Fishery Management Under Multiple Uncertainty

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Abstract

Among others who point to environmental variability and managerial uncertainty as causes of fishery collapse, Roughgarden and Smith (1996) argue that three sources of uncertainty are important for fisheries management: variability in fish dynamics, inaccurate stock size estimates, and inaccurate implementation of harvest quotas. We develop a bioeconomic model with these three sources of uncertainty, and solve for optimal escapement based on measurements of fish stock in a discrete-time model. Among other results we find: (1) when uncertainties are high, we generally reject the constant-escapement rule advocated in much of the existing literature, (2) inaccurate stock estimation affects policy in a fundamentally different way than the other sources of uncertainty, and (3) the optimal policy leads to significantly higher commercial profits and lower extinction risk than the optimal constant-escapement policy (by 42% and 56%, respectively).
1 Introduction

Fishery collapse is an increasingly common phenomenon worldwide. Though the Gordon-Schaeffer type models suggest management can overcome the economic (and indeed biological) consequences of unregulated fisheries, many managed fisheries have collapsed. Suggested causes include habitat destruction (World Resources Institute 2001), reduced recruitment levels in the face of environmental variability (Pearcy and Shoener 1987; Barber and Chavez 1983), fishery overcapitalization (Food and Agriculture Organization 1999; Grafton, Squires, and Kirkley 1996), and a lack of political will to impose quotas that will ensure sustainability (Johnson and Libecap 1982; Homans and Wilen 1997). Despite spatial and temporal differences in fisheries, excessive harvest is widely accepted as a major contributor to declines.

Roughgarden and Smith (1996) note that the inherent problem of over-fishing is exacerbated by uncertainty in fish stock size and dynamics. Among other factors, they attribute fishery collapse to uncertainty in marine environments, and suggests that ignoring uncertainty can lead to excessive harvest. “In reality, if stocks are seen to grow, then quotas are usually increased, resulting eventually in a quota that exceeds production and extinguishes the stock” ((Roughgarden and Smith 1996), page 5079).

This paper focuses on the implications of environmental and managerial uncertainties for the management of fisheries and formally addresses, in an economic framework, the issue posed by Roughgarden and Smith. The model they outline involves three sources of uncertainty: (1) environmental variability that influences the growth of fish stocks, (2) stock measurement error, and (3) inaccurate implementation of harvest quotas. The first two sources of stochasticity have been discussed in the literature, while the third is a novel element that they introduce. While the authors discuss how these growth shocks and error sources affect fishery management they do not offer an optimal solution to the manager’s problem. This paper frames and solves this fishery management problem under uncertainty. Among other implications for fishery policy, we find that uncertainty in measurement may have the greatest potential to affect policy. When uncertainty is sufficiently high, we find a general rejection of
the “constant-escapement” policy suggested by deterministic models (Gordon 1954), models of only growth uncertainty (Reed 1979), and other more heuristic approaches to fishery modeling from the biological community (Roughgarden and Smith 1996). In particular, when uncertainty in measurement is high, the optimal escapement policy increases in the measured stock.

The paper is organized as follows. In section 2 we provide a background of economic and biological models of fishery management under uncertainty. We then turn to our approach of integrating the two types of models, and discuss its relevance in section 3. The formal characterization is presented in section 4, and results in section 5. We conclude in section 6.

2 Background

There exists a large economic and biological modeling literature addressing management of renewable resources under uncertainty (Reed 1978; Clark and Kirkwood 1986; Roughgarden and Smith 1996; Costello, Polasky, and Solow 2001; Weitzman 2002). Economists typically pose allocation questions, pushing biological realism only to the extent that it permits clean analytical solutions. Although this approach often reveals the salient characteristics of a system, biological modelers often criticize such models for their inadequate treatment of realistic biological uncertainties. Although they incorporate more biological realism, biological models are often criticized by economists because they either ignore economic considerations such as harvest costs, prices, or time preference, or because they fail to solve for an optimal intertemporal allocation. We briefly describe the main approaches in each discipline, and then present our method of integrating the important elements of each into a stochastic dynamic decision-making framework.

2.1 Economic Models

The work of Beverton and Holt (1957) and Schaefer (1957) assumed a deterministic environment and provided analytically tractable models of renewable resource
exploitation. In this framework, the optimal policy is “bang-bang”: the optimal escapement level occurs where the rate of return from harvesting the last fish and investing the money from doing so just equals the rate of return from letting that last fish grow to the next period. If the stock is lower than the optimum level for any reason, the fishery is closed until it builds itself up. Whenever the stock is higher than the optimum level, the catch quota is simply the difference between the current and the optimum levels. Though this simple framework provides numerous useful insights, it has been criticized on several grounds. Most notably, it ignores the inherent environmental variability and managerial uncertainty faced by fishery managers.

The main results in the economics literature on stochastic fishery management have been developed in two papers: Reed (1979) and Clark and Kirkwood (1986). Both of these papers model one source of uncertainty for purposes of tractability. Reed introduces unpredictable environmental variability into a model of fishery management by recognizing stochastic fluctuations in recruitment. He assumes the stock of fish from one period to the next is governed by a deterministic, “compensatory” growth function, but that a random multiplicative shock disturbs this growth every period. The manager knows the stock at the beginning of the period and she chooses the level of escapement, but because of a random growth shock at the end of the current period she does not know future stock. Reed assumes a constant price per unit harvest, and a marginal cost function which decreases in stock. Reed concludes that despite uncertainty about future stocks, the optimal policy is to allow a constant escapement every period, regardless of stock at the beginning of the period (provided the initial stock is larger than the desired escapement). Since the stock of fish is known every period, management policy can never lead to accidental extinction in this model.

