A Coasian Model of International Production Chains

Thibault Fally and Russell Hillberry*

October 2017

Abstract

International supply chains require coordination of numerous activities across multiple countries and firms. We adapt a model of supply chains and apply it to an international trade setting. In each chain, the measure of tasks completed within a firm is determined by transaction costs and the cost of coordinating more activities within the firm. The structural parameters that govern these costs explain variation in supply-chain length and gross-output-to-value-added ratios, and determine countries' comparative advantage along and across supply chains. We calibrate the model to match key observables in East Asia, and evaluate implications of changes in model parameters for trade, welfare, the length of supply chains and countries' relative position within them.

Keywords: Fragmentation of production, Transaction costs, Trade in intermediate goods, Boundary of the firm
JEL Classification: F10, L23

*Thibault Fally: UC-Berkeley and NBER; Russell Hillberry: Purdue University. We would like to thank Pol Antràs, Arnaud Costinot, Ben Faber, Rob Feenstra, Tomoo Kikuchi, Aart Kraay, Phil Luck, Ezra Oberfield, Andres Rodriguez-Clare, Ina Simonovska and seminar participants at UC Davis, UC Berkeley, Dartmouth College, Federal Reserve Bank of Philadelphia, Federal Reserve Board of Governors, Drexel University, Columbia University, John Hopkins University, UC Santa Cruz, Barcelona GSE, Penn State University, Temple University, University of Arkansas, ITAM, Purdue University, CEPR/CAGE/ECARES Conference on Global Fragmentation of Production, West Coast Trade Workshop at SIEPR/Stanford, NBER ITI Spring meeting, and the SED meetings for helpful comments and discussions. Erik Johnson provided excellent research assistance. Research for this paper has been supported in part by the governments of Norway, Sweden, and the United Kingdom through the Multidonor Trust Fund for Trade and Development. Contact: fally@berkeley.edu, Department of Agricultural and Resource Economics, University of California, Berkeley, CA 94720-3310, USA.

1 Introduction

The nature of international trade changed dramatically in recent decades, as vertically integrated production processes spread across international borders, increasing trade in parts and components along the way.[1] This phenomenon raises a number of important questions for economic policy: How large are the gains from international fragmentation, and how are they distributed across countries? How do changes in trade costs affect trade flows and the distribution of value added across countries? How has China's entry into the world trading system affected the international fragmentation of production? Answers to these questions require quantitative models that can represent the complexities of international production chains in a tractable form.

Recent evidence has documented substantial variation in the length of supply chains.[2] Even within chains, firms vary in their contribution to value added.[3] The length of supply chains and the degree to which they are internationalized are difficult to separate from decisions that determine firm scope. The Ford Model T, for example, was produced in a single plant, while the production of modern day automobiles can involve a myriad of heterogeneous suppliers scattered across multiple countries. International fragmentation is limited, ultimately, by the extent of fragmentation at the firm level. Yet the literature lacks a unified treatment that can explain endogenous firm boundaries within chains, formalize endogenous chain lengths, and determine comparative advantage within and across chains.

We offer a framework that accomplishes these goals. In the model, an optimal allocation of tasks determines jointly the scope of sequentially-arranged firms of varying size, the length of chains and the sequence of countries in production. We calibrate the model using key moments from input-output tables on East Asia and the United States.[4] Our focus on East Asia reflects the importance of international fragmentation in that region. Based on our calibration, we are able to quantify the impact on intermediate and final goods trade, fragmentation and welfare, of changes in: 1) international trade costs, 2) productivity in China, 3) transaction costs in China, and 4) a reduction in bilateral trade costs between the US and China.

[1]Baldwin (2012) surveys these developments and provides insights into how they should affect our thinking about the economics of international trade and trade policy.

[2]See Antràs et al. (2012) and Fally (2012), who calculate a (dollar value-weighted average) number of plants through which an industry's output travels before reaching consumers. We model an endogenous measure of firms involved in a sequential production chain and link our theory to the indices derived in Antràs et al. (2012) and Fally (2012).

[3]See Kraemer et al. (2011), who illustrate the distribution of value added for Apple iPhones and iPads.

[4]Such tables have been used to quantify the extent of fragmentation and the allocation of value across countries. Johnson (2014) surveys a series of papers that use such tables to calculate value added trade. Koopman et al. (2010) use Chinese tables to calculate the domestic content of China's exports. Antras et al. (2012) derive indices of supply chain length from input-output tables.

A central theoretical contribution is the development of a tractable framework in which supply chain length is endogenous. Supply chains vary in the number of participating firms because of endogenous differences in firm scope.[5] We modify the supply chain model of Kikuchi et al. (forthcoming), in which firm scope is determined by a Coasian tradeoff between the cost of coordinating tasks inside the firm and the costs of conducting market transactions.[6] Within countries, outcomes are driven by two key parameters: one that governs coordination costs within the firm and one that summarizes domestic transaction costs. Our continuous representation of a firm allows us to derive strong and transparent links between structural parameters and observables in the data. Specifically, the gross-output-to-value-added ratio at any point in the chain is equal to the ratio of the Coasian parameters that govern coordination costs and transaction costs, respectively.

We allow these Coasian parameters to vary across countries and develop implications for international trade. Vertical specialization in our model is tightly related to firm scope. Because inter-firm transactions are more costly downstream, equilibrium firm scope is larger downstream. In turn, this pattern affects the sorting of countries along the chain. Within a given chain, the most downstream countries are those in which firms are most able to displace transaction costs by expanding firm scope. Transaction costs affect absolute, not comparative, advantage within a given chain, but transaction costs have an indirect effect on countries' average position in chains. Countries with high transaction costs are more likely to participate in chains for which the country has low coordination costs, which means that such countries tend to be positioned downstream.[7]

We examine the effect of trade costs on trade and fragmentation in this setting. Trade tends to increase the extent of fragmentation along several dimensions. A reduction in trade costs between two countries affects fragmentation at all stages along chains and decreases firm scope even for firms that do not directly offshore production but are related to firms that do. The reduction in firm scope along the chain is associated with the decrease in average costs, especially downstream, and contributes to the reduction in final goods prices. We derive analytical results that reveal transparent links between the impact of fragmentation on firm

[5] In the model atomistic firms make optimal scope decisions given the prices of factors, inputs and outputs, and their location in the production chain. The equilibrium can also be interpreted as the optimum plan solved by a single social planner, i.e. a multinational firm allocating tasks and quantities per unit of final output across plants to produce the final good at minimum cost.

[6] Kikuchi et al. (forthcoming) establish the existence of a discrete firm partial equilibrium within a single country. We develop a continuous firm treatment that facilitates tractable analytical solutions and calibration, and extend it to a multi-country general equilibrium setting. We also introduce a shadow market for tasks that formalizes the Coasian tradeoffs facing a chain of sequential firms.

[7] Countries with a high value of both γ and θ for a given variety may find it unprofitable to participate in the chain at all.

scope, final goods prices and a shadow cost of tasks that governs firm scope along the chain.

As trade costs decrease, countries tend to move downstream along chains and to enter new chains. We illustrate our finding in a partial-equilibrium setting (holding the set of participating countries and their labor cost constant) and in a two-country general equilibrium setting. In the latter, we also use our framework to examine the response of trade flows to trade costs, both in gross flows and value added content, and the welfare gains from trade. We compare our results to a single-stage Eaton and Kortum (2002) model: the gains from trade are relatively larger in countries that tend to specialize downstream and smaller in countries that tend to specialize upstream.

In order to explore the quantitative implications of our framework, we calibrate a numerical version of our model to match key features of input-output relationships in East Asia. This exercise relies on international input-output tables produced by IDE-JETRO. These data cover the US and nine East Asian countries. This region is interesting because production fragmentation there has grown quickly and is highly prevalent. The IDE-JETRO data are unique in that they track flows in four dimensions: from the making industry in the origin country to the using industry in the destination country.[8] To illustrate our findings, we adapt recently developed quantitative measures of firm position (i.e. upstreamness) to a multi-country setting and track border crossings. Our calculations indicate increasing international fragmentation over time, especially in key industries like electronics.

While the model has rich implications for trade and the fragmentation of production, its relative parsimony is useful for the purpose of calibration. We calibrate our model by targeting key moments such as GDP per capita, value added, countries' average position in international supply chains and gross-output-to-value-added ratios. All these moments imply large cross-country differences in productivity, transaction costs and coordination costs.

We use the calibrated model to conduct counterfactual exercises regarding changes in key structural parameters. We first examine what happens when cross-border trade costs decrease by 10%. In this counter-factual simulation, we tend to find larger gains from trade than predicted by Arkolakis, Costinot and Rodrigues-Clare (2012)'s formula based on imported final goods, but smaller gains than in Costinot and Rodriguez-Clare (2014) with input-output loops. We also examine the response in terms of the VAX ratio (the value-added content of exports) as defined by Johnson and Noguera (2012). We find that a decrease in trade costs leads to a decrease in the VAX ratio in all countries, which can be interpreted as an increase in cross-border fragmentation. In subsequent counterfactual exercises, we simulate a 10% increase in productivity in China and a 10% decrease in Chinese transaction costs. Both

[8]Other data that report such figures, like the World Input-Output Database (WIOD), impute these values assuming proportional treatments.

the productivity and transaction cost shocks produce similar changes in Chinese welfare, but the shocks have different implications for the organization of international production and other countries' welfare. The productivity shock causes China to move downstream while the rest of the world moves upstream. Instead, the reduction in Chinese transaction costs causes a relative move upstream by China. Reducing transaction costs in China also lengthens global supply chains. This is entirely consistent with the model's qualitative predictions, but the calibration teaches us that the shock to Chinese transaction costs has relatively larger effects outside of China. Finally, we simulate a 10% reduction in trade costs between China and the US. In this counterfactual, we find that fragmentation leads to larger trade responses.

Relationship to the literature: The paper contributes to the literature in two broad ways: 1) we develop a model of international production sharing that formalizes a role for firms (as distinct from tasks) in a sequential, multi-country general equilibrium with endogenous chain length, 2) we calibrate our model and provide quantitative implications using input-output tables for East-Asian production. We discuss the literature surrounding each of these contributions in turn.

1. Models of production chains. An important question in the literature on international production chains is the spatial organization of production across countries. We contribute to this literature in two ways: by endogenizing the extent of fragmentation across firms and by endogenizing the relative position of countries along the chain.

In recent work, Costinot *et al.* (2012) derive an explicitly sequential multi-country model in which mistakes can occur with given probability and these mistakes destroy all accumulated value. They show that countries with relatively high probabilities of mistakes are situated upstream. The intuition for this result broadly follows Kremer (1993), that higher rates of mistakes do less damage if they occur upstream. Since tasks are indistinguishable from firms in their model, Costinot *et al.* (2012) cannot inform questions related to the scope of firm activities or the number of firms participating in a chain.

We maintain the continuum of sequential tasks, but we also formalize the firm's internalization decision and endogenize the range of firms involved in the chain. The motivation for this follows Coase (1937), and our mathematical framework is inspired by Kikuchi et al (forthcoming), who show how Coase's insights can be applied to production chains. Kikuchi et al (forthcoming) solve their model in a closed-economy partial equilibrium setting, and employ discrete firms. We adapt their framework to a continuum of firms in a multi-country setting where countries differ in key parameters governing transaction costs and diseconomies of scope.

As in Costinot *et al.* (2012), we examine how countries specialize along the chain, but the patterns of specialization are now driven by interactions between firm scope, transaction costs and ad-valorem trade costs affecting cross-border transactions. In addition, we offer explicit links between the Coasian structural parameters in our model and empirical objects that can be observed or constructed from input-output tables. These links make calibration of the model relatively straightforward compared to other models in the literature.

Several models fix the number of production stages by assumption (Krugman and Venables 1996, Hillberry and Hummels 2002, Yi 2003, 2010, Johnson and Moxnes 2013). The focus of this literature is often the geographic location of each production stage, relative to the other(s), and so a finite and countable number of stages is useful for analytical purposes.[9] Relative to our work, these models avoid the question of the allocation of activities or tasks across stages, and focus on the extensive margin of completing a specific stage in a certain location.

These models are also silent about why some countries specialize upstream while others are downstream. In Yi (2010) and in Johnson and Moxnes (2013), for example, the specialization of countries along the chain is driven by exogenous productivity shocks and trade costs. This literature makes important insights about non-linear responses of trade to trade costs and differences between gross and VA trade. Our model also contains these forces, but we introduce intra-firm coordination costs and inter-firm transaction costs as additional sources of cross-country heterogeneity. One goal of our paper is to understand the robustness of these insights to the richer theoretical structure we offer, where the extent of fragmentation and the specialization of countries along the chain are endogenous.[10]

2. Quantitative implications. We contribute to a recent quantitative literature on value chains by using new indexes to calibrate our model of cross-border fragmentation and examine the effect of trade costs on the organization of production chains, trade and welfare.

Our quantification exercise relies on input-output matrices that we exploit in a new way. Input-output matrices and direct requirement coefficients are traditionally taken as an exogenous recipe that is essentially determined by technology. Instead, we argue that input-output matrices reflect transactions in intermediate goods between firms that are themselves endogenous economic outcomes. We show that these tables can be informative about the position of firms within supply chains that link firms both within national borders and across them. Unlike previous papers, our theory determines the allocation of tasks across firms and the length of production chains endogenously, and can thus shed some light on equilibrium input-output relationships when fragmentation is endogenous, both across and within countries.

[9] Antràs and Chor (2013) offer a different perspective by taking the location and length of production chains as exogenous but examining the optimal allocation of ownership along the chain.

[10] In an earlier version of the paper we showed that the model can be adapted to include a capstone assembly sector that uses the output of several optimized chains as inputs. Assembly adds realism, but it also requires additional parameters that complicate calibration of the model.

Under the assumption that IO tables effectively summarize plant-to-plant movements for a representative firm in each industry, matrix algebra can be used to calculate, for each industry in the table, two numerical values: i) a measure of the industry's "distance" from final demand (where distance is a count of the number of plant boundaries that will be crossed prior to final consumption) and ii) the average number of stages embodied in an industry's production.[11] We also examine more traditional indexes of fragmentation such as gross-output-to-value-added ratios and the share of intermediate goods in trade. We show that, within our framework, we can map each of these indexes to structural parameters and key summary statistics of our model. These mappings are useful when we calibrate the model to data on interregional input-output relationships in East Asia.

A key purpose of this exercise is to offer a model comparison vis-a-vis other papers in the literature. A prominent literature has emphasized that intermediate goods trade magnifies the effect of trade costs on trade. Yi (2010) and Johnson and Moxnes (2013) focus on the response of trade to trade cost shocks, whereas Krugman and Venables (1996), Hillberry and Hummels (2002), Yi (2010) and Johnson and Noguera (2014) link the spatial clustering of activities to trade costs and intermediate goods trade.[12] Clustering also occurs in our model, with sequential activities locating so as to avoid trade costs. Our calibrated model can be used to investigate the response of trade to trade cost shocks, as in Johnson and Moxnes (2013) or Yi (2010).

We also contribute to the recent literature on the welfare implications of trade cost change. Arkolakis, Costinot and Rodriguez-Clare (2012) show that a broad class of models imply the same response of welfare to trade costs, provided that the models are calibrated to generate the same trade response to trade cost change. Costinot and Rodriguez-Clare (2014) and Melitz and Redding (2014) show that welfare effects are magnified when intermediate goods trade is involved. Like other papers in the literature, these presume an explicit input-output relationship that governs supply chain length, in contrast to the endogenous length in our model. The Armington framework used in these papers also precludes movement along the extensive margin (in terms of countries involved in supply chains), while our theory allows this. Our calibrated model implies larger gains than in standard trade models like those described by Arkolakis et al (2012), especially for countries that tend to be downstream, but smaller gains than Costinot and Rodriguez-Clare (2014) or Melitz and Redding (2014).

[11]The first is described as "distance to final demand" in Fally (2012) and "upstreamness" in Antras et al (2012). The second is developed in Fally (2012). Both indexes are computed using the BEA input-output tables for the US. Here we extend these indicators to multi-rational input-output tables.

[12]More recently, Kee and Tang (2013), Bernard et al. (2014) and Antràs et al. (2014) have used firm-level data to examine both intensive and extensive margins in import decisions. Firm-level data, however, do not allow a full consideration of supply chains over several countries. Multi-country input-output tables are more suitable for exercises like ours.

2 Model setup

We develop a model where the production of each variety of final good requires a continuum of tasks and firms organized across countries. We describe, in turn, consumers' preferences in final goods, tasks and firms involved in the production of each good, the forces shaping firm scope and firm entry along the chain, differences between varieties and the labor market.

Preferences: Consumers have identical Cobb-Douglas preferences over varieties of final goods indexed by ω:

$$U_i = \int_{\omega \in \Omega_i} \alpha_i(\omega) \log y_i(\omega)\, d\omega \tag{1}$$

where Ω_i denotes the set of varieties available to consumers in i (fixed), $y_i(\omega)$ denotes quantities of final goods and $\alpha_i(\omega)$ is a constant term such that $\int_{\omega\in\Omega_i} \alpha_i(\omega) d\omega = 1$. We obtain that expenditures in each variety ω equals:

$$p_i^Y(\omega) y_i(\omega) = \alpha_i(\omega) L_i w_i \tag{2}$$

where $L_i w_i$ refers to income and p_i^Y to final good prices.

Tasks and firms along the chain: In order to produce the final good of variety ω, a range $[0, 1]$ of tasks must be performed sequentially. These tasks may be performed across different firms and different countries.

Firms are arranged sequentially along the chain to produce each good ω. A chain is specific to each variety ω of the final good and the location of final producers. On each chain, we assume that there is a continuum of firms indexed by f. Firms may be located in different countries. For each chain, we rank countries along the chain and index by $i(n, \omega)$ the n^{th} country, with $i(1, \omega)$ indicating the most downstream country and $i(N, \omega)$ the most upstream country along the chain.

We denote by $F_n(\omega)$ the range of firms involved in the chain in the n^{th} country $i(n)$. An elementary firm df performs a range $s_{nf}(\omega)$ of tasks. Both the range of firms $F_n(\omega)$ and firm scope $s_{nf}(\omega)$ are endogenous, but the range of tasks performed across all firms must sum up to one to obtain a final good:

$$\sum_n \int_{f=0}^{F_n(\omega)} s_{nf}(\omega) df = 1 \tag{3}$$

Denoting $S_n(\omega) = \int_{f=0}^{F_n(\omega)} s_{nf}(\omega) df$ the total range of tasks to be performed in country n, the last constraint can be rewritten:

$$\sum_n S_n(\omega) = 1$$

for all chains ω.

Coordination costs: There are costs and benefits to fragmenting production across firms and countries. Fragmentation across firms reduces total costs because of diseconomies of scope. As firms must manage employees across different tasks and perform tasks that are away from their core competencies, unit costs increase with the scope of the firm. We will refer to these costs as "coordination costs" that occur within the firm.

Formally, we assume that an elementary firm df in country i requires one unit of intermediate goods and $c_i(s,\omega)df$ units of labor which is a function of firm scope s. The cost of labor is w_i in country i and labor is the only production input besides intermediate goods. We assume that c_i is convex in firm scope s, thus generating gains from fragmentation across firms.

In particular, we specify the following labor requirements:

$$c_i(s,\omega) = a_i(\omega)\frac{s^{\theta_i(\omega)+1}}{\theta_i(\omega)+1}. \tag{4}$$

where $a_i(\omega)$ and $\theta_i(\omega)$ are specific parameters for each country i for variety ω.[13] The marginal cost of performing additional tasks within the firm increases with s. This follows recent work on the division of labor (the specification is similar to Chaney and Ossa, 2013), and in this context can represent the productivity loss associated with movement away from the firm's core competencies. $\theta_i(\omega)$ parameterizes "coordination costs" and governs the convexity of the cost function. The higher is $\theta_i(\omega)$, the greater the increase in costs when firms need to manage a larger range of tasks.[14] Accounting for the unit cost of labor w_i in country i, the cost function for value added by an elementary firm df is $w_i c_i(s,\omega)df$.

