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**ESTIMATING COKE AND PEPSI'S PRICE  
AND ADVERTISING STRATEGIES**

(formerly Estimating Firms' Mixed Price and Advertising Strategies: Coke and Pepsi)

by

Amos Golan, Larry S. Karp and Jeffrey M. Perloff

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**California Agricultural Experiment Station  
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## **Estimating Coke and Pepsi's Price and Advertising Strategies**

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**Table of Contents**

1. INTRODUCTION	1
2. OLIGOPOLY GAME	3
2.1 Strategies	4
2.2 Econometric Implications	5
3. GENERALIZED-MAXIMUM-ENTROPY ESTIMATION APPROACH	7
3.1 Background: Classical Maximum Entropy Formulation	7
3.2 Incorporating Sample Information	9
3.3 Incorporating the Non-Sample (Game-Theoretic) Information	11
3.4 Properties of the Estimators and Normalized Entropy	14
4. COLAS	15
4.1 Cola Estimates	17
4.2 Tests	20
4.3 Lerner Measures	22
4.4 Effects of the Exogenous Variables	23
5. CONCLUSIONS	24
REFERENCES	26
Appendix 1: The GME-Nash Estimator	30

## **Abstract**

A semi-parametric, information-based estimator is used to estimate strategies in prices and advertising for Coca-Cola and Pepsi-Cola. Separate strategies for each firm are estimated with and without restrictions from game theory. These information/entropy estimators are consistent and efficient. These estimates are used to test theories about the strategies of firms and to see how changes in incomes or factor prices affect these strategies.

KEYWORDS: strategies, noncooperative games, oligopoly, generalized maximum entropy, beverages

JEL: C13, C35, C72, L13, L66

## 1. INTRODUCTION

This paper presents two methods for estimating oligopoly strategies. The first method allows strategies to depend on variables that affect demand and cost. The second method adds restrictions from game theory. We use these methods to estimate the pricing and advertising strategies of Coca-Cola and Pepsi-Cola.

Unlike most previous empirical studies of oligopoly behavior, we do not assume that firms use a single pure strategy nor do we make the sort of ad hoc assumptions used in conjectural variations models.<sup>1</sup> Both our approaches recognize that firms may use either pure or mixed (perhaps more accurately, distributional) strategies.

In our application to Coca-Cola and Pepsi-Cola, we assume that the firms' decision variables are prices and advertising. We divide each firm's continuous price-advertising action space into a grid over prices and advertising. Then we estimate the vector of probabilities — the mixed or pure strategies — that a firm chooses an action (a rectangle in the price-advertising grid). We use our estimates to calculate the Lerner index of market structure and examine how changes in exogenous variables affect strategies.

The main advantages of using our method are that we can flexibly estimate firms' strategies subject to restrictions implied by game theory and test hypotheses based on these estimated strategies. The restrictions we impose are consistent with a variety of assumptions regarding the information that firms have when making their decisions and with either pure or mixed strategies.

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<sup>1</sup> Bresnahan (1989) and Perloff (1992) survey conjectural variations and other structural and reduced-form "new empirical industrial organization" studies.

For example, suppose that a firm's marginal cost in a period is a random variable observed by the firm but not by the econometrician. Given the realization of marginal cost, the firm chooses either a pure or a mixed strategy, which results in an action: a price-advertising pair. The econometrician observes only the firm's action and not the marginal cost. As a consequence, the econometrician cannot distinguish between pure or mixed strategies. If both firms in a market use pure strategies and each observes its rival's marginal cost, each firm can anticipate its rival's action in each period. Alternatively, firms might use pure strategies and know the distribution but not the realization of their rival's cost. Due to the randomness of the marginal cost, it appears to both the rival and the econometrician that a firm is using a mixed strategy. The equilibrium depends on whether firms' private information is correlated.

All of these possibilities — firms have only public information, firms observe each other's private information but the econometrician does not, or a firm only knows that its private information is correlated or uncorrelated with its rival's — lead to restrictions of the same form. For expositional simplicity, we concentrate on the situation where firms have private, uncorrelated information about their own - but not their rival's - marginal costs (or some other payoff-relevant variable). Firms choose either pure or mixed strategies.

There have been few previous studies that estimated mixed or pure strategies based on a game-theoretic model. These studies (Bjorn and Vuong 1985, Bresnahan and Reiss 1991, and Kooreman 1994) involve discrete action spaces. For example, Bjorn and Vuong and Kooreman estimate mixed strategies in a game involving spouses' joint labor market participation decisions using a maximum likelihood (ML) technique. Our approach differs

from these studies in three important ways. First, they assume that there is no exogenous uncertainty. Second, they allow each agent a choice of only two possible actions. Third, in order to use a ML approach, they assume a specific error distribution and likelihood function. Despite the limited number of actions, their ML estimation problems are complex.

Our problem requires that we include a large number of possible actions in order to analyze oligopoly behavior and allow for mixed strategies. Doing so using a ML approach would be difficult if not impossible. Instead, we use a generalized-maximum-entropy (GME) estimator. An important advantage of our GME estimator is its computational simplicity. Using GME, we can estimate a model with a large number of possible actions and impose inequality and equality restrictions implied by the equilibrium conditions of the game. In addition to this practical advantage, the GME estimator does not require the same strong, explicit distributional assumptions used in standard ML approaches. A special case of our GME estimator is identical to the ML multinomial logit estimator (when the ML multinomial logit has a unique solution), which indicates that those restrictions to the GME model is identical to the distributional assumption of the standard approach.

In the next section, we present a game-theoretic model of firms' behavior. In the third section, we describe a GME approach to estimating this game. The fourth section contains estimates of the strategies of Coke and Pepsi. In the final section, we discuss our results and possible extensions.

## **2. OLIGOPOLY GAME**

Our objective is to determine the strategies of oligopolistic firms using time-series data on prices, advertising, quantities, and variables that affect cost or demand, such as input

prices or seasonal dummies. We assume that two firms,  $i$  and  $j$ , play a static game in each period of the sample.

The econometrician observes payoff-relevant public information, such as demand and cost shifters,  $\underline{z}$ , but does not observe private information known only to the firms. Firm  $i$  (and possibly Firm  $j$ ), but not the econometrician, observes Firm  $i$ 's marginal cost or some other payoff-relevant random variable  $\varepsilon^i(t)$  in period  $t = 1, \dots, T$ . Where possible, we suppress the time variable  $t$  for notational simplicity. The set of  $K$  possible realizations,  $\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_K\}$ , is the same every period for both firms. The distributions are constant over time but may differ across firms. The firms, but not the econometrician, know these distributions. To simplify the description of the problem, we assume that  $\varepsilon^i$  and  $\varepsilon^j$  are private, uncorrelated information.

### 2.1 Strategies

The set of  $n$  possible actions (price-advertising pairs) for Firm  $i$  is  $\{x_1^i, x_2^i, \dots, x_n^i\}$ . We now describe the problem where the random state of nature is private information and uncorrelated across firms.

The profit of Firm  $i$  in a particular time period is  $\pi_{rsk}^i(\underline{z}) = \pi^i(x_r^i, x_s^j, \varepsilon_k^i, \underline{z})$ , where  $r$  is the action chosen by Firm  $i$  and  $s$  is the action chosen by Firm  $j$ . In state  $k$ , Firm  $i$ 's strategy is  $\underline{\alpha}_k^i(\underline{z}) = (\alpha_{k1}^i(\underline{z}), \alpha_{k2}^i(\underline{z}), \dots, \alpha_{kn}^i(\underline{z}))$ , where  $\alpha_{kr}^i(\underline{z})$  is the probability that Firm  $i$  chooses action  $x_r$  given private information  $\varepsilon_k^i$  and public information  $\underline{z}$ . If Firm  $i$  uses a pure strategy,  $\alpha_{kr}^i(\underline{z})$  is one for a particular  $r$  and zero otherwise.

Firm  $j$  does not observe Firm  $i$ 's private information, so it does not know the conditional probability  $\alpha_{kr}^i(\underline{z})$ . Firm  $j$  knows, however, the distribution of Firm  $i$ 's private informa-

tion. The Nash assumption is that Firm  $j$  knows the unconditional probability of Firm  $i$  using action  $r$ . This probability is the expectation over Firm  $i$ 's private information:  $\alpha_r^i(\underline{z}) = E_k \alpha_{kr}^i(\underline{z})$ , where  $E_k$  is the expectations operator. Similarly Firm  $i$  knows the unconditional probability  $\alpha_s^j(\underline{z})$  of Firm  $j$ .

In state  $k$ , Firm  $i$  chooses  $\alpha_k(\underline{z})$  to maximize expected profits,  $\sum_s \alpha_s^j(\underline{z}) \pi_{rsk}(\underline{z})$ , where the expectation is taken over its rival's actions. If  $Y_k^i(\underline{z})$  is Firm  $i$ 's maximum expected profits given  $\epsilon_k^i$  and  $\underline{z}$ , then Firm  $i$ 's expected loss from using action  $x_r$  is

$$(2.1) \quad L_{rk}^i(\underline{z}) \equiv \sum_s \alpha_s^j(\underline{z}) \pi_{rsk}^i(\underline{z}) - Y_k^i(\underline{z}) \leq 0,$$

which is non-positive. If it is optimal for Firm  $i$  to use action  $r$  with positive probability, the expected loss of using that action must be 0. Hence, optimality requires that

$$(2.2) \quad L_{rk}^i(\underline{z}) \alpha_{rk}^i(\underline{z}) = 0.$$

The equilibrium to this game may not be unique. Our estimation method selects the pure or mixed strategy equilibrium that is most consistent with the data.