Clark and Kirkwood (1986) modify Reed’s model by noting that fishery managers can rarely measure stock levels accurately and typically have confidence intervals of ±50%. With this practical motivation in mind, Clark and Kirkwood alter Reed’s model by changing the timing of the shock. In their model, the manager knows the escapement in the previous period but is uncertain about stock in the current period.
To simplify the analysis, Clark and Kirkwood eliminate fishing costs and price from the profit expression, since costs and prices have no qualitative effect on results in which they are interested. Clark and Kirkwood obtain results very different from those of Reed. When managers cannot perfectly measure current stock, the optimal policy is no longer one of constant-escapement. Perhaps somewhat surprisingly, the optimal policy is less cautious (for some levels of expected initial stock) than the constant-escapement policy. Furthermore, in rare cases it turns out to be optimal to harvest the population to extinction. This result is in direct contradiction to the constant escapement policy which, provided the intrinsic growth rate is sufficiently high, guards the population from extinction in perpetuity.

2.2 Management With Multiple Uncertainty

Roughgarden and Smith (1996) approach the issue of uncertainty from a biological perspective. Their model is motivated by the danger facing many of the world’s fisheries, and the belief that harvesting as if the resource growth and measurement were deterministic, when in fact it is stochastic, can lead to unintended extinction. With this observation in mind, Roughgarden and Smith extend Reed’s and Clark and Kirkwood’s models by introducing two additional sources of uncertainty, stock measurement error and harvest implementation error. In their model the fishery manager enters the period and measures the stock with some error. She must then make a harvest announcement knowing that the true harvest will be imprecisely implemented. Like the previous literature, Roughgarden and Smith’s model allows the multiplicative growth shock to vary from year to year. Within a year, however, growth occurs on a daily basis, where daily growth shocks are equal every day within a year. A stock measurement is made once every year and an annual quota is announced. Harvest occurs throughout the year so that daily harvest equals \( \frac{1}{365} \) of the annual harvest, which may deviate from the quota because of implementation error.

The significant increase in complexity of the Roughgarden and Smith model as compared to the Clark and Kirkwood model preclude them from obtaining analytical results. Although they frame their problem as a bioeconomic exercise of net revenue
maximization, to solve it they employ a rule-of-thumb. The authors locate the constant ‘target stock’ that maximizes the fishery value, where the target stock may be thought of the end-of-season stock in the absence of uncertainty. In a discrete-time model such as ours, the target-stock policy is equivalent to a constant-escapement policy. The authors provide no evidence that the optimal policy, that is, the one that maximizes expected present value of the fishery, would lie within the class of constant target stock (or constant escapement) policies.

3 This Research

From the perspective of the fishery manager, both biology and economic behavior appear to be stochastic processes that complicate decision-making. Biologically, the importance of different forms of uncertainty has to do largely with the risk of extinction. In the Reed model, the population cannot go extinct, unless by human design, because the manager always knows the stock level and chooses escapement precisely. In the Clark and Kirkwood model, extinction is only possible in the unlikely case in which there is such extreme miscalculation in stock measurement that the manager sets the harvest too high and drives it to extinction. Under assumptions in the Roughgarden and Smith model, extinction is much more likely because of the implementation error (now creating the possibility of both over-estimating the stock size and over-harvesting in the same period). Economically, the increase in uncertainty affects the optimal decision-making by the manager; as the number of sources of uncertainty increase, the manager must base her expectations on less precise information.

We solve for optimal fishery management under uncertainties in growth, measurement, and implementation. In the spirit of the questions posed by fishery modelers and managers, we address the following questions within the context of our multiple uncertainty model:

1. Given three sources of stochasticity (growth, measurement, implementation), how should the total allowable catch announcement depend on the stock mea-
surement in any given period?

2. What are the implications of optimal, and suboptimal, management for stock survival over a fixed period, say 50 years?

3. How important is each individual source of uncertainty in determining optimal management?

4. How should management optimally respond to changes in the magnitude of each source of uncertainty?

The next section describes the model we develop to answer these questions.

4 The Model

In this section we state our assumptions, formalize the optimization problem, and describe the solution technique. The general model and method below is appropriate for setting up and solving any stochastic dynamic programming problem with Markovian transitions. ¹

4.1 Assumptions

We make the following assumptions:

1. There are three random variables underpinning the uncertainty in period $t$: $z_{it}^g$, $z_{it}^m$, and $z_{it}^i$ ($t = 0, 1, 2, ..., \infty$), which affect growth, measurement, and implementation, respectively. These variables are independent (of each other and of calendar time $t$).² The manager knows the statistical distribution for each of these random variables. While we recognize a conceptual difference between $z_{it}^g$

¹The Markovian property is satisfied when the density of the state variable next period depends only on the the current value of the state and control variables. In other words, past realizations of the state and the control have no bearing on next period’s state.

