrarily large but finite number of periods can be made arbitrarily close to one. Since the regulator discounts the future, this fact means that there is only a small loss from ignoring the possibility that the quota is slack. Therefore, we view the assumption that the quota is always binding as an approximation.

If the regulator uses a tax  $p_t$ , firms in each period maximize the difference between the savings in abatement cost and the tax payment:

$$\text{Max}_{x_t} \Pi_t = B(x_t, \theta_t) - p_t x_t = \left[ f + (a + \theta_t)x_t - \frac{b}{2}x_t^2 \right] - p_t x_t.$$

The firms' first order condition implies<sup>5</sup>

$$x_t^* = \frac{a - p_t}{b} + \frac{\theta_t}{b}. \quad (4)$$

When the regulator uses a tax, the flow of emissions and the evolution of the pollutant stock  $S_t$  are stochastic. At time  $t$ , the regulator knows the actual level of emissions and the tax at time  $t - 1$ , and (using Equation (4)) he is able to infer the value of  $\theta_{t-1}$ . The tax-setting regulator does better using a feedback rather than an open-loop policy because he learns about the cost variable and the pollution stock, and he conditions his policy on this information.

Define  $z_t^i$  as the regulator's expected emission given the tax  $p_t$ , for  $i = \text{OL}$  (open-loop) or  $i = \text{FB}$  (feedback). Under the open-loop tax policy,

$$z_t^{\text{OL}} = E_0 x_t^* = \frac{a - p_t}{b} + \frac{1}{b} E_0 \theta_t = \frac{a - p_t}{b} + \frac{1}{b} \rho^t \bar{\theta}_0.$$

Under the feedback tax policy,

$$z_t^{\text{FB}} = E_t x_t^* = \frac{a - p_t}{b} + \frac{1}{b} E_t \theta_t.$$

The regulator's expectation of the cost variable (under the feedback policy) is  $E_t \theta_t = \bar{\theta}_0$  when  $t = 0$  and  $E_t \theta_t = \rho \theta_{t-1}$ , when  $t \geq 1$ . Choosing a tax  $p_t$  is equivalent to choosing expected emissions  $z_t^i$ . The firm's actual level of emissions is

$$x_t^*(z_t, \theta_t) = \begin{cases} z_t^{\text{OL}} + \frac{1}{b}(\theta_t - \rho^t \bar{\theta}_0) & (\text{open-loop}) \\ z_t^{\text{FB}} + \frac{1}{b}(\theta_t - E_t \theta_t) & (\text{feedback}). \end{cases} \quad (5)$$

Hereafter we model the tax-setting regulator as choosing  $z_t^i$ ; we drop the superscript  $i$  ( $i = \text{OL}$  or  $i = \text{FB}$ ) where the meaning is clear.

### 3. The Intuition for Policy Ranking

The policy ranking obtained in subsequent sections depends on four considerations. The first two, which we refer to as the flexibility effect and the stochasticity effect, form the basis for policy ranking in Weitzman (1974)'s static model, and in all of dynamic models previously cited. Under taxes, emissions and marginal abatement costs are positively correlated. This flexibility increases expected cost saving, favoring taxes. However, taxes result in a stochastic stock of pollution. Stochastic stocks increase expected damages because damages are convex in stocks. The stochasticity effect favors quotas.

The third and fourth considerations are related to the autocorrelation parameter  $\rho$ . The change in the stock over several periods approximately equals the sum of flows during that time. (A positive decay rate means that the stock change does not exactly equal the sum of flows.) Other things equal, the variation in the stock is smaller when the flows are negatively autocorrelated – as occurs under taxes when costs are negatively correlated. Thus, negative autocorrelation of costs reduces the characteristic (stochasticity of stocks) that tends to make taxes unattractive. Similarly, positive autocorrelation of costs increases the characteristic that tends to make taxes unattractive. This relation – “the stock correlation effect” – explains why the preference for quotas is monotonically increasing in  $\rho$  under an open-loop policy.

The stock correlation effect exists but is less important under the feedback tax policy, because the regulator is able to adjust the policy in every period to accommodate the previous shock. In choosing the current tax, he need only consider next-period stock variability.

The fourth consideration is the learning effect. A higher *absolute value* of  $\rho$  means that knowledge of the previous cost provides more information about the current cost. By observing the level of lagged emissions and taxes, the tax-setting regulator learns the previous cost variable under a feedback policy. Under the feedback quota, the regulator learns the previous cost variable only if quotas are traded.

When quotas are traded, the learning effect is the same under taxes and quotas. This symmetry means that the only difference (related to  $\rho$ ) between taxes and quotas is the stock correlation effect. Consequently, a higher value of  $\rho$  favors (tradable) quotas under feedback policies.

When quotas are not traded, the learning effect and the stock correlation reinforce each other for  $\rho < 0$ , so a smaller value of  $\rho$  favors taxes; when  $\rho > 0$  the two effects tend to counteract each other. The interplay of the two effects explains why the ranking of feedback taxes and non-tradable feedback quotas may be non-monotonic in  $\rho$ .

#### 4. Open-Loop Policy

With an open-loop policy, the regulator chooses an infinite sequence of policy levels,  $\{x_t\}_{t=0}^{\infty}$  under quotas and  $\{z_t\}_{t=0}^{\infty}$  under taxes, based on the information he has at time 0. This decision depends on the variance and covariance of the costs shocks, conditional on the information at time 0.

Conditional on information at time 0,  $\lim_{t \rightarrow \infty} \text{var}(\theta_t) = \frac{\gamma^2}{1-\rho^2}$ . If the initial conditional variance,  $\sigma_0^2$ , also equals  $\frac{\gamma^2}{1-\rho^2}$  (as in Newell and Pizer 2003), the regulator's open-loop problem is stationary. For any other value of  $\sigma_0^2$ , the problem is non-stationary. We consider the general case (an arbitrary value of  $\sigma_0^2$ ) in order to be able to compare the policy ranking in the open-loop and feedback settings. In the feedback setting, the regulator would have the prior  $\sigma_0^2 = \frac{\gamma^2}{1-\rho^2}$  after the first period only if he begins with these beliefs and uses a non-tradable quota. This prior is therefore not useful for comparing the open-loop and feedback settings.<sup>6</sup>

The regulator wants to maximize the expectation of the present value of the difference between abatement cost saving and pollution damages

$$E_0 \sum_{t=0}^{\infty} \beta^t \{B(x_t, \theta_t) - D(S_t)\}.$$

$\beta$  is a constant discount factor. The expectation is taken over the sequence of random variables  $\{\theta_t\}_{t=0}^{\infty}$  with respect to the information available at  $t = 0$ . Define  $T^{\text{OL}}(S_0)$  as the maximized expected value of the regulator's open-loop (OL) program when he uses taxes and the initial stock of pollutant is  $S_0$ . Define  $Q(S_0)$  as the maximized expected value of the regulator's program when he uses a quota. (Since – as we explain in section 5 – this value is the same under open-loop and feedback quotas without trading, we do not use a superscript on the function  $Q(S)$ .)

The expectation of the trajectories of the flow and the stock of pollution are the same for every scenario that we consider: open-loop and feedback, taxes and quotas, with and without quota trading. This fact is a consequence of the Principle of Certainty Equivalence of the linear-quadratic model with additive uncertainty.<sup>7</sup> Consequently, the policy ranking depends on the second moment of the random variable.

The payoff difference under taxes and quotas, given the open-loop policy, is

$$\Psi_0^{\text{OL}} \equiv T^{\text{OL}}(S_0) - Q(S_0) = \frac{1}{2b(1-\rho^2\beta)} \left( \sigma_0^2 + \frac{\gamma^2\beta}{1-\beta} \right) \left\{ 1 - \frac{g}{b} \frac{\beta}{1-\beta\Delta^2} \frac{1+\rho\beta\Delta}{1-\rho\beta\Delta} \right\}. \quad (6)$$

(Details of this and other derivations, discussions of tangential issues, and some technical proofs, are contained in an appendix that is available upon request.) Since the term outside the curly brackets is always positive, the policy ranking is independent of the regulator's priors  $(\bar{\theta}_0, \sigma_0^2)$ ; that is, Newell

and Pizer's (2003) criterion for ranking policies is correct for all priors. We restate this result as:

*Remark 1.* The preference for quotas under the open-loop policy is monotonically increasing in  $g$ ,  $\beta$ ,  $\Delta$ ,  $\rho$ , and monotonically decreasing in  $b$ .