Transaction costs: Fragmenting production across firms incurs transaction costs. We model transaction costs like iceberg transport costs in standard trade models. Let $q_{i,f}(\omega)$ be the quantity of an input for variety ω produced by firm f in country i. A transaction in country i with an elementary firm df involves losing a fraction $\gamma_i df$ of the good when upstream firm $f+df$ sells to firm f.

$$q_{i,f+df}(\omega) \;=\; q_{i,f}(\omega)\,(1+\,\gamma_i\,df) \tag{5}$$

Within each country, quantities thus follow a simple evolution depending on transaction costs γ_i and the position on the chain f. As we go upstream, quantities increase exponentially with

[13] $a_i(\omega)$ and $\theta_i(\omega)$ are constant along the chain (for a given country).

[14] Note that we assume diseconomies of scope but constant returns to scale in production. This differs from Chaney and Ossa (2013) and more closely follows Kikuchi et al (forthcoming). In keeping with Kikuchi et al (forthcoming), this framework allows us to examine patterns of fragmentation across firms while keeping a perfectly-competitive framework where the competitive allocation of tasks across firms is optimal.

the number of firms f participating in the chain:

$$q_{i,f}(\omega) = \mathrm{e}^{\gamma_i f} q_{i,0}(\omega) \tag{6}$$

Since part of the production is lost when transactions occur, upstream firms must produce larger quantities. The necessary increase in quantities is starker when transaction costs are high and when the chain is more fragmented.

In a similar fashion, a *cross-border* transaction between two consecutive countries $i = i(n)$ and $j = i(n+1)$ along the chain involves an iceberg trade cost $\tau > 1$ such that:

$$q_{j,0}(\omega) \; = \; \tau \, q_{i,F_i}(\omega) \tag{7}$$

where $q_{j,0}(\omega)$ denotes the quantities produced by the most downstream plant in the upstream country j and $q_{i,F}(\omega)$ denotes quantities produced by the next plant, i.e. the most upstream plant $f = F_i$ in the next country i along the chain, going downstream. For simplicity, we assume away geographical elements other than borders and impose a common border cost.[15] Cross-border trade costs τ also apply to trade in the final good, between the most downstream firm and final consumers if those are located in different countries.

Market structure: We assume perfect competition. Since we have constant returns to scale in quantities,[16] the price of each variety in each location equals its unit cost of production. Consistent with the perfect competition assumption, we impose free entry and zero profits. Imposing the zero profit condition everywhere along the chain, and for the chain as a whole, implies that the least cost solution to the problem is consistent with perfect competition as we will show in Lemma 1. The zero profit constraint will hold at optimized values of s_{if}, which will also be incentive compatible in equilibrium for every firm in the chain.[17]

Prices along the chain: The price of intermediate goods at each step along the chain is equal to their unit cost of production. Here, this cost accounts for all transaction costs and labor

[15] In our setting, there is a continuum of firms but only a discrete number of countries involved sequentially. When crossing a border, the unitary transaction cost is $\tau + \gamma_j df$ but $\gamma_j df$ is infinitesimally small relative to τ with a continuum of firms.

[16] There are decreasing returns to scope in tasks, s_{if}, but constant returns in terms of quantities, $q_{i,f}$.

[17] In the competitive equilibrium, tasks can be allocated across firms through a series of contracts. For instance, one can have a recursive contracting process: each firm f takes as given the measure of tasks $\bar{S}_{if}$ that must be completed before selling the good to the next firm, but choses the measure of tasks to complete in-house $s_{if}df$ and the measure of completed tasks to be required of the subsequent upstream firm, $\bar{S}_{i,f+df}$, such that $\bar{S}_{if} = \bar{S}_{i,f+df} + s_{if}df$. Conversely, one can think of a forward contracting process: firms take as given the range of tasks $\bar{S}_{i,f+df}$ performed by upstream firms and chooses the range of tasks $\bar{S}_{if} = \bar{S}_{i,f+df} + s_{if}df$ to be completed before selling to the next downstream firm. Both approaches lead to the same outcome. We develop this point in more length in the Appendix (proof of Lemma 1).

costs incurred by all upstream firms. Within country borders, the price of intermediate goods satisfies the following differential equation which describes its evolution along the chain:

$$p_{if}(\omega) = w_i c_i(s_{if})df + (1 + \gamma_i\, df)\, p_{i,f+df}(\omega) \tag{8}$$

where $c_i(s_{if})$ denotes the cost of performing a range s_{if} of tasks at stage f in country i as specified above. This equation is similar to its counterpart in Costinot, et al. (2013) and also features increasing intermediate goods prices as we go downstream. A key difference, however, is that the labor share is endogenous since s_{nf} is endogenous and thus not simply driven by differences in input prices along the chain. In particular, the cost of inputs per unit of labor is no longer necessarily larger for downstream firms. Many of the results in Costinot et al (2012) are driven by this feature and thus no longer hold in our framework.

Across borders, the price is simply multiplied by the international trade cost τ:

$$p_{j,F}(\omega) = \tau\, p_{i,0}(\omega) \tag{9}$$

for cross-border transactions from the most downstream plant in j to the most upstream plant in i. This arbitrage condition also applies to final goods.

Industry heterogeneity: While the previous assumptions are sufficient to generate interesting patterns of specialization along a particular chain, we still need to specify how chains vary across varieties. Following Eaton and Kortum (2002), we assume that labor efficiency is a random variable drawn independently across varieties and countries. Specifically, we assume that the labor cost parameter $a_i(\omega)$ is drawn from a Weibull distribution as in Eaton and Kortum (2002). For each country i, the cumulative distribution function for a_i is:

$$Proba(a_i < a) = 1 - \mathrm{e}^{-T_i a^{\xi}} \tag{10}$$

where T_i parameterizes the country average productivity and where ξ is inversely related to productivity dispersion. Note that $a_i(\omega)$ is thus constant along the chain for a specific country and variety ω. Unlike Yi (2003, 2010), Rodriguez-Clare (2010) and Johnson and Moxnes (2013), our framework does not require $a_i(\omega)$ to differ across tasks along the chain to generate trade in intermediate goods. Another component of the cost function is $\theta_i(\omega)$. We will explore different settings. In section 4.1, we do not impose any restriction on $\theta_i(\omega)$. In section 4.2 we only consider two countries U and D: one where $\theta_U(\omega) = \theta_U$ across all varieties, and another country with $\theta_D(\omega) = \theta_D < \theta_U$ across all varieties. In section 5 (the calibration exercise), we allow $\theta_i(\omega)$ to vary across countries and varieties, assuming that $\theta_i(\omega)$ is log-normally distributed with a country-level shifter $\bar{\theta}_i$ and a common standard deviation.

Labor supply: Finally, to close the model, we assume that workers are homogeneous and perfectly mobile within each country, with an inelastic supply of labor L_i in country i.

Labor demand corresponds to unit labor requirement at each stage, multiplied by output $q_{i,f}$, summing across all varieties and all stages performed in the country. Factor market clearance sets labor demand equal to the value of fixed labor supply.

$$\int_\omega \int_f q_{i,f}(\omega) c_{i,f}(s_{if}, \omega) = L_i \tag{11}$$

By Walras' law, trade is balanced.

Equilibrium can then be characterized as:

Definition 1 *For each variety of good ω, a partial equilibrium is a competitive equilibrium taking wages w_i and final consumption y_i given, defined as an allocation of tasks s_{if} to firms $f \in [0, F_i]$ and countries i to rank positions $n = 1, ...N$, a set of quantities q_{if} and intermediate prices p_{if}, such that: only the lowest-price chain produces (with free entry of chains and firms); prices equal marginal costs all along the chain; all tasks are performed (3); prices and costs satisfy (4), (8) and (9), and quantities satisfy (6) and (7).*

Definition 2 *General equilibrium is defined as a set of wages w_i to satisfy the labor market clearing condition (11), a set of final demands $y_i(\omega)$ as in (2) and production chains in competitive equilibrium as described in Definition 1.*

3 Partial equilibrium: optimal organization of chains

In this subsection, we take wages w_i as given and focus on the optimal fragmentation and location of production for a specific chain corresponding to a final good variety ω. For the sake of presentation, we drop the index ω. The reader should keep in mind, however, that the optimal fragmentation and allocation of value across firms, as well as costs parameters a_i and θ_i, are all specific to each variety of final good ω. For a given chain, we can reformulate the equilibrium as the solution to a social planner's problem.[18] Given our assumption of perfect competition and

[18]One could also view the partial equilibrium as the solution to a cost minimization problem for a large integrated multinational corporation allocating tasks across plants. The firm's price and quantity choices in final goods markets would not affect the organization of the optimal chain.

constant returns to scale, prices equal unit costs and the competitive equilibrium corresponds to the social optimum. This result is standard with a discrete number of firms and it holds here with a continuum of tasks and firms along each chain. In particular, we show in the appendix that the first-order conditions associated with the social planner's problem correspond to the free entry conditions and firm scope choices in the competitive equilibrium.

Lemma 1 *Taking wages as given (Definition 1), a competitive equilibrium is unique and corresponds to the social planner's solution, i.e. it minimizes the cost of producing final goods subject to the full range of tasks to be performed sequentially along the chain.*

Let us denote by $i(n)$ the ranking of countries along the chain, with $i(1)$ being the most downstream country and $i(N)$ the most upstream country, assuming that N countries are involved in the chain. One should keep in mind that the ranking of countries is an equilibrium outcome that we will characterize subsequently.

As expressed in Lemma 1, equilibrium can be summarized by the following optimization problem:

$$\min P_1 \tag{12}$$

$$\begin{aligned} \textit{over:} \quad & i(n)\ ,\ s_{nf}\ ,\ F_n\ ,\ S_n\ ,\ P_n \\ \textit{under the constraints:} \quad & P_n = \left[\int_{f=0}^{F_n} \mathrm{e}^{\gamma_{i(n)} f} c_{i(n)}(s_{nf}) df \ + \ \mathrm{e}^{\gamma_{i(n)} F_n} \tau P_{n+1} \right] \\ & S_n = \int_{f=0}^{F_n} s_{nf} df \\ & \sum_{i=1}^{N} S_n = 1 \end{aligned}$$

where N is the optimal number of countries involved in the chain and $P_n \equiv p_{0,n}$ denotes the price at the most downstream stage in country $i(n)$ at the n^{th} position. Recall that exponential terms $\mathrm{e}^{\gamma_{i(n)} f}$ reflect the evolution of quantity requirements along the chain as described in equation (6). The transaction cost parameter $\gamma_{i(n)}$ and the cost function $c_{i(n)}(s)$ are indexed by $i(n)$ because they depend on which country $i(n)$ is at the n^{th} position upstream. As an abuse of notation, P_{N+1} refers to the price of the most upstream good and is set to zero.[19] The solution to the model in autarky occurs when each country i produces all tasks $S_i = 1$ purchases inputs at price $P_{i+1} = 0$.

[19]Alternatively, we could set an exogenous price $P_{N+1} = \bar{p}$ of the most upstream good reflecting the price of primary commodity such as oil and minerals available from an outside economy that trades for final goods.

The optimization problem described in (12) can be formulated as a nested optimization problem. In the inner nest, firms in a specific country $i(n)$ are organized to minimize the price P_n of goods exported by this country, conditional on a given measure of tasks S_n to be performed and the price P_{n+1} of the imported intermediate good. The outer nest allocates measures of tasks to be completed to each country, and determines the import and export prices of participating countries, conditional on a sequential ranking of the countries. The solution must also provide a mapping of countries i to their rank order position $n = 1, ..., N$ and determine the number of countries N that participate in each chain. Our solution method is to first solve the within-country problem; then solve the global problem for any given ranking of countries. The optimal rank order and the choice of N are be determined by comparing minimized prices of each chain.[20]

3.1 Fragmentation of production within countries

Before turning to the cross-border organization of chains, we focus on the within-country problem. In this problem we allocate tasks s_{if} across firms $f \in [0, F_i]$ to minimize country i's last-stage (export) price P_i, given a measure of tasks to be completed and an import price.[21] P_i can be expressed as the solution of the following optimization:

$$\widetilde{P}_i(S_i, P_i^M) = \min_{s_{if}, F_i} \left[\int_{f=0}^{F_i} \mathrm{e}^{\gamma_i f}\, w_i c_i(s_{if}) df \;+\; \mathrm{e}^{\gamma_i F_i} P_i^M \right] \tag{13}$$

under the constraint:

$$\int_{f=0}^{F_i} s_{if} df = S_i \tag{14}$$

To examine the optimal allocation of tasks across firms and the optimal range of firms, it is useful to introduce the Lagrange multiplier λ_i associated with the constraint $\int_0^{F_i} s_f df = S_i$.

The first-order conditions of this planning program are:

$$\text{For } s_{if}: \quad e^{\gamma_i f} w_i c_i'(s_{if}) \;=\; \lambda_i \tag{15}$$

$$\text{For } F_i: \quad e^{\gamma_i F_i} w_i c_i(s_{iF_i}) \;+\; e^{\gamma_i F_i} P_i^M \gamma_i \;=\; s_{iF_i}\, \lambda_i \tag{16}$$

These conditions help us solve for firm scope (s_{if}) and the number of firms involved in the chain (F_i). Both s_{if} and F_i depend on λ_i, the shadow cost of a task.

Equation (15) defines a shadow market for tasks. All firms in the chain provide a measure

[20] Proposition 1 identifies parametric restrictions on rank orderings of countries that allow us to rule out most of the possible rank orders as optimal solutions. This substantially eases our computation of the optimum.

[21] The autarky solution of the model is the solution to the following problem for $S_i = 1$ and $P_i^M = 0$.

of tasks s_{if} such that their marginal cost of tasks equals the shadow price of a task, λ_i. In this way, the conditions that determine the scope of individual firms also define the allocation of tasks across firms that minimizes the cost of producing a measure of tasks S_i in country i.[22]

Condition (15) offers an additional insight about the relationship between firm heterogeneity and relative position along the chain. A move upstream (i.e. towards higher index f) increases required quantities $e^{\gamma_i f}$, which must be balanced by a reduction in the marginal cost $c_i'(s_{if})$. Hence, with convex costs, condition (15) implies that more upstream firms have smaller firm scope s_{if} and provide less value added. We can be more explicit about this using our parameterization: $c_i' = a_i s_{if}^{\theta_i}$, which implies that firm scope is log-linear in upstreamness f:

$$\frac{\partial \log s_{if}}{\partial f} = -\frac{\gamma_i}{\theta_i} < 0 \tag{17}$$

From a broader perspective, λ_i also links firm scope decisions across countries, a relationship we develop further in the following section of the paper. For those relationships it is helpful to recognize that $\lambda_i = \frac{\partial \widetilde{P}_i}{\partial S_i}$.

In an appendix we solve for s_{if} and F_i as a function of λ_i. We apply these in turn to the constraint $\int_0^{F_i} s_{if} df = S_i$ and derive an explicit solution for the shadow cost of fragmentation.

$$\lambda_i = w_i a_i \left[\frac{\gamma_i S_i}{\theta_i} + \left(\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i} \right)^{\frac{1}{\theta_i+1}} \right]^{\theta_i} \tag{18}$$

λ_i increases with all cost parameters a_i, θ_i and γ_i, with the price of intermediate goods P_i^M and with the range of tasks to be performed S_i. Having solved for the shadow cost of fragmentation, we can now solve for the price of the last-stage goods P_i, the extent of fragmentation F_i in country i and firm scope s_{if} across all firms f within the country. We also examine the (endogenous) intermediate goods intensity at each stage.

Firm scope: The model is tractable enough to solve for firm scope s_{if} all along the chain. Firm scope s_{i,F_i} for the most upstream firm is:

$$s_{iF_i} = \left[\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i} \right]^{\frac{1}{\theta_i+1}} \tag{19}$$

[22] Our Lagrangian formulation in (15) generalizes the condition $\delta c'(s_{f+1}) = c'(s_f)$ in Kikuchi et al (forthcoming) that links the marginal costs of tasks between (discrete) firms f that neighbor one another in the chain.

while the most downstream firm has scope :

$$s_{i,0} = \frac{\gamma_i S_i}{\theta_i} \; + \; s_{i,F_i} \tag{20}$$

Using expression (17), scope at intermediate positions corresponds to: $\log s_{if} = -\frac{\gamma_i}{\theta_i} f + \log s_{i,0}$. Note again that firms are *ex ante* homogeneous but end up with different firm scope due to their position on the chain. The difference $\frac{\gamma_i}{\theta_i} S_i$ between the scope of the most downstream and upstream firms in country i is illustrative of this within-country heterogeneity in firm scope. Heterogeneity is rising in S_i because more tasks produced in country i implies more firms, and thus more room for heterogeneity, conditional on θ_i and γ_i. Larger values of transaction costs γ_i imply more heterogeneity in firm scope because upstream firms must reduce s_{if} relatively more to satisfy equation (15). Larger values of θ_i imply that scope remains more uniform across firms.

Of further interest is the relationship between firm scope and the price of intermediate goods relative to labor costs $\frac{P_i^M}{a_i w_i}$. The scope of both the most upstream and downstream firms are rising in this ratio. The intuition is that when the price of intermediates is relatively high, the cost of outsourcing is relatively higher and firms will choose to add more value in-house. Conversely, when labor costs are high, firms will produce relatively few stages before outsourcing to upstream firms.

Length of the chain: The number (mass) of firms involved sequentially in production is a key measure of fragmentation of the chain. Here, since the range of tasks performed by each firm is endogenous, the length of the chain also becomes endogenous and is no longer proportional to S_i. For a given price P_i^M of imported intermediate goods and range S_i of tasks to be performed, the mass of sequential suppliers is:

$$F_i = \frac{\theta_i}{\gamma_i} \log \left[1 \; + \; \frac{S_i}{\theta_i + 1} \left(\frac{A_i w_i}{P_i^M} \right)^{\frac{1}{\theta_i + 1}} \right] \tag{21}$$

The mass of suppliers depends negatively on the price of intermediate goods because more expensive components make transactions more costly, which leads to less fragmentation. The number of suppliers also depends negatively on transaction costs and positively on θ_i, the parameter for diseconomies of scope.

Aggregate price: After solving for firm scope s_{if} and the number of firms F_i, we find that the price of the most downstream good in country i, i.e. the solution of the minimization program (13), is:

$$P_i = \widetilde{P}_i(S_i, P_i^M) = \left[\frac{S_i}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}}\right]^{\theta_i+1} \tag{22}$$

expressed as a function of the synthetic parameter A_i:

$$A_i = a_i \left(\gamma_i \frac{\theta_i+1}{\theta_i}\right)^{\theta_i} \tag{23}$$

This A_i depends on exogenous country-specific parameters θ_i, a_i and γ_i, and reflects the effective labor productivity in country i. Note that, conditional on A_i, prices no longer depend on transaction costs γ_i. The price mimics a CES cost function with two inputs: imported intermediate goods and labor, where the weight for labor depends on the range of tasks, productivity, transaction costs and coordination costs. The apparent elasticity of substitution is θ_i+1. When coordination costs θ_i are larger, production has to be more fragmented and there is a larger amount of production lost in transaction costs. These costs are larger when the price of intermediate goods P_i^M is high.

Labor vs. imported intermediate goods demand: Each unit of the final-stage good produced in country i also generates a demand $e^{\gamma_i F_i}$ for the most upstream intermediate goods, i.e. intermediate goods imported from the next country in the chain. In terms of value rather than quantities, we obtain that the share of imported inputs in the total cost of production in country i is:

$$\frac{P_i^M q_{i,F_i}}{P_i q_{i,0}} = \frac{\partial \log \widetilde{P}_i}{\partial \log P_i^M} = \frac{\left(P_i^M\right)^{\frac{1}{\theta_i+1}}}{\frac{S_i}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} + (P_i^M)^{\frac{1}{\theta_i+1}}} \tag{24}$$

Using this expression, we can retrieve the demand for local labor in country i. The share of local demand in the production of country i has a simple interpretation: it corresponds to the value-added content of exports for country i in that chain. As with the price of the produced good, this expression mimics a CES cost function. The share of labor (one minus the above expression) depends positively on the range of tasks to be performed as well as the price of intermediate goods. The elasticity of substitution between imported inputs and local labor is in turn endogenously determined by diseconomies of scope at the firm level.