²In principle, one could include nonzero covariance between variables of each type. We do not explore correlated uncertainty of that type here.
(which reflects uncontrollable environmental variability) and \( z^m_i \) and \( z^i_i \) (which reflect potentially controllable human error), we refer to all three as sources of “uncertainty”.

2. In each period, the growth of the stock of fish is as follows:

\[
x_t = z^i_i G(s_{t-1}),
\]

where \( G(s_{t-1}) \) gives the stock \( x_t \) as a function of the previous period escapement, \( s_{t-1} \), in the deterministic case.\(^3\)

3. Stock measurement, \( m_t \) is as follows:

\[
m_t = z^m_i x_t.
\]

The manager uses only the current measurement to form beliefs about the current stock, \( x_t \).

4. Given an announced quota, \( q_t \), the “attempted” harvest is \( a_t \equiv z^i_i q_t \), and the true harvest is:

\[
h_t = \min(x_t, a_t)
\]

5. The price of fish is \( p \) and the unit harvest cost is constant, \( c \). In the earlier part of the analysis, we assume that \( p - c \) is normalized to unity. However, we later allow costs to depend on the stock.

While some of these are innocuous, some are more restrictive in the sense that relaxing them would either make our model hard to implement or significantly affect our results. For example, we assume full knowledge of the density functions of the different types of uncertainty, as well as the parameter values associated with those. We also assume independence among the three types of uncertainty, and the absence

\(^3\)This is identical to the way uncertainty enters the growth function in Reed (1979). It differs from Roughgarden and Smith, who model a shock to the parameters of the growth function.

\(^4\)Note that this distribution of the implementation error assumes the absence of strategic behavior on the part of fishermen, and that their attempts are randomly distributed around the seasonal quotas. This assumption is consistent with that made by Roughgarden and Smith (1996).
of strategic behavior on the part of both the manager and the fishermen. While we recognize the pitfalls of making these important assumptions, our present aim is simply to discover the optimal policy and investigate its properties and compare these with the properties of a constant escapement policy.

We should also note the somewhat misleading use of the term “optimal policy” in this context. Specifically, Assumption 3 states that the manager uses only the current measurement when forming expectations. This assumption implies that the current measurement is the only state variable for the manager’s problem. Past measurements may contain some information about the current stock; a more sophisticated manager would use that information in forming expectations. There are three reasons why we assume that the manager ignores this information. The first is based on modelling choice and the second two are based on practical considerations.

First, one of our objectives is to determine the robustness of previous results to the inclusion of different (and multiple) sources of uncertainty. In order for this comparison to be as clear as possible, we want to use a model that nests the previous models in a simple manner. In particular, we want the nature of the optimization problem to remain the same. In previous models, the manager conditions her decision on a single state variable. We want to preserve this feature, because we want to be able to compare the decision rules. This comparison would be ambiguous if, for example, one decision rule was a function of a single piece of information and another was a function of many pieces of information.

Second, the optimization problem for a manager who uses past measurements is very complicated. Since all past measurements might contain some information, we would have to make one of two choices: we could include all past measurements in the state vector or we could treat the conditional probability distribution as a “state variable”. In order to implement either of these methods, we would have to use an approximation, such as truncating the history of observations to obtain a finite state vector, or approximating the distribution (e.g. by using a finite number of moments, and treating the moments as state variables).
Third, Assumption 3 provides a lower bound for the value of the fishery. A manager who uses more information could expect a higher payoff. Since we demonstrate the advantage of following a non-constant escapement policy, the use of greater information would only strengthen our results.

4.2 The Fishery Manager’s Problem

The fishery manager’s infinite horizon problem can be stated as follows:

\[
\max_{\{q_t\} \geq 0} \mathbb{E} \left\{ \sum_{t=0}^{\infty} \alpha^t h_t \right\}
\]

subject to

\[
x_t = \frac{z^g}{z^m} G(s_{t-1}) \\
\begin{align*}
st_t &= x_t - h_t \\
m_t &= z^m x_t \\
h_t &= \min(x_t, z^i q_t).
\end{align*}
\]

where \(\mathbb{E}\) is the expectations operator and the discount factor is \(\alpha\). We use the numerical technique of value function iteration to maximize (4) subject to (5).\(^5\) The fishery manager measures the stock each period and announces the seasonal quota on that basis. The dynamic programming equation (DPE) for this problem is as follows:

\[
J_t(m_t) = \max_{q_t \geq 0} \mathbb{E} \left\{ h_t + \alpha J_{t+1} \left( \frac{z^m}{z^i} z^g_{t+1} G(x_t - h_t) \right) \right\}.
\]

To numerically solve for the value function, we evaluate \(J_t(m_t)\) over an evenly spaced discrete grid of 200 measurements. We then employ cubic spline interpolation to generate what is essentially a continuous state-space representation of \(J(\cdot)\) and use this to solve for the optimal quota announcement. For a given measurement, the optimal announcement is the one that maximizes the expected fishery value over an infinite horizon.

The maximand in equation (6) has two terms. The first of these (\(\mathbb{E} h_t\)) represents the expected harvest in the current period, conditioned on the stock measurement in

\(^5\)For more information on this technique, see Judd (1998).
the current period. The second term is the expected future value of stock measurement in the following period, conditioned on the current stock measurement, \( m_t \), and the current quota announcement, \( q_t \). We discuss below how each of these is computed.