## 2.2 *Econometric Implications*

Our objective is to estimate the firms' strategies subject to the constraints implied by optimization, Equations 2.1 and 2.2. We cannot use these constraints directly, however, because they involve private information  $\epsilon_k^i$ . By taking expectations, we eliminate these unobserved variables and obtain usable restrictions.

We define  $Y^i(\underline{z}) \equiv E_k Y_k^i(\underline{z})$  and  $\pi_{rs}^i(\underline{z}) \equiv E_k \pi_{rsk}^i(\underline{z})$ . Taking expectations with respect to  $k$  of Equations 2.1 and 2.2 and using the previous definitions, we obtain

$$(2.3) \quad \sum_s \alpha_s^j(\underline{z}) \pi_{rs}^i(\underline{z}) - Y^i(\underline{z}) \leq 0,$$

$$(2.4) \quad \left( \sum_s \alpha_s^j(\underline{z}) \pi_{rs}^i(\underline{z}) - Y^i(\underline{z}) \right) \alpha_r^i(\underline{z}) + \delta_r^i(\underline{z}) = 0,$$

where  $\delta_r^i \equiv \text{cov}(L_{rk}^i, \alpha_{rk}^i) \geq 0$ . For each Firm  $i = 1, 2$ , we can estimate the unobservable strategies  $\underline{\alpha}^i(\underline{z})$  subject to the conditions implied by Firm  $i$ 's optimization problem, Equations 2.3 and 2.4.<sup>2</sup>

Firms may use approximately optimal decisions due to bounded rationality, or there may be measurement error. Therefore, we treat Equation 2.4 as a stochastic restriction and include additive errors in estimation. Equation 2.4, however, already has an additive function,  $\delta(\underline{z})$ , which we cannot distinguish from the additive error in 2.4. Thus,  $\delta(\underline{z})$  is the only "error term" we include in this equation.

If we tried to estimate this model (Equations 2.3 - 2.4) using traditional techniques, we would run into two problems. First, imposing the various equality and inequality restrictions

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<sup>2</sup> If  $\varepsilon^i$  and  $\varepsilon^j$  are correlated or observed by both firms, the restrictions are slightly more complicated. If information is correlated, it would be reasonable to suppose that Firm  $i$ 's beliefs about  $j$ 's actions depend on the realization of  $\varepsilon^i$ , so that  $\alpha_s^j$  is replaced by  $\alpha_{ks}^j$ . If information is observed by both firms, Firm  $i$ 's beliefs would also be conditioned on the realization of  $\varepsilon^j$ . In both cases, we can take expectations with respect to the private information and obtain equations analogous to 2.3 and 2.4. However, with either generalization, we would have an additional additive term in 2.3, say  $\theta$ , and the definition of  $\delta$  would be changed. The signs of both  $\theta$  and  $\delta$  would be indeterminate. In our empirical application to the cola market, all the estimated  $\delta$  are positive, which is consistent with the model in the text where  $\varepsilon^i$  and  $\varepsilon^j$  are uncorrelated.

from our game-theoretic model would be very difficult if not impossible with standard techniques. Second, as the problem is ill posed in small samples (there may be more parameters than observations), we would have to impose additional assumptions to make the problem well posed. To avoid these and other estimation and inference problems, we propose an alternative approach.

### **3. GENERALIZED-MAXIMUM-ENTROPY ESTIMATION APPROACH**

We use generalized maximum entropy (GME) to estimate the firms' strategies. In this section, we start by briefly describing the traditional maximum entropy (ME) estimation procedure. Then, we present the GME formulation as a method of recovering information from the data consistent with our game. Our GME method is closely related to the GME multinomial choice approach in Golan, Judge, and Perloff (1996 — henceforth GJP). Unlike ML estimators, the GME approach does not require explicit distributional assumptions, performs well with small samples, and can incorporate inequality restrictions.

#### *3.1 Background: Classical Maximum Entropy Formulation*

The traditional entropy formulation is described in Shannon (1948), Jaynes (1957a; 1957b), Kullback (1959), Gokhale and Kullback (1978), Levine (1980), Jaynes (1984), Shore and Johnson (1980), Denzau, Gibbons, and Greenberg (1989), Skilling (1989), Csiszár (1991), Soofi (1992, 1994) and Golan, Judge, and Miller (1996). In this approach, Shannon's (1948) entropy is used to measure the uncertainty (state of knowledge) we have about the occurrence of a collection of events. Letting  $x$  be a random variable with possible outcomes  $x_s$ ,  $s = 1, 2,$

...,  $n$ , with probabilities  $\alpha_s$  such that  $\sum_s \alpha_s = 1$ , Shannon (1948) defined the *entropy* of the distribution  $\underline{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n)'$ , as

$$(3.1) \quad H \equiv -\sum_s \alpha_s \ln \alpha_s,$$

where  $0 \ln 0 \equiv 0$ . The function  $H$ , which Shannon interprets as a measure of the uncertainty in the mind of someone about to receive a message, reaches a maximum when  $\alpha_1 = \alpha_2 = \dots = \alpha_n = 1/n$ . To recover the unknown probabilities  $\underline{\alpha}$ , Jaynes (1957a; 1957b) proposed maximizing entropy, subject to available sample-moment information and adding up constraints on the probabilities.

The frequency that maximizes entropy is an intuitively reasonable estimate of the true distribution when we lack any other information. If we have information from the experiment, such as the sample moments, or non-sample information about the random variable, such as restrictions from economic theory, we want to alter our "intuitively reasonable" estimate. The method of Maximum Entropy proceeds by choosing the distribution that maximizes entropy, subject to the sample and non-sample information.

In our game, the firms' price-advertising decisions are the random variables that correspond to  $\underline{x}$  in the previous example. We want to estimate the firms' strategies, which are their probability distributions over their actions. The next two subsections explain how we incorporate sample and non-sample (theory) information. In our application, the sample information for cola manufacturers consists of time series of price-advertising pairs for each firm, quantities sold, and time series of exogenous variables that affect demand (a seasonal

dummy and income) and cost (an interest rate, a wage rate, and the price of sugar). The game-theoretic restrictions, Equations 2.3 - 2.4, contain all the non-sample information.

### 3.2 Incorporating Sample Information

We incorporate the sample information into our GME estimator of the strategies,  $\underline{\alpha}^i$ , by maximizing the entropy of  $\underline{\alpha}^i$  subject to the moment or consistency conditions that contain sample information. We can use either of two approaches (as GJP shows). If we require that the moment restrictions hold exactly, we derive a ME estimator, which is identical to the ML multinomial logit estimator (when the ML estimate is unique). If we view the moment conditions as stochastic restrictions, we obtain a GME estimator, which is a generalization of the multinomial logit. With either the ME or GME approaches, we obtain estimates of the probabilities  $\underline{\alpha}^i$  as a function of the public information,  $\underline{z}$ .

In our problem, there are  $n$  actions, which are price-advertising pairs. The variable  $y_{tr}^i$  equals one in period  $t$  if action  $r$  is observed and zero otherwise. The variable  $y_{tr}^i$  is a function of the public information:

$$(3.2) \quad y_{tr}^i = G(\underline{z}_t' \underline{\zeta}_r^i) + e_{tr}^i = \alpha_{tr}^i + e_{tr}^i,$$

for  $i = 1, 2$ , where  $y_{tr}^i$  and  $\underline{z}_t$  are observed and  $\alpha_{tr}^i$ ,  $e_{tr}^i$ , and  $\underline{\zeta}_r^i$  are unknown parameters to be estimated.

By eliminating  $e_{tr}^i$  from Equation 3.2 and assuming that  $G(\cdot)$  is a known cdf such as the logistic or the normal, we can estimate this model using maximum likelihood multinomial logit or probit. To avoid having to assume a specific cdf, we follow GJP and relate the set of covariates  $\underline{z}_t$  to the data  $y_{tr}^i$  and the unknown  $\alpha_{tr}^i$  and  $e_{tr}^i$ . We multiply Equation (3.2) by each

covariate variable,  $z_{tl}$ , and sum over observations to obtain the stochastic sample-moment (data consistency) restrictions:

$$(3.3) \quad \sum_t y_{tr}^i z_{tl} = \sum_t \alpha_{tr}^i z_{tl} + \sum_t e_{tr}^i z_{tl},$$

for  $l = 1, \dots, L$ , which is the number of covariates in  $\underline{z}$ , and  $r = 1, \dots, n$ . We obtain the basic GME estimator by maximizing the sum of the entropy corresponding to the strategy probabilities,  $\underline{\alpha}^i$ , and the entropy from the noise,  $\underline{e}^i$ , subject to that data consistency condition (3.3).

As we discussed in Section 3.1, the arguments of the Shannon's entropy measures must be probabilities. The elements of  $\underline{\alpha}^i$  are probabilities, but the elements of  $\underline{e}^i$  range over the interval  $[-1, 1]$ . To determine the entropy of  $\underline{e}^i$ , we reparameterize its elements using probabilities. We start by choosing a set of discrete points, called the support space,  $\underline{v}^i = [v_1^i, v_2^i, \dots, v_M^i]'$  of dimension  $M \geq 2$ , that are at uniform intervals, symmetric around zero, and span the interval  $[-1/\sqrt{T}, 1/\sqrt{T}]$ , where  $T$  is the number of observations in the sample. Each error term  $e_r^i$  has corresponding unknown weights  $\underline{w}_r^i = [w_{r1}^i, w_{r2}^i, \dots, w_{rM}^i]'$  that have the properties of probabilities:  $0 \leq w_{rm}^i \leq 1$  and  $\sum_m w_{rm}^i = 1$ .