Gross-output-to-value-added ratio: We define gross output as: $GO_i = \int_0^{F_i} p_{if} e^{\gamma_i f} df$ by integrating the value of all transactions along the chain, while total value added by country i corresponds to: $VA_i = \int_0^{F_i} c_i(s_{if}) e^{\gamma_i f} df$. The ratio of these two variables has a useful empirical counterpart since it is readily available in input-output tables provided by statistical agencies.

Here, we find that the GO-VA ratio equals:

$$\frac{GO_i}{VA_i} = \frac{\theta_i}{\gamma_i} \tag{25}$$

Strikingly, this result also holds at the firm level. To be more precise, the ratio of price to cost at each stage is constant and equal to:

$$\frac{p_{if}}{w_i c_i(s_{if})} = \frac{\theta_i}{\gamma_i} \tag{26}$$

We can interpret this ratio as an index of fragmentation at the firm level. In particular, this ratio reflects the two key forces present in our model: stronger diseconomies of scope (coordination costs) θ_i lead to more fragmentation while larger transaction costs γ_i lead to less fragmentation. As seen in equations (17) and (20), this ratio also dictates the difference in scope between upstream and downstream firms.

The relationship between the structural parameters and summary measures of fragmentation are summarized in the following lemma:

Lemma 2 *Production fragmentation within countries – captured either by the GO/VA ratio or by the range F_i of firms involved in the chain – increases with coordination costs θ_i and decreases with transaction costs γ_i. In particular, the GO/VA ratio equals $\frac{\theta_i}{\gamma_i}$.*

Free entry and cost decomposition: We can also use (26) to better understand the link between our model, perfect competition and the shadow market for tasks. Perfect competition implies that firms' average and marginal costs of performing tasks will be equalized along the chain. If average costs exceed marginal costs, firms can reduce costs by expanding their scope. If marginal costs exceed average costs, there will be entry and firms will reduce their scope. Applying (26) and (15), we can equate average and marginal cost of performing tasks for firm f, and link these to the shadow cost λ_i:

$$\frac{w_i c_i(s_{if}) + \gamma_i p_{if}}{s_{if}} = \frac{(1+\theta_i)\, w_i c_i(s_{if})}{s_{if}} = w_i c_i'(s_{if}) = \lambda_i e^{-\gamma_i f}. \tag{27}$$

It is also useful to decompose the sources of costs in the left-hand-side term of (27). Average cost has two components: labor costs associated with producing tasks inside the firm, $w_i c_i(s_{if})$, and transaction costs linked to shipments between firms, $\gamma_i p_{if}$. A decomposition exercise highlights the central role of the coordination cost parameter θ_i, and will be useful in a later discussion of comparative advantage. Using (26), we solve for changes in average cost as we move along the implicit price function.

$$\frac{\gamma_i p_{if}}{w_i c_i(s_{if}) + \gamma_i p_{if}} = \frac{\theta_i}{\theta_i + 1} \tag{28}$$

The contribution of input prices to total cost growth is solely a function of θ. The share of labor costs is, by implication: $\frac{1}{\theta_i+1}$. A notable outcome in this calculation is the absence of a role for γ_i in this decomposition, which arises because firms react to higher values of γ_i by bringing more stages inside the firm. We revisit this issue when we describe comparative advantage within the supply chain.

3.2 Cross-border fragmentation

Now that we have described the allocation of tasks along the chain within borders, we turn to the optimal allocation of tasks and firms across borders. In particular, we need to characterize the ordering of countries $i(n)$ on the chain, with $i(1)$ being the most downstream and $i(N)$ the most upstream country.[23]

Given the optimal fragmentation of production across firms in each country $i = i(n)$, summarized by the price function from equation (22), $\widetilde{P}_i(S, P^M)$, the optimal global value chain corresponds to the following minimization program:

$$\min_{\{S_n, P_n\}} P_1 \tag{29}$$

under the constraints:

$$P_n = \widetilde{P}_{i(n)}(S_n, \tau P_{n+1}) \tag{30}$$

and

$$\sum_{i=n}^{N} S_n = 1 \tag{31}$$

where the function $\widetilde{P}_i(S, P^M)$ is the solution of the optimization described in equation (22) in the previous section.

For a given sequence of countries $i(n)$, we can go quite far in characterizing prices, ranges of tasks completed and labor demand along the chain. First, it is useful to explicitly express

[23]Recall that we drop for now the variety subscript ω while most parameters vary across varieties.

the Lagrangian:

$$\mathcal{L} = P_1 - \sum_{n=1}^{N} q_n \left[P_n - \widetilde{P}_{i(n)}(S_n, \tau P_{n+1})\right] - \lambda_G \Big[\sum_{n=1}^{N} S_n - 1\Big] \tag{32}$$

The Lagrange multipliers associated with price equations correspond to quantities required for each unit of final good. To be more precise, q_n correspond to quantities $q_{i(n),0}/q_{i(1),0}$ required at the most downstream task performed in the n^{th} country $i(n)$ per unit of final good $q_{i(1),0}$. The first-order condition $\frac{\partial \mathcal{L}}{\partial P_{n+1}} = 0$ is equivalent to imposing $q_{n+1} = \tau q_n e^{\gamma_{i(n)} F_n}$ (using the price derivative described in equation 24).

The first-order condition $\frac{\partial \mathcal{L}}{\partial S_n} = 0$ reflects the optimal allocation of tasks across countries. At the optimum, the marginal cost of completing another task should be equalized across all countries on the chain, up to quantities q_n produced by country n: $\lambda_G = q_n \frac{\partial \widetilde{P}_{i(n)}}{\partial S_n} = q_n \lambda_n$. This implies:

$$q_i \lambda_i = q_j \lambda_j \tag{33}$$

for any pair of countries i and j along the chain, where λ_i is the shadow cost of fragmentation within country i (per unit of goods exported by the country). The tight links between the Lagrange multipliers in successive countries serves to link the shadow cost of stages across markets.

Since a move upstream along the chain increases quantities (because of transaction costs and cross-border trade costs), the shadow cost $\lambda_{i(n)} > \lambda_{i(n+1)}$ must decrease. Concretely, a first implication is that firm scope tends to decrease as we go upstream, not just within countries but also across countries. The F.O.C. in S_i implies the following expression which generalizes equation (15) across countries along the chain:

$$q_n e^{\gamma_{i(n)} f} w_{i(n)} c'_{i(n)}(s_{nf}) = \lambda_G \tag{34}$$

where $q_n e^{\gamma_{i(n)} f}$ corresponds to the quantities of intermediate goods required for each unit of final good. Since the latter increases with upstreamness, we obtain that firm scope s_{nf} would be smaller if a country $i = i(n)$ specializes upstream than if it specializes downstream. Therefore, a country with large within-firm coordination costs would have a relatively larger cost downstream than upstream compared to a country with low coordination costs.

This feature has important implications for the sorting of countries along the chain. Because firm scope is smaller upstream, diseconomies of scope have a smaller impact on upstream stages than downstream stages. Hence, we should then expect countries with high-θ to specialize upstream while low-θ countries tend to specialize downstream. Formally, we can confirm this insight by examining second-order conditions of the optimization problem described in Equation (29), which yields the following Proposition:

Proposition 1 *Let us denote by $i(n)$ the ranking of countries involved in the same production chain, $i(1)$ being the most downstream and $i(N)$ the most upstream country. In equilibrium, the relative position of countries along the chain is fully determined by coordination costs θ_i; countries with smaller coordination costs specialize downstream:*

$$\theta_{i(1)} < \theta_{i(2)} < ... < \theta_{i(N)}$$

Proposition 1 describes comparative advantage within a supply chain, conditional on a country's participation in the chain.[24] Two implications are of primary interest here: the central role of θ_i in determining within-chain comparative advantage, but also the absence of a role for the transaction cost parameter γ_i. The lack of a role for γ_i would seem to run counter to Costinot et al. (2013), where cross-country differences in the rates of mistakes in production drive comparative advantage within the chain. The closest counterpart in our model to the mistakes in Costinot et al. (2013) is the γ_i parameter.[25]

In both models cross-country sorting of sequential activities mitigates the effects of firm-to-firm transaction costs on the price of the completed good. In Costinot et al. (2013) countries with low transaction costs produce downstream in equilibrium because they impose the least "melt" on goods that are nearing completion. In our model firms offset transaction costs by expanding firm scope. Because offsetting such costs is more valuable downstream, the countries in which firms can most easily expand firm scope, the low-θ countries, locate downstream.

Another way to see this is to exploit the insights in Costinot (2009), who links comparative advantage to the mathematics of log super-modularity. The accumulation of value added along the chain insures that the cost of intermediate goods is rising along the chain. This means that if production costs are log-supermodular in a parameter and input prices, then countries that have low values of that parameter will locate downstream. Using previous results on cost decomposition (equation 28), we find for any country i and stage f:

$$\frac{\partial \log \left\{ \frac{w_i c_i(s_{if}) + \gamma_i p_{if}}{s_{if}} \right\}}{d \log p_{if}} = \frac{\gamma_i p_{if}}{w_i c_i(s_{if}) + \gamma_i p_{if}} = \frac{\theta_i}{1 + \theta_i}. \tag{35}$$

[24]Countries with large values γ_i, w_i, a_i and/or θ_i may not participate at all in an equilibrium chain. Proposition 1 describes the sorting of countries that do participate in an equilibrium chain.

[25]Costinot et al. (2013) offer cross-country differences in contract enforcement as a rationale for differences in the rates of mistakes. Here, the parameter most closely related to contract enforcement is clearly γ_i.

In Costinot et al. (2013), s_{if} is fixed, so average cost is log supermodular in p_{if} and γ_i. The middle term $\frac{\gamma_i p_{if}}{w_i c_i(s_{if})+\gamma_i p_{if}}$ would be increasing in γ_i if firm scope were fixed. This implies that transaction costs γ_i have the least impact on average cost when p_{if} is low (i.e. early in the chain), so that lower transaction costs create a comparative advantage in downstream tasks.

In contrast, γ_i does not appear in (35), and thus does not affect comparative advantage within the chain. The simple explanation is that in our model, firm scope is endogenous to changes in γ_i; in countries with larger transaction costs, firms will endogenously increase firm scope to mitigate the role of higher transaction costs. These endogenous responses nullify the role that transaction cost would otherwise play if firm scope were exogenous. Instead, θ_i plays a singular role in determining countries' positions within the chain. As shown in equation (35), countries with higher coordination costs θ_i should specialize upstream to mitigate the effect of input prices on value added.[26] A related implication is that there will be no international fragmentation without cross-country variation in θ_i. Proposition 1 also implies that there is no back-and-forth trade along a specific chain in equilibrium, except when a final good is shipped back to be consumed in an upstream country.

Equilibrium allocation of tasks across countries: Given the ranking of countries described in Proposition 1, we now describe the range of tasks performed by each one. Using marginal conditions imposed by the optimization problem, we can also determine prices and firm scope along the chain depending on wages and relative productivity. Specifically, the first-order conditions determine the c.i.f. price between consecutive countries $i(n)$ and $i(n+1)$. First-order conditions between three consecutive countries $i(n-1)$, $i(n)$ and $i(n+1)$ then yield the range of tasks performed in $i(n)$. Denoting A_n, w_n and θ_n the productivity, wages and the coordination cost parameter in the n^{th} country $i(n)$ along the chain, we obtain:

$$\left\{ \begin{array}{rcl} \tau P_{n+1} & = & (A_n w_n)^{\frac{\theta_{n+1}+1}{\theta_{n+1}-\theta_n}} (\tau A_{n+1} w_{n+1})^{-\frac{\theta_n+1}{\theta_{n+1}-\theta_n}} \\ \frac{S_n}{\theta_n+1} & = & \left(\frac{A_{n-1}w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \end{array} \right. \quad (36)$$

where τ is the trade cost between any two countries.

Given the range of tasks performed in country $i(n)$, it is interesting to derive the share of

[26] Our specific choice of functional form removes γ_i altogether from the determination of within-chain comparative advantage. It is difficult to derive analytical solutions for other functional forms, and the effects of γ_i on within-chain comparative advantage may not be completely offset or may be more-than-offset by endogenous responses to θ for other functional forms of $c(s, \theta)$. The intuition nonetheless goes through for cost functions where the convexity is governed by parameter θ. Countries with low values of θ are better able to reduce inter-firm transaction costs through expansion and thus tend to locate downstream. Computational experiments with alternative functional forms (e.g. $a\left(e^{\theta s}-1\right)$ as in Kikuchi et al. forthcoming) confirm that θ is the main determinant of comparative advantage within the chain.

local labor in exports to the next country in the chain. Because ours is a single-factor model, this corresponds to the value added locally in the exports of country $i(n)$, a key statistic for economic policy.[27] Here, we find that the demand for labor (in value) in country $i(n)$ per dollar of good exported to the next country $i(n-1)$ in the chain is:

$$\frac{w_n l_n}{P_n} = 1 \; - \; \left(\frac{w_n A_n}{\tau w_{n+1} A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \left(\frac{\tau w_n A_n}{w_{n-1} A_{n-1}}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \tag{37}$$

Intuitively, the share of local value added in exports is higher when the relative labor cost is lower, as lower labor costs allows the country to serve as the low cost location for a larger measure of stages. This operates through margins that are both up- and down-stream. A lower labor cost makes country $i(n)$ more competitive at the margin than the previous upstream country $i(n+1)$ as well as the next country $i(n-1)$ downstream.

The effect of trade costs on this statistic also operates through two channels: higher trade costs reduce the contribution of country $i(n)$ in the downstream country $i(n-1)$ operations, but they also reduce the upstream country's contribution. For a country in the middle of the chain, trade costs have a positive effect on local labor content only if there are stronger complementarities with downstream rather than upstream countries, i.e. when the differences in θ_n are larger with the downstream country than with the upstream country: $\theta_n - \theta_{n-1} > \theta_{n+1} - \theta_n$.

Conditional on the set of countries participating (with θ_n increasing with n along the chain), we can go further and obtain a simple expression for the price of final goods (i.e. price of downstream goods in country 1) as a function of costs parameters A, θ and wages w. Conditional on the set of countries, we can also derive simple expressions for the share of labor costs from a specific country.

Lemma 3 *Conditional on the set of countries $n = 1, 2, etc.$ participating (with θ_n increasing with n along the chain), the price of the final good is:*

$$P_1 \; = \; \frac{A_1 w_1}{(\theta_1 + 1)^{\theta_1 + 1}} \; \Theta\big(\mathbf{wA}, \tau\big) \tag{38}$$

where $\Theta\big(\boldsymbol{wA}, \tau\big) < 1$ captures the gains from fragmentation for the chain:

$$\Theta\big(\mathbf{wA}, \tau\big) = \left[1 \; - \sum_{n=1}^{N-1} (\theta_{n+1} - \theta_n) \left(\frac{w_n A_n}{\tau w_{n+1} A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]^{\theta_1 + 1}$$

[27] For example Koopman et al. (2010) investigate the share of domestic value added in China's exports.

Moreover, country $i(n)$'s contribution to each dollar of final good being produced is equal to:

$$\frac{l_n w_n}{P_1} = \frac{d\log P_1}{d\log w_n} = \frac{d\log\Theta}{d\log w_n} = \frac{\left(\frac{w_{n-1}A_{n-1}}{\tau w_n A_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{w_n A_n}{\tau w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}}{\left(\frac{P_1}{A_1 w_1}\right)^{\frac{1}{\theta_1+1}}} \tag{39}$$

In the expression for the final good price above, the first term $\frac{A_1 w_1}{(\theta_1+1)^{\theta_1+1}}$ is the cost of production in country 1 if there is no possibility to fragment production across countries, while the second term $\Theta\big(\mathbf{wA},\tau\big)$ is the price reduction obtained from fragmenting production across countries. We can verify that this term increases with trade costs. It also increases with labor requirements A in each upstream country.

Reductions in trade costs allow chains to reorganize some of the tasks abroad, which in turn has an effect on all other firms along the chain. Equation (34) shows that the marginal cost of increasing firm scope has to be equalized across all stages. A decrease in trade costs leading to a decrease in the final good's price also lead to a decrease in firm scope at other stages. The price of the final good is itself tightly linked to the shadow cost of fragmentation:

$$\lambda_G \;=\; \frac{A_1 w_1}{(\theta_1+1)^{\theta_1}}\,\Theta\big(\mathbf{wA},\tau\big)^{\frac{\theta_1}{\theta_1+1}} \tag{40}$$

As expressed with the Θ term, there is a tight connection between the gains from fragmentation (which reduces the final good's price) and the shadow cost of fragmentation. Any increase in fragmentation and decrease in the final good price follows: $d\log\lambda_G \;=\; \frac{\theta_1}{\theta_1+1}\,d\log\Theta$.

A change in trade costs and wages along the chain has implications for firm scope everywhere on the chain. Each firm equalizes the cost of the marginal task and the shadow cost λ_G of performing the task somewhere else. Hence, a change in the shadow cost of fragmentation λ_G has implications for firm scope everywhere along the chain. In particular, the marginal cost of increasing firm scope in the most downstream firm in the most downstream country, $w_1 c'(s_{1,f=0})$ is equal to λ_G, which implies:

$$s_{1,f=0} = \frac{\gamma_1}{\theta_1}\,\Theta\big(\mathbf{wA},\tau\big)^{\frac{1}{\theta_1+1}} \tag{41}$$

Hence: $d\log s_{1,f=0} \;=\; \frac{1}{\theta_1+1}\,d\log\Theta$, which formalizes how a change in fragmentation and trade costs (changes in Θ) affects firm scope for the last firm in the chain, the one that produces the finished good.

Note, however, that the scope of the average firm in an upstream country $i(n)$ (with $n>1$) does not decrease with trade costs. As trade costs decrease, a country moves downstream where firms tend to be larger. Upstream firms, which tend to be smaller in scope, exit or relocate. More specifically, we find that both the size of the most downstream and the most upstream firm within a country increase as trade costs decrease:

$$s_{n,0} = \frac{(\theta_n+1)\gamma_n}{\theta_n}\left(\frac{A_{n-1}w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \qquad \text{and:} \qquad s_{n,F_n} = \frac{(\theta_n+1)\gamma_n}{\theta_n}\left(\frac{A_n w_n}{\tau A_{n+1}w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \tag{42}$$

Proposition 2 below summarizes the effect of trade costs on a chain in partial equilibrium (exogenous wages) for a given set of countries involved in the chain:

Proposition 2 *Holding wages constant, a decrease in cross-border trade costs leads to:*
i) a decrease in the price of the final good;
ii) an increase in the value share of imported inputs at any stage of the chain;
iii) an increase in the range of tasks being offshored;
iv) a decrease in the shadow cost of fragmentation λ_G*;*
v) a decrease in firm scope $s_{n,f}$ *at a given stage* f*;*
vi) an increase in average firm scope in upstream countries $n > 1$*.*

While the ranking of countries along the chain (from downstream to upstream stages) is dictated by the ranking in θ_i (Proposition 1), it is more difficult to characterize the participation of a specific country in the chain. Expression (36) for S_i can be used to obtain a necessary condition for $S_i > 0$, but cannot be used to derive a sufficient condition for country i to participate in the chain. Moreover, the reader should keep in mind that we have dropped the variety subscript ω to simplify the notation, but the costs parameters A_i and θ_i are assumed to be specific to a particular variety of final good ω. Hence, the organization of the chains across firms and countries is specific to each variety and country of final destination.

In the next section, we address this problem in a two-country case with trade costs and heterogeneous chains.[28] In Section 5, we examine numerically a ten-country case calibrated using input-output data. Using expressions (38) and (39) from Lemma 2, we can dramatically reduce the complexity of the numerical problem and reformulate the problem into a simpler linear programming problem that allows us to solve for large economies with a large number of final goods.

[28] In the working paper version (NBER working paper 21520), we also examine a fully tractable case as in Costinot et al (2012) with more than two countries, symmetric chains and frictionless trade.

4 General equilibrium

The discussion so far has described the production structure of the equilibrium chain. As described in Definition 2, general equilibrium requires labor markets and the final goods market to clear.