### 4.2.1 Expected Value of One-Period Return

The first term in the maximand of the dynamic programming equation (6) is the expected current period harvest. Given any stock measurement, \( m \), and any quota announcement \( q \), the expected harvest is:

\[
E\{h_t|q_t=q, m_t=m\} = \int_0^\infty f^A(a|q) \left[ \int_0^a x f^X(x|m)dx + a \int_a^\infty f^X(x|m)dx \right] da \tag{7}
\]

where \( f^A(a|q) \) is the conditional density of attempted harvest of \( a \) given an announced quota of \( q \) and \( f^X(x|m) \) is the conditional density of true stock of \( x \) given a measurement of \( m \).

The term \( \int_0^a x f^X(x|m)dx \) reflects the fact that an attempted harvest of \( a_t \) does not guarantee a harvest of precisely \( a_t \) since there may not be a sufficiently large stock of fish available (\( x_t < a_t \)). The term \( a \int_a^\infty f^X(x|m)dx \) reflects the case in which the stock is at least as great as the attempt (\( x_t \geq a_t \)). Thus, the term in the brackets represents the expected harvest when the attempt is \( a \) and the measured stock is \( m \).

To obtain the expected harvest when the measurement is \( m \) and the announcement is \( q \), we multiply this term by \( f^A(a|q) \) and integrate over all levels of attempted harvest, \( a \).

### 4.2.2 Expected Value of Future Returns

To obtain the optimal quota announcement we need to calculate the expectation of the value of future returns – the second term on the right side of equation (6). Here we provide the formula for the conditional expectation of the future payoff, given that the function \( J_{t+1}(m_{t+1}) \) is known. This expectation is taken with respect to the unobserved variables \( x_t \) and \( h_t \), and is conditioned on the measurement \( m_t \) and the
current decision $q_t$. The expectation is

$$\mathbb{E}\{J_{t+1}(m_{t+1})|q_t = q, m_t = m\} =$$

$$\int_0^\infty (J_{t+1}(y)|q_t = q, m_t = m) \int_0^\infty f^S(s|q, m) \int_0^\infty f^G(x|s) f^M(m|x) dx ds dy$$  \quad (8)

where $f^S(s|q, m)$ is the density of escapement of $s$ conditional on a quota of $q$ and a measurement of $m$, $f^G(x|s)$ is the density of true stock of $x$ conditional on previous period escapement of $s$, and $f^M(m|x)$ is the density of measurement of $m$ conditional on true stock of $x$. Interpretation of equation 8 is aided by first interpreting the underbraced terms. Term 1 is the conditional probability of next period’s measurement given a current period escapement, while Term 2 is the conditional probability of next period’s measurement given both the current quota and the current measurement. Thus, the product of Term 2 and $(J_{t+1}(y)|q_t = q, m_t = m)$ gives the contribution to next period value when next period’s measurement is $y$. Integrating over $y$ gives the desired expected value of future returns.

### 4.3 Numerical Computation of the Optimal Policy

The manager’s problem can be solved by adding the current period value and the future value of making an announcement for each stock measurement. The optimal announcement maximizes this value for each measurement. The first order condition for this problem equates the marginal expected value of the current harvest to the expected discounted value of the shadow value of the state (the stock measurement) in the next period.

We carry out the value function iteration procedure by assuming we know the value function, $J(m)$. Given a particular policy, $q(m)$, we can determine the conditional distribution of a measurement the next period given the current measurement, $m$. Thus, we can choose the optimal policy $q^*(m)$ for the assumed value function, $J(m)$. For an infinite horizon problem, the value of the program evaluated at $q^*(m)$ equals the value function, $J(m)$. The processes of updating of the value function by selecting
\( q^*(m) \) is repeated until the function \( J(m) \) converges. Convergence requires that \( J(m) \) for iteration \( i \) equals \( J(m) \) for iteration \( i + 1 \) for all \( m \). The function to which this procedure converges is the unique value function for this infinite horizon problem. The policy function associated with the unique value function is the optimal policy.\(^6\) The next section explains our choices of functional forms and parameter values.

4.4 Parameter Values and Functional Forms

For our base case numerical computation, we adopt the following parameter values and functional forms:

1. Fish growth follows the logistic curve:

\[
G(s) = rs \left(1 - \frac{s}{K}\right) + s \tag{9}
\]

where \( r (=1) \) is interpreted as the intrinsic growth rate and \( K (=100) \) is the carrying capacity of the environment.\(^7\)

2. \( z_t^g, z_t^m, \) and \( z_t^i \) are independent, stationary, uniformly distributed random variables of the following form:

\[
z_t^\xi = 1 + (2\hat{u}_t^\xi - 1)\varepsilon^\xi \quad \xi = \{g, m, i\} \tag{10}
\]

where \( \hat{u}_t^\xi \) is drawn from a uniform distribution lying between zero and one, and \( \varepsilon^\xi \) are parameters (larger \( \varepsilon^g \), larger variance of the distribution of \( z_t^g \)). For example, if \( \varepsilon^m = 0.5 \), \( z_t^m \) is distributed uniformly between 0.5 and 1.5 for all \( t \). Furthermore, if stock is 100, this implies measured stock is a uniform random variable with support \([50,150]\). The corresponding coefficient of variation is

\(^6\)In practice, one iterates until the difference between successive \( J \)'s is smaller than a predetermined tolerance level. We ran the model until the largest absolute difference in the value function vector was smaller than one-thousandth the carrying capacity from one iteration to the next. For our numerical results, this implied that we stopped when the value function had changed by less than 1.5e-003%.