Since the possibility of learning about cost shocks depends on the value of  $\rho$ , we emphasize the role of this parameter.

**Proposition 1.** *Under an open-loop policy, the preference for quotas is monotonically increasing in  $\rho$ . Taxes dominates quotas iff*

$$\rho \leq \frac{1 - \beta\Delta^2 - \beta\frac{g}{b}}{\beta\Delta(1 - \beta\Delta^2 + \beta\frac{g}{b})}.$$

## 5. Feedback Policy without Quota Trading

The assumption that the optimal quota is always binding means that the quota-setting regulator learns nothing about the previous cost shock and also means that the evolution of the stock of pollution is non-stochastic. Since no new information becomes available, the open-loop and feedback quota policies (and payoffs) are identical when quotas are not traded. When the regulator uses taxes, the evolution of  $S_t$  is stochastic. By observing the firms' response to the tax, the regulator learns the value of the random cost. With taxes the regulator obtains information over time, so a feedback tax policy results in a higher payoff than does an open-loop policy.

We compare the payoffs under the two policies by comparing their respective value functions. With correlated costs, there are two state variables, the stock of pollution  $S_t$  and the current expected value of the cost variable  $y_t \equiv E_t\theta_t$ . When necessary to avoid confusion, we use superscripts to distinguish state variables under tax or quota policies, e.g.,  $y_t^{\text{tax}}$  and  $y_t^{\text{quota}}$ .

The value functions under both taxes and quotas are quadratic in the state, i.e., they both have the form

$$V_{0,t}^i + \underbrace{(v_1 \ v_2)}_{V_1} \begin{pmatrix} S_t \\ y_t \end{pmatrix} + \frac{1}{2} (S_t \ y_t) \underbrace{\begin{pmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{pmatrix}}_{V_2} \begin{pmatrix} S_t \\ y_t \end{pmatrix}. \quad (7)$$

The first term of the value function  $V_{0,t}^i$ ,  $i = \text{tax, quota}$ , depends on  $t$  because  $\text{Var}_t(\theta_t)$  changes exogenously in the first period. (See Equation (29) in the Appendix.)

The Appendix contains explicit expressions for the parameters of the value functions. The matrices  $V_1$  and  $V_2$  are the same under taxes and quotas. For a given state, the optimal control,  $z_t^*$  under taxes and  $x_t^*$  under quotas are equal and are given by

$$\frac{a(1 - \beta\Delta) - \beta c}{(b - \beta V_{11}) - b\beta\Delta} + \frac{1 - \rho\beta\Delta}{b(1 - \rho\beta\Delta) - \beta V_{11}} y_t + \frac{\beta\Delta V_{11}}{b - \beta V_{11}} S_t, \quad (8)$$

with

$$V_{11} = \frac{-(b\beta\Delta^2 + \beta g - b) - \sqrt{(b\beta\Delta^2 + \beta g - b)^2 + 4\beta g b}}{2\beta} < 0. \quad (9)$$

The function  $V_{11}$  is independent of  $\rho$ ; the correlation parameter affects only the slope of the control rule with respect to the state  $y_t$ . Equation (8) implies that an increase in current expected costs,  $y_t$ , increases the current (expected) flow of pollution, as in the static model.

The values of  $z_t^*$  and  $x_t^*$  differ over time because the actual trajectories of the state vector differ under taxes and quotas. However, as we remarked above, the first moments of pollution flows and stocks are the same in all of our scenarios, because of the Principle of Certainty Equivalence. Taking expectations at time 0, we have  $E_0 y_t^{\text{tax}} = E_0(\rho \theta_{t-1}) = \rho^t \bar{\theta}_0 = y_t^{\text{quota}}$ . This relation and Equation (8) imply

$$E_0 z_t^* = x_t^*. \quad (10)$$

The expected values of the programs are different because of the second moments of the cost shocks. It makes sense to compare the values of these programs only under the same information set, e.g., at time  $t = 0$ . The same comparison holds at an arbitrary time, provided that the regulator evaluates the two policy instruments using the same information set.

In the initial period,  $y_0^{\text{tax}} = y_0^{\text{quota}} = \bar{\theta}_0$ . The payoff difference under taxes and quotas, given the feedback policy, is due to the difference in the term  $V_{0,t}^i$  (evaluated at  $t = 0$ ) in equation (7) :

$$\Psi_0^{\text{FB}} \equiv T^{\text{FB}}(S_0, \bar{\theta}_0) - Q(S_0, \bar{\theta}_0) = \frac{1}{2b(1-\rho^2\beta)} \left( \sigma_0^2 + \frac{\gamma^2\beta}{1-\beta} \right) \frac{(1-\frac{\beta}{b}V_{11}) \{ (1-\rho\beta\Delta)^2 - \frac{\beta^2}{b^2} V_{11}^2 (1-\rho^2\beta) \}}{(1-\rho\beta\Delta - \frac{\beta}{b}V_{11})^2}. \quad (11)$$

The ranking of feedback tax and quota policies depends only on the sign of the term in curly brackets. Hoel and Karp (2002) show that for  $\rho = 0$  and  $\sigma_0^2 = \gamma^2$ , the parameters  $b$ ,  $g$ ,  $\beta$  and  $\Delta$  have qualitatively the same effect on the policy ranking under both open-loop and feedback policies. The comparative statics of the ranking with respect to  $b$ ,  $g$ , and  $\Delta$  are unchanged when  $\rho \neq 0$ . *Remark 2.* The preference for quotas under the feedback policy is monotonically increasing in  $g$ ,  $\Delta$ , and monotonically decreasing in  $b$ .

However, the effect of  $\rho$  and  $\beta$  is different in the open-loop and feedback setting. Equation (11) implies

**Proposition 2.** *Feedback taxes dominate non-tradable feedback quotas iff*

$$f(\rho) \equiv \rho^2 \beta^2 \left( \Delta^2 + \frac{\beta}{b^2} V_{11}^2 \right) - 2\rho\beta\Delta + 1 - \frac{\beta^2}{b^2} V_{11}^2 \geq 0. \quad (12)$$

The function  $f(\rho)$  is convex in  $\rho$  and for some parameter values is non-monotonic in  $\rho$  over  $\rho \in (-1, 1)$ . For such parameter values, the preference for quotas is non-monotonic in  $\rho$ .

The function  $f(\rho)$  reaches a minimum at  $\rho \geq 0$ ; the minimum occurs at  $\rho = 0$  if and only if  $\Delta = 0$ , i.e., for a flow externality. We define  $\rho_1$  and  $\rho_2$  as the smaller and the larger roots of  $f(\rho) = 0$ , provided that these roots are real. If both of these roots are in the interval  $(-1, 1)$ , then for low values of  $\rho$  an increase in  $\rho$  makes quotas more attractive, and the opposite holds for high values of  $\rho$ . The following are sufficient conditions for either taxes or quotas to dominate:

**Corollary 1.** *A sufficient condition for taxes to dominate quotas is*

$$\frac{1 - \beta\Delta^2}{\beta} > \frac{g}{b}.$$

*A sufficient condition for quotas to dominate taxes is  $\rho_1 < -1$  and  $\rho_2 > 1$ .*

Under an open-loop policy, a larger value of  $\beta$  favors the use of quotas, because a larger  $\beta$  increases the importance of the future stock variability arising from the current flow variability. Under feedback policies, the comparative statics of  $\beta$  is ambiguous. We have:

**Proposition 3.** *Under a feedback policy, a higher discount factor favors quotas (i.e., it decreases  $\Psi_0^{FB}$ ) iff*

$$\frac{-(\Delta^2 + \frac{g}{b})\frac{\beta}{b}V_{11} + \frac{g}{b}}{\sqrt{(\beta\Delta^2 + \beta\frac{g}{b} - 1)^2 + 4\beta\frac{g}{b}}} \geq \frac{\rho(\rho^2\beta\Delta - 2\Delta + \rho)}{2(1 - \rho^2\beta)^{\frac{3}{2}}}. \quad (13)$$

*A sufficient condition for inequality equation (13) is  $\beta \leq \frac{2\Delta - \rho}{\rho^2\Delta}$  and  $\rho \geq 0$ . These two inequalities are satisfied if  $0 \leq \rho \leq \Delta$ .*

A higher discount factor favors the use of quotas if the gain from the informational advantage under taxes is not great enough to offset the higher expected damage from future stock variability. The sufficient condition  $0 \leq \rho \leq \Delta$  means that equation (13) is very likely to hold for stock pollutants that decay slowly. For example, when a period is one year a half-life of 15 years corresponds to  $\Delta = 0.9548$ ; greenhouse gases have a half-life of over 80 years, for  $\Delta = 0.99$  (see section 8).