Using Lemma 3 (equation 39) for each individual chain and our expression for labor demand per unit of final output, it is now easier to compute aggregate labor market demand. As in equation (11), factor market clearance sets labor demand equal to the value of fixed labor supply:

$$\sum_{j,n} \int_{\omega \in \Omega_j} \alpha_j(\omega) L_j w_j \frac{l_{i(n)}(\omega)}{P_{i(n)}(\omega)} \, d\omega \; = \; L_i \tag{43}$$

where $l_{i(n)}$ and $P_{i(n)}$ are the values of unit labor demands and export prices that apply to region i when it takes the n^{th} position in an equilibrium supply chain for variety ω that is completed in country j. $\alpha_j(\omega)L_j w_j$ corresponds to expenditures on the final good (equation 2).

Two-country case: trade elasticity and welfare

A central question in the trade literature is the link between trade and welfare. Arkolakis et al. (2012) show that several theoretical models summarize the trade welfare link with a simple formula requiring only the home trade share and a trade elasticity parameter. Our model of sequential production does not provide such a simple summary of trade and welfare links, but we are able to make a direct analytical comparison of our model's framework, relative to the Arkolakis et al. (2012) benchmark, in a two-country setting.

We consider country D and country U. In country D, we assume that all $\theta_D(\omega)$ equal θ_D. In country U, all $\theta_U(\omega)$ equal θ_U. To justify these country names, we assume that $\theta_U > \theta_D$. This implies that country U is always upstream and country D downstream when there is production sharing.

The first relationship we study is the response of trade to trade costs. While the previous sections examine the fraction of value-added by a country along a given chain (intensive margin), here we examine how trade costs affect the fraction of varieties that a country sources from another country depending on trade costs.

Country D, in particular, relies on imports from U to produce some goods that it exports back to U. As in Yi (2010), this back-and-forth trade generates a higher trade elasticity. There are two reasons for that. When trade costs increase by 1%, the price of goods imported by U from D increases by more than 1% since the production of final goods in D relies itself on goods imported from U (a double penalty). The second reason is that, even if there were no double penalty of trade costs, labor costs in country D would need to be strictly more than

1% lower to offset a 1% increase in the price of its exports when its labor only contributes a fraction of the value of the good. Considering the extensive margin, this implies a larger decline in the fraction of goods sold by D when trade costs increase. Combining these two effects, we find that the trade elasticity, on the extensive margin, is larger than without fragmentation of production across countries. For any two countries $i \neq j$, we have:

$$\frac{d \log \left(\frac{\pi_{ij}}{\pi_{ii}}\right)}{d \log \tau} \quad \leq \quad -\xi$$

where π_{ij} is the share of products from country j among final goods purchased by consumers in country i.[29] Moreover, as we describe in Appendix, lower trade costs generate more fragmentation of production and therefore increase the trade elasticity. Because lower trade costs lead to more fragmentation, the foreign labor content embodied in the marginal variety increases. When trade becomes frictionless, the foreign labor content for this marginal variety converges to unity and the trade elasticity can, in theory, go to infinity.

We also examine Johnson and Noguera (2012a)'s "Value-added-to-export" (VAX) ratio, which compares a country's value added embodied in its exports to its gross export value.[30] A decrease in the VAX ratio reflects an increase in fragmentation across borders, because embodied import value accounts for a larger share of gross export value (Johnson 2014). In our two country model, the VAX ratio is below unity in both countries. The upstream country sells a combination of intermediate and final goods, while the downstream country adds value to the upstream country's intermediates. In the Appendix, we show that this back-and-forth trade grows faster than other trade flows as trade costs decrease, which implies that the VAX ratio for country U decreases as trade costs decrease. These results are both intuitive and supported by recent empirical evidence. In particular, Johnson and Noguera (2012b, 2013) use multi-country input-output tables to show that the VAX ratio has decreased over the past decades and that the bilateral VAX ratio depends positively on bilateral trade costs.

We now turn to the distributional question of how trade affects welfare in upstream and downstream countries. A key policy question is whether a country is affected differently depending on its position in international production chains. To examine this question, we derive an exact expression for the price index and the gains from trade relative to autarky and compare it to standard models without cross-border fragmentation of production. In particular, we use the formula developed by Arkolakis et al. (2012) as a benchmark. Arkolakis et al. (2012) show that welfare gains from trade equal $\Delta \log \left(\frac{w_i}{P_i}\right) = -\frac{1}{\xi} \log \pi_{ii}$ in a wide set of models, including

[29] Recall that ξ is the parameter that defines the dispersion of productivity draws in each country.

[30] As noted in Fally (2012), the inverse of the VAX ratio for the world can also be interpreted as the embodied number of border crossings.

Eaton and Kortum (2002, henceforth EK), when there is no fragmentation of production.

These calculations show that welfare changes from international integration are larger than the simple EK model for D, and smaller than the EK model for U (explicit formulas are provided in Appendix). The brief intuition on this point is as follows. In a simple EK model, or in an EK model with fully-domestic production chains, each country benefits from trade in final goods because trade allows both countries to purchase rather than produce the varieties for which they have comparative disadvantage. When we allow international integration of production chains, D benefits in this way, but additional gains from trade arise from D's ability to offshore those tasks for which it is has comparative disadvantage (within the set of varieties in which it has comparative advantage in the final good). By contrast, country U's gains from integration are smaller than they would be in an EK model because in the fully integrated equilibrium country U produces (the early, low-productivity stages of) varieties for which it has comparative disadvantage in final goods.

We summarize the results on the trade cost elasticity, VAX ratio and welfare in the following Proposition:

Proposition 3 *With two countries, the effect of trade costs on trade is such that:*
i) The elasticity of trade in final goods to trade costs is higher than without fragmentation;
ii) This elasticity is larger when trade costs are smaller;
iii) The value-added content of trade decreases as trade costs decrease.
iv) Welfare gains from trade $\Delta \log \left(\frac{w_i}{P_i} \right)$ *are larger than* $-\frac{1}{\xi} \log \pi_{ii}$ *for country D and smaller than that for country U.*

While these results are shown here only for a two-country case, our counterfactual simulations in Section 5 suggest that these insights hold more generally. In what follows, we calibrate our model and compute gains from trade, trade elasticities and VAX ratios by using input-output tables and information on domestic and foreign labor content which, as shown above, are crucial to obtain a more adequate measure of the gains from trade when production is fragmented across borders.

5 Quantitative analysis

5.1 Data

Our main sources of data are the Asian input-output tables developed by IDE-JETRO. These tables provide information on gross output, value-added, and (most importantly) input purchases by product, parent industry (downstream industry), source country and destination country. For instance, the data report the amount of metals purchased from China by the auto industry in Japan. These 4-dimensional input-output tables, are, as far as we know, the only tables that track international transactions directly, rather than imputing them from trade flows. This is an exceptional data set for investigating the organization and evolution of international production fragmentation in a region of the world where fragmentation is an important feature of international trading relationships.[31]

The dataset covers 9 Asian countries and the US.[32] Our analysis mostly focuses on the year 2000 (with most disaggregated product classification), but we also compare our results to IDE-JETRO data from 1975 and 1990. This period marks a time in which the region began to emerge as an important location for internationally fragmented production (see Baldwin and Lopez-Gonzalez (2015) for example).

Information on input purchases and production is disaggregated at the 76-sector level in 2000. For the sake of comparison to previous input-output tables (1975 and 1990), we also construct a more aggregated 46-sector classification to obtain harmonized product categories across years. The sector classification is far more detailed for manufacturing goods and commodities than services (among the 46 sectors, only 5 of them are service industries). We thus mostly restrict our attention to tradable goods: commodities and manufacturing goods.

The information provided in the IDE-JETRO tables goes beyond a simple aggregation of country-level input-output tables. Besides the harmonization of product categories, input flows by parent and source countries are estimated using supplementary surveys about firms' input choices. This supplementary information informs deviations from the proportionality assumption, which, according to Puzzello (2012), is rejected in these data.[33] This constitutes an important advantage of using the IDE-JETRO input-output compared to previous attempts at constructing input-output tables based on the proportionality assumption (as in Johnson and Noguera 2012, for example).[34]

[31]Like other international IO tables such as the World Input-Output Database or Global Trade Analysis Project database, the Asian IO tables are constructed by merging harmonized national IO tables with international trade statistics. The Asian tables supplement these sources of information with a survey of input users, who report the specific country-of-origin of their inputs. See Meng et al. (2013) for details.

[32]The countries in the data base are the US, Japan, China, Taiwan, Korea, Singapore, Malaysia, Thailand, Indonesia and the Philippines.

[33]The proportionality assumption is made to construct input purchase by source country and parent industry when only partial information is provided. For instance, traditional country-level input-output tables describe how much steel is used by the auto industry in each country. Using trade flow data (which describe how much steel is imported from a particular country), previous international input-output tables have been constructed by allocating the use of input across source countries on a proportional basis.

[34]Ideally, one would want to use firm-level data tracking chains across multiple countries in order to study why countries specialize at different positions along chains. While recent research has examined the structure of chains by matching suppliers and buyers within a country (e.g. Bernard et al. (2016)) or between two countries

5.2 Measuring the position along the chain: indexes N, D and DX

To better understand the degree of fragmentation in vertical production chains we adopt four indexes that generalize the two indexes proposed in Fally (2012) and applied there to US data. The indexes are designed to describe industries' position in vertical production chains by exploiting information about relationships in the input-output table. The 'D' index measures an industry's weighted average distance to final demand, where distance is a measured by the apparent number of plants visited by the industry's output before reaching consumers.[35] The 'N' index calculates, for each industry, the number of stages that are embodied in each industry's production. These two calculations are distinct for each industry, and in the US data there is only a weak correlation between them.

Distance to final demand or "upstreamness": We turn to a formal representation of the two indices. Consider a variable D_{ik}, which is intended to measure the distance of a product k from final demand. Some part of product k's sales will be intermediate trade purchased by downstream industries, so the industry in question's distance measure will depend upon which industries buy its output, and in turn how far those downstream industries are from final demand. Because an industry's sales go to several industries, which will vary in their respective measures of D, the industry measure must be weighted, and it must also be defined recursively. Let D_{ik} indicate the distance measure in region i for product k. We define D_{ik} as:

$$D_{ik} \; = \; 1 \; + \; \varphi_{ikik} D_{ik} \; + \sum_{(j,l) \neq (i,k)} \varphi_{ikjl} D_{jl}$$

where φ_{ikjl} denotes the share of output from sector k in country i that is used in sector l in country j. The entire system of equations that includes a D_{ik} for each industry and country can be solved to produce a measure for each sector-country pair.

As shown in Antras, et al. (2012), this index can be interpreted as the average number of stages of production an industry's output passes through before reaching final consumers. Using the input-output matrix, we can decompose the different trajectories taken by the good across and within industries. Each trajectory is associated with a specific number of transactions across or within industries. Index D would then correspond to the average number of transactions weighted by the fraction of output corresponding to each trajectory. Notice that D_{ik} does not only rely on inter-industry linkages but also depends on the extent of fragmenta-

(e.g. Bernard, Moxnes and Ulltveit-Moe, 2016), firm-level data of these kinds do not provide information that allow us to track chains on their entire length across multiple countries. These input-output tables allow us to do so at a more aggregate level.

[35] The measure is equivalent to the 'upstreamness' measure in Antras, et al. (2012).

tion within each industry. If an industry's production is partly used an input by other firms in the industry (e.g. electronic parts are used as inputs into other electronic parts within the same country), the coefficient φ_{ikik} would be strictly positive and would contribute to a higher index D_{ik} (since it would also correspond to a higher number of transactions).[36]

How to interpret 'D' in the model? First, suppose that products k correspond to stages f. When f is strictly positive, i.e. when it does not refer to the most downstream stage in the n^{th} country $i(n)$, then all sales are made to the next plant $f - df$ in the chain:

$$D_{i(n),f+df} = df + D_{i(n),f}$$

If $f = 0$ and $i(n)$ is not the most downstream country $i(1)$, then all sales go towards the most upstream firm in the next country in the chain. After integrating, we obtain that the model counterpart of D_{if} corresponds to the total range of firms located downstream:

$$D_{i(n),f} = f + \sum_{n'<n} F_{i(n')}$$

summing across all downstream countries $i(n')$ with $n' < n$.

In terms of the model, we can also interpret $D_{i,k}$, for any country i, as a semi-elasticity of required quantities w.r.t. to transaction costs. Formally, D_{if} corresponds to:

$$D_{i,f} = \sum_j \frac{\partial \log q_{i,f}}{\partial \gamma_j}, \qquad (44)$$

which follows from (6). Because γ_j governs proportional iceberg "melt" that arises in firm-to-firm transactions, it summarizes the degree to which the additional quantities required of upstream firms rise with the upstreamness of their position.

Embodied stages: The N_{ik} index captures a weighted average of the number of plants involved sequentially in the production of good k in country i. It is defined recursively by:

$$N_{ik} \;=\; 1 \;+\; \mu_{ikik} N_{ik} + \sum_{(j,l)\neq(i,k)} \mu_{ikjl} N_{jl}$$

where μ_{ikjl} denotes the amount of input l from country j used to produce one dollar of product k in country i. This is a single equation, but, as with the D index, the system of equations can

[36] One may argue that the industry classifications are too aggregated and create biases in computing D_{ik} and N_{ik} compared to what would be obtained with more precise data. Fally (2012) examines the aggregation properties of indexes D and N and shows that aggregating industries does not much affect the average of D and N across industries.

be solved to produce a measure of N for each sector-country pair. As shown in Fally (2012),[37] this index can also be expressed as a weighted average of the number of stages required to produce good k in country i, weighted by how much each stage of production contributes to the final value of that good.

We can also interpret N in light of our theoretical framework. In the model, the amount of input purchased by other firms corresponds to the price of the good minus the labor cost incurred at each stage, i.e.: $\frac{w_i c_i(s_{if})df}{p_{if}}$, where p_{if} denotes the price of the good in country i at stage f. The model counterpart of index N would thus correspond to the following recursive definition:

$$N_{i,f} = df + \left(1 - \frac{c_i(s_{if})df}{p_{if}}\right) N_{i,f+df}$$

with a similar equation when the chain crosses a border. The solution to this differential equation equals the average of the number of production stages required to produce a good at stage f in country i.

There is a strong connection between the N and D index. Since the number of stages between firm f' in the m^{th} country $i(m)$ and firm f in the n^{th} country $i(n)$ corresponds to $D_{i(m),f'} - D_{i(n),f}$, we obtain formally:

$$N_{i(n),f} = \frac{1}{q_{i(n),f} p_{i(n),f}} \left[\int_{(m,f')>(n,f)} (D_{i(m),f'} - D_{i(n),f})\, q_{i(m),f'} c_{i(m)}(s_{i(m),f'}) \right]$$

where the integral is taken across all upstream firms either in $i(n)$ at a more upstream stage $f' > f$ or in more upstream countries $i(m)$ with $m > n$, and where the price p_{if} can be itself re-expressed as the sum of all costs incurred in upstream stages, adjusting for quantities: $q_{i(n),f} p_{i(n),f} = \int_{(m,f')>(n,f)} q_{i(m),f'} c_{i(m)}(s_{i(m),f'})$.

The connection between the two indexes N and D is clearest if we look at the most downstream stage. For the most downstream country $i = 1$ and the most downstream firm $f = 0$ in the country, index N corresponds to a weighted average of D:

$$N_{i(1),f=0} = \frac{1}{q_{1,0} p_{1,0}} \left[\sum_j \int_{f'=0}^{F_j} D_{j,f'}\; q_{if'} c_j(s_{jf'})\, df' \right]$$

with the price $p_{i=1,f=0} = \sum_j \int_{f'=0}^{F_j} q_{if'} c_j(s_{jf'}) df'$ being the sum of all upstream costs.

As for D, we can also use the model to interpret N_{ik} as a semi-elasticity of w.r.t. to

[37] See Proposition 1 in Fally (2012).

transaction costs, looking at prices instead of quantities. Formally, N_{if} corresponds to:[38]

$$N_{if} = \sum_j \frac{\partial \log p_{if}}{\partial \gamma_j}$$

Aggregation: Using the IDE-JETRO data, the D and N statistics are calculated at the level of country-industry (i, k) pairs. For the calibration exercise that follows a country-level statistic will be useful so as to better describe countries' average position in global supply chains. A weighted average across statistics is most suitable, although there are several options for defining weights, including value added- or export-weighting for D and output-weighting for N. As argued in Fally (2012), a natural weight for the upstreamness index D is value added, and final production by sector-country for index N.

In order to best capture the relative position of countries on international production chains, the aggregate we use in calibration is an export-weighted average of index D_{ik}. This export-weighted average is the statistic calculated in Antràs et al. (2012) to document countries' comparative advantage along production chains. Formally, we define DX_i by country with the following:

$$DX_i = \frac{\sum_k X_{ik} D_{ik}}{\sum_k X_{ik}} \tag{45}$$

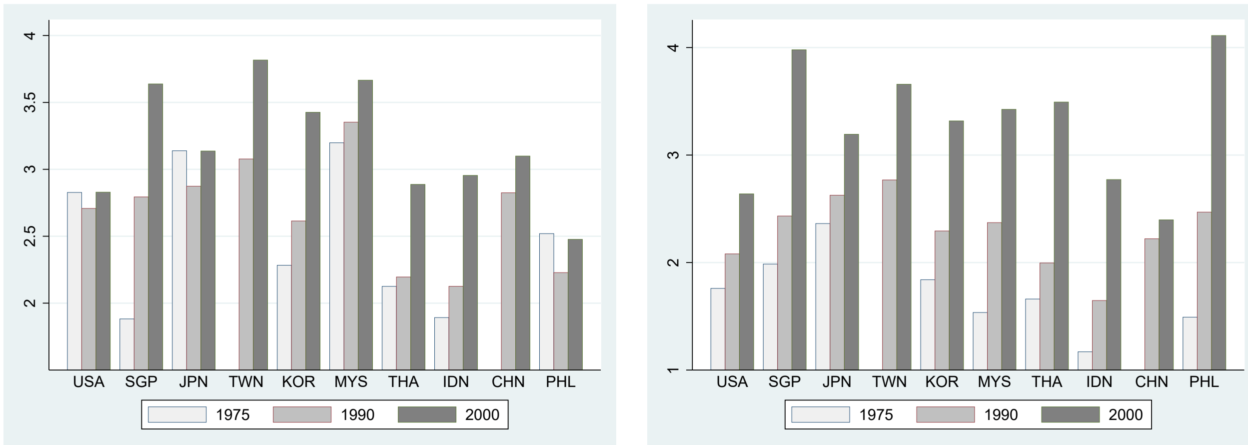
where X_{ik} represents country i's exports of product k.

Descriptive statistics: We calculate these indices using the IDE-JETRO data from 1975, 1990 and 2000. First, we examine at upstreamness, weighted by value-added, across countries and years. Figure 1(a) exhibits the results for all tradable goods. There is some variation in the levels and trends of upstreamness index D across countries. This index increases over time for most countries (with Japan and the US as notable exceptions), which suggests that chains have become longer or that the countries in question have moved into upstream positions along production chains.

In this graph, countries are sorted by their per capita GDP's. One can see that there is no monotonic relationship between per capita GDP and average upstreamness. Countries at both end of the spectrum tend to be downstream while middle-income countries are relatively more upstream. This is not in line with the model developed by Costinot et al (2012) where more productive countries tend to be located downstream.

[38]Based on the results in Fally (2012), we should also note that N_{if} equals the aggregate gross-output-to-value-added ratio across upstream activities. It can therefore be interpreted as a weighted average of $\frac{\theta_i}{\gamma_i}$ across upstream activities.

Figure 1: Average upstreamness index D by country for 1975, 1990 and 2000



(a) Manufacturing

(b) Electronics

In Figure 1(b), we report results of our calculations for the electronics sector only, by country and by year. The electronics sector is particularly interesting over this period. Complex international production chains are, anecdotally, an important phenomenon in East Asian manufacturing. This is even more notably so within the electronics sector. Moreover, there has been important growth in the region's trade in electronics, which constituted only 8% of Asian exports in 1975, and 34% in 2000. For electronics, there has been a sharp upward movement in index D for most countries, which is consistent with increasing fragmentation of production chains in Asia in the electronic industry. Some countries such as China and the US remain downstream while other countries such as the Philippines have moved upstream. But again, there is no clear monotonic relationship between upstreamness and GDP per capita as predicted by Costinot et al (2012).