\(^7\)See section on Sensitivity Analysis for results based on alternative growth functions and the different values of the parameter \( r \).
0.29. We will refer to \( \varepsilon^g \), \( \varepsilon^m \), and \( \varepsilon^g \) as the “uncertainty” in growth, measurement, and implementation, respectively. We will refer to a realization of \( z_t^g \), \( z_t^m \), or \( z_t^i \) as a “shock”.

3. Each continuous state and control space variable can take on any value between 0 and its upper bound. For example, with carrying capacity \( K = 100 \), and the growth uncertainty \( \varepsilon^g = 0.5 \), the stock in a given season will lie between 0 and 150 \( (= K \times (1 + \varepsilon^g)) \). Likewise, if the measurement uncertainty \( \varepsilon^m = 0.5 \), the measurement lies between 0 and 225 \( (= K \times (1 + \varepsilon^g) \times (1 + \varepsilon^m)) \).

5 Results

In the results that follow, we explore various combinations of the level and source(s) of uncertainty. We refer values of \( \varepsilon \) of .1 and .5 as “low” and “high” uncertainty, respectively. These refer to uniform random deviations of \( \pm 10\% \) and \( \pm 50\% \) around the mean. Given our assumptions relating to the distribution of the random variables, this uncertainty translates into coefficients of variation of 0.0577 and 0.2887 respectively.

Our references to \( \pm 50\% \) deviations as “large” deviations are, at least roughly, empirically based. For example, the growth \( (\varepsilon^g) \) in a fishery with notoriously high variability, the Southeast Pink Salmon fishery, expresses a fluctuation about the mean of about 53\% (calculations based on Quinn and Deriso (1999)). Measurement errors

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8 The manager’s stock measurement is a signal received by her, and does not represent her beliefs. Therefore, when a manager receives a measurement signal that is above 150, she interprets it as such and updates her prior beliefs using Bayes’ rule. Given the information she has, both her prior and posterior beliefs about the stock are bounded above by 150.

9 In reality, the fish stock can take extremely large values (including values above well above the carrying capacity) although it does so with very small probability. A possible modeling approach would be to define the state space to include all values that we think are possible. That alternative requires that the distribution of the measurement error is a function of the measurement. For example, for very large measurements, most of the weight of the distribution is below one. That generalization would greatly complicate the model. Our approach is to define the state space to exclude extremely unlikely values, thereby making it possible to use a measurement error that is independent of the measurement.
of as large as $\pm 50\%$ were considered also by Clark and Kirkwood (1986). Because of sparse data, we have no sound empirical basis for referring to uncertainty of $\pm 50\%$ in harvest implementation. But by considering a range of coefficients of variation, we hope to capture the magnitudes of the shocks that are likely in many fisheries.

### 5.1 Previous Results

For purposes of comparison, we begin by presenting the results obtained by authors of previous work. In the presence of only growth uncertainty and when present stocks can be measured accurately, the optimal policy is one of constant-escapement. This result is due to Reed (1979) and is presented in figure 1. Clark and Kirkwood (1986) note that if escapement can be measured with precision, but stock cannot, the policy function entails non-constant escapement (see figure 1).

![Figure 1: Optimal Policy with Growth Shocks.](image)

There are two interesting properties of this result. First, higher uncertainty increases the optimal expected escapement for large measured stocks.\textsuperscript{11} Second, higher

\textsuperscript{10}Here ‘very large’ uncertainty refers to a uniform random deviation of $\pm 90\%$ around the mean, the coefficient of variation of which is 0.577.

\textsuperscript{11}This result breaks down if the uncertainty is excessively high; for the growth function employed
uncertainty decreases the measured stock at which the fishery should be closed to harvest.

When we restrict attention to the class of constant escapement policies in the presence of all three sources of uncertainty, we find that the optimal constant escapement level is about 48% of the carrying capacity when the level of uncertainty ($\varepsilon$) is 0.1, and 69% of carrying capacity when the level of uncertainty is 0.5; these policies are shown in Figure 2. The former is almost identical to the deterministic optimum escapement level (47.5% of carrying capacity). By contrast, Roughgarden and Smith recommend that, in the face of these three sources of uncertainty, the target stock be set at 75% of carrying capacity in order to create a buffer away from extinction in the face of all the uncertainty. Under the constraint that a constant escapement policy must be used, we obtain results similar to those obtained by Roughgarden and Smith – an escapement level of 69% versus an escapement level of 75%. From the perspective of economic optimality however, both policies are significantly inferior to the optimal policy.