In the limiting case where  $\Delta = 0$ , the externality is a flow pollutant. Define  $\tilde{g} = \beta g$ , the present value of the slope of marginal damages and set  $\sigma_0^2 = \gamma$ , the variance of the innovation to costs. The difference between payoffs under taxes and quotas for a flow pollutant is

$$\Psi_0^{\text{FB}} = \frac{\sigma_0^2}{2b(1-\beta)} \left[ \frac{\beta\rho^2}{(1-\beta\rho^2)(1+\frac{\tilde{g}}{b})} + \left(1 - \frac{\tilde{g}}{b}\right) \right].$$

This expression shows how the learning that is made possible by a non-zero value of  $\rho$  favors the use of taxes for the case of a flow pollutant. When  $\rho = 0$ , we obtain Weitzman's (1974) criterion.

## 6. Ranking with Quota Trading

Previous sections assume that all firms are identical, so firms have no incentive to trade emissions permits. When firms are heterogeneous, emissions trade increases efficiency and also reveals industry-wide costs with a one-period lag. The informational advantage of taxes disappears in this case.

Suppose there are  $n$  firms, where  $n$  is large. Let  $x_{i,t}$  be firm  $i$ 's emissions at time  $t$ . The benefit for firm  $i$  of emitting  $x_{i,t}$  is

$$B_i(x_{i,t}, \theta_t, \epsilon_{i,t}) = \frac{f}{n} + (a + \theta_t + \epsilon_{i,t})x_{i,t} - \frac{bn}{2}x_{i,t}^2, \quad (14)$$

where  $\epsilon_{i,t}$  is the firm-specific deviation from the industry-wide cost  $\theta_t$ . These firm-specific deviations are *i.i.d* over time with mean 0 and constant variance  $\sigma_\epsilon^2$ . They are uncorrelated with each other and are independent of the industry-wide average  $\theta_t$ , which follows the *AR*(1) process defined in Equation (3).

We use  $p_t$  ( $p_t^{\text{tax}}$  or  $p_t^{\text{quota}}$ ) to denote either the tax level or the quota price from trading. The first order condition to firm  $i$ 's problem gives its emission response as

$$x_{i,t}^* = \frac{a - p_t}{bn} + \frac{\theta_t + \epsilon_{i,t}}{bn}. \quad (15)$$

Summing over  $x_{i,t}^*$  gives the aggregate industry level emission

$$x_t = \sum_{i=1}^n x_{i,t}^* = \frac{1}{b} \left( (a - p_t) + \theta_t + \frac{\sum \epsilon_{it}}{n} \right). \quad (16)$$

The last term in Equation (16) is *iid*( $0, \frac{\sigma_\epsilon^2}{n}$ ); since  $n$  is large, we replace  $\frac{\sigma_\epsilon^2}{n}$  with 0. Thus, once the regulator knows  $p_t$  and  $x_t$ , he knows  $\theta_t$ . Under quotas the regulator chooses  $x_t$ , and  $p_t$  is endogenous; under taxes the regulator chooses  $p_t$ , and  $x_t$  is endogenous.

*Quotas.* Under quotas, in each period the aggregate emissions, and consequently the pollutant stock, are deterministic. Equation (16) implies the equilibrium quota price  $p_t^{\text{quota}} = a + \theta_t - bx_t$ . By observing  $p_t^{\text{quota}}$ , the regulator learns  $\theta_t$ . Substituting this price into Equation (15) gives firm  $i$ 's

emission level as  $\frac{x_t}{n} + \frac{\epsilon_{i,t}}{bn}$ . Substituting this expression into Equation (14), summing over  $i$ , and taking expectations gives the aggregate expected cost saving under quotas:<sup>8</sup>

$$E_t \sum_{i=1}^n B_i(x_{i,t}^{\text{quota}}, \theta_t, \epsilon_{i,t}) = f + (a + E_t \theta_t)x_t - \frac{b}{2}x_t^2 + \frac{1}{2b}\sigma_\epsilon^2. \quad (17)$$

*Taxes.* A tax policy results in a stochastic level of aggregate emissions. Choosing a tax is equivalent to choosing the expected aggregate emissions  $z_t \equiv E_t x_t = \frac{1}{b}(a - p_t^{\text{tax}}) + \frac{1}{b}E_t \theta_t$ . The actual level of aggregate emissions is  $x_t = z_t + \frac{1}{b}(\theta_t - E_t \theta_t)$ . By observing  $x_t$ , the regulator learns  $\theta_t$ . Firm  $i$ 's emission level is  $\frac{1}{n}z_t + \frac{1}{bn}(\theta_t - E_t \theta_t) + \frac{1}{bn}\epsilon_{i,t}$ . Substituting this expression into Equation (14), summing over  $i$ , and taking expectations gives the aggregate expected cost saving under taxes:

$$E_t \sum_{i=1}^n B_i(x_{i,t}^{\text{tax}}, \theta_t, \epsilon_{i,t}) = f + (a + E_t \theta_t)z_t - \frac{b}{2}z_t^2 + \frac{1}{2b}\sigma_\epsilon^2 + \frac{1}{2b}\text{Var}_t(\theta_t). \quad (18)$$

*Taxes vs. Quotas.* The firms' individual deviations have the same effect on the aggregate expected cost saving under taxes and quotas. These firm-specific deviations do not affect the policy ranking.

As before, the regulator maximizes the expectation of the difference between firms' aggregate cost saving from polluting and environmental damage:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{i=1}^n B_i(x_{i,t}, \theta_t, \epsilon_{i,t}) - D(S_t) \right\}.$$

Under quotas, the control variable is  $x_t$  and the expected aggregate benefit is Equation (17). Under taxes, the control variable is  $z_t$  and the expected aggregate benefit is Equation (18). The expectation is taken over sequences of random variables  $\{\theta_t\}_{t=0}^{\infty}$  and  $\{\epsilon_{i,t}\}_{t=0}^{\infty}$  with respect to the information available at the current time,  $t = 0$ .

With the open-loop policy, the regulator decides his future policy trajectory at the initial period  $t = 0$  and commits to it. There is no learning. All of the conclusions in section 4 also hold when quotas are traded. The payoff difference under taxes and quotas,  $\Psi_0^{\text{OL+Trade}}$ , is the same as in Equation (6).

With the feedback policy, the regulator adjusts the instrument level as he learns about cost shocks. Under both taxes and quotas, the regulator's posterior is

$$\theta_t \sim (\rho\theta_{t-1}, \gamma^2), \quad \forall t \geq 1.$$

The corresponding law of motion for  $y_t \equiv E_t \theta_t$ , under both quotas and taxes is  $y_{t+1} = \rho y_t + \rho \mu_t$ . (Without quota trading,  $y_{t+1} = \rho y_t$ ; see Appendix.)

This difference identifies the informational advantage of tradeable quotas (relative to non-traded quotas).

The optimal control,  $z_t$  under taxes and  $x_t$  under quotas, obeys the linear control rule (8). The regulator has the same amount of information about costs under taxes and quotas, although in general the realization of  $S$  is different under the two policies. Consequently, the optimal controls are identical, for a given value of  $S$ :

$$z_t^* = x_t^*. \quad (19)$$

Note the qualitative difference between equations (10) ( $E_0 z_t^* = x_t^*$ ) and (19) ( $z_t^* = x_t^*$ ). By observing the price of emissions permits, the regulator has the same information about costs under taxes and quotas in every period, not just at time  $t = 0$ .

Although – as previously noted – the first moment of stocks and flows are the same under taxes and quotas, their second moments differ, leading to different expected payoffs:

$$\begin{aligned} \Psi_0^{\text{FB+Trade}} &= J_0^T(S_0, \bar{\theta}_0) - J_0^Q(S_0, \bar{\theta}_0) \\ &= \frac{1}{2b} \left( \sigma_0^2 + \frac{\gamma^2 \beta}{1-\beta} \right) \frac{(1-\frac{\beta}{b}V_{11})(1-\rho\beta\Delta+\frac{\beta}{b}V_{11})}{1-\rho\beta\Delta-\frac{\beta}{b}V_{11}}. \end{aligned} \quad (20)$$

$V_{11} < 0$  is given in equation (9).

