

Lecture 3a:

Specific-factor Model

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C181 – International Trade

Spring 2018

- **CHAPTER 2: Ricardian model:**
 - Only one factor of production: labor
 - Labor is mobile across sectors
 - ➔ Everyone gains from trade

The next model relaxes these assumptions:

- **CHAPTER 3: But what if:**
 - We have more than one factor of production?
 - What if these factors are NOT mobile across sectors?
 - ➔ Then there may be losers and winners!
(unequal effects of globalization)

- **CHAPTER 3: Road map:**
 - Setting up the specific factor model
 - Change in production and employment
 - Aggregate gains from trade
 - Effect on labor wages
 - Effect on returns to K and Land

1 Setup of Factor-Specific Model

Setup

- Two countries: Home and Foreign.
- Two sectors: Manufacturing and Agriculture
- Manufacturing uses labor and **capital**
- Agriculture uses labor and **land**.

1 Setup of Factor-Specific Model

Setup

- Two countries: Home and Foreign.
- Two sectors: Manufacturing and Agriculture
- Manufacturing uses labor and **capital**
- Agriculture uses labor and **land**.
- *Diminishing returns* for labor in each industry:
The marginal product of labor declines if the amount of labor used in the industry increases.

1 Setup of Factor-Specific Model

Alternative interpretation

NOTE:

We can also use the same model and interpret “capital” and “land” as fixed labor:

Capital: equivalent to Labor that is stuck in manufacturing

Land: equivalent to Labor that is stuck in Agriculture

Labor: Labor that is mobile across industries

→ Three types of labor depending on its mobility

(for the lecture, we'll keep talking about capital and land as it's easier to follow)

1 Setup of Factor-Specific Model

Production function with Constant Returns to Scale:

- Manufacturing output: $Y = F(K, L)$

such that: $F(\lambda K, \lambda L) = \lambda F(K, L)$

1 Setup of Factor-Specific Model

Production function with Constant Returns to Scale:

- Manufacturing output: $Y = F(K, L)$

such that: $F(\lambda K, \lambda L) = \lambda F(K, L)$

- This implies *decreasing* returns to scale if we focus on one input:

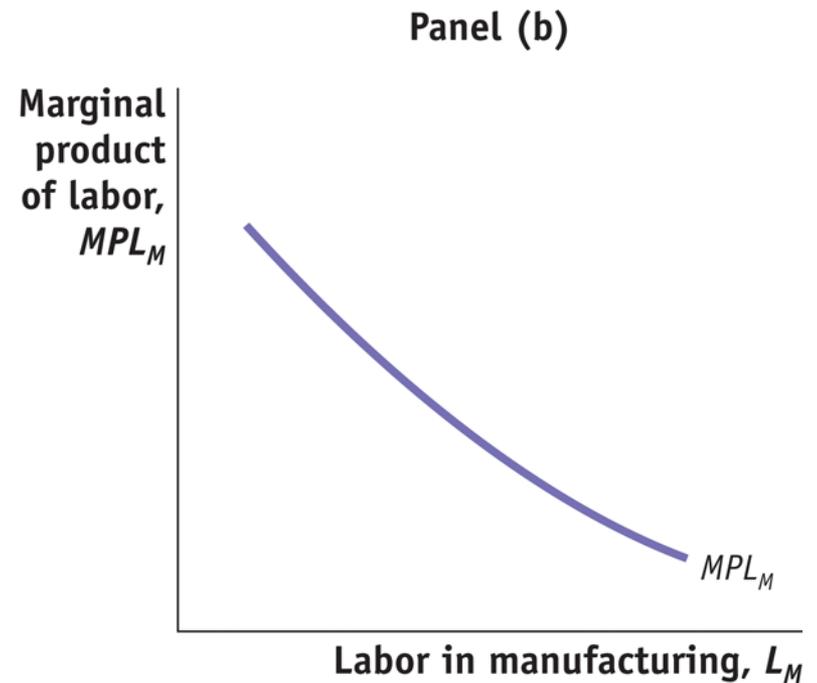
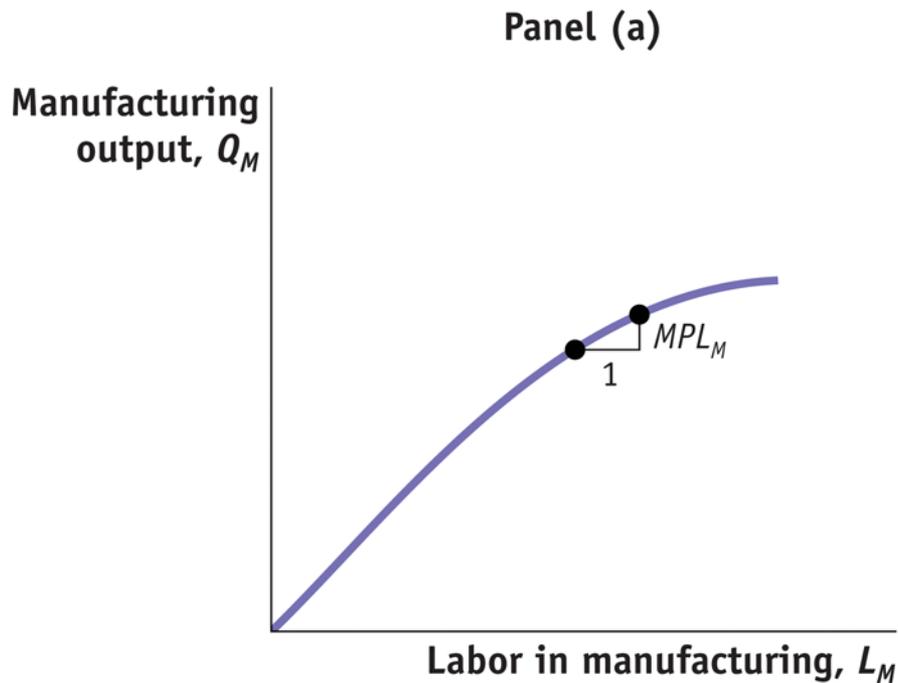
$$F(K, \lambda L) < F(\lambda K, \lambda L)$$

$$\rightarrow F(K, \lambda L) < \lambda F(K, L)$$

- In each industry: $\frac{\partial MPL}{\partial L} < 0$

1 Setup of Factor-Specific Model

Diminishing returns for labor in each industry:



(same for Agriculture: MPL decreases with production)

1 Setup of Factor-Specific Model

Example of production function:

- Manufactures: $Y_M = a_M K^{1/3} L_M^{2/3}$
- Agriculture: $Y_A = a_A T^{1/3} L_A^{2/3}$

1 Setup of Factor-Specific Model

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→ Marginal product of Labor:

- MPL in Manufactures: $MPL_M = \frac{2}{3} a_M (K/L_M)^{1/3}$
- MPL in Agriculture: $MPL_A = \frac{2}{3} a_A (T/L_A)^{1/3}$

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→ Marginal product of Labor:

- MPL in Manufactures: $MPL_M = \frac{2}{3} a_M (K/L_M)^{1/3}$
Increases with K/L_M

- MPL in Agriculture: $MPL_A = \frac{2}{3} a_A (T/L_A)^{1/3}$
Increases with T/L_A

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→ Marginal product of Capital and Land:

- MPK in Manufactures: $MPK = \frac{1}{3} a_M (L_M / K)^{2/3}$
- MPT in Agriculture: $MPT = \frac{1}{3} a_A (L_A / T)^{2/3}$

1 Setup of Factor-Specific Model

Example of production function:

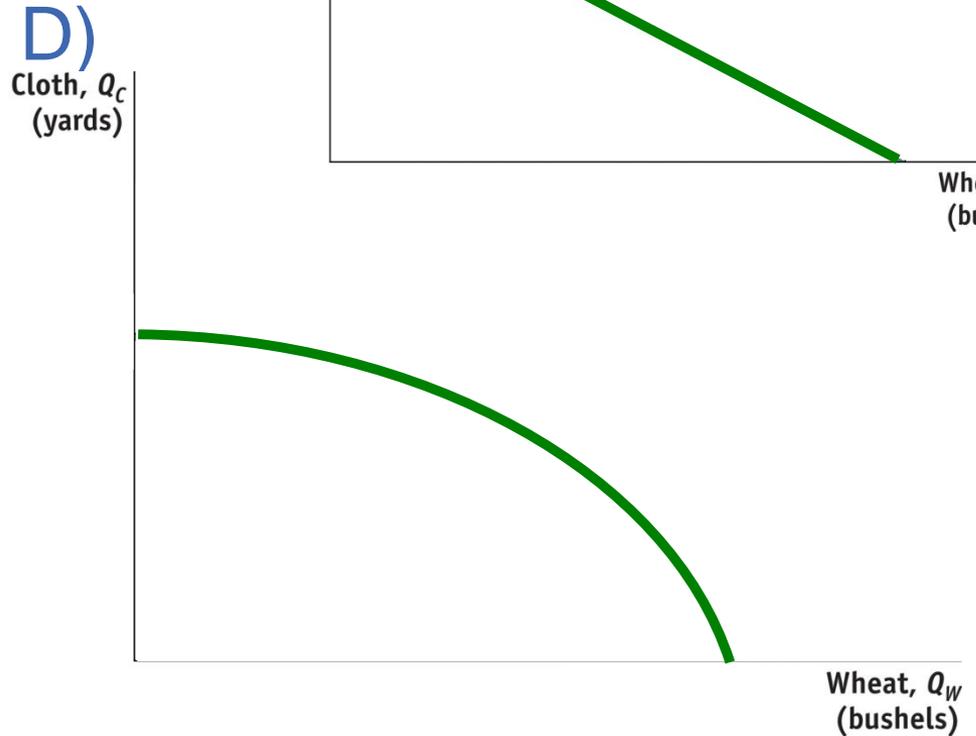
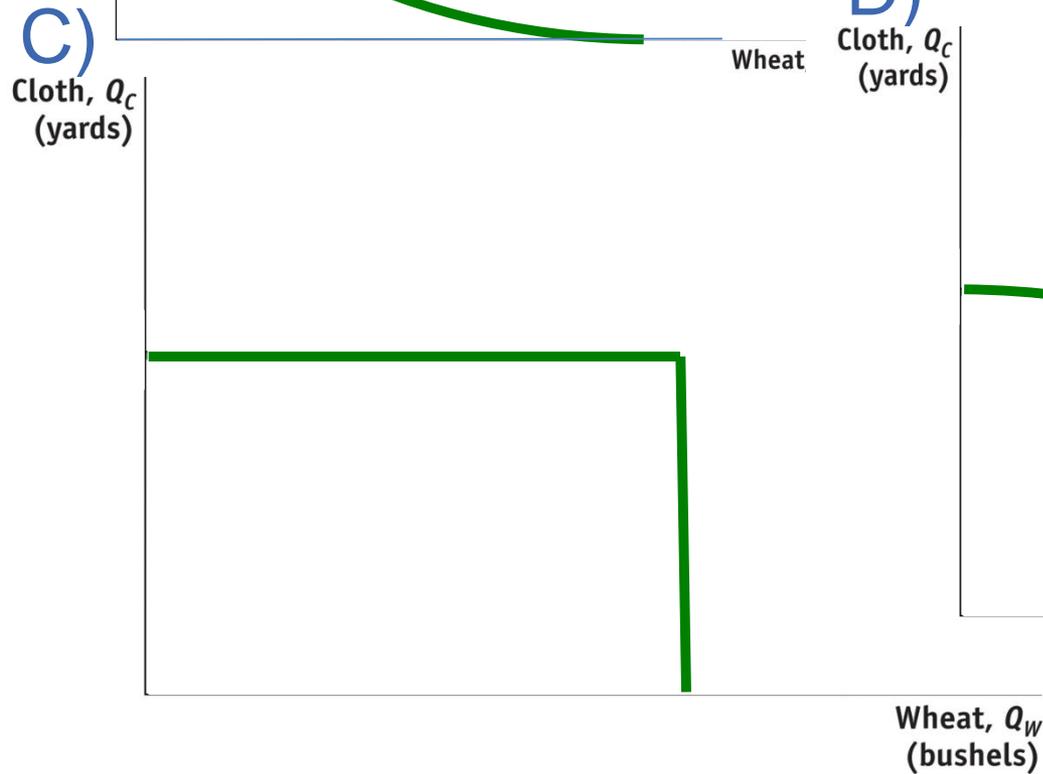
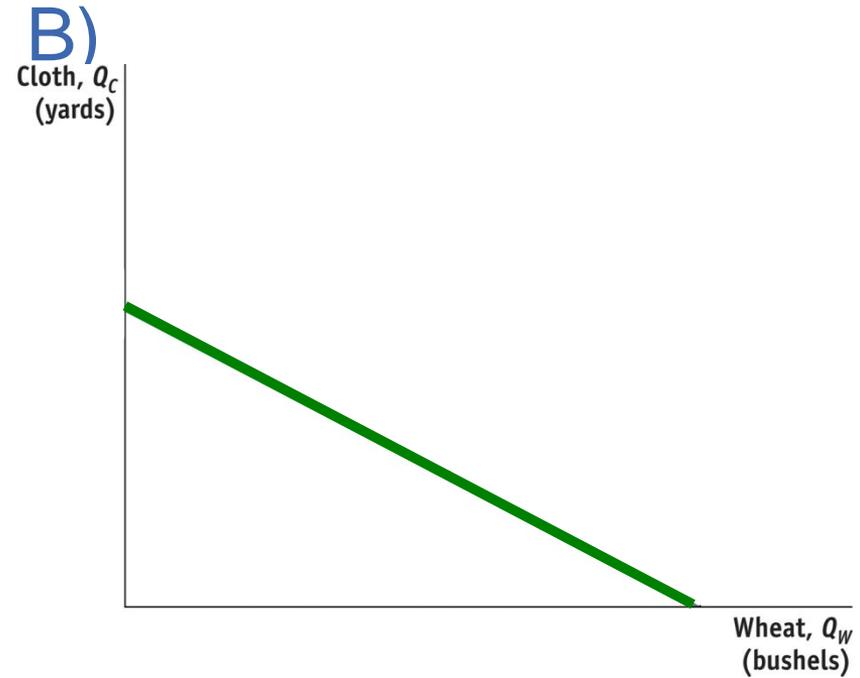