5.2 Our Results

In this section we explore various combinations of the levels and sources of uncertainty and present numerical results and graphs of the associated optimal policy functions. Recall that the optimal policy function exhibits the optimal announced escapement (measured stock minus announced quota) as a function of measured stock. In particular, we explore the following cases: low uncertainty (section 5.3), a single source of high uncertainty (section 5.4), and multiple sources of high uncertainty (section 5.5). In all cases the optimal policy function is a line of slope 1 for sufficiently low measured stock which indicates fishery closure for low stock measurements.

by Clark and Kirkwood, optimal expected escapement is non-monotonic when the coefficient of variation of the uncertainty is 0.46. Sethi (2001) lays down the conditions under which the optimal control is non-monotonic. With our growth function we obtain a similar result for a larger coefficient of variation viz. 0.52.
5.3 Small Uncertainty

In the absence of any stochasticity, the optimal policy is a “bang-bang” solution with constant escapement level at the point where the discount rate equals the slope of the logistic growth curve. As may be expected, small variations in the random variables - whether considered individually or all together - lead to a policy that is not significantly different from the deterministic rule. The solid line in figure 3 shows the graph of the optimal policy function when all uncertainty levels are low. This policy suggests that for low uncertainty, not only is the deterministic rule qualitatively appropriate (i.e. it suggests a constant-escapement policy), but it is also quantitatively appropriate since the escapement target is approximately equal to the deterministic policy.

5.4 Single Source of High Uncertainty

How does the optimal policy change when one of the sources of uncertainty is high, while the others are low? The answer depends on which variable is highly uncertain. The dashed line in figure 3 confirms Reed’s result that high uncertainty in growth does
Figure 3: Optimal Fishing Policy Under Multiple Small/Single Large Uncertainty.

not alter the optimality of a constant-escapement policy. This result is qualitatively similar to the one where only the implementation uncertainty is high. These results suggest that if only the growth or implementation sources of uncertainty is high, a constant-escapement policy is qualitatively appropriate since the slopes of these policies beyond the shutdown point are virtually flat.

There is a significant change in the policy, however, when measurements are highly uncertain (see figure 3). While higher measurement error unambiguously lowers the level at which the fishery is shut down, optimal escapement changes ambiguously as a function of measured stock. For example, compare the scenario of high measurement error with the deterministic case. For measurement levels just below 47, a higher measurement error implies a lower expected escapement, whereas for stock measurements above $K$ the optimal policy is associated with higher expected escapements with respect to the deterministic case.

It may seem counter-intuitive that a measurement error causes lower expected escapements below the deterministic fishery closure threshold. In order to simplify our explanation, we consider the case where there is no implementation error. As a further...
simplification, we compare the case with 0 measurement error and positive measurement error regarding the stock. The increase in measurement error has two distinct types of effects on the regulator’s optimization problem. First, higher measurement error in the current period changes the current decision for a given continuation value; second, higher measurement error in the future changes the continuation value of the stock.

To make this distinction precise, define \( \tilde{J}(z^gG(x-q)) \) as the continuation value in the absence of measurement error. In addition, define

\[
B(q, x) \equiv E_q(q + \alpha \tilde{J}(z^gG(x-q)) ,
\]

the expectation of the value of the fishery given the current stock and the current harvest, in the absence of future measurement error. We first compare the decision problems with and without current measurement error, in the absence of future measurement error. That is, we compare the solutions to the following two optimization problems

\[
\max_{x \geq q \geq 0} B(q, x) \quad \text{and} \quad \max_{x \geq q \geq 0} E \{B(q, x)\}.
\]

In the absence of current measurement error, the stock at which it is optimal to close the fishery, \( x^* \), is the solution to \( B_q(0, x) = 0 \). With current measurement error, the measurement at which it is optimal to close the fishery, \( m^* \), is the solution to \( E_{x|m} \{B_q(0, x)\} = 0 \). A sufficient condition for \( m^* < x^* \) is that \( B_q(0, x) \) is a convex function of \( x \). In this case we have

\[
E_{x|m} \{B_q(0, x)\} > B_q\left(0, E_{x|m} x\right) > B_q(0, m) \implies m^* < x^*. \quad (11)
\]

The first inequality follows from the (assumed) convexity of \( B_q(0, x) \) and Jensen’s inequality. The second inequality follows from two facts. First, our model states that \( x = m/z \), so \( x \) is a convex function of the random variable \( z \); this implies (again, by Jensen’s inequality) that \( E_{x|m} x > m \), i.e., the current measurement is a downwards biased estimate of the current stock. The second fact is that \( B_q(0, x) \) is an increasing function of \( x \), at least in the neighborhood of \( x^* \). If we had written the model in a slightly different manner, so that the current measurement is an unbiased estimate of the stock, convexity of \( B_q(0, x) \) would be both necessary and sufficient for \( m^* < x^* \).
Establishing the convexity of $B_q(0, x)$ involves evaluating a function of the first three derivatives of both $G(\cdot)$ and $\tilde{J}(\cdot)$, which is analytically intractable. However, we have the following intuitive explanation for why a small amount of measurement uncertainty would be likely to decrease the stock at which the fishery is closed, even when $m$ is an unbiased estimate of $x$. Consider a manager who faces no uncertainty at all and makes a stock measurement just under the kink (around 45 in figure 3). In a deterministic world, she would harvest nothing since the deterministic solution entails fishery closure below a stock of 47.5 (at a 5% discount rate). Now suppose she makes the same measurement but believes her measurements are prone to errors. In this case, the true stock is distributed around the measured stock which creates a positive probability of stocks for which the optimal quota is positive. Thus, in expectation, a measurement error implies a positive optimal quota for stock levels to the left of the kink of the deterministic policy. This result is consistent with Clark and Kirkwood because it is driven by fundamentally the same factor: imprecise knowledge about the fish stock when the manager sets the seasonal quota.