The payoff under taxes is higher than under quotas if and only if

$$1 - \rho\beta\Delta + \frac{\beta}{b}V_{11} \geq 0.$$

This inequality implies:

*Remark 3.* With efficient quota trading, changes in parameters  $b$ ,  $g$ ,  $\beta$ ,  $\Delta$ , and  $\rho$  have qualitatively the same effect on the policy ranking under open-loop and feedback policies.

The qualitative differences in policy ranking between open-loop and feedback policies depend only on the informational advantage of taxes under feedback policies. Emissions trading eliminates this informational advantage. Again, we emphasize the effect of cost autocorrelation:

**Proposition 4.** *With quota trading, more positively autocorrelated cost shocks (higher  $\rho$ ) favors the use of quotas under both open-loop and feedback strategies. Feedback taxes dominate tradeable feedback quotas iff  $\rho \leq \frac{1}{\beta\Delta} \left( 1 + \frac{\beta}{b}V_{11} \right)$ .*

## 7. Sensitivity of the Ranking

This section identifies the region of parameter space where the open-loop and feedback assumptions lead to different policy rankings. The *difference* in

policy ranking arises because the informational differences between taxes and quotas varies over the three scenarios (open-loop, feedback with quota trade, and feedback without quota trade). Table I summarizes these informational differences for  $\rho \neq 0$ . Rows 2 and 3 of the first column give the two state variables, the pollution stock and the lagged cost shock. For example, the entry “no/yes” in the second column and second row indicates that for the open-loop tax policy, the regulator does not condition the policy level in future periods on the value of the stock in those periods (“no”). In contrast, for the open-loop quota policy, it is *as if* the regulator does condition the policy level in future periods on the value of the stock in those periods (“yes”). Under quotas the stock trajectory is non-stochastic, so the regulator at time 0 knows all future stocks, given the quota trajectory.

We define the critical ratio of  $\frac{g}{b}$  as the value of the ratio that makes the regulator indifferent between taxes and quotas. The critical ratio is obtained by setting the differences in payoffs equal to 0 and solving for  $\frac{g}{b}$ . The preference for quotas is monotonically increasing in  $\frac{g}{b}$  under both open-loop and feedback policies, so the quota is the right instrument if and only if the actual value of the ratio of slopes exceeds the critical value.

The critical values in the open-loop and feedback cases are respectively

$$\begin{aligned} \left(\frac{g}{b}\right)^{*OL} &= \left(\frac{g}{b}\right)^{*OL+Trade} = \left(\frac{1 - \rho\beta\Delta}{1 + \rho\beta\Delta}\right) \left(\frac{1}{\beta} - \Delta^2\right), \\ \left(\frac{g}{b}\right)^{*FB} &= \frac{(1 - \rho\beta\Delta) \left\{ (1 - \rho\beta\Delta) + (1 - \beta\Delta^2) \sqrt{1 - \rho^2\beta} \right\}}{\beta \left\{ (1 - \rho^2\beta) + (1 - \rho\beta\Delta) \sqrt{1 - \rho^2\beta} \right\}}, \\ \left(\frac{g}{b}\right)^{*FB+Trade} &= (1 - \rho\beta\Delta) \left(\frac{1}{\beta} - \frac{\Delta^2}{2 - \rho\beta\Delta}\right). \end{aligned}$$

In the static model, the critical ratio of  $\frac{g}{b}$  is 1. When both  $\rho = 0$  and  $\Delta = 0$ , the critical ratio under both open-loop and feedback policies is  $\beta^{-1}$  rather than 1, since (by assumption) the current flow of pollution causes damages in

Table I. The different information in different scenarios. (Under quotas the stock is non-stochastic, so the regulator always knows the current stock.)

Policy conditioned on	Open-loop tax/quota	Feedback, homogenous firms tax/quota without trade	Feedback, heterogenous firms tax/quota with trade
(i) Current pollution stock	no/yes	yes/yes	yes/yes
(ii) Lagged cost shock	no/no	yes/no	yes/yes

the next period. The following Propositions describe the relation between the critical ratio of  $\frac{g}{b}$  and  $\rho$ .

**Proposition 5.** *The critical ratio of  $\frac{g}{b}$  is monotonically decreasing in  $\rho$  under both an open-loop policy and a feedback policy with tradable quotas. Under a feedback policy without tradable quotas, the critical ratio of  $\frac{g}{b}$  is non-monotonic in  $\rho$ :*

$$\frac{\partial(\frac{g}{b})^{*FB}}{\partial\rho} \begin{cases} < 0, & \text{if } \rho < \Delta \\ = 0, & \text{if } \rho = \Delta \\ > 0, & \text{if } \rho > \Delta. \end{cases}$$

**Proposition 6.** *Without efficient quota trading, the critical ratio of  $\frac{g}{b}$  under feedback policies is always greater than or equal to the open-loop level:*

$$\left(\frac{g}{b}\right)^{*FB} \geq \left(\frac{g}{b}\right)^{*OL}, \quad \forall \rho. \tag{21}$$

*With efficient quota trading, the critical ratio of  $\frac{g}{b}$  under feedback policies is greater than the open-loop level only for a subset of parameters:*

$$\left(\frac{g}{b}\right)^{*FB+Trade} \geq \left(\frac{g}{b}\right)^{*OL+Trade}, \quad \text{iff } \rho \geq \tilde{\rho}; \tag{22}$$

where  $\tilde{\rho}$  is a function (of only  $\beta$  and  $\Delta$ ) that satisfies  $-1 \leq \tilde{\rho} < 0$ .

The equality in Equation (21) holds only if  $\rho = \Delta = 0$ , i.e., with a flow externality and uncorrelated cost shocks. The function  $\tilde{\rho}(\beta, \Delta)$  used in Equation (22) equals  $-1$  only if  $\beta = \Delta = 1$ , i.e., where both discount rate and stock decay rate are zero. The proposition implies:

**Corollary 2.** *Taxes will necessarily be the right instrument choice in the feedback setting if taxes dominate quotas in the open-loop setting, provided that*

- *there is no quota trading; or*
- *there is quota trading, and cost shocks are non-negatively autocorrelated.*

With negatively correlated shocks and quota trading, there exist parameter values such that it is optimal to choose taxes under the open-loop policy but quotas under the feedback policy – or vice versa.

Figure 1 plots the critical ratios of  $\frac{g}{b}$  against  $\rho$ , holding  $\beta = 0.9512$  and  $\Delta = 0.9548$ . Since a period is 1 year, these values imply a continuous discount rate of 0.05 and a half-life of 15 years. The left panel shows the case without quota trading, and the right panel shows the case with quota trading. In both panels, the solid curve graphs the critical ratio under the feedback policy and the dotted curve graphs the ratio under the open-loop policy. The tax is better than the quota if and only if the actual ratio of  $\frac{g}{b}$  lies below the critical ratio. The left panel of Figure 1 illustrates Equation (21) and the right panel illustrates Equation (22). Table II summarizes the different ranking possibilities.

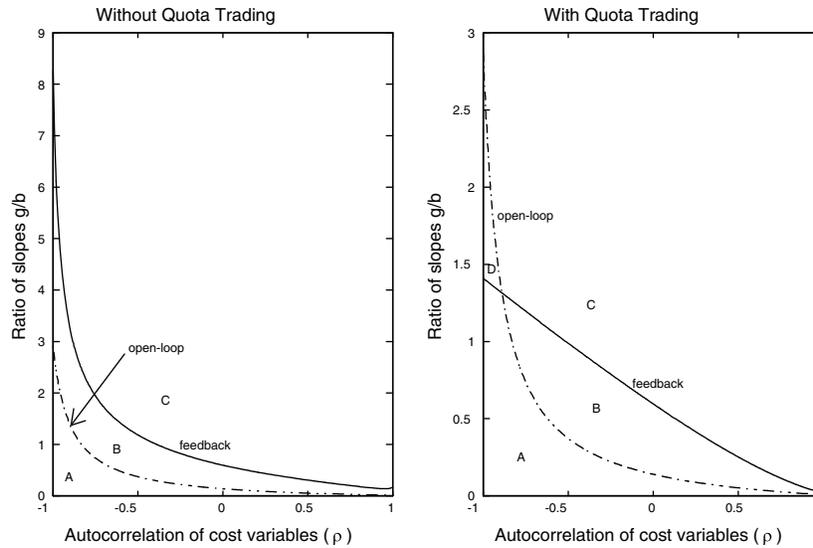


Figure 1. Critical ratios  $\frac{g}{b}$  without quota trading (left panel) and with quota trading (right panel). Solid curve identifies feedback, dashed curve identifies open-loop.