We rewrite each error element as  $e_r^i \equiv \sum_m v_m^i w_{rm}^i$ . For example, if  $M = 3$ , then  $\underline{v}^i = (-1/\sqrt{T}, 0, 1/\sqrt{T})'$ , and there exists  $w_1^i, w_2^i$ , and  $w_3^i$  such that each noise component can be written as  $e_r^i = -w_{r1}^i/\sqrt{T} + w_{r3}^i/\sqrt{T}$ . Using this parameterization, we represent the GME consistency conditions, Equation 3.3, as

$$\begin{aligned}
(3.4) \quad \sum_t y_{tr}^i z_{tl} &= \sum_t \alpha_{tr}^i z_{tl} + \sum_t e_{tr}^i z_{tl} \\
&= \sum_t \alpha_{tr}^i z_{tl} + \sum_t \sum_m w_{trm}^i v_m^i z_{tl}^i.
\end{aligned}$$

For notational simplicity, we now drop the firm superscript. If we assume that the actions,  $\underline{x}$ , and the errors,  $\underline{e}$ , are independent and define  $\underline{w}$  as the vector which contains the elements  $w_{trm}$ , the GME problem for each firm is

$$(3.5) \quad \max_{\underline{\alpha}, \underline{w}} H(\underline{\alpha}, \underline{w}) = -\underline{\alpha}' \ln \underline{\alpha} - \underline{w}' \ln \underline{w},$$

subject to the GME consistency conditions, Equation 3.4, and the normalization constraints

$$(3.6) \quad \underline{1}' \underline{\alpha}_t = 1,$$

$$(3.7) \quad \underline{1}' \underline{w}_{ts} = 1.$$

for  $s = 1, 2, \dots, n$  and  $t = 1, 2, \dots, T$ . GJP shows how to estimate this model and demonstrates that the GME problem can be viewed as a generalized logit likelihood function, which includes the traditional logit as a special case.

Henceforth we refer to the GME estimator that uses only sample information as "the GME" estimator. When using the GME estimator, we may estimate  $\underline{\alpha}^i$  and  $\underline{\alpha}^j$  separately.

### 3.3 Incorporating the Non-Sample (Game-Theoretic) Information

We obtain the "GME-Nash" estimator by adding the game-theoretic restrictions, Equations 2.3 and 2.4, to the GME estimator. The GME-Nash estimator, and its application to

data, comprise the contributions of the current paper. To explain how to implement this estimator, we first proceed as if we knew the parameters of the profit function,  $\pi_{rs}^i$ . This assumption allows us to concentrate on the game-theoretic restrictions. We then consider the more realistic case where we need to estimate the parameters of the profit function.

To use the GME-Nash estimator, we need to estimate  $\underline{\alpha}^i$  and  $\underline{\alpha}^j$  jointly because both strategy vectors appear in Equation 2.4. [In contrast, as we noted above, the strategies can be estimated separately when using the GME estimator.] Further, we also have to estimate  $\underline{\delta}^i(\underline{z})$ ,  $i = 1, 2$ , from Equation 2.4. As we discussed in Section 2.2,  $\underline{\delta}^i(\underline{z})$  is nonzero if the econometrician does not observe firms' private information or if firms make mistakes in optimization.

Our first step is to reparameterize  $\underline{\delta}^i(\underline{z})$  using probabilities. Let  $\underline{y}^a$  be a vector of dimension  $J^a \geq 2$  with corresponding unknown weights  $\underline{\omega}_r^a$  such that

$$(3.8) \quad \sum_j \omega_{rj}^a = 1,$$

$$(3.9) \quad \underline{y}^{a'} \underline{\omega}_r^a = a_r,$$

for  $a = \underline{\delta}^1, \underline{\delta}^2$ . The support spaces  $\underline{y}^a$  are defined to be symmetric around zero for all  $a_r$ .<sup>3</sup>

Let

$$\underline{y} = (\underline{y}^{i'}, \underline{y}^{j'})', \quad \underline{\alpha} = (\underline{\alpha}^{i'}, \underline{\alpha}^{j'})', \quad \underline{w} = (\underline{w}^{i'}, \underline{w}^{j'})', \quad \text{and} \quad \underline{\omega} = (\underline{\omega}^{\delta^{i'}}, \underline{\omega}^{\delta^{j'}})'$$

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<sup>3</sup> We do not have natural boundaries for  $\underline{\delta}^i$ , so we use the "three-sigma rule" (Pukelsheim, 1994; Miller 1994; Golan, Judge, and Miller 1996) to choose the limits of these support spaces, where sigma is the larger of the empirical standard deviation of the discrete action space of prices or advertising.

As above, we assume independence between the actions and the errors. The GME-Nash problem is

$$(3.10) \quad \underset{\underline{\alpha}, \underline{w}, \underline{\omega}}{\text{Max}} H(\underline{\alpha}, \underline{w}, \underline{\omega}) = -\underline{\alpha}' \ln \underline{\alpha} - \underline{w}' \ln \underline{w} - \underline{\omega}' \ln \underline{\omega}$$

subject to the data consistency conditions 3.4 for each firm, the necessary economic conditions 2.3 and 2.4 for each firm, and the adding-up conditions for  $\underline{\alpha}$ ,  $\underline{w}$ , and  $\underline{\omega}$ . The terms  $\delta_{\mathbb{F}}^i$  in Equation 2.4 are defined by Equations 3.8 and 3.9. Solving the problem (3.10) yields the estimates  $\tilde{\underline{\alpha}}$ ,  $\tilde{\underline{w}}$ , and  $\tilde{\underline{\omega}}$ .

In order to write the Lagrangean for this problem, we need to determine how to impose the game-theoretic restrictions. Ideally, we would require that the game-theoretic restrictions hold for all possible values of  $\underline{z}$ . We cannot impose these restrictions for all values because we cannot write  $\underline{\alpha}(\underline{z})$  in closed form independent of the unknown Lagrangean multipliers. Instead, we impose the weaker condition that the game-theoretic restrictions, Equations 2.3 and 2.4, hold at some or all of the values of  $\underline{z}$  in our sample. Because of the large number of restrictions for each value of  $\underline{z}$  in our application, we impose the restrictions for only a subset of the observations. These restrictions, however, affect the  $\underline{\alpha}$  for all observations through the Lagrangean multipliers corresponding to the sample information (the moment conditions Equation 3.2).

We now turn to the more realistic case where we do not know the parameters of the profit functions. We simultaneously estimate the strategies and the parameters of the profit function, which depends on the demand and cost functions. We do not have observations on cost, but we observe Firm  $i$ 's output,  $q^i$ , and some factor-cost (wages, price of sugar, and

interest rate) variables,  $\underline{z}_c$ . We assume that Firm  $i$ 's cost is given by a cost function  $C^i(q^i, \underline{z}_c; \eta^i)$ , where  $\eta^i$  are parameters. Similarly, we assume that the demand for Firm  $i$ 's product is  $q^i(x^i, x^j, \underline{z}_d; \phi^i)$ , where  $\underline{z}_d$  are demand shifters (income and a seasonal dummy) and  $\phi^i$  are parameters. We substitute these functions in the Constraints 2.3 and 2.4 and estimate  $\eta^i$  and  $\phi^i$  jointly with the other parameters.

Because we observe demand (but not cost) we have an additional set of data consistency (sample) restrictions in the form of demand equations for each Firm  $i$ ,

$$q^i = q^i(x^i, x^j, \underline{z}_d; \phi^i) + u^i,$$

where  $u^i$  is an error term. We estimate the parameters  $\phi^i$  and  $\eta^i$  using the same method described in the previous subsection for estimating parameters that are not probabilities.<sup>4</sup>

That is, we choose a support for each such parameter and estimate the probability distribution over that support. We perform this estimation by maximizing the sum of all the entropy measure in equation 3.10 plus the entropy associated with the unknown demand and cost parameters.

### 3.4 Properties of the Estimators and Normalized Entropy

Both the GME and GME-Nash estimators are consistent, but they differ in efficiency and information content. GJP shows that the GME estimator is consistent given an appropriate choice of the bounds of the error term in the data consistency constraint 3.4. Under the assumption that a solution to the GME-Nash estimation problem exists for all samples, Appendix 1 shows that the GME-Nash estimator is consistent.

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<sup>4</sup> A formal derivation is available from the authors.

GJP show that the GME estimates of  $\underline{\alpha}$  have smaller variances than the ME-ML multinomial logit estimates. The possible solution space for the GME-Nash estimate of  $\underline{\alpha}$  is a subset of the solution space of the GME estimate of  $\alpha$ . Thus, we conjecture that the GME-Nash estimator has a smaller variance than the GME.<sup>5</sup>

We can quantify the added information contained in the game-theoretic restrictions by comparing the normalized entropy of  $\underline{\alpha}$  with and without the restrictions. The normalized entropy measure is  $S(\underline{\alpha}) = -(\sum_r \alpha_r \ln \alpha_r)/(\ln n)$ . The normalized entropy measure is  $S(\underline{\alpha}) = 1$  if all outcomes are equally likely, and is  $S(\underline{\alpha}) = 0$  if we know which action will be taken with certainty. The magnitude of the change in normalized entropy from imposing the game-theoretic restrictions provides a measure of the information they contain. See Soofi (1992) and Appendix 1 for a derivation of the properties and inferences results for this estimator.