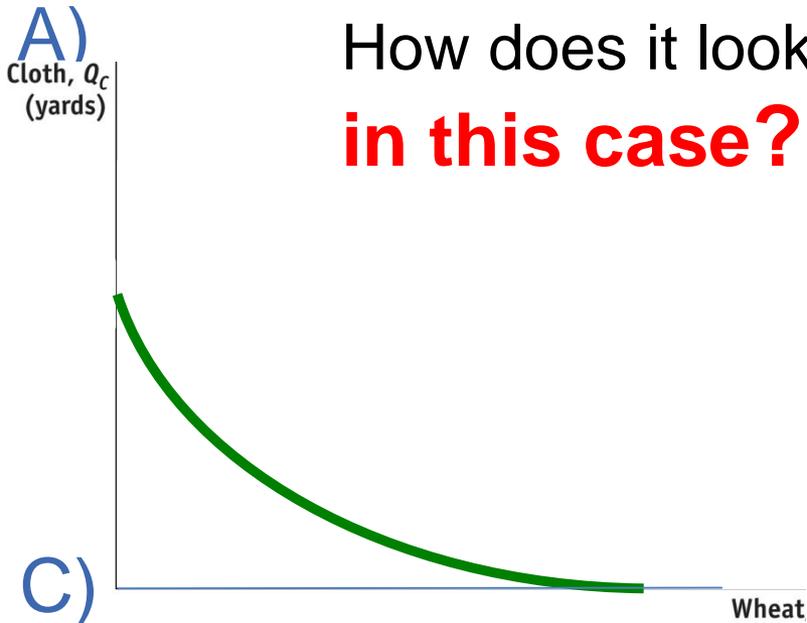
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→ Marginal product of Capital and Land:

- MPK in Manufactures: $MPK = \frac{1}{3} a_M (L_M / K)^{2/3}$
Decreases with K / L_M
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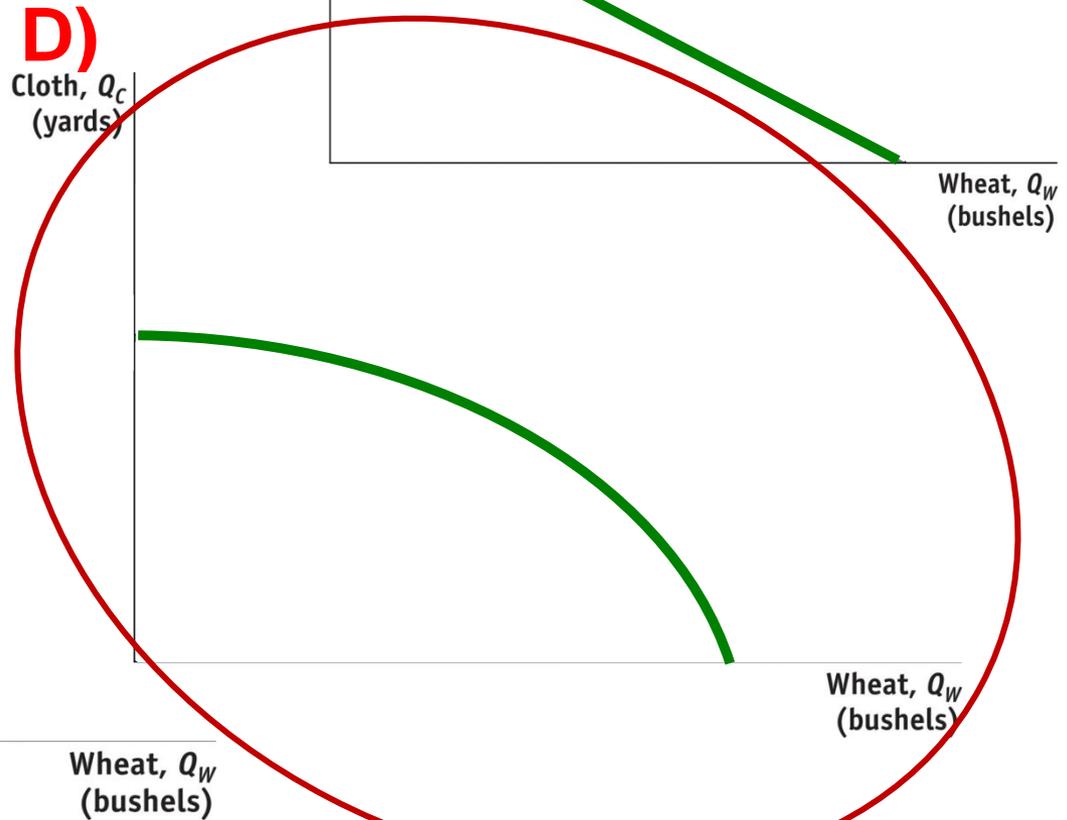