Table II. Feedback vs. open-loop optimal instrument choice

Optimal instrument choice	Open-loop	Feedback
Without quota trading		
A	tax	tax
B	quota	tax
C	quota	quota
With quota trading		
A	tax	tax
B	quota	tax
C	quota	quota
D	tax	quota

## 8. An Application to Global Warming

Current estimates of greenhouse gas-related abatement costs and environmental damages are largely speculative. This absence of (reasonably) precise estimates makes it particularly important to use a parsimonious and simple model of global warming, i.e., one for which the relation between economic assumptions and model parameters is transparent. If we had a good knowledge of the physics and economics of global warming, it would be worth constructing complex models. At this stage we know little more than

that there is a probable connection between greenhouse gases and global warming, and that the economic consequences of this relation may be important. The scientific uncertainty and disagreement makes the simplicity of the linear–quadratic model very attractive. With this model, we can easily see how policy conclusions depend on beliefs about the unknown parameters. For example, we can determine whether a particular conclusion would change if we increase our estimate of environmental damages by a factor of 10 or 100. Of course, since these experiments maintain the assumption of the linear–quadratic structure, they tell us nothing about whether the policy conclusions are sensitive to functional form.

A number of papers use the linear–quadratic structure, together with existing estimates of costs and benefits of greenhouse gas abatement, to examine particular policy issues. We briefly review these papers. Under the assumption of zero autocorrelation of cost shocks, Hoel and Karp (2002) show that taxes dominate quotas even if environmental damages associated with greenhouse gases are much more severe than is commonly believed. This conclusion holds under open-loop and feedback policies, with a period of commitment of anywhere from 1 to 10 years. Newell and Pizer (2003) find that taxes dominate quotas under open loop policies with positive autocorrelation. Hoel and Karp (2001) compare feedback taxes and quotas with multiplicative (rather than additive) and serially uncorrelated cost shocks. The multiplicative structure means that the Principle of Certainty Equivalence does not hold, and thus raises issues that are not present in the previous papers. Subsequent papers examine more complicated (feedback) models, where the regulator learns about environmental damages (Karp and Zhang 2002b) or where there is endogenous investment that reduces abatement costs (Karp and Zhang 2002a). Those papers review previous integrated assessment models that have been used to study greenhouse gas abatement.

We use a time period of one year. Many economic studies (Nordhaus 1994); (Kolstad 1996) use an annual decay rate of 0.0083 (a half-life of 83 years) for atmospheric concentrations of the primary greenhouse gas, CO<sub>2</sub>,

*Table III.* The critical ratio  $g/b$

$\rho$	Open-loop	Feedback and homogenous firms (no trade)	Feedback and heterogenous firms (with trade)
–0.99	1.9427	8.2154	1.3619
–0.5	0.1343	1.1344	0.9393
0	0.0470	0.5388	0.5388
0.5	0.0165	0.2470	0.1987
0.99	0.0011	0.0471	0.0043

implying  $\Delta = 0.9917$ . We set  $\beta = 0.9704$  (a continuous discount rate of 0.03). Using these values, Table III shows the critical values of the ratio  $\frac{g}{b}$  for five values of  $\rho$ , under three models: open-loop and feedback policies with either homogenous or heterogenous firms (i.e., without quota trade and with quota trade).<sup>9</sup> In interpreting this table, it is important to keep in mind the informational differences, summarized in Table I, between the three models.

Table III illustrates two important characteristics of the ranking. First, the criterion for ranking policies is sensitive to whether the regulator uses an open-loop or a feedback policy. For our purposes, the economically most relevant situation is where  $\rho \geq 0$ . Here, the critical ratio  $\frac{g}{b}$  differs by a factor of between 4 and 12, even when there is quota trade. (The difference is larger without quota trade.) A primary motivation for this paper is that correlated abatement cost shocks may be important in regulating stock pollutants, and that an open-loop policy is not reasonable in the presence of this correlation. (With an open-loop policy, the regulator does not use the potential information that arises as a consequence of the correlation.) The analytic results reported in sections 4–6 show that the qualitative difference between open-loop and feedback policies is minor. Table III shows that the quantitative difference is significant. For empirical purposes, the feedback model is an important improvement over the open-loop model.

Table III also shows that for  $|\rho| \approx 1$ , the ranking is very sensitive to whether the feedback quota policy enables the regulator to learn the lagged cost shock. With regard to the cost shock, neither policy has an informational advantage under either the open-loop structure or under the feedback structure with traded quotas. (With the open-loop structure, the regulator does not use the lagged cost shock for either the tax or the quota; for the feedback structure and traded quotas, the regulator uses the lagged cost shock for both the tax and the quota.) However, for large  $|\rho|$ , taxes have an important informational advantage over quotas in the feedback structure where quotas are not traded.

The survey in Hoel and Karp (2002) suggests a point estimate of  $\frac{g}{b} = 1.4062 \times 10^{-5}$  (based on the estimate that doubling carbon stocks causes a 5% reduction in Gross World Product (GWP) and that a 50% reduction in emissions causes a 1% reduction in GWP).<sup>10</sup> The estimated value of  $\frac{g}{b}$  is linearly related to the estimate of environmental damages. (A 10-fold increase in the estimated reduction in GWP associated with a doubling stocks leads to a ten-fold increase in the estimate of  $\frac{g}{b}$ .) This evidence suggests that in the case of greenhouse gases, the serial correlation of cost shocks does not overturn the preference for taxes. This conclusion does not depend on whether the regulator uses feedback policies and learns about the cost shock.

## 9. Summary

This paper provides a criteria for ranking taxes and quotas for the control of a stock pollutant in a linear–quadratic model. We extended previous results by including serially correlated abatement costs under either open-loop or feedback policies, with or without trading in emissions quotas.

Under a tax policy, or under a quota policy with efficient quota trading among polluting firms, the regulator learns about the industry's abatement cost schedule by observing the aggregate emission response. The feedback strategy, unlike the open-loop strategy, enables the regulator to use new information to adjust his policy level, leading to higher welfare.

With feedback policies, the ranking of taxes and non-traded quotas may be non-monotonic with respect to both the autocorrelation parameter and the discount factor. In contrast, the effects of the other parameters (the relative slopes of marginal costs and damages, and the decay rate) on the ranking are monotonic and are qualitatively the same under the open-loop and feedback assumptions. A large absolute value of autocorrelation increases the potential for learning, thus increasing the advantage of feedback taxes relative to both the non-traded quota and to open-loop taxes. When quota trading occurs, the feedback tax loses its informational advantage over feedback quotas, and more autocorrelated cost shocks favor the use of quotas.

Without quota trading, taxes will certainly be the right instrument choice in the feedback setting if taxes dominate quotas in the open-loop setting. However, this conclusion does not hold if firms can trade quotas. With tradeable permits, endogenous learning occurs under quotas. In this case, a regulator who is required to use an open-loop policy might want to use one instrument, where a regulator who is able to use a feedback policy would use the other instrument. Using estimates of greenhouse gas-related damages and abatement costs, we provide evidence that taxes dominate quotas for the control of greenhouse gases regardless of the opportunities for learning.

## Notes

1. There is a large literature that examines other aspects of the problem of choosing policies under asymmetric information. Important contributions to this literature include Dasgupta, Hammond and Maskin (1980), Kwerel (1977), Malcomson (1978), Robert and Spence (1976), Stavins (1996) and Watson and Ridker (1984). These papers are concerned with the problem of flow rather than stock externalities.  
Non-linear policies can achieve higher payoffs than linear taxes or quotas. Robert and Spence (1976) points out that a non-linear tax, with the marginal tax rate equals the marginal damage, achieves the first-best outcome in regulating a single firm and is superior to a quantity policy.