#### 4. COLAS

Using quarterly data for 1968-1986, we estimate the price and advertising strategies for Coca-Cola and Pepsi-Cola using the GME and GME-Nash approaches. The Coca Cola Company and Pepsico, Inc. dominate the cola and soft-drink markets.<sup>6</sup> We use quarterly data for 1968-86, which were obtained from a variety of secondary sources and are described in Gasmi (1988), Gasmi and Vuong (1991), and Gasmi, Laffont, and Vuong (1992).<sup>7</sup>

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<sup>5</sup> Monte Carlo sampling experiment that support this conjecture are available from the authors.

<sup>6</sup> In 1981 for example, Coca-Cola's share of colas was 44.4% and its share of the national carbonated soft-drink market was 27.8% (according to the *Beverage Industry Annual* for 1986). The corresponding shares for Pepsi were 34.6% and 21.6%.

<sup>7</sup> The data were generously provided by these authors. We especially thank Farid Gasmi for patiently describing the data and making suggestions about the specification of our model.

We assume that firms set prices and advertising and use the demand specification from these earlier studies:

$$(4.1) \quad q_t^i = \phi_0^i + \phi_1^i p_t^i + \phi_2^i p_t^j + \phi_3^i (A_t^i)^{1/2} + \phi_4^i (A_t^j)^{1/2} + \phi_5^i d + \phi_6^i I + u^i,$$

where  $i = 1, 2, i \neq j$ ;  $q^i$  is the quantity sold,  $p^i$  is the real price charged, and  $A^i$  is the real advertising by Firm  $i$ ;  $d$  is a seasonal dummy;  $I$  is income;  $u^i$  is an error term;  $\phi_1^i$  is negative;  $\phi_2^i$  and  $\phi_3^i$  are positive. We reparameterize 4.1 so that it can be estimated along with the other parameters in the GME-Nash model. We assume that the marginal and average cost of Firm  $i$  is  $c^i = \eta_0^i + \eta_1^i \times \text{real price of sugar} + \eta_2^i \times \text{real unit cost of labor in the nondurable manufacturing sector} + \eta_3^i \times \text{real yield on a Moody's AAA corporate bond}$ , where  $\eta_0, \eta_1, \eta_2, \eta_3 \geq 0$ .<sup>8</sup>

Both Gasmi, Laffont, and Vuong and we chose a demand curve that does not include lagged values of decision variables (prices and advertising) or lagged functions of those variables (lagged quantity). This restrictive functional form is necessary given our economic model in which firms play a repeated (static) game and both price and advertising are decision variables. Were the demand curve to depend on lagged values of decision variables, firms would realize that choices today influence profits in the future and hence would not act as though they were playing a repeated static game.

It would be possible to alter our model to allow for lagged advertising by assuming that the firms' only decision variable is price and that advertising is only a demand shifter

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<sup>8</sup> The earlier studies did not include a constant term. Moreover, some of them used separate interest rates for the two companies. Because the correlation between these two interest rate measures is 0.99, we use only one.

(like income). Though this approach would lead to a much simpler estimation problem than the one we examine, we do not believe that it is reasonable to view advertising as anything but a decision variable.

Another approach we could have taken would be to include lagged sales (arguing that consumers develop brand loyalties) and then estimated a dynamic oligopoly model. There have been a number of efforts to estimate dynamic models of this sort, including Erickson (1992), Karp and Perloff (1989) and Roberts and Samuelson (1988). Erickson, for example, estimates a game in which Coke and Pepsi's current market share depends on the lagged market share in addition to current advertising, so that a stock effect exists. In principle our estimation methods can be applied to dynamic games, although the problem becomes difficult due to data and computational limitations.

#### *4.1 Cola Estimates*

For both the GME and GME-Nash models, firms have 35 possible actions in each period.<sup>9</sup> We divide the range of possible prices into seven intervals and the range of possible advertising levels into five intervals.<sup>10</sup>

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<sup>9</sup> If we reduce the number of possible actions, we estimate strategies distributions that are always single peaked (unlike the results reported below where Pepsi has double-peaked mixed strategies). This smoothing effect is the standard result from reducing the number of categories in a histogram. Smoothing aside, our estimates did not vary greatly as we experimented with other action spaces. For example, the strategy mode still occurs at the same area of price-advertising space.

<sup>10</sup> Within the sample, the prices range between \$10.886 and \$17.790 for Coca-Cola and between \$6.646 and \$9.521 for Pepsi-Cola. This difference in price levels is apparently due to the greater use of Coke syrup at fountains. Advertising expenditures range between 5.726 and 71.966 for Coca-Cola and from 7.058 to 50.814 for Pepsi-Cola.

To estimate the GME-Nash model, we impose sign restrictions from economic theory on both the cost (all cost coefficients are non-negative) and demand parameters (demand falls with a firm's own price and rises with the other firm's price and its own advertising) and the game-theoretic restrictions in 15 periods (every fifth quarter starting with the third quarter). By only imposing the restrictions in about one-fifth of the periods, we greatly reduce the size of the estimation problem.<sup>11</sup>

The Coca-Cola demand coefficients are  $\phi_0 = 4.549$ ,  $\phi_1 = -1.079$ ,  $\phi_2 = 2.137$ ,  $\phi_3 = 0.741$ ,  $\phi_4 = -0.232$ ,  $\phi_5 = 7.730$ ,  $\phi_6 = 0.737$ . The corresponding demand coefficients for Pepsi-Cola are  $-20.021$ ,  $-1.596$ ,  $0.582$ ,  $0.808$ ,  $-0.211$ ,  $5.592$ , and  $2.056$ . The correlation coefficients between observed quantities and those predicted by the demand equation are 0.93 for Coke and 0.94 for Pepsi.

For the GME-Nash, the estimated cost parameters are  $\eta_0 = 13.482$ ,  $\eta_1 = \eta_2 = 0$  (due to the theoretical restriction that the coefficient be non-negative), and  $\eta_3 = 0.208$  for Coca-Cola. The corresponding cost coefficients for Pepsi-Cola are  $7.251$ ,  $0$ ,  $0$ , and  $0$ .

Table 1 shows the GME estimates of Coca-Cola's coefficients on the exogenous variables,  $\underline{z}$ .<sup>12</sup> From the estimated coefficients, we can calculate the strategy probabilities,

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<sup>11</sup> We examined the sensitivity of our results to this assumption. We compared the estimate here where we imposed the theoretical restrictions on every fifth period (the frequency) starting with the third quarter to one with the same frequency where we started with the second or fourth quarter and found that the results were virtually identical. We also found that our estimates were not very sensitive to reducing the frequency.

<sup>12</sup> To save space, we do not report the coefficients for Pepsi or the two tables for the GME-Nash. These tables are available from the authors. Due to an arbitrary normalization convention, the GME (and GME-Nash) estimates have the opposite sign of the coefficients that would be produced by the roughly comparable ML multinomial logit approach.

$\underline{\alpha}$ , for each period. We show the estimates for the first quarter of 1977, near the midpoint of the sample, for Coca-Cola in Figure 1 and Pepsi-Cola in Figure 2. In both figures, panel a shows the GME estimates and panel b shows the GME-Nash estimates.

For both companies, the GME probability estimates are more uniform (reflect greater entropy) than the GME-Nash estimates. These figures illustrate that the game-theoretic conditions contain additional information beyond that in the data alone. If this theoretical information is true, it improves our estimates.

The corresponding marginal distributions for price and advertising strategies for both the GME and GME-Nash models are shown in Figure 3 for Coke and in Figure 4 for Pepsi. The GME-Nash marginal distributions put more weight on the category with the largest probability than do the GME marginal distributions.

This pattern is repeated in virtually all periods. We can compare the different estimators empirically using the normalized entropy (information) measure  $S(\underline{\alpha})$ . The normalized entropy measures for the GME, 0.66 (Coke) and 0.73 (Pepsi), are closer to one (the upper bound of entropy) than are the corresponding GME-Nash measures, 0.31 and 0.41. These numbers show the extent to which the game theoretic restrictions bind: They measure the amount of additional information contained in the restrictions. Similarly, the pseudo- $R^2$ , which is the expected value of  $1 - S(\cdot)$  for both firms (see Appendix 1), is 0.31 for the GME model and 0.64 for the GME-Nash model.

The GME-Nash model is flexible enough to allow for both pure and mixed strategies. Out of the 76 periods of the sample, there are three periods for each firm where it uses a pure strategy.

## 4.2 Tests

We now test whether our theory is consistent with the firms' behavior (data), using a variety of tests (see Appendix 1, Kullback, 1959, and Gokhale and Kullback, 1978). The entropy-ratio test statistic (which is analogous to the likelihood-ratio test from classical statistics) is  $[2H(\text{GME}) - 2H(\text{GME-Nash})] = 359.38 < \chi_{1050, 0.01}^2$ , where  $H(\cdot)$  is the optimal value of the objective function. Thus, we conclude that the economic theory represented by the set of conditions (2.3) and (2.4) is *consistent* with the data.

We now compare the strategies (estimated  $\underline{\alpha}$ ) of the GME and the GME-Nash models using first cross-entropy  $\chi^2$  tests (Appendix 1) and then Kolmogorov-Smirnov (KS) tests.<sup>13</sup> On the basis of a cross-entropy  $\chi^2$  test, we reject the null hypothesis that the GME and GME-Nash estimated strategies are identical in 34 periods (out of the 76 total periods) for Coke and in 25 periods for Pepsi. Thus, we conclude that the profit-maximizing, Nash restrictions are consistent with the data and contain useful information, so that imposing these restrictions affects our estimates of the strategies.