For other indexes, we refer to Table 2 and 3 (indexes DX, GOVA and N for 2000). In an appendix, we describe the variations in D and N across industries, showing that the two indexes are not strongly correlated. As one could expect, primary commodities such as ores and feeds are associated with higher upstreamness while finished goods tend to have lower upstreamness index values.

Correlation between upstreamness, value-added content and transaction costs: The model predicts that: i) countries with high coordination costs θ should specialize upstream while countries with low θ should specialize downstream (Proposition 1); ii) the gross-output-to-value-added (GO/VA) ratio increases with coordination costs θ (Lemma 2); iii) the GO/VA ratio increases with transaction costs γ (Lemma 2).

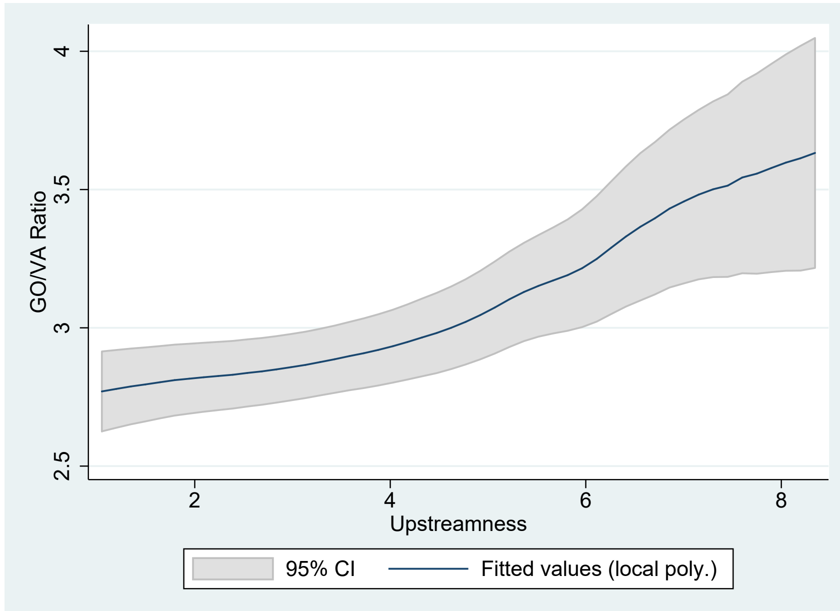


Figure 2: Gross-output-to-value-added ratio as a function of upstreamness D_{ik}

By combining predictions i) and ii), we should observe a positive correlation between upstreamness and the GO/VA ratio. This correlation should hold after controlling for transaction costs γ_i. Moreover, we should observe a negative correlation between the GO/VA ratio and proxies for transaction costs γ_i. Here we use the "cost of enforcing contracts" from the Doing Business database (World Bank), but similar results obtain with measures from the Doing Business database on the "time to enforce contracts" or the recovery rate in insolvency proceedings.

Figure 2 shows a strong positive correlation between upstreamness and the GO/VA ratio. This finding supports the prediction that firm scope tends to be smaller upstream than downstream. If firm scope were fixed and held constant along production chains, the GO/VA ratio would be negatively correlated with upstreamness because a constant measure of value added per firm would accrue to increasing levels of gross output as production moved downstream. Our model with endogenous firm scope generates equilibria with relatively more value added by downstream firms, thereby allowing it to replicate the positive relationship observed in Figure 2.

In Table 1, we confirm the result from Figure 2 by regressing the gross-output-to-value-added ratio on upstreamness. We find a significant and positive correlation between the two, whether we include country fixed effects (column 1), industry fixed effects (column 2) or both (column 3). Consistent with the model, the correlation is the strongest when most of the variation is driven by cross-country variation (column 2), i.e. when we include industry fixed effects but no country fixed effects. Even assuming our model structure, the correlation might not be expected to emerge if the structural parameters γ_i and θ_i were strongly correlated across countries. Nevertheless, we do see the positive relationship. Moreover, the correlation is robust to controlling for γ_i using proxies for transaction costs (as in column 6).

As shown in columns (3) to (6) of Table 1, our results also corroborate another prediction of the model, that the GO/VA ratio decreases with transaction costs γ_i. This result is intuitive: higher transaction costs lead to fewer transactions and a larger range of activities performed internally, hence a higher value-added content and a lower GO/VA ratio.

Table 1: GO/VA ratio, upstreamness D_{ik} and contract enforcement

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent var.:	GO/VA ratio					
Upstreamness	0.132	0.244	0.161			0.226
	[0.031]**	[0.069]**	[0.060]**			[0.069]**
Cost of enforcing contracts				-0.385	-0.376	-0.329
				[0.136]**	0.083]**	[0.082]**
Industry FE	No	Yes	Yes	No	Yes	Yes
Country FE	Yes	No	Yes	No	No	No
R2	0.16	0.49	0.59	0.02	0.48	0.50
N	344	344	344	344	344	344

Notes: Notes: OLS regression with robust s.e.; by country and sector in 2000, excluding services and trimming outliers with upstreamness above 10; the cost of enforcing contracts is from the Doing Business Database; * significant at 5%; ** significant at 1%.

5.3 Calibration

Our calibration exercise focuses on the 10 countries that are covered by the IDE-JETRO input-output tables. The general equilibrium model described above is calibrated so as to reproduce key features of the data. The parsimony of the model allows us to consider only a small number of parameters to calibrate, those listed in the left column of Table 2.

Thanks to Lemma 2 and the analytical results described in section 3.2, we can reduce the optimization problem described in equation (12) to a linear programming problem for each chain. We consider chains of varying length, and identify the low cost chain completed in each destination country. The supplier of final goods to each market is the country with the lowest delivery cost, gross of trade costs.[39] Numerical simulations are performed in Matlab. We approximate a continuum of varieties by assuming 1,000,000 different final goods.

We now describe each calibrated parameter and its targeted moment, as described respectively in the left and right columns of Table 2.

Labor supply: Each country is endowed with an exogenous supply of factors. In the benchmark case, we consider only one factor of production: labor. For each country, we choose the labor force L_i to match aggregate value-added in tradeable goods sectors (i.e. excluding services) divided by the cost of labor (proxied by income per capita in our benchmark simulation).

Labor productivity: Labor productivity is calibrated such that labor demand equals labor supply, while wages are set equal to per capita income, obtained from the Penn World Tables for the year 2000.

As in Eaton and Kortum (2002), average labor productivity across all varieties is equal to $\bar{A}_i \equiv T_i^{-\frac{1}{\xi}}$ where T_i is the shift parameter for the Weibull distribution of $A_i(\omega)$ in country i (equation 10). The dispersion parameter $\xi = 5$ is calibrated based on recent estimates such as Simonovska and Waugh (2014). Conditional on wages and calibrated parameters, we can compute labor demand for each country. Equality between aggregate labor demand and labor supply is attained by adjusting labor productivity $\bar{A}_i$ (or, equivalently, by adjusting T_i). Note that there is a tight link between wages and implied labor productivity in country i. As shown in Table 2, there is a nearly log-linear (downward-sloping) relationship between wages w_i and $\bar{A}_i$ in our benchmark calibration.

Coordination costs: As shown in Proposition 1, the coordination cost parameter θ_i is a key determinant of the position of a country along the chain, downstream or upstream. A country tends to export final goods when coordination costs are low and export intermediate goods when these costs are higher. Since all countries export a mix of final and intermediate goods, we assume that θ_i is heterogeneous across varieties ω, as discussed in section 4.3. We assume that it is log-normally distributed. In calibration countries are allowed a different shift parameter $\bar{\theta}_i$ and a different standard deviation σ_θ. We use DX as the primary moment to calibrate $\bar{\theta}_i$, and calibrate σ_θ to fit countries' intermediate share in total exports. While the correlation of the $\bar{\theta}_i$ and DX_i in Table 2 is weak (0.22), the prediction of the model is confirmed: countries that are more upstream have higher average values of $\bar{\theta}_i$. Most countries share similar values of DX_i, with China and, to a lesser degree, Thailand positioned relatively downstream, while Indonesia is relatively upstream.

[39] In general, if trade costs are sufficiently high, the final good associated with a single a variety may be produced for export by one country and for domestic consumption by one or more other countries.

Transaction costs: Another key parameter of the model is γ_i, the cost of transactions between two firms. This cost is assumed to be positive even for transactions that occur within borders. Transaction costs are difficult to estimate in practice, but our model indicates that the gross-output-to-value-added ratio equals the ratio of coordination and transaction costs parameters $\frac{\theta_i}{\gamma_i}$ and thus can be used to retrieve an estimate of γ_i once we know $\bar{\theta}_i$ (on average). Results are provided in Table 2. As a credibility check we compare our results to plausible real-world counterparts of the γ_i parameter using the Doing Business Database (World Bank). Reassuringly, we find expected correlations of our calibrated γ_i variables with the costs of enforcing a contract claim (0.69), the time to enforce contracts (0.29), and the recovery rate in insolvency proceedings (-0.39). We note that Singapore has unusually low transaction costs in the calibration, which is consistent with its high gross output to value added ratio. At the other end of the spectrum, Indonesia has the highest transaction costs, which is consistent with the Doing Business indicators and recent literature (Olken and Barron, 2009).

Trade costs: In addition to transaction costs that are also incurred within borders, cross-border transactions face an additional burden. The nature and size of the extra cost affecting international transactions is the matter of an extensive debate in the trade literature (transport costs, asymmetric information, marketing cost, technical barriers, or cultural differences are plausible sources of such costs). There is however a consensus that these costs are large and have a large effect on cross-border trade. While distance usually plays a key role in explaining the pattern of international trade (see Disdier and Head, 2008), distance is not as crucial for trade among IDE-JETRO countries.[40] We therefore assume that cross-border trade costs are uniform across all country pairs in our sample.[41] We fit the trade cost parameter by asking the model to replicate the global ratio of trade to output.

Note that, as a consequence of trade costs, not all countries participate in a given production chain. In our fitted model, less than one percent of the chains involve more than three countries.

Simulation results on other moments: Before turning to the counterfactual results we briefly describe the benchmark equilibrium. By construction, our model is able to reproduce key indicators of fragmentation such as the gross-output-to-value-added ratio and values of DX. Alternatively, we can examine how the fitted model fares in terms of other indexes such as indexes D, N and the share of intermediate goods in exports, M-share.

[40] In a standard gravity equation of trade using the IDE-JETRO data, the coefficient for distance is not significant while the estimated border effect is large and significant, economically and statistically. Of course, this is a small sample of only 10 countries.

[41] Antràs and de Gortari (2017) build a general equilibrium model of international value chains assuming heterogeneous trade costs linked to geography. They show that more geographically central regions are more likely to host final production. In our model, geography would affect participation in the chain but not the vertical ordering of countries.

Table 2: Parameter choice and moments to match

Parameters:			*Moments to match:*		
Average $\bar{A}_i = T^{-\frac{1}{\xi}}$	USA	1.000	GDP per capita	USA	35,080
by country	SGP	1.284	(PWT)	SGP	32,808
(relative to the US)	JPN	0.484		JPN	26,721
	TWN	1.463		TWN	21,891
	KOR	1.434		KOR	17,208
	MYS	2.644		MYS	7,917
	THA	2.847		THA	5,178
	IDN	3.182		IDN	2,549
	CHN	3.193		CHN	2,442
	PHL	3.680		PHL	2,210
Labor supply	USA	53,551	Total value-added	USA	1,878.6
in tradeable goods	JPN	41,665	in tradeable goods	JPN	1,113.3
(x1000 workers)	SGP	735	(in $M)	SGP	24.1
	TWN	3,889		TWN	85.1
	KOR	10,491		KOR	180.5
	MYS	5,637		MYS	44.6
	THA	10,410		THA	53.9
	IDN	36,585		IDN	93.3
	CHN	266,707		CHN	651.3
	PHL	13,618		PHL	30.1
Average coordination costs $\bar{\theta}_i$	USA	0.875	DX Index	USA	2.923
by country	SGP	0.384	(Export weighted)	SGP	2.879
	JPN	0.760		JPN	2.663
	TWN	0.444		TWN	2.853
	KOR	0.560		KOR	2.892
	MYS	0.390		MYS	2.679
	THA	0.455		THA	2.439
	IDN	0.679		IDN	2.957
	CHN	0.530		CHN	2.009
	PHL	0.515		PHL	2.776
Transaction costs γ_i	USA	0.378	aggregate GO / VA ratio	USA	2.599
by country	SGP	0.209		SGP	3.842
	JPN	0.336		JPN	2.745
	TWN	0.216		TWN	3.865
	KOR	0.276		KOR	3.224
	THA	0.210		THA	3.718
	MYS	0.276		MYS	2.801
	IDN	0.460		IDN	2.161
	CHN	0.249		CHN	3.049
	PHL	0.359		PHL	2.434
Simple average border cost	All	15%	Trade/output ratio	All	26%

Table 3 compares indexes from the model vs. data. In broad terms the magnitudes of D and N are consistent with the data, even though they are constructed in very different ways. These indexes from the data are computed at the industry level then averaged across industries. Index N in the benchmark calibration is computed for the most downstream firm in the chain while index D is a weighted average across firms weighted by value-added at each stage. The levels of D and N are approximately the same in model and data, and the cross-country correlations are high.

The calculated share of intermediates in exports is generally lower in the calibration than in the model. Recall that our model has no scope for back and forth trade in intermediates. Neither does our model have an explicit role for assembly nor multiple sources of inputs ("spiders"). Any of these features would lead real world data to report higher shares of intermediates in exports.[42] We nonetheless find M-share useful for model analytics in subsequent counterfactual analysis and so report it here for consistency.

Table 3: Fragmentation indexes: model vs. data

Index	D		N		M share	
	Data	Model	Data	Model	Data	Model
USA	2.829	2.666	3.397	2.871	0.649	0.365
SGP	3.638	3.842	3.833	3.976	0.690	0.240
JPN	3.137	2.735	3.152	3.053	0.596	0.301
TWN	3.817	3.805	3.691	3.997	0.711	0.238
KOR	3.426	3.312	3.565	3.472	0.676	0.280
MYS	3.666	3.654	3.453	3.897	0.640	0.216
THA	2.888	2.869	3.432	3.156	0.609	0.216
IDN	2.955	2.786	2.642	2.579	0.675	0.353
CHN	3.099	2.771	3.255	3.360	0.439	0.162
PHL	2.477	2.824	2.725	2.798	0.692	0.280
Correl. with data		0.882		0.843		0.506

5.4 Counterfactual simulations

East-Asian economies have been the setting for tremendous changes in recent decades. Arguably the most significant changes are the increased fragmentation of production, China's opening to international trade and its subsequent rapid economic growth. In our theory these phenomena can certainly be related, as China's opening to trade could have facilitated fragmentation along

[42]An earlier draft of this paper showed that a capstone assembly sector can easily be added to our model, see NBER working paper version 21520.

chains in which it is now involved. Rapid economic growth may be associated with trade-related increases in productivity, but multi-factor productivity growth not specifically related to trade might also have been important. With a calibrated model at hand, we can now examine various counterfactual simulations to study how structural changes would affect economic outcomes such as output, trade, welfare and the fragmentation of production.

We see at least four experiments that would provide interesting insight into the reorganization of supply chains in Asia:

- Counterfactual 1): Trade costs have fallen significantly over the past decades and their reduction is cited as the most likely source of the increased fragmentation of production in Asia. Trade costs may decline even further in the near future, as there is still room to improve trade agreements, especially on a multi-lateral basis (Baldwin, 2008). We model this structural change with a 10% reduction in cross-border trade costs.

- Counterfactual 2): Arguably the most dramatic economic change in Asia over the past two decades has been the very high rates of growth in the Chinese economy, along with its opening to trade. With GDP growth rates reaching 10%, the Chinese economy is inducing very large changes in how production in Asia is organized. To examine the role of China in light of our model, we shock the productivity parameter there. We first simulate a 10% productivity increase in China. Formally, this corresponds to a 10% increase in $T_{CHN}^{\frac{1}{\xi}}$.

- Counterfactual 3): We simulate a reduction in the transaction costs γ_{CHN} for China. This scenario could be used to understand growing transparency in contractual disputes, for example. Reduced transaction costs should encourage relatively more domestic outsourcing in China, and raise the share of domestic value added in production/exports.[43]

- Counterfactual 4): Finally, we consider a bilateral trade cost reduction. This allows us to offer a local estimate of the trade elasticity. In particular we are interested in a quantitative evaluation of the claim in Proposition 3, that the elasticity of final goods trade to trade cost changes is larger in the presence of fragmentation. In order to do this, we reduce trade costs between China and the US.

5.4.1 Reduction in trade costs

Our first scenario is a 10% reduction in international trade costs τ. These results are reported in Table 4. For comparison purposes, we provide welfare results for two versions of an Eaton

[43] Kee and Tang (2013) and Li and Liu (2014) provide evidence that the share of domestic value added in Chinese exports has been growing over time.

and Kortum model, a one-stage model without intermediate goods and an alternative model featuring intermediate production (EK-loop). In the EK-loop model, all goods are both final good and intermediate goods. Production costs are a Cobb-Douglas function of labor costs and good price indexes in each country. We calibrate the share of labor such that it equals the value-added-to-gross-output ratio in each country. Hence, all varieties are traded internationally multiple times and welfare gains from trade are magnified by a factor equal to the gross-output-to-value-added ratio.[44]

In level terms the calculated welfare gains in our model are much more similar to the standard EK model than to the EK-loop model. As illustrated in Proposition 3, our supply chain model does not necessarily yield larger welfare gains than one-stage models, especially for countries that tend to be upstream. On average our simulations yield relatively larger gains for downstream countries such as Japan and China, and lower gains for some upstream countries such as Singapore, compared to the one-stage model. In the EK-loop model, all of value added is exposed to trade costs, repeatedly, so the welfare costs of trade cost changes are significantly higher. The data have GO/VA ratios between two and three, so welfare gains in the EK-loop model are multiplied by more than a factor of two. This leads to systematically larger gains from trade then our sequential production chain model.

Table 4: Counterfactual 1): 10% decrease in border trade costs

(10 x change)	Fragmentation					Welfare		
Country	DX	D	N	M share	VAX	Model	EK 1-stage	EK loop
USA	-0.075	0.007	0.008	-0.002	-0.076	0.060	0.054	0.147
SGP	-0.441	-0.383	-0.193	-0.067	-0.099	0.209	0.240	0.537
JPN	-0.197	-0.046	0.019	-0.025	-0.077	0.094	0.069	0.209
TWN	-0.277	-0.197	-0.089	-0.031	-0.083	0.186	0.186	0.480
KOR	-0.238	-0.103	-0.023	-0.028	-0.080	0.170	0.156	0.398
MYS	-0.263	-0.217	-0.052	-0.028	-0.085	0.202	0.222	0.439
THA	-0.249	-0.199	0.004	-0.033	-0.092	0.196	0.198	0.408
IDN	-0.224	-0.022	0.061	-0.050	-0.078	0.189	0.180	0.325
CHN	-0.084	-0.112	0.006	-0.003	-0.084	0.117	0.085	0.288
PHL	-0.210	-0.112	0.024	-0.034	-0.087	0.197	0.202	0.379

Reductions in VAX indicate a lower value added content of trade, which occurs because a larger fraction of exported goods now rely on imported intermediate goods. This change is consistent with Hummels, Ishii and Yi (2001) and Johnson and Noguera (2012), who document a decrease in the VAX ratio over the decades. This does not imply, however, that trade

[44] Costinot and Rodriguez-Clare (2014) model a similar loop with multiple industries.

is growing faster for upstream goods. Actually, our simulations indicate that trade grows faster for downstream stages: both the share of intermediate goods trade (M-share) and trade-weighted upstreamness (DX_i) decrease for all 10 countries when trade costs fall.[45] This finding illustrates that final goods trade is more sensitive than intermediate goods trade to reduced trade costs (see Proposition 3): trade flows increase relatively more for goods that embody small shares of domestic labor and larger shares of traded intermediate goods. In a similar fashion, value-added-weighted upstreamness (D_i) tends to decrease with trade.