2. The effect of the length of a period is the same in the more general setting discussed in this paper (where costs are serially correlated). Details are available upon request.
3. The parameters satisfy  $f = -\frac{b}{2}x^2 +$  (terms involving  $\tilde{\theta}$ ). We hereafter ignore the effect of  $\tilde{\theta}$  on  $f$  since  $f$  has no effect on the regulator's control.
4. Costello and Karp (2004) study a dynamic model with flow pollution, in which the possibility that the quota is not binding enables the regulator to learn about the firm's cost. Brozovic, Sunding and Zilberman (2002) point out that even in the simplest static problem, the regulator's payoff might not be globally concave. In this case, a binding quota might be a local but not a global maximum.
5. Our model treats technology as fixed. In particular, firms do not invest in abatement capital. Therefore, firms solve a succession of static optimization problems. The exclusion of endogenous technical change clearly reduces the realism of our model, and therefore limits its policy relevance. However, any tractable model has to ignore some important features of the real world. In a separate paper, Karp and Zhang (2002a), we study the case where firms make investment decisions which affect their future abatement costs. That model requires the solution of a dynamic game, even though firms are non-strategic.
6. An earlier version of this paper explains a second reason for considering general priors. The variance of costs changes in a predictable manner, raising the possibility that the choice of policy instruments (as distinct from the choice of policy levels) might be time-inconsistent. Our earlier paper shows that this type of time-inconsistency does not occur.
7. This Principle states that in the linear-quadratic control problem with additive random variables, the optimal control rule does not depend on second moments of the random variable. Hoel and Karp (2001) examine the case, where the slope rather than the intercept of firms' marginal abatement cost is uncertain. In that case uncertainty is multiplicative, and the Principle of Certainty Equivalence does not hold.
8. If firms are heterogeneous and each firm receives the same allocation of quotas, but there is no trade in quotas, the expected single period benefit of emissions is given by  $f + (a + E_0\theta_t)x_t - \frac{b}{2}x_t^2$ . The difference between this expression with the right side of Equation (17) identifies the informational advantage of trade (the fact that with trade we have  $E_t\theta_t$  rather than  $E_0\theta_t$ ) and the allocative efficiency (the presence of the term  $\frac{1}{2b}\sigma_\epsilon^2$ ).
9. Recall that in the absence of trade there is no allocative inefficiency because (by assumption) firms are homogenous; the quota is always efficient. The reader interested in comparing taxes and inefficient quotas can carry out the calculations using the single period benefit function described in note 8. This calculation requires an additional parameter that measures the extent of firm heterogeneity, given by  $\sigma_\epsilon^2$ .
10. We cannot assess the plausibility of these estimates. A reader who thinks that the damage estimate understates actual damages by, for example, a factor of 10, should magnify the point estimate of  $\frac{g}{b}$  by a factor of 10. The lack of "reliable" estimates of costs and damages is, as we have emphasized, one of the main attractions of using a simple linear-quadratic model.

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## A. Appendix

### A.1. GENERAL SOLUTION FOR A LINEAR QUADRATIC DYNAMIC PROGRAMMING PROBLEM

We set up the dynamic programming equations in general matrix notation as:

$$\begin{aligned}
 J_t(X_t) = \text{Max}_{Y_t} & \left\{ U_0 + \frac{1}{2} X_t' Q X_t + Y_t' W X_t + M' X_t + U_1' Y_t - \frac{1}{2} Y_t' U_2 Y_t \right. \\
 & \left. + U_3 \text{Var}_t(\theta_t) + \beta E_t J_{t+1}(X_{t+1}) \right\}, \\
 \text{s.t.} \quad & X_{t+1} = A X_t + B Y_t + C \mu_t + D.
 \end{aligned} \tag{23}$$

The subscript  $t$  in  $J_t$  denotes the change in  $\text{Var}_t(\theta_t)$ .  $\text{Var}_t(\theta_t) = \sigma_0^2$ , when  $t = 0$ ; and  $\text{Var}_t(\theta_t) = \sigma_\mu^2$ , when  $t > 0$ .

$X_t$  is a  $n \times 1$  vector of state variables;  $Y_t$  is a  $m \times 1$  vector of control variables;  $\mu_t$  is a white noise. Dimensions for those coefficient matrices are:  $Q$  is  $n \times n$  symmetric;  $W$  is  $m \times n$ ;  $M$  is  $n \times 1$ ;  $U_0$  is  $1 \times 1$ ;  $U_1$  is  $m \times 1$ ;  $U_2$  is  $m \times m$ ;  $U_3$  is  $1 \times 1$ ;  $A$  is  $n \times n$ ;  $B$  is  $n \times m$ ;  $C$  is  $n \times 1$ ;  $D$  is  $n \times 1$ .

Given the quadratic value function  $J_t(X_t) = V_{0,t} + V_1'X_t + \frac{1}{2}X_t'V_2X_t$ , the first order condition with respect to  $Y_t$  is

$$WX_t + U_1 - U_2Y_t + \beta E_t[B'V_1 + B'V_2(AX_t + BY_t + C\mu_t + D)] = 0.$$

The optimal feedback control rule is

$$Y_t^* = (U_2 - \beta B'V_2B)^{-1}[U_1 + \beta B'(V_1 + V_2D) + (W + \beta B'V_2A)X_t], \quad (24)$$

a linear function of state variables. Substituting  $Y_t^*$  back into the dynamic programming equation and equating coefficients gives the algebraic Riccati matrix equation for  $V_2$ :

$$V_2 = Q + \beta A'V_2A + (W' + \beta A'V_2B)(U_2 - \beta B'V_2B)^{-1}(W + \beta B'V_2A). \quad (25)$$

$V_2$  is a  $n \times n$  symmetric negative-semidefinite matrix. After obtaining  $V_2$ , we can solve for the  $n \times 1$  coefficient matrix  $V_1$ :

$$\begin{aligned} V_1 &= \left[ I - \beta A' - \beta(W' + \beta A'V_2B)(U_2 - \beta B'V_2B)^{-1}B' \right]^{-1} \\ &\quad \times \left[ M + \beta A'V_2D + (W' + \beta A'V_2B)(U_2 - \beta B'V_2B)^{-1} \right. \\ &\quad \left. \times (U_1 + \beta B'V_2D) \right], \end{aligned} \quad (26)$$

and the constant term  $V_{0,t}$

$$\begin{aligned} V_{0,t} &= \eta_t \left( U_3 + \frac{\beta}{2}C'V_2C \right) + \frac{1}{1-\beta} \left\{ U_0 + \beta V_1'D + \frac{\beta}{2}D'V_2D \right. \\ &\quad \left. + \frac{1}{2}[U_1 + B'(V_1 + V_2D)]'(U_2 - \beta B'V_2B)^{-1}[U_1 + B'(V_1 + V_2D)] \right\} \end{aligned} \quad (27)$$

where  $\eta_t$  depends on the second moment of cost shocks:  $\eta_t = \sigma_0^2 + \frac{\beta}{1-\beta}\sigma_\mu^2$  when  $t = 0$ , and  $\eta_t = \frac{1}{1-\beta}\sigma_\mu^2$  when  $t > 0$ . The first moment of the cost shock affects both the value function and the optimal control, but the second moment affects only the constant term of the value function.

## A.2. FEEDBACK POLICY

First we write the law of motion for  $y_t \equiv E_t(\theta_t)$ . For the feedback quota policy, no new information becomes available over time, and

$$y_{t+1} = E_{t+1}\theta_{t+1} = E_0(E_{t+1}\theta_{t+1}) = E_0\theta_{t+1} = \rho^{t+1}\bar{\theta}_0 = \rho y_t.$$

Under the feedback tax policy, the regulator infers  $\theta_t$  by observing the firm's emissions, and

$$y_{t+1} = E_{t+1}\theta_{t+1} = E_{t+1}(\rho\theta_t + \mu_{t+1}) = \rho y_t = \begin{cases} \rho\theta_0 = \rho y_0 + \rho\mu_0, & t=0 \\ \rho(\rho\theta_{t-1} + \mu_t) = \rho y_t + \rho\mu_t, & t \geq 1 \end{cases} \quad (28)$$

with  $\mu_0 \equiv \theta_0 - \bar{\theta}_0$ . The distribution of cost shocks in the initial and subsequent periods have different variances:

$$Var_t(\theta_t) = \begin{cases} \sigma_0^2, & t=0 \\ Var_t(\mu_t) = \gamma^2, & t \geq 1. \end{cases} \quad (29)$$

Under both feedback taxes and quotas, the equation of motion for the state variable  $y_t \equiv E_t\theta_t$  is independent of the regulator's actions. However, there is a qualitative difference in  $y_t$  under taxes and quotas because of endogenous learning.