Next, we compare the strategies of the two firms for the GME-NASH model. We reject the null hypothesis that the two sets estimated strategies are identical in 74 out of the 76 periods. That is, the firms use *different* strategies.

For example, by comparing Figures 1b and 2b, we see that Coke and Pepsi had very different strategy distributions in the middle of the sample. Coke had a single-modal strategy distribution with most weight on a moderate price-moderately intense advertising strategy,

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<sup>13</sup> These test results are for a 0.05 significance level. As there are seven support points for the price strategy and five for the advertising strategy, there are  $(7 - 1) \times (5 - 1) = 24$  degrees of freedom.

whereas Pepsi had a bimodal distribution with the most weight on a high price-intensive advertising strategy.

We obtain similar results using the Kolmogorov-Smirnov (KS) test to examine whether the estimated GME and GME-Nash distributions differ systematically. On the basis of a KS test at a 5% significance level, we cannot reject the hypothesis that the GME and GME-Nash distributions and marginal distributions, averaged over all periods, are identical.<sup>14</sup> If, however, we examine the hypothesis that the distributions are the same period by period, we can reject the hypothesis for Coke in 66 (out of 76) periods and for Pepsi in 62 periods.

Next, we investigate the significance of the individual covariates,  $l =$  income, price of sugar, wage, and bond rate. That is, we test whether the estimated coefficient is zero ( $H_0: z_l = 0$ ) or nonzero ( $H_1: z_l \neq 0$ ). The  $\chi^2$  test-statistic values are 54.41, 34.38, 78.07, and 56.51 for income, price of sugar, wage, and bond rate respectively. As a result, we reject the null hypothesis for *all* the covariates at the  $\alpha = 0.01$  level. Thus, though the factor prices do not greatly affect the marginal costs of the firms, they do affect the strategies the firms use (see Section 4.4).

These estimators fit the data reasonably well, as Table 2 shows for both the GME model and for the GME-Nash model. For example, the GME-Nash estimator correctly predicted which of the seven price categories Coke chose in 55% of the periods. Moreover,

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<sup>14</sup> The ML multinomial logit (or ME model) for the entire sample ignoring  $\underline{z}$  is the same as the empirical distribution (the frequencies) of the actions. We cannot reject the hypothesis that that estimate is the same as the average of the GME and GME-Nash estimates based on KS tests.

the model missed by more than one category in only 11% of the periods (this fact is not shown in the table). In this study, the GME predictions are in each case more accurate than those of the GME-Nash: For example Coke's price category is correctly predicted in 70% of the periods by the GME model and only 55% by the GME-Nash.<sup>15</sup>

### 4.3 Lerner Measures

A standard measure of market power is the Lerner index, which is the percentage by which price is set above marginal cost. Usually, the Lerner index ranges between zero (competition) and one.

We use our estimates of probabilities to calculate the expected Lerner index,  $E[(p^i - c^i)/p^i] = \sum_r \alpha_r^i [(p_r^i - c^i)/p_r^i]$ , where  $c^i$  is our estimate of Firm  $i$ 's marginal cost.<sup>16</sup> We suppress the dependence of all functions on the public information,  $\underline{z}$ , and hold  $\underline{z}$  constant for purposes of this discussion. In our study, the average adjusted Lerner index is 0.24 for Coke and 0.27 for Pepsi.

For comparison, we also calculated the Lerner index for the Bertrand-Nash model using the coefficients from Model 1 of Gasmi, Laffont, and Vuong (1992). Averaged over the sample, the index for Coke is 0.42 and for Pepsi is 0.45. Thus, the GME-Nash estimates

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<sup>15</sup> Based on our Monte Carlo simulations (available from the authors), the GME-Nash can predict better than the GME. If the GME-Nash restrictions are correct, we expect the GME-Nash to have lower mean squared errors than the GME.

<sup>16</sup> A formal derivation of the relevant equations is available from the authors.

indicate that firms have less market power than do ML estimates of a Bertrand-Nash equilibrium.<sup>17</sup>

#### 4.4 *Effects of the Exogenous Variables*

Using our estimated models, we can calculate the effect of a change in each of the exogenous  $z$  variables on the strategy probabilities,  $\alpha$ , using the same approach as is used with logit and probit models. Table 3 shows the average strategy elasticities using the GME-Nash estimates (the percentage change in expected action divided by the percentage change in a  $z$  variable).<sup>18</sup> Some of these elasticities are large in absolute value because the corresponding probabilities are close to zero.

By inspection of Table 3, we see that an increase in income, which shifts out demand, increases the probability that Coke, and to a lesser extent Pepsi, charge higher prices. The elasticity of Coke's expected price with respect to income is 0.154 and the corresponding elasticity for Pepsi is 0.0013.

An increase in income (and demand) spreads a unit cost of advertising over a greater volume of sales, so we expect higher income to shift the distribution for advertising to the right. Pepsi's advertising strategy does shift to the right, but for Coke more probability

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<sup>17</sup> These differences in the Lerner indexes are largely due to differences in the estimates of costs. Our GME-Nash cost estimates are substantially higher than their ML-Bertrand estimates.

<sup>18</sup> First, we calculate the derivatives of probabilities and average them over the 59 periods where we do not impose the game-theoretic constraints. Then, we average the probabilities for these periods. Using these averages, we summed over categories to compute the marginals of the averages, and used the results to calculate the elasticities of the marginals. By using the periods where we do not impose the constraints, we are able to use an explicit formula for probabilities when calculating the derivatives. Had we used the periods where the game theoretic constraints are imposed, we would have had to calculate the derivatives of a system of 70 implicit equations.

weight is shifted to both tails. The elasticity of expected advertising with respect to income is  $-0.097$  for Coke and  $0.03$  for Pepsi.

We can calculate similar elasticities with respect to the other exogenous variables. According to our estimates, the corporate bond rate does not directly affect Pepsi's costs, and it has a negligible effect on Pepsi's strategy. Despite the absence of a direct effect (via costs), the bond rate might indirectly affect Pepsi's strategy, possibly because Pepsi thinks that it alters Coke's strategy. Coke's costs increase with the interest rate. For Coke, an increase in the interest rate changes the mix of probabilities of charging a high price. An increase in the interest rate shifts more weight to the tails of Coke's advertising strategy. The elasticities with respect to the corporate bond rate are  $0.004$  for Coke's expected price,  $0.032$  for Coke's expected advertising,  $0.004$  for Pepsi's expected price, and  $0.0005$  for Pepsi's expected advertising. Thus, for changes in either income or bond rates, Coke responds more than does Pepsi.

## 5. CONCLUSIONS

We developed two methods of estimating the strategies of firms, which are the probabilities of taking particular actions. In our application to the cola market, the actions are price-advertising pairs. Both methods are free of parametric assumptions about distributions and ad hoc specifications such as those used in conjectural-variations models. Unlike previous studies of oligopoly behavior that only allowed for pure strategies, we allow for both pure and mixed strategies.

Our simplest approach is to use generalized maximum entropy (GME) to estimate the strategies for each firm using only sample information. This method is more flexible and

efficient than the standard maximum likelihood multinomial logit (ML) estimator. Both the traditional ML and the GME estimators ignore restrictions imposed by economic theory and some information about demand and costs.

Our generalized-maximum-entropy-Nash (GME-Nash) approach estimates firms' strategies consistent with the underlying data generation process and the restrictions implied by game theory. The application to the cola market demonstrates that both the GME and GME-Nash models can be used practically.

Tests show that the profit-maximizing, Nash restrictions are consistent with the data but that, because they contain information, alter our estimates of firms' strategies. We are able to use our estimates to show how changes in exogenous variables such as income or factor prices affect the firms' strategies.

Our GME and GME-Nash approaches to estimating games can be applied to many problems in addition to oligopoly, such as wars and joint decisions by husbands and wives. To do so only requires replacing profits with an appropriate alternative criterion.

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## Appendix 1: The GME-Nash Estimator

### A2.1 Consistency

Call the GME-Nash estimates of the strategies  $\underline{\tilde{\alpha}}$ , the GME estimates  $\underline{\check{\alpha}}$ , and the ME-ML estimates  $\underline{\hat{\alpha}}$ . We make the following assumptions:

*Assumption 1:* A solution of the GME-Nash estimator  $(\underline{\tilde{\alpha}}, \underline{\tilde{w}}, \underline{\tilde{\omega}})$  exists for any sample size.

*Assumption 2:* The expected value of each error term is zero, its variance is finite, and the error distribution satisfies the Lindberg condition (Davidson and MacKinnon, 1993, p. 135).

*Assumption 3:* The true value of each unknown parameter is in the interior of its support.

We want to prove

*Proposition:* Given assumptions 1-3, and letting *all* the end points of the error support spaces  $\underline{y}$  (for each firm) be normed by  $\sqrt{T}$ ,  $\text{plim}(\underline{\tilde{\alpha}}) = \text{plim}(\underline{\check{\alpha}}) = \underline{\alpha}$ .

This result holds even when the profit parameters are unknown.

According to this proposition, the GME-Nash estimates,  $\underline{\tilde{\alpha}}$ , and the GME basic estimates,  $\underline{\check{\alpha}}$ , are equal to each other and to the true strategies in the limit as the sample size becomes infinite,  $T \rightarrow \infty$ . That is, all the estimators are consistent.