5.4.2 Increasing labor productivity in China

Table 5 reports results from a 10% shock to labor productivity in China, $T_{CHN}^{\frac{1}{\xi}}$. This is a uniform shock that improves labor productivity at all points in the chain. Our interest is in seeing how such shocks affect China's relative position in chains, and the degree to which such shocks spill over into other countries.

Table 5: Counterfactual 2): 10% increase in labor productivity in China

(10 x change)	DX	D	N	M share	VAX	Welfare
CHN	-0.322	-0.281	-0.171	-0.038	0.000	0.933
USA	0.182	0.075	0.011	0.038	0.008	0.022
SGP	0.177	0.159	0.109	0.022	0.006	0.031
JPN	0.200	0.104	0.024	0.039	0.010	0.021
TWN	0.185	0.172	0.096	0.023	0.007	0.036
KOR	0.234	0.195	0.063	0.036	0.012	0.026
MYS	0.198	0.185	0.053	0.027	0.012	0.028
THA	0.101	0.099	-0.003	0.014	0.007	0.036
IDN	0.262	0.237	0.051	0.045	0.017	0.029
PHL	0.224	0.199	0.028	0.035	0.014	0.037

The welfare changes reported in column 2 show that the vast majority of the welfare gains accrue to China, which sees a 9.33% increase in welfare from a 10% productivity shock. The gains elsewhere are limited, and reasonably similar across countries. The most notable changes have to do with changes in relative position in supply chains. Changes in China's DX, and D indices indicate that the technology shock moves Chinese production significantly closer to final demand, while the other countries move upstream. China's move downstream can also be seen in the fourth column (M share), which shows a reduction in the intermediate share of China's exports. This market size effect is similar to the overshooting effect in Baldwin and

[45] Fally (2012, Figure 4) documents reductions in trade-weighted upstreamness over a period (1962-1996) when trade costs arguably fell substantially.

Venables (2012). With a larger fraction of world income spent by consumers in China and a larger fraction of tasks being performed in China, other vertically-related tasks are also more likely to be performed there to save on trade costs.

5.4.3 Reduced transaction costs in China

When compared with the productivity shock, the welfare effects of a 10% reduction in transaction costs in China are smaller for China and relatively larger for other countries. In the case of a shock to internal transaction costs, China's production moves upstream, as indicated by the movements in DX, D and the share of exports in intermediate goods. Similarly, the increase in N reflects greater fragmentation of production within China. Other countries also move upstream because falling Chinese transaction costs lead to longer chains. This counterfactual also leads to a higher value-added export to gross export ratio (VAX) for China, and heterogeneous effect on other countries' VAX ratio. In broad terms, a move upstream by China is consistent with the evidence presented in Kee and Tang (2013), who find that Chinese exporters have been shifting their purchases of inputs from foreign to domestic sources.

Table 6: Counterfactual 3): 10% decrease in Chinese transaction costs γ_i

(10 x change)	DX	D	N	M share	VAX	Welfare
CHN	1.418	4.646	3.832	0.095	0.090	0.667
USA	0.223	0.130	0.195	-0.003	0.015	0.029
SGP	0.062	0.106	0.155	-0.002	-0.014	0.032
JPN	0.139	0.114	0.190	-0.004	0.006	0.028
TWN	0.067	0.089	0.078	-0.005	-0.005	0.040
KOR	0.166	0.174	0.126	0.007	0.002	0.032
MYS	0.044	0.071	0.024	-0.008	-0.003	0.036
THA	0.081	0.096	0.081	0.003	-0.004	0.038
IDN	0.229	0.250	0.123	0.009	0.001	0.035
PHL	0.183	0.217	0.079	0.001	0.000	0.039

5.4.4 Reduced trade costs between China and the US

As noted above and in Yi (2010), international fragmentation raises the elasticity of trade to trade costs. In order to explore the quantitative magnitude of this effect we shock trade costs for a single country pair (US and China) and measure trade responses. We compare results in our model to those in the calibrated EK model described above (note that one-stage EK and EK-loop models yield the same elasticities). Recall that our two-country model showed a higher elasticity for final goods trade than in the standard EK model. We see that final goods trade is indeed more responsive to trade cost changes in our model, as is the elasticity of total trade.

Table 7: Counterfactual 4): 10% bilateral decrease in US-China trade costs

	Importer-exporter pair	With cross-border fragmentation		EK
		All trade	*Final goods*	*All trade*
$\Delta \log \pi_{ni}$	USA-CHN	0.063	0.065	0.050
	CHN-USA	0.059	0.057	0.043
Trade cost elasticity	USA-CHN	5.510	5.727	4.589
	CHN-USA	6.170	6.201	5.272

6 Concluding remarks

Recent empirical work has documented sizable differences in supply chain length. In this paper we attempt to explain such variation in an integrated framework that links the internalization decisions of firms within a supply chain to the organization of the chain across countries. We develop a continuous firm representation of the optimal organization of a multi-country supply chain, with an endogenous allocation of tasks across countries and firms. We derive formal and intuitive representations of the gains from fragmentation within a chain and relate these to the implicit price of tasks and the price of the final good.

In this Coasian setting, we show that the same parameters that shape the boundaries of firms also determine comparative advantage within international supply chains. Conditional on participation in a supply chain, the lower a country's coordination costs the more downstream it will be. Low within-firm coordination costs also imply an ability to host larger firms. By contrast, countries with high transaction costs will tend to participate downstream because their disadvantages can only be offset in chains for which they have low within-firm-coordination costs and larger firms.

In order to link the model to the prominent literature on the welfare gains of trade we use a conventional Ricardian framework to produce a general equilibrium model with multiple chains, and with exogenous productivity shocks *across* chains. We derive implications for trade elasticities and welfare, relative to standard theoretical benchmarks (Arkolakis et al 2012). Among a number of theoretical results we show that the elasticity of final goods trade to trade costs is larger in the presence of fragmentation. Relative to the Arkolakis et al (2012) formula without fragmentation, we show that welfare effects are smaller for upstream countries and larger for downstream countries in the presence of fragmentation.

To illustrate the quantitative implications of the model we conduct a calibration exercise with counterfactual simulations. In our model, the Coasian structural parameters determine the gross-output-to-value added ratio, a fact that facilitates calibration. We shock international trade costs and find numerical evidence that is consistent with our theoretical results. We find that shocks to Chinese productivity and to the coordination cost parameter in China to highlight different implications for welfare, spillover to other countries and specialization along production chains.

References

Antràs, Pol and Alonso de Gortari, "On the Geography of Global Value Chains," *Working paper*, 2017.

_ **and Davin Chor**, "Organizing the Global Value Chain," *Econometrica*, 2013, *81* (6), 2127–2204.

_ **,** _ **, Thibault Fally, and Russell Hillberry**, "Measuring the Upstreamness of Production and Trade Flows," *American Economic Review Papers and Proceedings*, 2012, *102* (3), 412–416.

_ **, Teresa C Fort, and Felix Tintelnot**, "The Margins of Global Sourcing: Theory and Evidence from US Firms," *Working paper*, 2014.

Arkolakis, Costas, Arnaud Costinot, and Andres Rodriguez-Clare, "New Trade Models, Same Old Gains?," *American Economic Review*, 2012, *102* (1), 94–130.

Baldwin, Richard, "Global Supply Chains: Why They Emerged, Why They Matter, and Where They Are Going," *C.E.P.R. Discussion Papers*, August 2012, *9103.*

_ **and Javier Lopez-Gonzalez**, "Supply-chain Trade: A Portrait of Global Patterns and Several Testable Hypotheses," *The World Economy*, 2015, *38* (11), 1682–1721.

Baldwin, Richard E., "Managing the Noodle Bowl: The Fragility of East Asian Regionalism," *The Singapore Economic Review (SER)*, 2008, *53* (03), 449–478.

Bernard, Andrew B., Andreas Moxnes, and Karen Helene Ulltveit-Moe, "Two-Sided Heterogeneity and Trade," *NBER Working Paper no. 20136*, 2014.

_ **,** _ **, and Yukiko U. Saito**, "Production Networks, Geography and Firm Performance," *Dartmouth Tuck manuscript*, 2016.

Chaney, Thomas and Ralph Ossa, "Market Size, Division of Labor, and Firm Productivity," *Journal of International Economics*, 2013, *90* (1), 177–180.

Costinot, Arnaud, "An Elementary Theory of Comparative Advantage," *Econometrica*, 2009, *77* (4), 1165–1192.

_ **and Andres Rodriguez-Clare**, "Trade Theory with Numbers: Quantifying the Consequences of Globalization," *Handbook of International Economics*, 2014, *4*, 197–261.

_ **, Jonathan Vogel, and Su Wang**, "An Elementary Theory of Global Supply Chains," *Review of Economic Studies*, 2013, *80*, 109–144.

Disdier, Anne-Celia and Keith Head, "The Puzzling Persistence of the Distance Effect on Bilateral Trade," *The Review of Economics and Statistics*, 2008, *90* (1), 37–48.

Dornbusch, Rudiger, Stanley Fischer, and Paul Anthony Samuelson, "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *The American Economic Review*, 1977, pp. 823–839.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography, and Trade," *Econometrica*, 2002, *70*, 1741–1780.

Fally, Thibault, "Production Staging: Measurement and Evidence," *Working paper*, 2012.

Hillberry, Russell and David Hummels, "Explaining Home Bias in Consumption: The Role of Intermediate Input Trade," *NBER working paper no. 9020*, 2002, pp. 1–33.

Johnson, Robert and Andreas Moxnes, "Technology, Trade Costs, and the Pattern of Trade with Multi-Stage Production," *Working Paper*, 2013.

_ **and Guillermo Noguera**, "Accounting for Intermediates: Production Sharing and Trade in Value Added," *Journal of International Economics*, 2012, *86* (2), 224–236.

Johnson, Robert C., "Five Facts about Value-Added Exports and Implications for Macroeconomics and Trade Research," *Journal of Economic Perspectives*, 2014, *28* (2), 119–42.

Johnson, Robert C and Guillermo Noguera, "Proximity and production fragmentation," *The American Economic Review*, 2012, *102* (3), 407–411.

_ **and** _ , "Fragmentation and Trade in Value Added over Four Decades," *Working paper*, 2013.

Kee, Hiau Looi and Heiwai Tang, "Domestic Value Added in Chinese Exports: Firm-level Evidence," *Working Paper*, 2013.

Kikuchi, Tomoo, Kazuo Nishimura, and John Stachurski, "Span of Control, Transaction Costs and the Structure of Production Chains," *Theoretical Economics*, forthcoming.

Koopman, Robert, Zhi Wang, and Shang-Jin Wei, "How Much of Chinese Exports is Really Made in China? Assessing Domestic Value-added when Processing Trade is Pervasive," *Working Paper*, 2010.

Kraemer, Kenneth L., Greg Linden, and Jason Dedrick, "Who Captures Value in the Apple iPad and iPhone?," *Working paper*, 2011.

Kremer, Michael, "The O-Ring Theory of Economic Development," *The Quarterly Journal of Economics*, 1993, *108* (3), 551–75.

Krugman, Paul and Anthony J Venables, "Integration, Specialization, and Adjustment," *European Economic Review*, 1996, *40* (3), 959–967.

Li, Ben G. and Yibei Liu, "Moving Up the Value Chain," *Boston College Working Paper*, 2014.

Melitz, Marc J. and Stephen J. Redding, "Missing Gains from Trade?," *American Economic Review*, 2014, *104* (5), 317–21.

Meng, Bo, Yaxiong Zhang, and Satoshi Inomata, "Applications of IDE-JETRO's International Input Output Tables," *Economic Systems Research*, 2013, *25* (1), 122–142.

Olken, Benjamin and Patrick Barron, "The Simple Economics of Extortion: Evidence from Trucking in Aceh," *Journal of Political Economy*, 2009, *117* (3), 417–452.

Puzzello, Laura, "A Proportionality Assumption and Measurement Biases in the Factor Content of Trade," *Journal of International Economics*, 2012, *87* (1), 105–111.

Rodriguez-Clare, Andres, "Offshoring in a Ricardian World," *American Economic Journal: Macroeconomics*, 2010, *2* (2), 227–258.

Simonovska, Ina and Michael E Waugh, "The Elasticity of Trade: Estimates and Evidence," *Journal of International Economics*, 2014, *92* (1), 34–50.

Yi, Kei-Mu, "Can Vertical Specialization Explain the Growth of World Trade?," *Journal of Political Economy*, 2003, *111* (1), 52–102.

_ , "Can Multistage Production Explain the Home Bias in Trade?," *American Economic Review*, 2010, *100* (1), 364–93.

A Mathematical Appendix

Proof of Lemma 1: First best

Sketch of proof. Given that we have constant returns to scale, prices equal unit costs and marginal costs in the competitive equilibrium. With free entry, firms in production chain have exactly zero profits all along the chain in equilibrium, and any chain associated with a lower final good price would have negative profits somewhere along the chain. As we will describe, the equilibrium chain corresponds to the chain that yields the lowest price of final goods under the constraints that: i) all firms along the chain choose their scope to maximize their profits; ii) profits are zero in equilibrium. What is not trivial is that firm scope in the social planner's solution maximizes profits of each firm along the chain. We need to show that, defining the price schedule as the cumulative cost along the chain, profit maximization for each firm along the chain leads to the same decision in firm scope as with the social planner's problem.

For this Lemma, we introduce two pieces of notation:

- We denote by S_{if} the amount of tasks embodied at stage f in country i. The planning problem's constraint $\sum_i \int_{f=0}^{F} s_{if} df = 1$ cannot be used directly in the competitive solution, because the requirement that the entire chain is completed cannot enter directly into an individual firm's problem. Instead, each firm takes as given its position on the chain and the range of tasks being performed by their suppliers.

- We denote by $p^C(S_{if})$ the sequence of prices associated with the range of tasks S_{if} in the competitive equilibrium. This price is imposed upon each firm f by its downstream buyer. Similarly, we denote by $p^W(S_{if})$ the sequence of prices associated with the range of tasks S_{if} in the social planner's solution.

We focus on a specific chain ω (hence we remove ω from the notation below for the sake of exposition).

Characterization of the competitive equilibrium. Given a sequence of prices $p^C(S_{if})_{f=0}^{F_i}$ (and wages, w_i which we subsume in $c_i(s_{if})$) and a required bundle of quantities $q_0 = \bar{q}_{if}$ and embodied stages $\bar{S}_{if}$ to be delivered to the next downstream firm, the problem facing each firm f in country i is to choose q_{if} and s_{if} to maximize profits π_{if} solving:

$$\pi_{if} = \max_{q_{if}, s_{if}} \quad q_{if}\left[p^C(S_{if}) - c(s_{if})df\right] - p^C(S_{i,f+df})q_{i,f+df} \tag{46}$$

$$\begin{aligned} \text{s.t.} \quad q_{if} &= q_{i,f-df}(1+\gamma_i df) \\ S_{i,f+df} + s_{if}df &= S_{if} \\ S_{if} &= \bar{S}_{if} \\ q_{if} &= \bar{q}_{if} \end{aligned}$$

The first constraint represents the goods market clearance condition for the output of firm f. The second constraint insures that the firm's choice of s_{if} together with the stages embodied in its own inputs are sufficient to meet the input requirements of the downstream firm, which demands S_{if} embodied stages. The final two constraints define the contractual requirements for output and tasks to be performed.

We can simply rewrite (46) as:

$$\pi_{if} = \max_{s} \quad q\left[p^C(S_{if}) - c_i(s)df - (1+\gamma_i df)p^C(S_{if} - sdf)\right] \tag{47}$$

Joint solution of the first order conditions (w.r.t. q and s) represent a solution to the competitive firm's problem. We link the solution to the firms' problem to the equilibrium price function in what follows.

First, the first-order condition in s yields:

$$c_i'(s)\ df = (1+\gamma_i df)\,\frac{dp_i^C}{dS}\ df \tag{48}$$

Taking the limit where df is infinitesimally small (i.e. ignoring second order terms in df^2), we simply obtain that the marginal cost of an additional task performed within the firm should be equal to the marginal price associated with this task along the chain for the equilibrium firm scope s_{if}:

$$c_i'(s_{if}) = \frac{dp_i^C}{dS} \tag{49}$$

Note that we would obtain exactly the same result if, instead of choosing their intermediate goods and how much to outsource, each firm were to choose how much to produce given the intermediate goods that they receive. The marginal cost would be $c_i'(s_{if})$ while the marginal gains would be $\frac{dp_i^C}{dS}$

In turn, the zero-profit condition (i.e. the first-order condition in q) implies that the output price for each firm equals its average cost. This leads to:

$$p^C(S_{if}) - c_i(s)df - (1+\gamma_i df)\,p_i^C(S_{if} - sdf) = 0 \tag{50}$$

Rearranging, this can be written:

$$p^C(S_{if}) - p^C(S_{if} - sdf) = c_i(s)df + \gamma_i df p^C(S_{if} - sdf)$$

Taking the limit where df is infinitesimally small (i.e. ignoring second-order terms in df^2), we obain:

$$\frac{dp^C}{dS} s_{if} = c_i(s_{if}) + \gamma_i p^C(S_{if}) \tag{51}$$

for the competitive price schedule and equilibrium firm size s_{if}. To summarize, the price schedule for the competitive equilibrium is characterized by the optimal firm scope in equation (49) and the free-entry condition (51).

Comparison with the social planner's solution. We have yet to show that the social planner's solution satisfies these two equations. To do so, we need to characterize the price in the social planner's solution associated with one unit of the intermediate good as a function of the range of tasks that has been completed. The first-best chain is the chain that minimize the price of the final good:

$$\min P_1 \tag{52}$$

$$\begin{aligned}
\textit{over:} \quad & i(n)\ ,\ s_{nf}\ ,\ F_n\ ,\ S_n\ ,\ P_n \\
\textit{under the constraints:} \quad & P_n = \left[\int_{f=0}^{F_n} \mathrm{e}^{\gamma_{i(n)} f} c_{i(n)}(s_{nf}) df \ + \ \mathrm{e}^{\gamma_{i(n)} F_n} \tau P_{n+1} \right] \\
& S_n = \int_{f=0}^{F_n} s_{nf} df \\
& \sum_{i=1}^{N} S_n = 1
\end{aligned}$$

First, our goal is to show that the allocation of tasks across firms within a given country satisfies the competitive market equilibrium conditions described above. Taking the sequence of countries $i(n)$ as given (we discuss below why the sequence of countries is identical to the one in the competitive equilibrium), we define $p^W(S)$ as the minimum unit cost to performed a range S of tasks, for each $S \in [S_{n+1}, S_n]$ and country $i = i(n)$ where S_n, P_{n+1} and $i(n)$ are the solution from above:

$$p^W(S) = \min_{s_f, F} \left[\int_f^F e^{\gamma_i f} c_i(s_f) df + e^{\gamma_i F} \tau P_{n+1} \right] \tag{53}$$

$$\text{s.t.} \quad S_{n+1} + \int_{f=0}^{F} s_f df = S$$

Notice that we can split the minimization problem in two parts. For any $S' \in (S_{n+1}, S)$, we obtain:

$$\begin{aligned}
p^W(S) &= \min_{s_f, F} \left[\int_f^F e^{\gamma_i f} c_i(s_f) df + e^{\gamma_i F} P_{n+1} \right] \qquad \text{s.t.} \int_{f=0}^{F} s_f df = S \\
&= \min_{s_f, F, s'_f, F'} \left\{ \int_f^F e^{\gamma_i f} c_i(s_f) df + e^{\gamma_i F'} \left[\int_{F'}^F e^{\gamma_i (f-F')} c_i(s_f) df + e^{\gamma_i (F-F')} P_{n+1} \right] \right\} \\
& \qquad \text{s.t.} \int_{f=0}^{F'} s_{if} df = S - S' \qquad \text{and} \qquad \int_{f=F'}^{F} s_f df = S' \\
&= \min_{s_f, F'} \left\{ \int_{f=0}^{F'} e^{\gamma_i f} c_i(s_f) df + e^{\gamma_i F'} p^W(S') \right\} \qquad \text{s.t.} \int_{f=0}^{F'} s_f df = S - S'
\end{aligned} \tag{54}$$

This implies that the optimal sequence of firm scope s_f is common across all price minimization $p^W(S)$ within a given country. In other words, the scope of firm after completing a range S of tasks is independent of what happens downstream (within a country).