Under feedback tax policies, using Equation (5), the expected benefit in period  $t$ , is

$$E_t\{B[x_t^*(z_t, \theta_t), \theta_t]\} = f + (a + E_t\theta_t)z_t - \frac{b}{2}z_t^2 + \frac{1}{2b}\text{Var}_t(\theta_t).$$

The regulator's value function under taxes,  $T^{\text{FB}}(S_t, y_t)$ , solves the dynamic programming equation

$$\begin{aligned} T^{\text{FB}}(S_t, y_t) &= \text{Max}_{z_t} \{E_t B[x_t^*(z_t, \theta_t), \theta_t] - D(S_t) + \beta E_t T^{\text{FB}}(S_{t+1}, y_{t+1})\} \\ &= \text{Max}_{z_t} \left\{ f + (a + y_t)z_t - \frac{b}{2}z_t^2 + \frac{1}{2b}\text{Var}_t(\theta_t) \right. \\ &\quad \left. - \left( cS_t + \frac{g}{2}S_t^2 \right) + \beta E_t T^{\text{FB}}(S_{t+1}, y_{t+1}) \right\} \\ \text{s.t.} \quad S_{t+1} &= \Delta S_t + z_t + \frac{1}{b}\mu_t \\ y_{t+1} &= \rho y_t + \rho\mu_t. \end{aligned}$$

Under quotas, the regulator's value function,  $Q(S_t, y_t)$ , solves the dynamic programming equation

$$\begin{aligned} Q(S_t, y_t) &= \text{Max}_{x_t} \{E_t B(x_t, \theta_t) - D(S_t) + \beta E_t Q(S_{t+1}, y_{t+1})\} \\ &= \text{Max}_{x_t} \left\{ f + (a + y_t)x_t - \frac{b}{2}x_t^2 - \left( cS_t + \frac{g}{2}S_t^2 \right) + \beta Q(S_{t+1}, y_{t+1}) \right\} \\ \text{s.t.} \quad S_{t+1} &= \Delta S_t + x_t \\ y_{t+1} &= \rho y_t. \end{aligned}$$

**Solving the Dynamic Programming Equations.** Using a two-dimensional state vector  $X_t = (S_t, y_t)'$ , the dynamic programming equations (DPE) can be written in general matrix notations as in the previous subsection. The control variable is  $z_t$  under taxes, and  $x_t$  under quotas. Those coefficients are

$$\begin{aligned} Q &= \begin{pmatrix} -g & 0 \\ 0 & 0 \end{pmatrix}, \quad W = (0 \ 1), \quad M = \begin{pmatrix} -c \\ 0 \end{pmatrix}, \\ A &= \begin{pmatrix} \Delta & 0 \\ 0 & \rho \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad C = \begin{cases} C_{\text{tax}} = (\frac{1}{b}, \rho)' & (\text{tax}) \\ C_{\text{quota}} = (0, 0)' & (\text{quota}) \end{cases}, \quad D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \\ U_0 &= f, \quad U_1 = a, \quad U_2 = b, \quad U_3 = \begin{cases} \frac{1}{2b} & (\text{tax}) \\ 0 & (\text{quota}) \end{cases}. \end{aligned}$$

We see the DPE for taxes and quotas only differ in  $U_3$  and  $C$ . Thus at any time with the same state vector  $X_t$ , the payoff difference between feedback taxes and quotas is

$$\begin{aligned} T^{\text{FD}}(X_t) - Q(X_t) &= V_{0,t}^{\text{tax}} - V_0^{\text{quota}} = \eta_t \left( U_3 + \frac{\beta}{2} C'_{\text{tax}} V_2 C_{\text{tax}} \right) \\ &= \begin{cases} \frac{1}{2b} \left( \sigma_0^2 + \frac{\gamma^2 \beta}{1-\beta} \right) (1 + b\beta C'_{\text{tax}} V_2 C_{\text{tax}}), & t = 0; \\ \frac{\gamma^2}{2b(1-\beta)} (1 + b\beta C'_{\text{tax}} V_2 C_{\text{tax}}), & t \geq 1. \end{cases} \end{aligned}$$

With  $C_{\text{tax}} = (\frac{1}{b}, \rho)'$ , expanding  $C'_{\text{tax}} V_2 C_{\text{tax}}$  leads to

$$T^{\text{FD}}(X_0) - Q(X_0) = \frac{1}{2b} \left( \sigma_0^2 + \frac{\gamma^2 \beta}{1 - \beta} \right) \left( 1 + \frac{\beta}{b} V_{11} + 2\beta\rho V_{12} + \beta b \rho^2 V_{22} \right). \quad (30)$$

where  $V_{11}$ ,  $V_{12}$ ,  $V_{22}$  are elements of  $V_2$  which can be solved for from Equation (25). Substituting  $Q$ ,  $A$ ,  $B$ ,  $W$  into Equation (25) and equating elements of matrices at different side of the “=” sign, we get the following system of equations for  $V_{11}$ ,  $V_{12}$ ,  $V_{22}$ .

$$V_{11} = -g + \beta\Delta^2 V_{11} + (b - \beta V_{11})^{-1} \beta^2 \Delta^2 V_{11}^2; \quad (31)$$

$$V_{12} = \rho\beta\Delta V_{12} + (b - \beta V_{11})^{-1} \beta\Delta(1 + \beta\rho V_{12}) V_{11}; \quad (32)$$

$$V_{22} = \rho^2 \beta V_{22} + (b - \beta V_{11})^{-1} (1 + \beta\rho V_{12})^2. \quad (33)$$

Given that the value function is quadratic in  $S_t$  and bounded above,  $V_{11} < 0$ .  $V_{22} < 0$  is not required since the equation of motion for  $y_t$  is not affected by the regulator's actions. Obtaining the negative root of Equation (31) and then solving the linear Equations (32) and (33) recursively lead to the expression for  $V_{11}$ ,  $V_{12}$ , and  $V_{22}$ :

$$\begin{aligned} V_{11} &= \frac{-(b\beta\Delta^2 + \beta g - b) - \sqrt{(b\beta\Delta^2 + \beta g - b)^2 + 4\beta g b}}{2\beta} < 0; \\ V_{12} &= \frac{\beta\Delta V_{11}}{b(1 - \rho\beta\Delta) - \beta V_{11}} < 0; \\ V_{22} &= \frac{\left(1 - \frac{\beta}{b} V_{11}\right)(1 - \rho\beta\Delta)^2}{b(1 - \rho^2\beta)\left(1 - \rho\beta\Delta - \frac{\beta}{b} V_{11}\right)^2} > 0. \end{aligned}$$

Similarly we obtain elements  $(v_1, v_2)$  of matrix  $V_1$  from Equation (29) by substituting in  $V_2$ ,  $M$ ,  $A$ ,  $B$ ,  $W$ :

$$\begin{aligned} v_1 &= \frac{-c + (b - \beta V_{11})^{-1} a\beta\Delta V_{11}}{1 - \beta\Delta - (b - \beta V_{11})^{-1} \beta^2 \Delta V_{11}} < 0; \\ v_2 &= \frac{1}{1 - \rho\beta} \times \frac{a(1 - \beta\Delta) - \beta c}{(b - \beta V_{11}) - b\beta\Delta} \times \frac{(1 - \rho\beta\Delta)(b - \beta V_{11})}{b - \beta V_{11} - b\rho\beta\Delta}. \end{aligned}$$

After obtaining  $V_2$  and  $V_1$ , we can get the constant term  $V_{0,r}$ . The constant term under quotas is

$$V_0^{\text{quota}} = \frac{1}{1 - \beta} \left\{ f + \frac{1}{2} \times \frac{[a(1 - \beta\Delta) - \beta c]^2}{(b - \beta V_{11}) - 2b\beta\Delta + (b - \beta V_{11})^{-1} b^2 \beta^2 \Delta^2} \right\}.$$

We obtain the optimal control rule (Equation (8) in the text) by substituting matrices  $V_1$ ,  $V_2$ ,  $A$ ,  $B$ ,  $W$  into Equation (24); and the payoff difference (Equation (11) in the text) by substituting  $V_{11}$ ,  $V_{12}$ ,  $V_{22}$  into Equation (30).

The procedure for solving the DPE with quota trading is similar. The only necessary change for the coefficient matrices is to replace  $C_{\text{quota}}$  by  $C_{q+\text{Trade}} = (0, \rho)'$ .