*Proof:*

i) The consistency of the GME estimator is proved in GJP. Let the end points of the error supports of  $\underline{v}$ ,  $v_1$  and  $v_m$ , be  $-1/\sqrt{T}$  and  $1/\sqrt{T}$  respectively. As  $T \rightarrow \infty$ ,  $\psi_s \rightarrow 1$  for all  $s$  in the dual-GME, Equation 3.12. Thus,  $\sum_s \ln \psi_s(\underline{\lambda}) \rightarrow 0$  and  $\text{plim } \underline{\alpha}_T = \underline{\alpha}$ .

ii) The GME-Nash with known profit parameters is consistent: By Assumption 1, after we have added the restrictions 2.3 and 2.4, we still have a solution. The argument in (i) together with Assumption 2 implies that  $\text{plim } \underline{\alpha}_T = \underline{\alpha}$ .

iii) The GME-Nash with unknown profit parameters is consistent. Given Assumption 3, the GME is a consistent estimator of  $\underline{\phi}$  in Equation A1.1 (Mittelhammer and Cardell, 1996):  $\text{plim } \underline{\phi}_T = \underline{\phi}$ . By the argument in (ii),  $\text{plim } \underline{\alpha}_T = \underline{\alpha}$ . These asymptotic properties can also be established via the empirical likelihood approach (Owen, 1990; Qin and Lawless, 1994; Golan and Judge, 1996).

## A2.2 Hypothesis testing

On the basis of the consistency of the estimators, we can define an "entropy ratio statistic" which has a limiting  $\chi^2$  distribution (see Kullback, 1959, Gokhale and Kullback, 1978, and Soofi, 1992). We use this statistic to test hypotheses. In general, let  $\underline{\lambda}^*$  be the vector of Lagrange multipliers for *all* the model's constraints. Let  $H_M(\underline{\lambda}_0^*)$  be the entropy value of the constrained problem where  $\underline{\lambda}^* = \underline{0}$ , or equivalently all the parameters (strategies as well as demand coefficients) are constrained to be zero (or at the *center* of their supports). Thus,  $H_M(\underline{\lambda}_0^*)$  is the maximum value of the joint entropies (objective function). It can be obtained by maximizing Equation (3.11) subject to no constraints (except for the requirement that all distributions are proper). Doing so yields the total entropy value of the three sets of

discrete, uniform distributions  $\underline{\alpha}$ ,  $\underline{w}$ , and  $\underline{\omega}$ . Now, let  $H_u(\tilde{\lambda}^*)$  be the objective (total entropy) value for the full GME-Nash model — the optimal value of Equation (3.14) — where  $\tilde{\lambda}^*$  is the set of estimated values (that is, they are not forced to equal zero).

The *entropy-ratio statistic* for testing the null hypothesis  $H_0$  that all parameters are zero is

$$\mathcal{E}(\text{parameters} = \underline{0}) = 2H_M(\text{parameters} = \underline{0}) - 2H_u(\tilde{\alpha}, \tilde{w}, \tilde{\omega}).$$

Under the mild assumptions we made above (or the assumptions of Owen, 1990 and Qin and Lawless, 1994),  $\mathcal{E}(\text{parameters} = \underline{0}) \rightarrow \chi_K^2$  as  $T \rightarrow \infty$  when  $H_0$  is true and  $K$  is the number of restrictions. The approximate  $\alpha$ -level confidence intervals for the estimates are obtained by setting  $\mathcal{E}(\cdot) \leq C_\alpha$ , where  $C_\alpha$  is chosen so that  $\Pr(\chi_K^2 < C_\alpha) = \alpha$ . Similarly, we can test any other hypothesis of the form  $H_0: \underline{\alpha} = \underline{\alpha}_0$  for all, or any subset, of the parameters. We use these entropy-ratio statistics to test whether the economic and Nash restrictions are consistent with the data.

Using the same line of reasoning as above (each constraint, or data point, represents additional potential information that may lower the value of the objective function but can never increase it), one can derive a "goodness of fit" measure for our estimator:

$$R^* = 1 - \frac{H_u(\tilde{\lambda}^*)}{H_M(\tilde{\lambda}^* = \underline{0})},$$

where  $R^* = 0$  implies no informational value of the data set, and  $R^* = 1$  implies perfect certainty or perfect in-sample prediction. This measure,  $R^*$ , is the same as the information index in Soofi, 1992.

The small-sample approximated variances can be computed in a number of ways. We discuss two simplest approaches here. First, for each equation (say the two sets of demand equations), we calculate

$$\hat{\sigma}_i^2 = \frac{1}{T} \sum_t \tilde{u}_{it}^2,$$

where  $\tilde{u}_{it} \equiv \sum_j \tilde{\omega}_{ij}^u v_j^u$  and  $\text{vâr}(\phi_k^i) \equiv \hat{\sigma}_i^2 (X'X)^{-1}$  for each parameter  $\phi_k^i$ . Similarly, for each

set of equations, the relevant  $\sigma^2$  is estimated.

Because our model is a system of a large number of equations, the elements of the asymptotic variance-covariance matrix,  $\Omega$ , for the error terms of the entire system are estimated in the traditional way, taking into account all the data and all the restrictions (Equations 2.3, 2.5, 2.6, and A.6).

Finally, we note the relationship between the entropy objective and the  $\chi^2$  statistic.

This relationship is used for comparison of various estimated strategies and various estimated distribution. The cross-entropy measures is defined as

$$(2.1) \quad I(\underline{\alpha}, \underline{\alpha}^o) = \sum_k \alpha_k \ln(\alpha_k/\alpha_k^o),$$

where  $\underline{\alpha}^o$  is a proper prior probability distribution. Now, let  $\{\alpha_k\}$  be a set of  $K$  observed frequencies (strategies) over a set of  $K$  observed prices. Let the null hypothesis be  $H_0: \underline{\alpha} = \underline{\alpha}^o$ , then

$$\chi_{(k-1)}^2 = \sum_k \frac{1}{\alpha_k^o} (\alpha_k - \alpha_k^o)^2.$$

A second-order approximation of (A2.1) is

$$I(\underline{\alpha}, \underline{\alpha}^o) \cong \frac{1}{2} \sum_k \frac{1}{\alpha_k^o} (\alpha_k - \alpha_k^o)^2,$$

which is the entropy-ratio statistic (for evaluating  $\underline{\alpha}$  versus  $\underline{\alpha}^o$ ) that we previous discussed.

We conclude by noting that two times the entropy-ratio statistic corresponds (at the limit) to

$$\chi_{(k-1)}^2.$$

Table 1

## GME Estimates of Coefficients for Coca-Cola

Price	Advertising	Constant	Seasonal Dummy	Income	Price of Sugar	Wage	Bond Rate
1	2	-46.738	1.851	0.832	1.207	49.393	-0.077
1	3	-43.588	-1.188	0.822	0.295	49.534	-0.115
1	4	1.441	0.017	-0.013	-0.098	-0.955	0.002
1	5	1.428	0.017	-0.013	-0.098	-0.940	0.002
2	1	77.525	0.834	-1.384	-0.041	-90.107	-0.052
2	2	10.040	0.896	-0.274	0.823	-11.263	-0.275
2	3	-20.309	-1.695	0.586	1.785	11.465	-0.396
2	4	51.699	-0.287	-0.860	-0.441	-60.512	-0.277
2	5	33.840	-1.141	-0.769	-0.266	-30.543	-0.024
3	1	1.428	0.017	-0.013	-0.098	-0.940	0.002
3	2	9.887	1.542	-0.196	-0.841	-12.846	-0.021
3	3	-13.102	-0.128	0.203	1.646	7.332	0.109
3	4	-61.368	-1.401	1.070	2.526	65.273	0.420
3	5	-0.719	-0.671	-0.190	1.200	5.090	0.357
4	1	21.993	0.756	-0.262	-2.909	-23.202	-0.168
4	2	-19.817	2.187	0.354	-2.500	25.788	-0.034
4	3	-5.802	0.697	-0.110	-0.903	11.068	0.291
4	4	59.688	-1.339	-1.201	-2.096	-59.023	0.281

4	5	1.428	0.017	-0.013	-0.098	-0.941	0.002
5	1	1.441	0.017	-0.013	-0.098	-0.955	0.002
5	2	1.425	0.017	-0.013	-0.098	-0.938	0.002
5	3	-22.607	-1.331	0.543	-0.379	23.203	-0.110
5	4	1.452	0.017	-0.013	-0.099	-0.966	0.002
5	5	1.462	0.017	-0.014	-0.099	-0.977	0.002
6	1	1.433	0.017	-0.013	-0.098	-0.947	0.002
6	2	1.438	0.017	-0.013	-0.098	-0.952	0.002
6	3	1.418	0.017	-0.013	-0.098	-0.930	0.002
6	4	-23.977	-1.188	0.479	0.901	24.449	0.037
6	5	1.453	0.017	-0.013	-0.099	-0.968	0.002
7	1	1.420	0.017	-0.013	-0.098	-0.932	0.002
7	2	-29.797	1.311	0.546	1.344	29.438	0.026
7	3	1.453	0.017	-0.013	-0.099	-0.968	0.002
7	4	1.418	0.017	-0.013	-0.098	-0.930	0.002
7	5	1.457	0.017	-0.013	-0.099	-0.972	0.002