Now we briefly consider the effect of future measurement error on the optimal closure level, $x^*$. Future measurement error leads to a reduction in $x^*$ if and only if this measurement error causes the function $B_q(0, x)$ to shift up. An upward shift occurs if and only if the future measurement error decreases the shadow value of the measured stock. Once again, we cannot analytically determine the effect of measurement error on this shadow value. However, our model here is a special case of Theorem 2, Chapter 2 in Laffont (1989) who shows that the direction of change in the optimal control depends on the third derivative of the objective function. Unfortunately, it is hard to provide much intuition for this result.

Interestingly, the optimal policy associated with the high measurement uncertainty is nearly linear beyond the threshold of 30. The marginal escapement rate beyond this point is approximately 30% of the measured stock.
5.5 Multiple Sources of High Uncertainty

The combination of high uncertainties both in growth and implementation does not lead to a significant change in the shape of the policy function (figure 4). However, when measurements are also highly uncertainty, the marginal escapement rate increases significantly beyond the kink (figure 4), even when compared to the situation where the only high uncertainty is in the measurement.

![Figure 4: Optimal Policy Under Combinations of High Uncertainty.](image)

This suggests that uncertainty about fish population growth and large random deviations in the implementation of the seasonal quotas - by themselves or together - do not imply a significant policy change. However, imprecise stock estimation affects fishery policy significantly, both by itself and in the presence of other sources of uncertainty. These results imply also that the optimal escapement level under uncertainty: (1) is lower than the optimal deterministic escapement level when measured stocks are sufficiently small and (2) exceeds the optimal deterministic escapement level when measured stocks are sufficiently large. These results also suggest that an increase in only the measurement error causes the optimal escapement level to fall, as shown in figure 3. However, an increase in both the measurement error and the
implementation error increases optimal escapement at large stocks, as seen in figure 4.

5.6 Sensitivity Analysis

The results obtained above are derived under specific assumptions about the functional forms and parameter values. To evaluate the sensitivity of these results to changes in model specification or inputs, we also considered the following.

- Alternative specifications of the growth function. We analyzed three popular forms viz. the logistic, Gompertz, and Ricker functions.\(^{12}\)

- Alternative values of the intrinsic growth parameter of the logistic growth function. The parameter \(r\) in the logistic growth function can be interpreted as the marginal growth rate at a stock level close to zero. This parameter governs the rate of change of stock away from equilibrium. For the purposes of sensitivity analysis, we calculate the optimal policy function for values of \(r = .5\) and \(r = .75\) in order to compare with the base case policy function assuming \(r = 1\).

- More realistic versions of the harvest cost function. To include a more realistic “stock effect” on costs, we assume that the cost function has a simple and intuitive form: the marginal harvest cost \(C\) is given by \(C = \theta/x\), where \(x\) is stock and \(\theta\) is a parameter of the cost function (see Reed, 1979). \(^{13}\)

In each of these cases, we assessed model sensitivity by comparing the optimal policy function generated by our model to the optimal policy function generated under the new assumption. In all cases, we found that the optimal policy is virtually insensitive to the above modifications.


\(^{13}\)To use a reasonable value for \(\theta\), we borrow from recent ecological evidence which suggests that less than 10% of large predatory fish (tuna, billfish, etc.) remain in the global ocean (Myers and Worm 2003). Given that fishermen continue to target many of these species for harvest, it is reasonable to conclude that at \(.1K\), harvest effort remains a profitable enterprise, that is, that \(\theta < 10\). For the purposes of this exercise, we assume \(\theta = 5\).
5.7 Properties of the Optimal Policy

With the specification of the optimal policies under various types and combinations of uncertainty, we are in a position to make comparisons of some of their salient properties. To obtain these results, we assume that the policies are implemented in a fishery where the initial measured stock equals the carrying capacity (100 in this analysis) and that the discount rate is 5%. For each of the infinite-horizon policies, the variables we report are (1) the net present value of the commercial fishery over a period of 50 years and (2) the probability of extinction within 50 years.\footnote{The stock level 0 is an absorbing state, and when uncertainty is sufficiently high, this state can be reached from any other state, depending on the harvest policy pursued by the regulator. In that case, the long run probability of extinction is 1. However, the extinction probability within a finite number of periods differs under different scenarios. To illustrate these differences, we report the probability of extinction within 50 years.}

<table>
<thead>
<tr>
<th>Level of Uncertainty</th>
<th>Type of Uncertainty</th>
<th>Optimal Policy</th>
<th>Optimal CE Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>n/a</td>
<td>506 (0%)</td>
<td>506 (0%)</td>
</tr>
<tr>
<td>Low</td>
<td>All</td>
<td>502 (0%)</td>
<td>502 (0%)</td>
</tr>
<tr>
<td>High</td>
<td>Growth (others low)</td>
<td>502 (0%)</td>
<td>500 (0%)</td>
</tr>
<tr>
<td></td>
<td>Implementation (others low)</td>
<td>490 (0%)</td>
<td>490 (0%)</td>
</tr>
<tr>
<td></td>
<td>Measurement (others low)</td>
<td>439 (0%)</td>
<td>417 (2%)</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>433 (0%)</td>
<td>306 (57%)</td>
</tr>
</tbody>
</table>

Table 1: Fishery profit (probability of extinction) over 50 years.