**Proof.** (Remark 2) Feedback emission taxes are preferred to quotas if and only if

$$(1 - \rho\beta\Delta)^2 - \frac{\beta^2}{b^2} V_{11}^2 (1 - \rho^2\beta) \geq 0. \quad (34)$$

Given  $V_{11} < 0$ , Equation (34) is equivalent to

$$\begin{aligned} \frac{V_{11}}{b} &\geq -\frac{1 - \rho\beta\Delta}{\beta\sqrt{1 - \rho^2\beta}} \\ \Leftrightarrow \frac{(\beta\Delta^2 + \beta_b^g - 1) + \sqrt{(\beta\Delta^2 + \beta_b^g - 1)^2 + 4\beta_b^g}}{2} &\leq \frac{1 - \rho\beta\Delta}{\sqrt{1 - \rho^2\beta}}. \end{aligned} \quad (35)$$

It is easy to see that the left hand side of the above inequality is monotonically increasing in  $g$ ,  $\Delta$ ,  $\beta$ , and monotonically decreasing in  $b$ . The right hand side of the above inequality is independent of  $g$  and  $b$ , and monotonically decreasing in  $\Delta$ . Hence, higher  $g$  and  $\Delta$  and lower  $b$  all make quotas more attractive, relative to taxes.

**Proof.** (Proposition 2) Equation (34) can be transformed to

$$f(\rho) \equiv \rho^2\beta^2 \left( \Delta^2 + \frac{\beta}{b^2} V_{11}^2 \right) - 2\rho\beta\Delta + 1 - \frac{\beta^2}{b^2} V_{11}^2 \geq 0. \quad (36)$$

$f(\rho)$  is a convex quadratic function in  $\rho$ , and symmetrical with respect to

$$\rho_0 = \frac{\Delta}{\beta \left( \Delta^2 + \frac{\beta}{b^2} V_{11}^2 \right)} \geq 0.$$

**Proof.** (Corollary 1) The inequality in Equation (36) will always hold if

$$\begin{aligned} (2\beta\Delta)^2 - 4\beta^2 \left( \Delta^2 + \frac{\beta}{b^2} V_{11}^2 \right) \left( 1 - \frac{\beta^2}{b^2} V_{11}^2 \right) &< 0 \\ \Leftrightarrow 4\beta^3 \frac{V_{11}^2}{b^2} \left( -1 + \beta\Delta^2 + \beta^2 \frac{V_{11}^2}{b^2} \right) &< 0 \\ \Leftrightarrow 4\beta^3 \frac{V_{11}^2}{b^2} \left( 1 - \frac{\beta}{b} V_{11} \right) \left( \beta\Delta^2 + \beta_b^g - 1 \right) &< 0 \\ \Leftrightarrow \frac{1 - \beta\Delta^2}{\beta} > \frac{g}{b}. \end{aligned}$$

Otherwise, there will be two real roots

$$\rho_{1,2} = \frac{\Delta \pm \sqrt{\frac{\beta}{b^2} V_{11}^2 \left( 1 - \frac{\beta}{b} V_{11} \right) \left( \beta\Delta^2 + \beta_b^g - 1 \right)}}{\beta \left( \Delta^2 + \frac{\beta}{b^2} V_{11}^2 \right)}$$

satisfying the equality, and the turning point

$$0 \leq \rho_0 = \frac{\Delta}{\beta \left( \Delta^2 + \frac{\beta}{b^2} V_{11}^2 \right)} \leq \Delta.$$

**Proof.** (Proposition 3) The effect of  $\beta$  on the right hand side of (35) is given by

$$\frac{\partial\left(\frac{1-\rho\beta\Delta}{\sqrt{1-\rho^2\beta}}\right)}{\partial\beta} = \frac{\rho}{2} \times \frac{\rho^2\beta\Delta - 2\Delta + \rho}{(1-\rho^2\beta)^{\frac{3}{2}}}$$

which is non-positive if  $\beta \leq \frac{2\Delta-\rho}{\rho^2\Delta}$  when  $\rho \geq 0$  or  $\beta > \frac{2\Delta-\rho}{\rho^2\Delta}$  when  $\rho < 0$ . Otherwise, higher  $\beta$  will increase both sides of the inequality (35) and make the effect of higher discount factor on the ranking more complicated. We obtain the left hand side of Equation (13) in the text by taking partial derivative of the left hand side of Equation (35) with respect to  $\beta$ . The condition  $\beta \leq \frac{2\Delta-\rho}{\rho^2\Delta}$  is satisfied if

$$0 \leq \rho \leq \Delta \implies 2\Delta \geq 2\rho \geq \rho + \rho^2\Delta \implies \frac{2\Delta - \rho}{\rho^2\Delta} \geq 1 \geq \beta.$$

**Proof.** (Remark 3) With quota trading, feedback taxes are preferred to quotas if and only if

$$1 - \rho\beta\Delta + \frac{\beta}{b}V_{11} \geq 0$$

$$\iff 1 - \rho\beta\Delta - \frac{(\beta\Delta^2 + \beta_b^g - 1) + \sqrt{(\beta\Delta^2 + \beta_b^g - 1)^2 + 4\beta_b^g}}{2} \geq 0. \quad (37)$$

It is easy to see that the left hand side of the above inequality is monotonically decreasing in  $g$ ,  $\Delta$ ,  $\beta$ , and monotonically increasing in  $b$ . Hence, higher  $g$ ,  $\beta$  and  $\Delta$  and lower  $b$  all make quotas more attractive relative to taxes, qualitatively the same as under open-loop policies.

**Proof.** (Proposition 4) It is straightforward from Equation (37).

### A.3. COMPARISON OF RANKING

**Proof.** (Proposition 5)

$$\frac{\partial\left(\frac{g}{b}\right)^{*OL}}{\partial\rho} = \frac{-2\beta\Delta}{(1+\rho\beta\Delta)^2} \left(\frac{1}{\beta} - \Delta^2\right) < 0.$$

$$\frac{\partial\left(\frac{g}{b}\right)^{*FB}}{\partial\rho} = \frac{2(\rho - \Delta)}{(1-\rho^2\beta)^{\frac{3}{2}}} \times \frac{\sqrt{(\beta\Delta^2 + \beta_b^g - 1)^2 + 4\beta_b^g}}{(\beta\Delta^2 + \beta_b^g + 1) + \sqrt{(\beta\Delta^2 + \beta_b^g - 1)^2 + 4\beta_b^g}}$$

with the sign depending on the sign of  $\rho - \Delta$ .

$$\frac{\partial\left(\frac{g}{b}\right)^{*FB+Trade}}{\partial\rho} = \frac{\Delta[\beta\Delta - (2 - \rho\beta\Delta)^2]}{(2 - \rho\beta\Delta)^2} < 0$$

since in general  $0 < \beta\Delta < 1 < 2 - \rho\beta\Delta$ .

**Proof.** (Proposition 6)

$$\begin{aligned}
\left(\frac{g}{b}\right)^{*FB} &\geq \left(\frac{g}{b}\right)^{*OL} \\
\iff \frac{(1 - \rho\beta\Delta) + (1 - \beta\Delta^2)\sqrt{1 - \rho^2\beta}}{(1 - \rho^2\beta) + (1 - \rho\beta\Delta)\sqrt{1 - \rho^2\beta}} &\geq \frac{1 - \beta\Delta^2}{1 + \rho\beta\Delta} \\
\iff (1 - \rho^2\beta^2\Delta^2) - (1 - \rho^2\beta)(1 - \beta\Delta^2) &\geq -2\rho\beta\Delta(1 - \beta\Delta^2)\sqrt{1 - \rho^2\beta} \\
\iff \rho^2\beta(1 - \beta\Delta^2) + \beta\Delta^2(1 - \rho^2\beta) &\geq -2\rho\beta\Delta(1 - \beta\Delta^2)\sqrt{1 - \rho^2\beta}
\end{aligned}$$

which holds in general.

$$\begin{aligned}
\left(\frac{g}{b}\right)^{*FB+Trade} &\geq \left(\frac{g}{b}\right)^{*OL+Trade} \\
\iff \rho^2\beta\Delta - 2(1 - \beta\Delta^2)\rho - \Delta &\leq 0 \\
\iff \tilde{\rho} \leq \rho \leq \bar{\rho}
\end{aligned}$$

with

$$\begin{aligned}
-1 \leq \tilde{\rho} &\equiv \frac{1 - \beta\Delta^2 - \sqrt{(1 - \beta\Delta^2)^2 + \beta\Delta^2}}{\beta\Delta} < 0, \\
\bar{\rho} &\equiv \frac{1 - \beta\Delta^2 + \sqrt{(1 - \beta\Delta^2)^2 + \beta\Delta^2}}{\beta\Delta} \geq 1.
\end{aligned}$$