Let us now examine optimal firm scope in the social planner's minimization problem. After completing a range S of tasks, the price minimization associated with $p^W(S)$ implies that the marginal cost $c'(s)$ equals the Lagrange multiplier associated with the constraint $\int_{f=0}^{F} s_{if} df = S$. Since the range S of tasks appears only in this constraint, the Lagrange multiplier (shadow cost of completing a task) also equals the derivative $\frac{dp^W}{dS}$. Hence, we obtain the same condition as (49) in the competitive equilibrium:

$$c_i'(s) = \lambda_i(S) = \frac{dp^W}{dS}$$

Next, taking equation (54) with $S' = S - sdf$, where s is the optimal scope at this stage f, we obtain:

$$p^W(S) = c_i(s_f)df + (1 + \gamma_i df)p^W(S - sdf)$$

Taking the limit where df is infinitesimally small (i.e. neglecting second order terms df^2) yields:

$$\frac{dp^W}{dS} = \frac{c_i(s_f) + \gamma_i p^W(S)}{s_f}$$

for the optimal firm scope s at this stage f. This is the same condition as the free-entry condition (51) in the competitive equilibrium within each country.

Finally, we argue that the allocation of tasks and firms across countries is identical. Obviously, the first-best solution corresponds to the sequence of countries and cross-country allocation of tasks S_n that yields the minimum final good price. The same applies to the competitive equilibrium. If a sequence of countries in a chain does not yield the lowest price, a lower-cost chain can enter (with a better sequence of countries) and capture all its consumers. The free-entry condition for chains is key to this argument.

Proofs for Section 3.1: Within-country fragmentation

FOCs: The first-order conditions of this planning program correspond to equations (15) and (16):

$$\begin{aligned} \text{For } s_{if}: &\quad e^{\gamma_i f} w_i c_i'(s_{if}) \;=\; \lambda_i \\ \text{For } F_i: &\quad e^{\gamma_i F_i} w_i c_i(s_{i,F_i}) \;+\; e^{\gamma_i F_i} P_i^M \gamma_i \;=\; s_{i,F_i}\, \lambda_i \end{aligned}$$

Using our parameterization of the cost function, the first-order condition for s_{if} can be rewritten:

$$e^{\gamma_i f} a_i w_i s_{if}^{\theta_i} = \lambda_i$$

which yields:

$$s_{if} = \left(\frac{\lambda_i}{a_i w_i}\right)^{\frac{1}{\theta_i}} e^{-\frac{\gamma_i f}{\theta_i}}$$

By combining the first-order condition in F_i and the first-order condition in s_{if}, we obtain:

$$e^{\gamma_i F_i} a_i w_i \frac{s_{i,F_i}^{\theta_i+1}}{\theta_i+1} + e^{\gamma_i F_i} P_i^M \gamma_i = s_{i,F_i} \,.\, e^{\gamma_i F_i} a_i w_i s_{i,F_i}^{\theta_i}$$

which can be simplified into:

$$\tfrac{w_i\, a_i\, \theta_i}{\theta_i+1}\, s_{i,F_i}^{\theta_i+1} = \gamma_i P_i^M$$

and thus:

$$s_{i,F_i} = \left[\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i}\right]^{\frac{1}{\theta_i+1}}$$

Lagrangian multiplier: It is the solution of:

$$\int_0^{F_i} s_{if} df = S_i$$

where s_{if} and F_i functions of the Lagrangian multiplier as shown above. The left-hand side can be rewritten:

$$\begin{aligned}
\int_0^{F_i} s_{if} df &= \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \int_0^{F_i} e^{-\frac{\gamma_i f}{\theta_i}} df \\
&= \frac{\theta_i}{\gamma_i} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \left[1 - e^{-\frac{\gamma_i F_i}{\theta_i}}\right] \\
&= \frac{\theta_i}{\gamma_i} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \left[1 - \left(\tfrac{\lambda_i}{w_i a_i}\right)^{-\frac{1}{\theta_i}} s_{i,F_i}\right] \\
&= \frac{\theta_i}{\gamma_i} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} - \frac{\theta_i s_{i,F_i}}{\gamma_i}
\end{aligned}$$

We obtain the following solution in λ_i such that the expression above equals S_i:

$$\lambda_i = w_i a_i \left[\frac{\gamma_i S_i}{\theta_i} + \left(\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i}\right)^{\frac{1}{\theta_i+1}}\right]^{\theta_i}$$

Final price: Expression for P_i:

$$
\begin{aligned}
P_i &= \int_{f=0}^{F_i} e^{\gamma_i f} w_i c_i(s_{if}) df \;+\; e^{\gamma_i F_i} P_i^M \\
&= \int_{f=0}^{F_i} e^{\gamma_i f} \frac{w_i a_i s_{if}^{\theta_i+1}}{\theta_i+1} df \;+\; e^{\gamma_i F_i} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} \int_{f=0}^{F_i} e^{-\frac{\gamma_i f}{\theta_i}} df \;+\; e^{\gamma_i F_i} P_i^M \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} \left[1 - e^{-\frac{\gamma_i F_i}{\theta_i}}\right] \;+\; e^{\gamma_i F_i} P_i^M \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} e^{-\frac{\gamma_i F_i}{\theta_i}} \;+\; \frac{w_i a_i}{\gamma_i} \left(\frac{\gamma_i P_i^M}{w_i a_i}\right) e^{\gamma_i F_i} \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{1}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \lambda_i s_{i,F_i} \;+\; \frac{wa}{\gamma} \tfrac{\theta}{\theta+1} s_{i,F_i}^{\theta_i+1} e^{\gamma_i F_i} \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{1}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left[\lambda_i s_{i,F_i} - w_i a_i s_{i,F_i}^{\theta_i+1} e^{\gamma_i F_i}\right] \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} \;+\; 0
\end{aligned}
$$

Using the expression above for λ_i, we obtain equations (22) and (23) in the text:

$$
P_i = \left[\frac{S_i}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}}\right]^{\theta_i+1}
$$

with

$$
A_i = a_i \left(\gamma_i \tfrac{\theta_i+1}{\theta_i}\right)^{\theta_i}
$$

It is also useful to note that:

$$
\lambda_i = (w_i A_i)^{\frac{1}{\theta_i+1}} (P_i)^{\frac{\theta_i}{\theta_i+1}} \tag{55}
$$

Labor demand: Each unit of last-stage good produced in country i generates the demand for labor $w_i L_i^D = \frac{\partial \log P_i}{\partial \log w_i}$ in country i. This yields:

$$
w_i L_i^D = \frac{S_i}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} \left[\frac{S_i}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}}\right]^{\theta_i} \tag{56}
$$

Prices along the chain: To obtain a simple expression for the value-added-to-gross-output ratio $\frac{c_i(s_{if})}{p_{if}}$, the first step is to compute p_{if} is the price along the chain.

$$\begin{aligned}
p_{if} &= \int_{f'=f}^{F_i} e^{\gamma_i(f'-f)} c(s_{if'})df' \ + \ e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} \int_{f'=f}^{F_i} e^{\gamma_i(f'-f)} \ s_{if'}^{\theta_i+1} df' \ + \ e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} \int_{f'=f}^{F_i} e^{-\gamma_i(F_i-f')} \ s_{if'}^{\theta_i+1} df' \ + \ e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} s_{i,F_i}^{\theta_i+1} \int_{f'=f}^{F_i} e^{-\gamma_i(F_i-f')} e^{\gamma_i\left(\frac{\theta_i+1}{\theta_i}\right)(F_i-f')} df' \ + \ e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} s_{i,F_i}^{\theta_i+1} \int_{f'=f}^{F_i} e^{\frac{\gamma_i(F-f')}{\theta_i}} df' \ + \ e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} e^{\gamma_i(F_i-f)} s_{i,F_i}^{\theta+1} \left[e^{\frac{\gamma_i(F_i-f)}{\theta}} - 1 \right] \ + \ e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} s_{if}^{\theta_i+1} \ - \ \frac{e^{\gamma_i(F_i-f)}}{\gamma_i} \left[\frac{w_i a_i \theta_i}{\theta_i+1} s_{i,F_i}^{\theta_i+1} \ - \ \gamma_i P_i^M \right] \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} s_{if}^{\theta_i+1} \ - \ 0 \\
&= \frac{\theta_i}{\gamma_i} \ w_i c_i(s_{if})
\end{aligned}$$

Hence the gross-output-to-value-added ratio at the firm level is:

$$\frac{p_{if}}{w_i c_i(s_{if})} = \frac{\theta_i}{\gamma_i}$$

We also obtain the same expression for the aggregate gross-output-to-value-added ratio. If we define gross output as the total value of all transactions:

$$GO_i = \int_0^{F_i} q_{if} p_{if} df$$

we obtain:

$$\frac{GO_i}{VA_i} = \frac{\int_0^{F_i} q_{if} p_{if} df}{\int_0^{F_i} q_{if} w_i c_i(s_{if}) df} = \frac{\int_0^{F_i} \frac{\theta_i}{\gamma_i} q_{if} w_i c_i(s_{if}) df}{\int_0^{F_i} q_{if} w_i c_i(s_{if}) df} = \frac{\theta_i}{\gamma_i}$$

Proof of Proposition 1

To simplify the exposition, we index countries by n, with $n = 1$ referring to the most downstream country and $n = N$ the most upstream country. The goal is to minimize:

$$\min P_1 \tag{57}$$

under the constraints:

$$P_{n+1} = \widetilde{P}_n(S_n, \tau P_{n+1}) \qquad \text{and} \qquad \sum_{n=n}^{N} S_n = 1$$

with:

$$\widetilde{P}_n(S, P^M) = \left[\frac{S}{\theta_n + 1} (A_n w_n)^{\frac{1}{\theta_n+1}} + \left(P^M\right)^{\frac{1}{\theta_n+1}} \right]^{\theta_n + 1}$$

Under which condition can country n be downstream from country $n+1$? Let us take as given the price in country $n+2$ and consider the following function:

$$m(x)^{\theta_n+1} = \widetilde{P}_n(S_n - x, \tau \widetilde{P}_n(S_{n+1} + x, \tau P_{n+2}))$$

This function $m(x)$ indicates by how much the price of output in n will increase if we shift a measure x of tasks from country n to country $n+1$.

$$m(x) = \frac{(S_n - x)}{\theta_n + 1} (A_n w_n)^{\frac{1}{\theta_n+1}} + \left[\frac{(S_{n+1} + x)}{\theta_{n+1} + 1} (A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}} \right]^{\frac{\theta_{n+1}+1}{\theta_n+1}}$$

If we are at equilibrium, the function $m(x)$ must be at its minimum at $x = 0$. The first-order condition imply that $m'(x) = 0$. We obtain that:

$$m'(x) = -\frac{(A_n w_n)^{\frac{1}{\theta_n+1}}}{\theta_n + 1} + \frac{(A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n + 1} \left[\frac{(S_{n+1} + x)}{\theta_{n+1} + 1} (A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}} \right]^{\frac{\theta_{n+1}-\theta_n}{\theta_n+1}} \tag{58}$$

must equal zero at $x = 0$.

More importantly, to prove Proposition 1, one needs to examine the second order condition, which imposes $m''(x) > 0$. If $m''(x)$ were negative, $x = 0$ would not be a local minimum and it would be more efficient to shift some tasks to either country n or $n+1$.

As one can see in equation (58), the right-hand-side term is increasing in x (i.e. $m''(x) > 0$) only if the exponent $\frac{\theta_{n+1}-\theta_n}{\theta_n+1}$ is positive. This proves that we must have $\theta_{n+1} > \theta_n$ at equilibrium.

Finally, it is not difficult to verify that two consecutive countries cannot have the same $\theta_n = \theta_{n+1}$ as long as we have non-zero trade costs $\tau - 1 > 0$.

Other proofs for Section 3.2

Prices along the chain: Using again equation (58), the first-order condition $m'(0) = 0$ implies:

$$\begin{aligned}
\frac{(A_n w_n)^{\frac{1}{\theta_n+1}}}{\theta_n + 1} &= \frac{(A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n + 1} \left[\frac{S_{n+1}}{\theta_{n+1} + 1} (A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}} \right]^{\frac{\theta_{n+1}-\theta_n}{\theta_n+1}} \\
&= \frac{(A_{n+1} w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n + 1} (P_{n+1})^{\frac{1}{\theta_n+1} \cdot \frac{\theta_{n+1}-\theta_n}{\theta_{n+1}+1}}
\end{aligned}$$

This yields expression (36) for the price of goods sold by country $n+1$:

$$\tau P_{n+1} = (A_n w_n)^{\frac{\theta_{n+1}+1}{\theta_{n+1}-\theta_n}} (\tau A_{n+1} w_{n+1})^{-\frac{\theta_n+1}{\theta_{n+1}-\theta_n}}$$

For country i, this gives:

$$P_n = (A_{n-1} w_{n-1}/\tau)^{\frac{\theta_n+1}{\theta_n-\theta_{n-1}}} (A_n w_n)^{-\frac{\theta_{n-1}+1}{\theta_n-\theta_{n-1}}}$$

Allocation of tasks across countries: The range of tasks performed by country i can then be obtained as:

$$\frac{S_n}{\theta_n+1} = (A_n w_n)^{-\frac{1}{\theta_n+1}} \left[P_n^{\frac{1}{\theta_n+1}} - (\tau P_{n+1})^{\frac{1}{\theta_n+1}} \right]$$
$$= (A_n w_n)^{-\frac{1}{\theta_n+1}} \left[(A_{n-1} w_{n-1}/\tau)^{\frac{1}{\theta_n-\theta_{n-1}}} (A_n w_n)^{-\frac{\theta_{n-1}+1}{(\theta_n+1)(\theta_n-\theta_{n-1})}} - (A_n w_n)^{\frac{\theta_{n+1}+1}{(\theta_n+1)(\theta_{n+1}-\theta_n)}} (\tau A_{n+1} w_{n+1})^{-\frac{1}{\theta_{n+1}-\theta_n}} \right]$$

which can be simplified into expression (36) given in the text:

$$\frac{S_n}{\theta_n+1} = \left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$

For the last country N in the chain, we obtain:

$$\frac{S_N}{\theta_N+1} = \left(\frac{A_{N-1} w_{N-1}}{\tau A_N w_N} \right)^{\frac{1}{\theta_N-\theta_{N-1}}}$$

Finally, the range of tasks performed by the last country in the chain is:

$$S_1 = 1 - \sum_{n=2}^{N-1} S_n$$
$$= 1 - (\theta_N+1) \left(\frac{A_{N-1} w_{N-1}}{\tau A_N w_N} \right)^{\frac{1}{\theta_N-\theta_{N-1}}} - \sum_{n=2}^{N-1} (\theta_n+1) \left[\left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}} \right]$$
$$= 1 - \sum_{n=2}^{N} (\theta_n+1) \left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} + \sum_{n=2}^{N-1} (\theta_n+1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$
$$= 1 - \sum_{n=1}^{N-1} (\theta_{n+1}+1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}} + \sum_{n=2}^{N-1} (\theta_n+1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$
$$= 1 - (\theta_1+1) \left(\frac{A_1 w_1}{\tau A_2 w_2} \right)^{\frac{1}{\theta_2-\theta_1}} - \sum_{n=1}^{N-1} (\theta_{n+1}-\theta_n) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$

Final good price: Using the above expressions for S_1 and P_2, we obtain the price of the final good:

$$\begin{aligned}
P_1 &= \left[\frac{S_1}{\theta_1+1}(A_1w_1)^{\frac{1}{\theta_1+1}} + (\tau P_2)^{\frac{1}{\theta_1+1}}\right]^{\theta_1+1} \\
&= \left[\frac{S_1}{\theta_1+1}(A_1w_1)^{\frac{1}{\theta_1+1}} + (A_1w_1)^{\frac{\theta_2+1}{(\theta_1+1)(\theta_2-\theta_1)}}(\tau A_2w_2)^{-\frac{1}{\theta_2-\theta_1}}\right]^{\theta_1+1} \\
&= \frac{A_1w_1}{(\theta_1+1)^{\theta_1+1}}\left[S_1 + (\theta_1+1)\left(\frac{A_1w_1}{\tau A_2w_2}\right)^{\frac{1}{\theta_2-\theta_1}}\right]^{\theta_1+1} \\
&= \frac{A_1w_1}{(\theta_1+1)^{\theta_1+1}}\left[1 - \sum_{n=1}^{N-1}(\theta_{n+1}-\theta_n)\left(\frac{A_nw_n}{\tau A_{n+1}w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]^{\theta_1+1} \\
&= \frac{A_1w_1}{(\theta_1+1)^{\theta_1+1}}\Theta\big(\mathbf{wA},\tau\big)
\end{aligned}$$

This corresponds to expression (38) in the text with the term in Θ reflecting gains from fragmentation:

$$\Theta\big(\mathbf{wA},\tau\big) = \left[1 - \sum_{n=1}^{N-1}(\theta_{n+1}-\theta_n)\left(\frac{w_nA_n}{\tau w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]^{\theta_1+1}$$

Demand for labor: By the envelope theorem, demand for labor in upstream countries can be obtained by:

$$\frac{l_nw_n}{P_1} = \frac{d\log P_1}{d\log w_n} = \frac{d\log\Theta}{d\log w_n}$$

This gives expression (39) in the text:

$$\frac{l_nw_n}{P_1} = \frac{\left(\frac{w_{n-1}A_{n-1}}{\tau w_nA_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{w_nA_n}{\tau w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}}{\left(\frac{P_1}{A_1w_1}\right)^{\frac{1}{\theta_1+1}}}$$

Lagrangian multiplier: The Lagrangian multiplier λ_G is equal to the Lagrangian multiplier λ_1 in the most downstream country (since $q_1 = 1$). Using (55), we obtain:

$$\lambda_G = (A_1w_1)^{\frac{1}{\theta_1+1}}P_1^{\frac{\theta_1}{\theta_1+1}} = \frac{A_1w_1}{(\theta_1+1)^{\theta_1}}\Theta\big(\mathbf{wA},\tau\big)^{\frac{\theta_1}{\theta_1+1}}$$

Firm scope: For the most downstream firm in the most downstream country, equation (34) becomes:

$$w_1c_1'(s_{1,f=0}) = \lambda_G$$

This gives:

$$w_1a_1s_{1,f=0}^{\theta_1} = \lambda_G$$

Using the expression above for λ_G, we obtain:

$$\begin{aligned} s_{1,f=0} &= \left(\frac{\lambda_G}{w_1 a_1}\right)^{\frac{1}{\theta_1}} \\ &= \frac{\gamma_1(\theta_1+1)}{\theta_1}\left(\frac{\lambda_G}{w_1 A_1}\right)^{\frac{1}{\theta_1}} \\ &= \frac{\gamma_1}{\theta_1}\Theta\big(\mathbf{wA},\tau\big)^{\frac{1}{\theta_1+1}} \end{aligned}$$

To obtain downstream firm scope for other countries (expression 42), we use the first-order condition for firm scope for $f=0$:

$$w_n c_n'(s_{n,f=0}) = \lambda_n$$

which gives:

$$w_n a_n s_{n,f=0}^{\theta_n} = \lambda_n$$

Using $\lambda_n = (w_n A_n)^{\frac{1}{\theta_n+1}}(P_n)^{\frac{\theta_n}{\theta_n+1}}$ (expression 55) together with the expression for P_n, we obtain:

$$\begin{aligned} s_{n,f=0} &= \left(\frac{\lambda_n}{w_n a_n}\right)^{\frac{1}{\theta_n}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{\lambda_n}{w_n A_n}\right)^{\frac{1}{\theta_n}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{P_n}{A_n w_n}\right)^{\frac{1}{\theta_n+1}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}(A_n w_n)^{-\frac{1}{\theta_n+1}}\left(A_{n-1}w_{n-1}/\tau\right)^{\frac{1}{\theta_n-\theta_{n-1}}}\left(A_n w_n\right)^{-\frac{\theta_{n-1}+1}{(\theta_n+1)(\theta_n-\theta_{n-1})}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{A_{n-1}w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \end{aligned}$$

We follow similar steps to find the scope of the most upstream firm in each country, s_{n,F_n}:

$$s_{n,F_n} = \frac{(\theta_n+1)\gamma_n}{\theta_n}\left(\frac{A_n w_n}{\tau A_{n+1}w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$

Proof of Proposition 2

Proposition 2: Results in Proposition 2 are obtained simply by taking the derivative of the expressions above w.r.t trade costs τ. In particular, we find:

$$\frac{\partial\Theta\big(\mathbf{wA},\tau\big)}{\partial\tau} > 0$$

which implies that: i) the price in the final good and iv) the shadow cost λ_G decrease when trade costs decrease. Given equation (34)

$$q_n e^{\gamma_n f} w_n c_n'(s_{nf}) = \lambda_G,$$

we obtain that a decrease in λ_G affects firm scope everywhere along the chain and leads to a decrease in s_{nf}, conditional on the position on the chain, wages and the set of countries involved in the chain (point v). As trade costs decrease, however, countries tend to move downstream. Since firms scope is larger downstream, moving up the chain implies larger average firm scope for each country $i > 1$ (except the most downstream one). This can be seen in expressions (42): firm scope at both end of the chain in country i is a decreasing function of τ (point vi in Proposition 2). Finally, we can also see above that S_1 is an increasing function of trade costs (conditional on wages), which proves point iii).