These results are presented in Table 1. The table confirms that under low levels of uncertainty, the constant escapement policy performs well - both in terms of commercial profits and extinction risk. The same conclusion can be drawn even when growth or implementation uncertainties are high. However, when only high measurement uncertainties exist, the optimal policy performs slightly better than the constant escapement policy - both in terms of commercial profits (a gain of 5%) and stock viability (a gain of 2%). Our analysis shows that when all uncertainties are high, following the optimal policy yields a commercial value of $433 compared with...
a commercial value of $306 if the constant escapement policy is followed, a gain of 42%. Further, when all uncertainties are high and the optimal constant-escapement rule is used, the extinction probability over a 50 year horizon is 57%. This result can also be seen by comparing the cumulative densities of measured stock after 50 years associated with each policy. Figure 5 shows these densities; the optimal policy is generally associated with high stocks, whereas the likelihood of ending up with small stocks is larger when the constant escapement policy is followed.

![Figure 5: 50-Year CDF of Measured Stock Under High Uncertainties.](image)

Also note that a 3/4ths escapement policy does better than the optimal constant escapement policy in terms of extinction risk (43%), but is (obviously) worse in terms of profitability ($299). More importantly, like the optimal constant-escapement policy, the 3/4th escapement policy does worse compared to the optimal policy on grounds of both profitability and extinction risk.

It is useful to say a word about why we get lower extinctions even as it might seem that the constant-escapement policy calls for a larger escapement. First, it is useful to partition the state space into three ranges: $0 \leq m \leq 30$, $30 < m \leq 80$, $80 < m \leq 225$. In the first of these, both policies suggest fishery closure. While the
optimal policy suggests lower escapement in the middle range, it advocates *higher* escapement in the last range (since the policy function is upward sloping).

Next, due to the fact that both policies shut the fishery down at low state values, and the fact that each of the uncertainty structures is multiplicative, one can never get extinction in this model for low fish stock. However, when the stock of fish is large, the potential measurement can be significantly larger due to its multiplicative nature. For large measurements, the optimal policy calls for a large escapement (or a small seasonal quota) while the constant-escapement policy calls for a large quota since, by definition, the escapement is insensitive to the measurement. It is precisely the large quota values associated with the constant-escapement policy that make extinctions more likely. Essentially, the strength of the optimal policy is that it recognizes that large measurements are either misleading (if they are over 150) or very likely are indicative of stocks above the carrying capacity, which would imply a decline in stock in the subsequent period. The constant-escapement policy ignores this information, making it inferior.

6 Concluding Remarks

This research addresses a global concern of increasing importance; numerous economically and culturally important fisheries around the world are threatened with collapse. While there are many causes, uncertainty facing fishery managers is a central concern. In general, the economic literature has focused on stylized models at the expense of biological realism. Biological models of fishery management are often not solved for allocative efficiency. Instead, rules-of-thumb are calculated, mistreating the inherent tradeoff in any allocation problem.

We frame an economic allocation problem that incorporates three sources of uncertainty that have been identified as playing a central role in management decisions – stock growth uncertainty, stock measurement uncertainty, and harvest implementation uncertainty. The model is solved through iteration on the value function from the dynamic programming equation. Results include the following insights about
management:

- Low uncertainty (±10%) has no significant effect on policy, profits, and extinction risk vis-a-vis the deterministic rule.

- Growth and implementation uncertainties have only a small effect on optimal policy, profits, and extinction risk - even when uncertainties are high.

- Measurement uncertainty has the largest impact on fishery policy, profits, and extinction risk - especially when combined with growth and implementation uncertainty. This result suggests that stock surveys are important. There is clearly scope for future work exploring the dynamics of fishery policy under measurement error.

- Under highly stochastic environments, our results suggest fishery closure for measurements of fewer than \( \frac{K}{3} \) fish. Larger measurements should give rise to marginal escapement levels of about 0.30-0.40 (rather than 0 as in the constant escapement case).

- When the manager faces high uncertainty on all three fronts, the optimal policy does better - from both the stock survival and the commercial fishery value perspectives - than the rule-of-thumb constant escapement policy previously proposed.

While our results appear to be robust to some key parameter and functional form assumptions, the conditions on which they are based must be kept in mind. We assume full knowledge of the density functions of the different types of uncertainty, as well as independence among these. We also ignore age, stage, and spatial structure of the stock, as well as other metapopulation dynamics, and treat growth as a function only of aggregate stock. We assume non-strategic behavior on the part of both the fishermen as well as the regulator. Most importantly, the results we present were derived using numerical techniques, and to the extent that we did not explore the entire parameter or model space, they may be specific to our assumptions. These
caveats notwithstanding, this model is a step towards exploring fuller, more realistic models of optimal resource policies in complex and potentially highly uncertain environments.
References