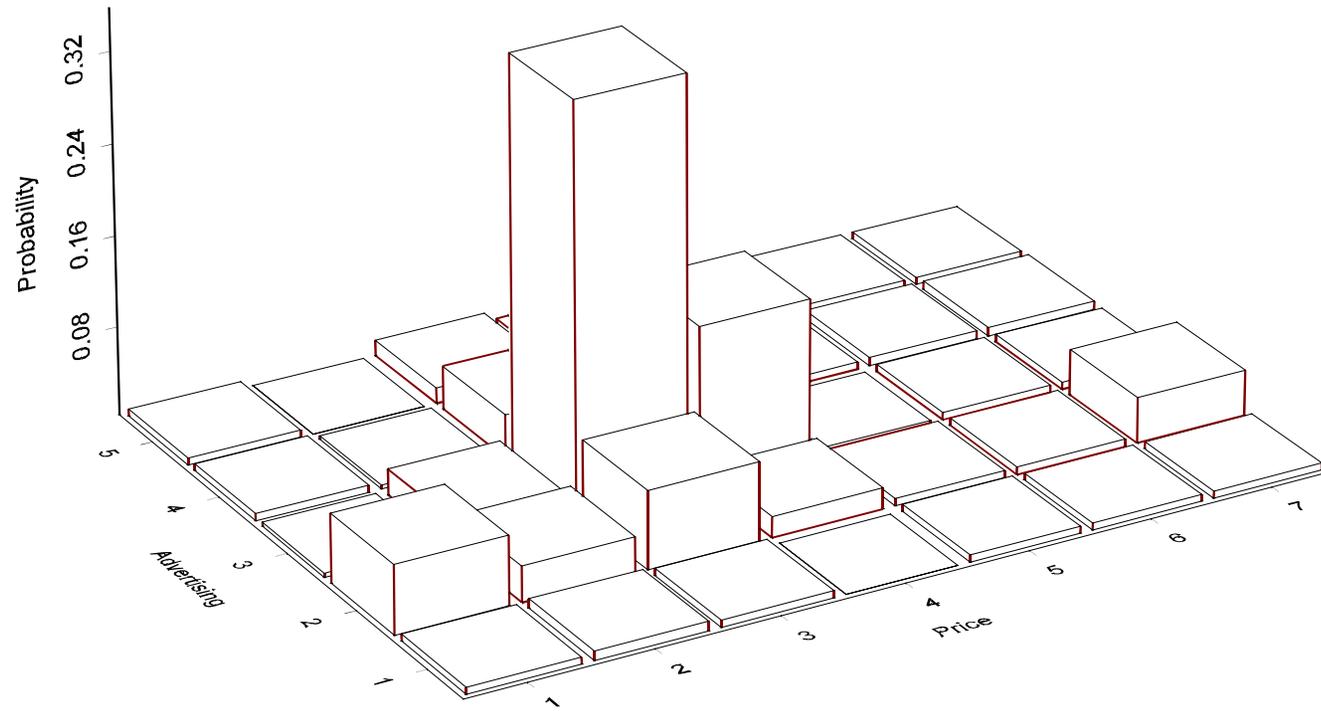
*Notes:* The first two columns show the price and advertising categories. The coefficients in the missing first row are normalized to zero.

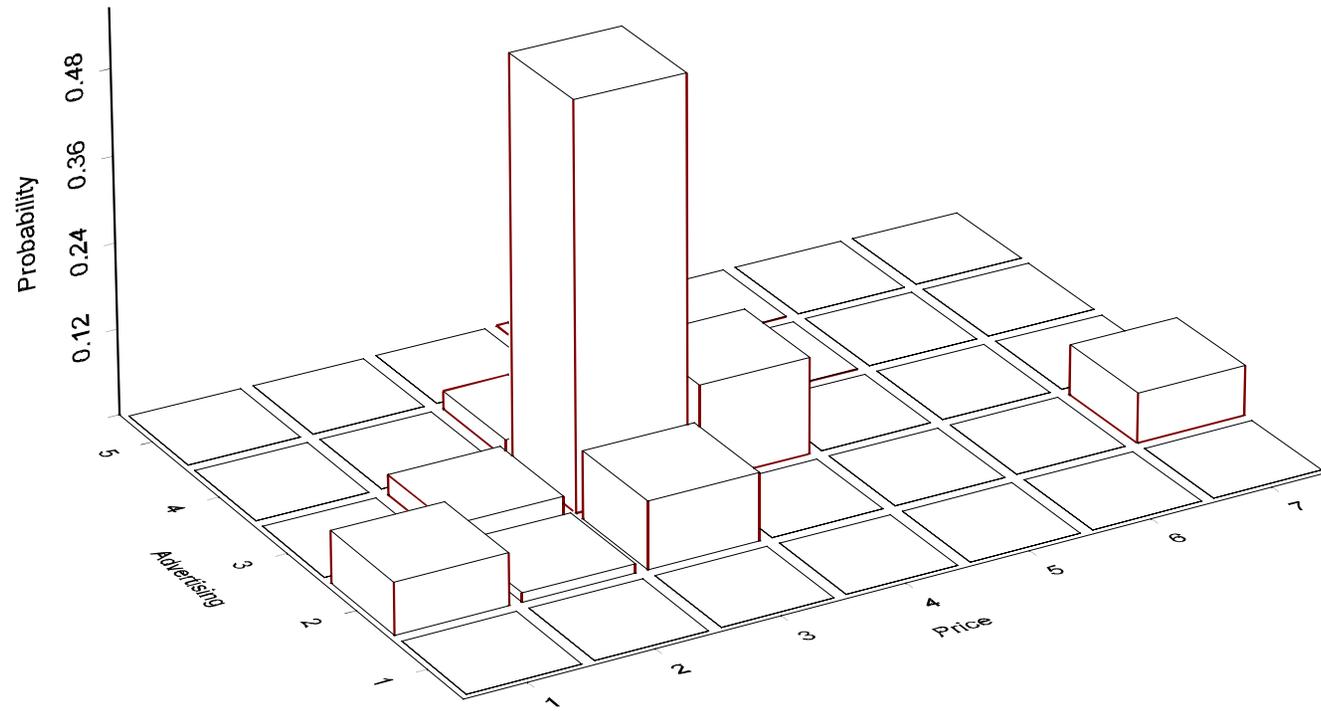
**Table 2:** Percent of Categories Correctly Predicted

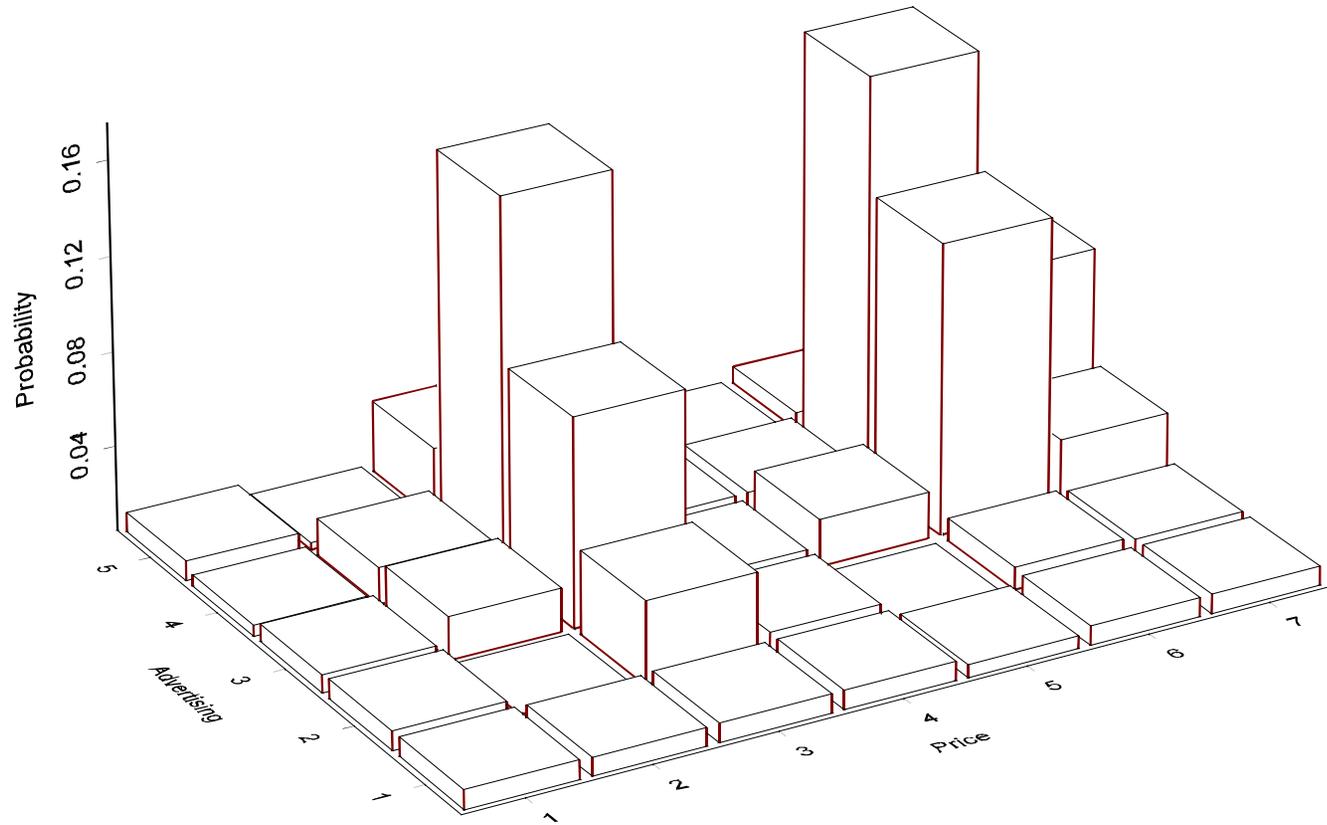
	GME		GME-Nash	
	Price	Advertising	Price	Advertising
Coke	70	68	55	63
Pepsi	61	33	54	32