Two-country case and proof of Proposition 3

Two-country setting: As stated in the text, we assume that $\theta_U > \theta_D$. As specified in equation (10) in section 2, labor efficiency $a_D(\omega)$ and $a_U(\omega)$ are distributed Weibull with coefficient T_D and T_U respectively for countries D and U. We make no assumption about the relative ranking of T_D and T_U. We also make no assumption about relative transaction costs γ_D and γ_U for countries D and U. Also, we normalize $w_D = 1$.

As shown in equation (23), it is useful to instead define an adjusted labor costs parameter $A_D(\omega) = a_D(\omega)\left(\gamma_D \frac{\theta_D+1}{\theta_D}\right)^{\theta_D}$ and $A_U(\omega) = a_U(\omega)\left(\gamma_U \frac{\theta_U+1}{\theta_U}\right)^{\theta_U}$. The effect of transaction costs is equivalent to a shift in labor productivity. The resulting $A_D(\omega)$ and $A_U(\omega)$ parameters also follow a Weibull distribution with adjusted shift parameters:[46]

$$\begin{aligned}\tilde{T}_D &= T_D\left(\gamma_D \frac{\theta_D+1}{\theta_D}\right)^{-\xi\theta_D}\\ \tilde{T}_U &= T_U\left(\gamma_U \frac{\theta_U+1}{\theta_U}\right)^{-\xi\theta_U}\end{aligned}$$

Following Dornbusch et al. (1977), we rank varieties ω between 0 and 1 and specify the following relative cost:

$$\frac{A_U(\omega)}{A_D(\omega)} = \left[\frac{\tilde{T}_D}{\tilde{T}_U}\left(\frac{\omega}{1-\omega}\right)\right]^{\frac{1}{\xi}} \equiv A(\omega) \quad (59)$$

where $A(\omega)$ is defined as the relative labor requirement in country U. This ordering implies that U has a comparative advantage in low-ω chains while country D has a comparative advantage in high-ω chains. For the sake of exposition, we normalize $A_D(\omega)$ to unity. It is otherwise equivalent to redefine all prices as relative to $A_D(\omega)$.

Sourcing patterns: As shown in Proposition 1, the ranking $\theta_D < \theta_U$ determines relative position on the chain. A chain that involves the two countries necessarily features country U

[46] Our parameter ξ is the same as the dispersion parameter θ in Eaton and Kortum (2002).

specializing upstream and country D specializing downstream. Some chains may also involve country U only. However, when country D produces the final good, we find that country U is also involved in the chain, at least for some of the most upstream tasks.

When country D produces the final good (with country U involved in upstream tasks), the price of the final good in D is:

$$P_D(\omega) = \frac{1}{(\theta_D+1)^{\theta_D+1}} \left[1 - (\theta_U - \theta_D)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D+1} \tag{60}$$

Consumers in U can also import these goods at a price $\tau P_D(\omega)$. When country U produces the entire range of tasks, the price of final goods in U is:

$$P_U(\omega) = \frac{w_U A(\omega)}{(\theta_U+1)^{\theta_U+1}} \tag{61}$$

while consumers in country D can also import these goods for a price $\tau P_U(\omega)$.

Given the patterns of labor costs across varieties, the ratio of prices $\frac{P_D(\omega)}{P_U(\omega)}$ strictly increases with ω. For each final destination $X \in \{D, U\}$, there is a unique threshold ω^*_X for which the two prices are equal. These thresholds ω^*_D and ω^*_U are implicitly defined by:

$$P_D(\omega^*_D) = \tau P_U(\omega^*_D) \tag{62}$$
$$\tau P_D(\omega^*_U) = P_U(\omega^*_U) \tag{63}$$

As in Dornbusch, Fisher and Samuelson (1977), these cutoffs ω^*_D and ω^*_U correspond to the goods for which consumers (resp. in D and U) are indifferent between purchasing locally or importing. There is no analytical solution for ω^*_U but it is easy to check the following solution in ω^*_D:

$$\omega^*_D = \frac{\tilde{T}_U \tau^{-\xi} w_U^{-\xi} (\theta_D+1)^{-(\theta_U-\theta_D)\xi}}{\tilde{T}_D + \tilde{T}_U \tau^{-\xi} w_U^{-\xi} (\theta_D+1)^{-(\theta_U-\theta_D)\xi}}$$

General equilibrium and wages: While production in U only relies on local labor, production in D relies on country U to perform upstream tasks. Using the results from Lemma 2, the demand for labor in U for each dollar of final goods produced in D (at a price P_D) equals:

$$\frac{w_U\, l_U(\omega)}{P_D(\omega)} = \frac{(\theta_D+1)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}{1 - (\theta_U-\theta_D)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}} \tag{64}$$

Trade balance imposes:

$$w_U L_U (1-\omega^*_U) \;=\; L_D \omega^*_D \;+\; L_D \int_{\omega=\omega^*_D}^{1} \frac{w_U\, l_U(\omega)}{P_D(\omega)} d\omega \;+\; w_U L_U \int_{\omega=\omega^*_U}^{1} \frac{w_U\, l_U(\omega)}{P_D(\omega)} d\omega \tag{65}$$

where the left-hand side correspond to exports of final goods by D and the right-hand side corresponds to exports of final and intermediate goods by U. Note that the marginal variety ω^*_D is such that: $\frac{w_U\, l_U(\omega^*_D)}{P_D(\omega^*_D)} = 1$. Hence the trade balance above is equivalent to a trade balance in value-added content:

$$w_U L_U \int_{\omega=\omega_U^*}^{1} \left(1 - \frac{w_U\, l_U(\omega)}{P_D(\omega)}\right) d\omega \;=\; L_D \int_{\omega=0}^{1} \min\left\{\frac{w_U\, l_U(\omega)}{P_D(\omega)}\,,\, 1\right\} d\omega \tag{66}$$

We prove here that τw_U decreases as trade costs τ decrease, which also implies that foreign labor content $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$ increases when τ decreases. To prove this result, we show that we arrive at a contradiction if we assume that τw_U increases when τ decreases. The right-hand-side term of expression (66) would decrease since it is a strictly decreasing function of τw_U: country D sources less from U if trade-cost-adjusted wages increase in U. On the other hand the term on the left would increase because of higher income $L_U w_U$, a lower import threshold ω_U^* (since goods from D would become relatively cheaper) and higher foreign value-added content $1 - \frac{w_U\, l_U(\omega)}{P_D(\omega)}$. Hence it must be that τw_U decreases when τ decreases.

Trade elasticity: For country D, it is easy to check that the elasticity is the same as in Eaton and Kortum (2002):

$$\frac{\omega_D^*}{1-\omega_D^*} = \frac{\tilde{T}_U \tau^{-\xi} w_U^{-\xi} (\theta_D + 1)^{-(\theta_U - \theta_D)\xi}}{\tilde{T}_D}$$

Hence:

$$\varepsilon_D^F \;\equiv\; \frac{d\log\left(\frac{\omega_D^*}{1-\omega_D^*}\right)}{d\log\tau} \;=\; -\xi$$

For country U, we take the derivative of $\tau P_D(\omega_U^*) = P_U(\omega_U^*)$ with respect to $\log\tau$, which gives:

$$1 + \frac{\partial \log P_D}{\partial \log \tau} \cdot \left[1 + \frac{d\log A(\omega_U^*)}{d\log\tau}\right] = \frac{\partial \log P_U}{\partial \log \tau}\; \frac{d\log A(\omega_U^*)}{d\log\tau}$$

In the expression above, the partial derivative $\frac{\partial \log P_U}{\partial \log \tau}$ is equal to unity. The partial derivative $\frac{\partial \log P_D}{\partial \log \tau}$ is lower than one and equals the share of value coming from U for the threshold variety ω_U^*:

$$\frac{d\log P_D}{d\log\tau} = \frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}$$

After solving for $\frac{d\log A(\omega_U^*)}{d\log\tau}$, we find:

$$\frac{d\log A(\omega_U^*)}{d\log\tau} = \frac{1 + \frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}{1 - \frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}$$

The trade elasticity in final goods for country U is then:

$$\varepsilon_U^F = \frac{d\log\left(\frac{1-\omega_U^*}{\omega_U^*}\right)}{d\log\tau} = \frac{1}{\xi}\;\frac{d\log A(\omega_U^*)}{d\log\tau} = \frac{1}{\xi}\;\frac{1 + \frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}{1 - \frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}$$

The lower the trade costs, the higher the trade elasticity. Because lower trade costs leads to more fragmentation, the foreign labor content for the marginal variety increases. Note that, when trade becomes frictionless, the foreign labor content for this marginal variety converges to unity and the trade elasticity ε_U^F goes to infinity.

Vertical specialization and the value-added content of trade: We focus here on Johnson and Noguera (2012a)'s "VAX ratio", the ratio of the value-added content of exports and gross exports. For country D, this corresponds to:

$$\begin{aligned} VAX_D &= \frac{1}{1-\omega_U^*}\int_{\omega_U^*}^1 \left(1-\frac{w_U\, l_U(\omega)}{P_D(\omega)}\right) d\omega \\ &= \frac{1}{1-\omega_U^*}\int_{\omega_U^*}^1 \left(1-\frac{(\theta_D+1)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}{1-(\theta_U-\theta_D)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}\right) d\omega \end{aligned}$$

where $1-\omega_U^*$ is the share of imported goods by consumers in U and where $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$ is the share of foreign labor in the production of variety ω in country D.

For a given ω, the term in the integral sum increases with trade-cost-adjusted wages τw_U, which itself increases with τ (larger domestic value added share as trade costs increase). This is a direct effect. There is also a composition effect: ω_U^* increases with τ, so that country D only exports high-value-added goods (varieties ω closer to one) when trade costs are higher. This second effect also leads to an increase in the VAX ratio when trade costs increase.

A similar intuition holds for the VAX ratio for country U, as described in the text. The VAX ratio for country U equal the one for country D in this two-country example because we have a trade balance in gross flows as well as in the value-added content of trade.

Gains from trade for country D: For country D, the wage and the price index under autarky are normalized to zero (in log). Hence the log of the price index with trade also reflects the gains from trade:

$$\Delta \log\left(\frac{1}{P_D}\right) = \log P_D = \left[\int_0^1 \log P_D(\omega) d\omega\right]$$

Using expressions above for prices, we obtain:

$$\Delta \log\left(\frac{1}{P_D}\right) = \int_0^{\omega_D^*} \log\left(\tau w_U A(\omega)\right) d\omega + \int_{\omega_D^*}^1 \log \Theta\left(\tau w_U A(\omega)\right) d\omega$$

with:

$$\Theta(\tau w_U A(\omega)) = \left[1-(\theta_U-\theta_D)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D+1}$$

and:

$$\Theta(\tau w_U A(\omega_D^*)) = \tau w_U A(\omega_D^*)$$

at the threshold ω_D^*.

The expression for the gains from trade can be rewritten:

$$\begin{aligned}\Delta \log\left(\frac{1}{P_D}\right) &= -\int_0^{\omega_D^*} \log\left(\tau w_U A(\omega)\right) d\omega - \int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega \\ &= -\int_0^{\omega_D^*} \log\left(\frac{A(\omega)}{A(\omega_D^*)}\right) d\omega - \int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega \\ &\quad -\omega_D^* \log\Theta\left(\tau w_U A(\omega_D^*)\right)\end{aligned}$$

There are three terms in the above formula. The first term $\frac{1}{\xi}\log\left(1-\omega_D^*\right)$ corresponds to the Arkolakis et al (2012) formula based on final demand trade: the log of the gains from trade are proportional to the log of the domestic content of consumption, where the proportionality coefficient is the inverse of the trade elasticity ξ.

After integrating by part, we can see that it equals the ratio of the change in domestic consumption share and the trade elasticity ξ for country D:

$$\begin{aligned}-\int_0^{\omega_D^*} \log\left(\frac{A(\omega)}{A(\omega_D^*)}\right) d\omega &= \int_0^{\omega_D^*} \frac{\partial \log A(\omega)}{\partial \log \omega} d\omega \\ &= \frac{1}{\xi}\int_0^{\omega_D^*} \frac{d\omega}{1-\omega} \\ &= -\frac{1}{\xi}\log\left(1-\omega_D^*\right)\end{aligned}$$

The second and third terms reflect additional gain from fragmentation:

$$\begin{aligned}-\int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega \; - \; \omega_D^* \log\Theta\left(\tau w_U A(\omega_D^*)\right) \; &= \; -\int_{\omega_D^*}^1 \frac{\partial \log\Theta}{\partial \log A}\frac{\partial \log A}{\partial \log \omega} d\omega \\ = \int_{\omega_D^*}^1 \frac{\partial \log\Theta}{\partial \log A}\frac{1}{\xi}\frac{1}{1-\omega} d\omega \; &= \; \frac{1}{\xi}\int_{\omega_D^*}^1 \frac{w_U\, l_U(\omega)}{P_D(\omega)}\frac{d\omega}{1-\omega} \; > \; 0\end{aligned}$$

using the equality between $\frac{\partial \log\Theta}{\partial \log A}$ and the foreign labor content $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$. Hence, for country D, Arkolakis et al (2012) formula underestimates the gains from trade.

Gains from trade for country U: For country U, the price index under autarky is:

$$\log P_U^{aut} = \int_0^1 \log\left(w_U^{aut} A(\omega)\right) d\omega$$

where w_U^{aut} denotes the wage in autarky. With trade, the price index is:

$$\log P_U = \int_0^{\omega_U^*} \log\left(w_U A(\omega)\right) d\omega \; + \; \int_{\omega_U^*}^1 \log \tau\Theta\left(\tau w_U A(\omega)\right) d\omega$$

where $\Theta\left(\tau w_U A(\omega)\right)$ is defined like above:

$$\Theta(\tau w_U A(\omega)) = \left[1 - (\theta_U - \theta_D)\left(\tau w_U A(\omega)\right)^{-\frac{1}{\theta_U - \theta_D}}\right]^{\theta_D+1}$$

with the following equality at the threshold ω_U^*:

$$\tau\Theta\left(\tau w_U A(\omega_U^*)\right) = w_U A(\omega_U^*)$$

Gains from trade can then be expressed as:

$$\begin{aligned}\Delta \log\left(\frac{w_U}{P_U}\right) &= \int_{\omega_U^*}^1 \log\left(w_U A(\omega)\right) d\omega - \int_{\omega_U^*}^1 \log \tau\Theta\left(\tau w_U A(\omega)\right) d\omega \\ &= \int_{\omega_U^*}^1 \log\left(\frac{A(\omega)}{A(\omega_U^*)}\right) d\omega - \int_{\omega_U^*}^1 \log\left(\frac{\Theta\left(\tau w_U A(\omega)\right)}{\Theta\left(\tau w_U A(\omega_U^*)\right)}\right) d\omega\end{aligned}$$

Like above, the first term corresponds to Arkolakis et al (2012) formula:

$$\int_{\omega_U^*}^1 \log\left(\frac{A(\omega)}{A(\omega_U^*)}\right) d\omega = -\frac{1}{\xi}\log\omega_U^*$$

The second term yields:

$$\begin{aligned}-\int_{\omega_U^*}^1 \log\left(\frac{\Theta\left(\tau w_U A(\omega)\right)}{\Theta\left(\tau w_U A(\omega_U^*)\right)}\right) d\omega &= -\int_{\omega_U^*}^1 \frac{\partial \log\Theta}{\partial \log A}\,\frac{\partial \log A}{\partial \omega}\,(1-\omega)\, d\omega \\ &= -\int_{\omega_U^*}^1 \frac{\partial \log\Theta}{\partial \log A}\,\frac{1}{\xi\omega}\, d\omega \\ &= -\frac{1}{\xi}\int_{\omega_U^*}^1 \frac{w_U\, l_U(\omega)}{P_D(\omega)}\frac{d\omega}{\omega} \;<\; 0\end{aligned}$$

This term is negative, which means that country U gains less than predicted by the Arkolakis et al (2012) benchmark.

Indexes D and N across industries

Figure 3 plots cross-country averages of index D_{ik} and N_{ik} in the IDE-JETRO data. As expected, primary commodities such as ores and metals tend to be upstream while finished goods tend to be downstream. Moreover, one can also see that indexes D and N are not strongly correlated and capture different moments and industry characteristics. More details on these indexes can be found in Fally (2012), who documents in particular the robustness of these indexes to aggregation biases. In Figure 4, we plot the upstreamness index D_{ik} evaluated for our five largest countries. Index D_{ik} is highly correlated across industries but there are still large differences across countries. Overall, we find that industry fixed effects can explain about 70% of the variance in D_{ik} across countries and industries.

Figure 3: Average of indexes D_{ik} and N_{ik} by industry

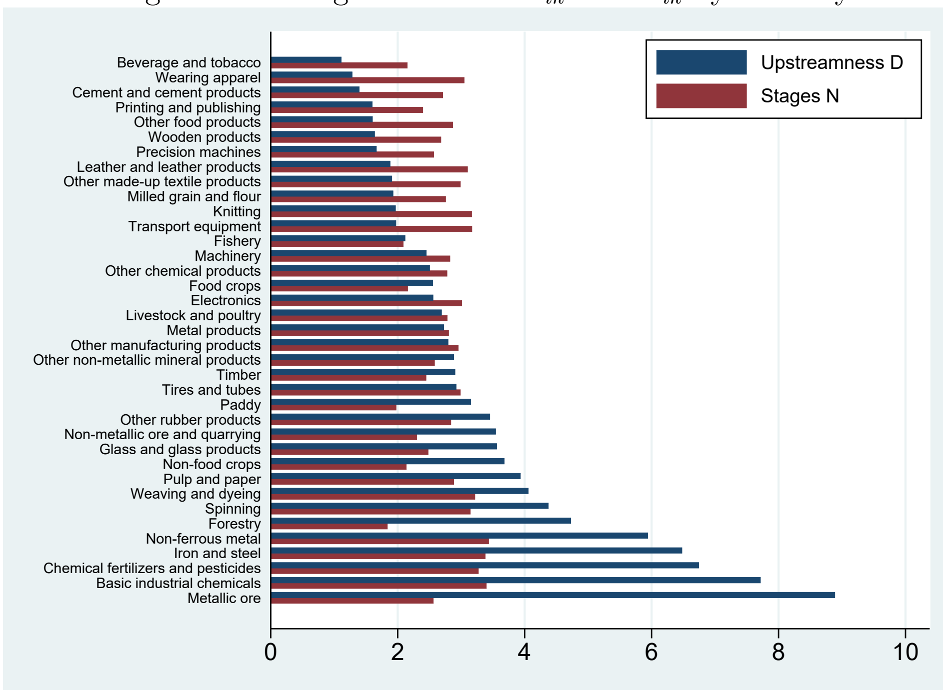


Figure 4: Index D_{ik} across industries for five largest countries