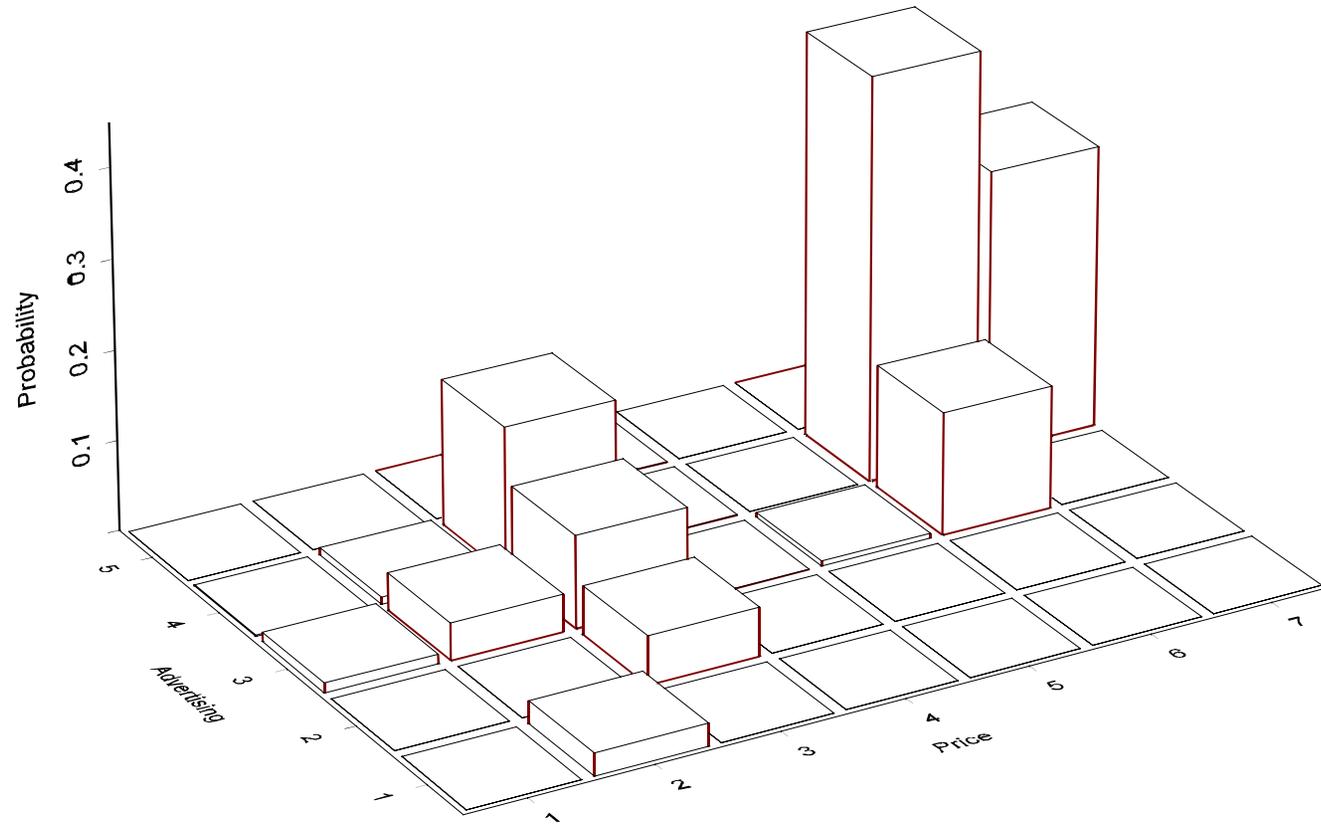
**Table 3: Strategy Elasticities**

	Categories						
	1	2	3	4	5	6	7
<b>Interest Rate</b>							
Coke Price	.000	.000	.000	.001	-2.920	2.998	-.192
Pepsi Price	.000	.000	-.003	.003	.000	.000	.000
Coke Advertising	.262	.000	-.005	-1.266	3.767		
Pepsi Advertising	-.004	.000	.000	.000	.003		
<b>Income</b>							
Coke Price	.000	.000	-.000	.015	-87.737	64.180	15.077
Pepsi Price	.000	.000	-.165	.204	.000	.001	.000
Coke Advertising	10.825	.000	.635	-32.756	75.448		
Pepsi Advertising	-.265	.000	.000	.000	.155		

**Figure 1a:** GME Estimates of Coke's Strategies (First Quarter 1977)

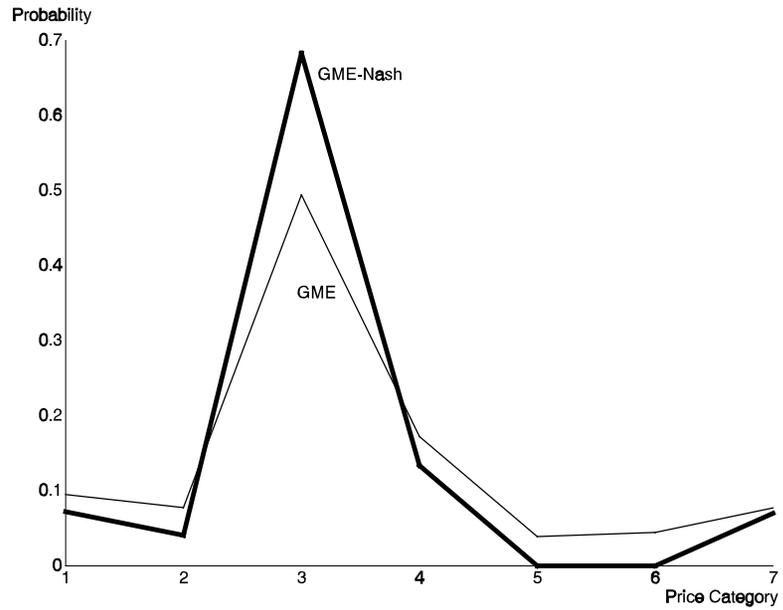
**Figure 1b:** GME-Nash Estimates of Coke's Strategies (First Quarter 1977)

**Figure 2a:** GME Estimates of Pepsi's Strategies (First Quarter 1977)

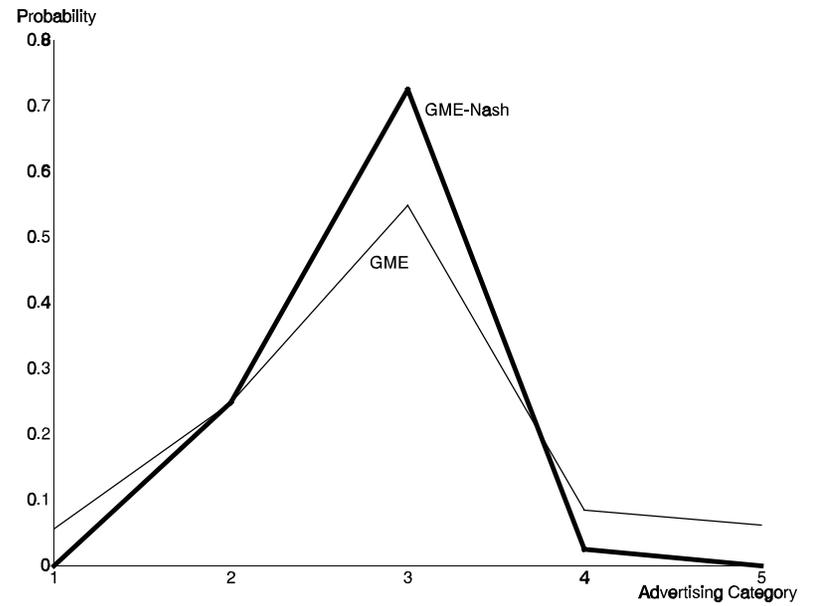
**Figure 2b:** GME-Nash Estimates of Pepsi's Strategies (First Quarter 1977)

**Figure 3:** GME and GME-Nash Marginal Strategy Distributions for Coke (First Quarter 1977)

a) Pricing Strategies

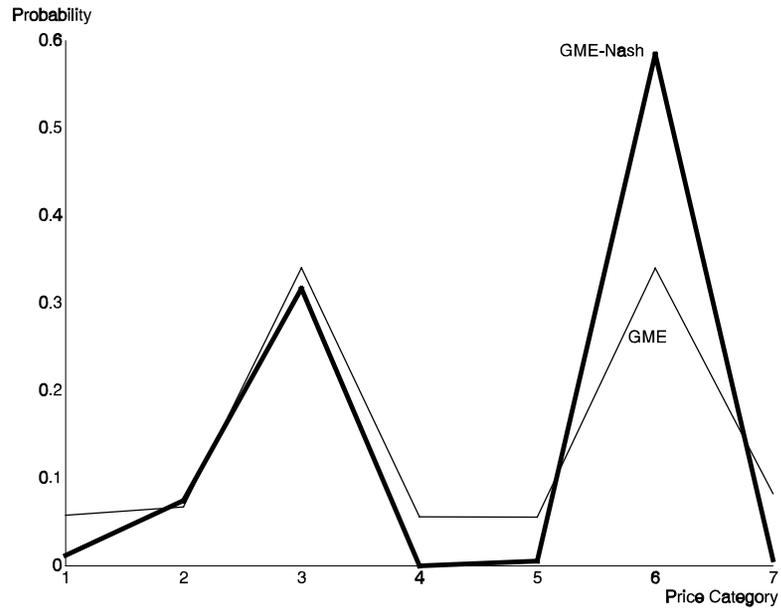


b) Advertising Strategies



**Figure 4:** GME and GME-Nash Marginal Strategy Distributions for Pepsi (First Quarter 1977)

a) Pricing Strategies



b) Advertising Strategies