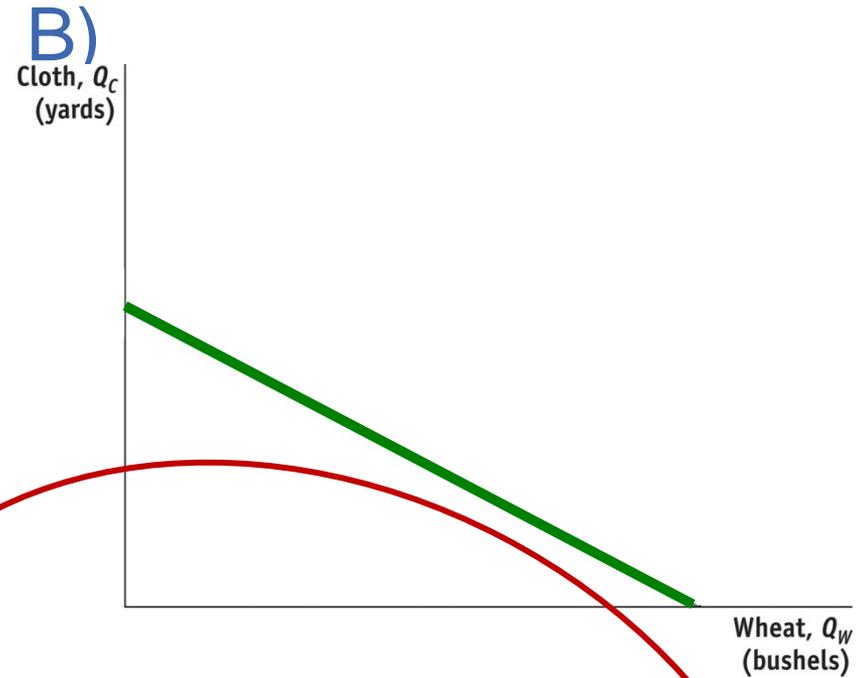
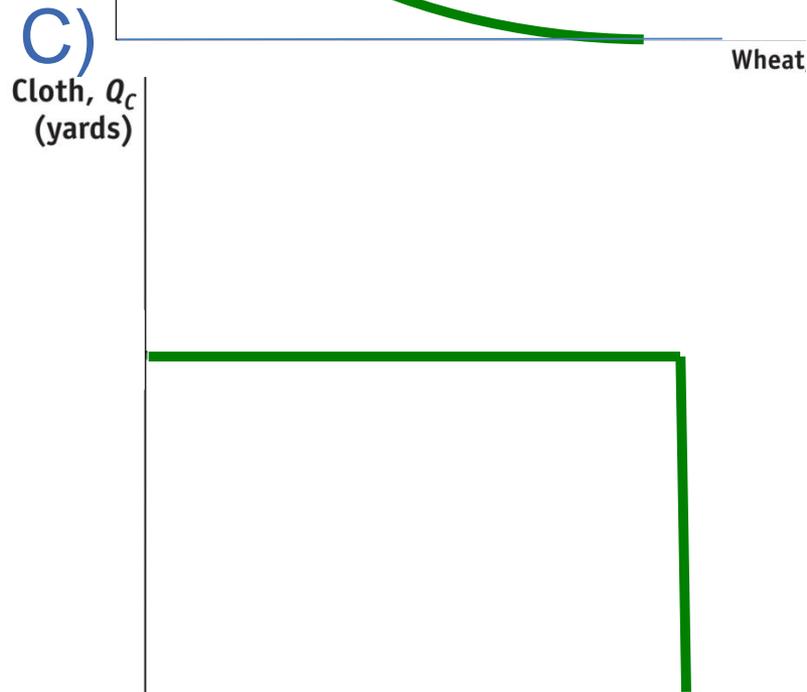
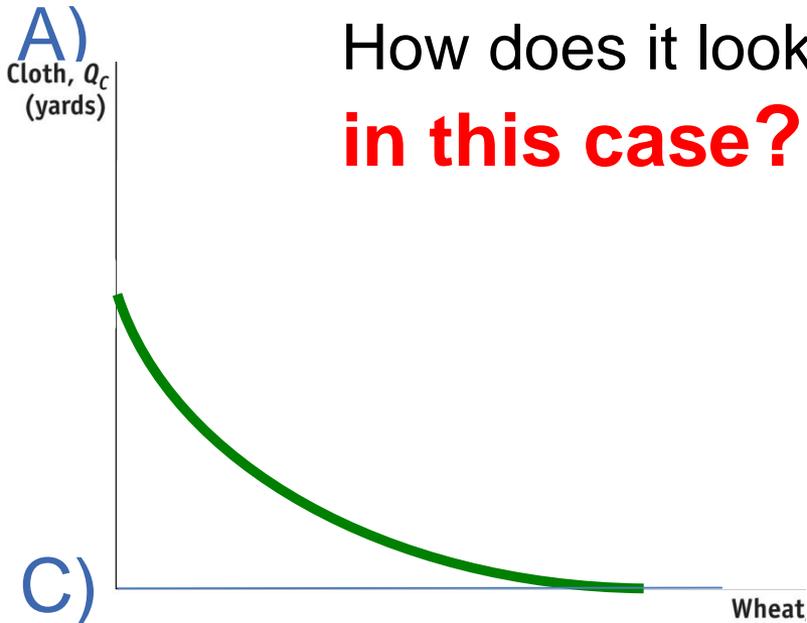
Production Possibility Frontier:

How does it look like
in this case?



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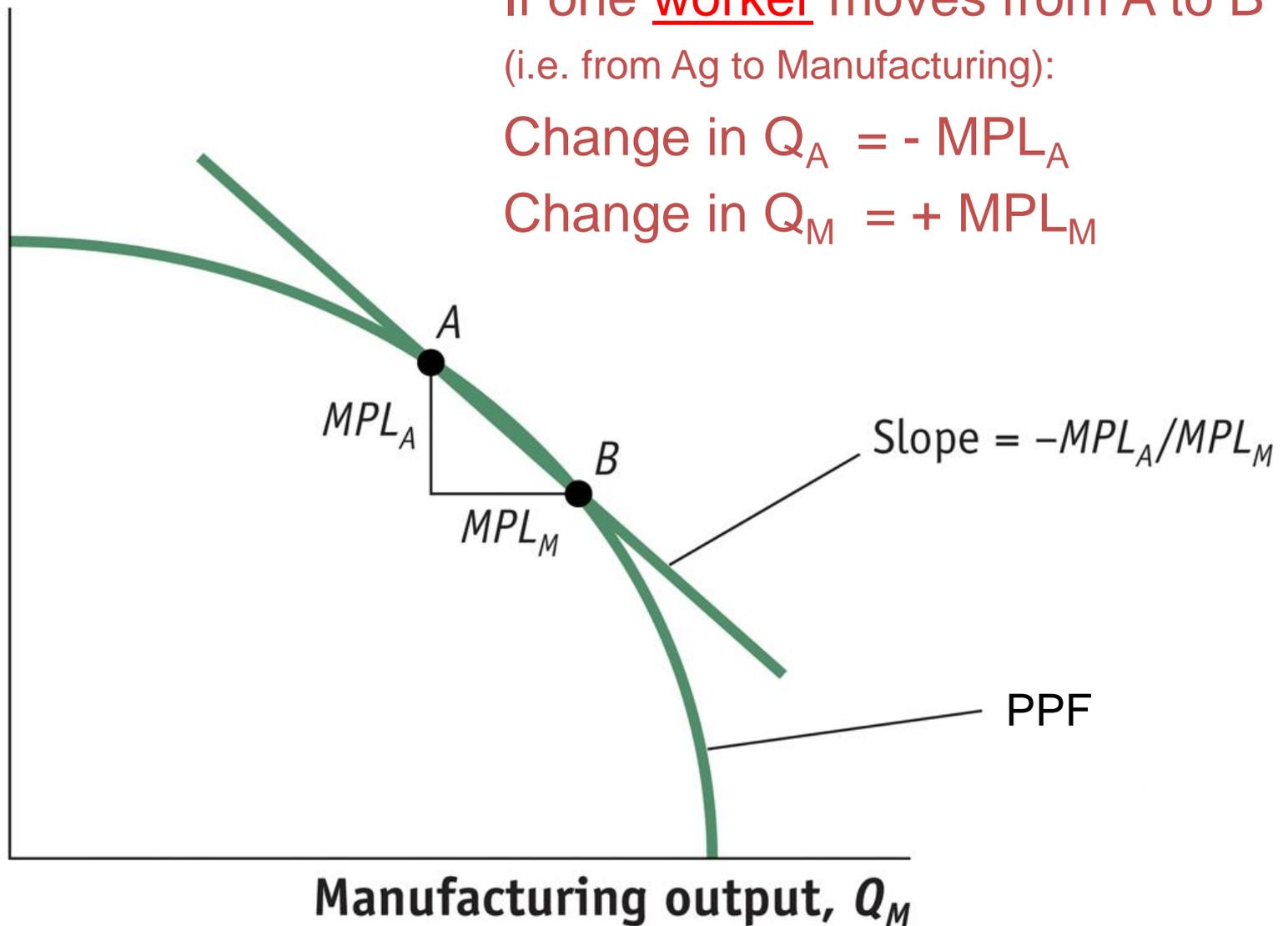
Slope of PPF reflects the opportunity cost of manuf. output:

Agriculture
output, Q_A

If one worker moves from A to B
(i.e. from Ag to Manufacturing):

$$\text{Change in } Q_A = -MPL_A$$

$$\text{Change in } Q_M = +MPL_M$$

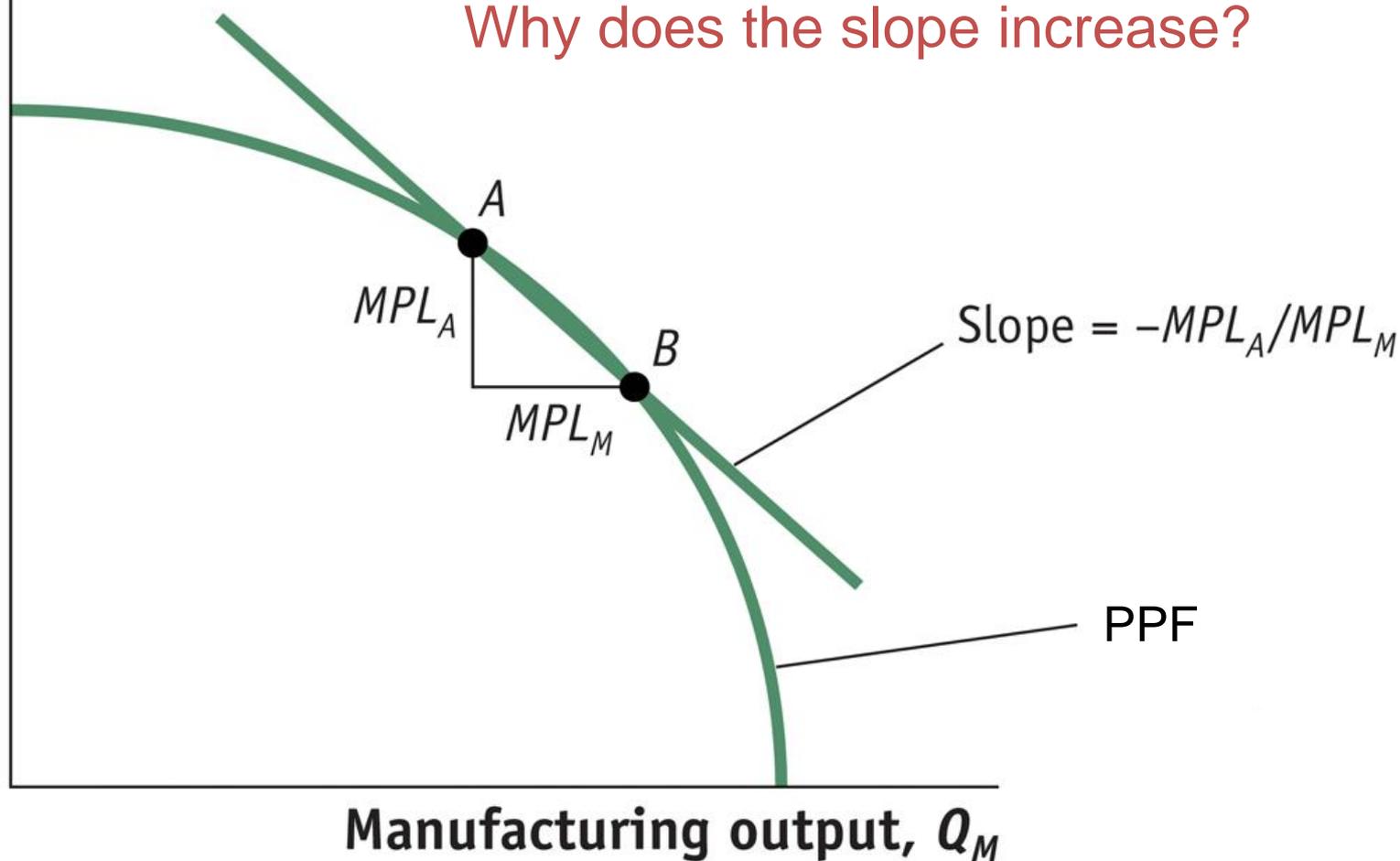


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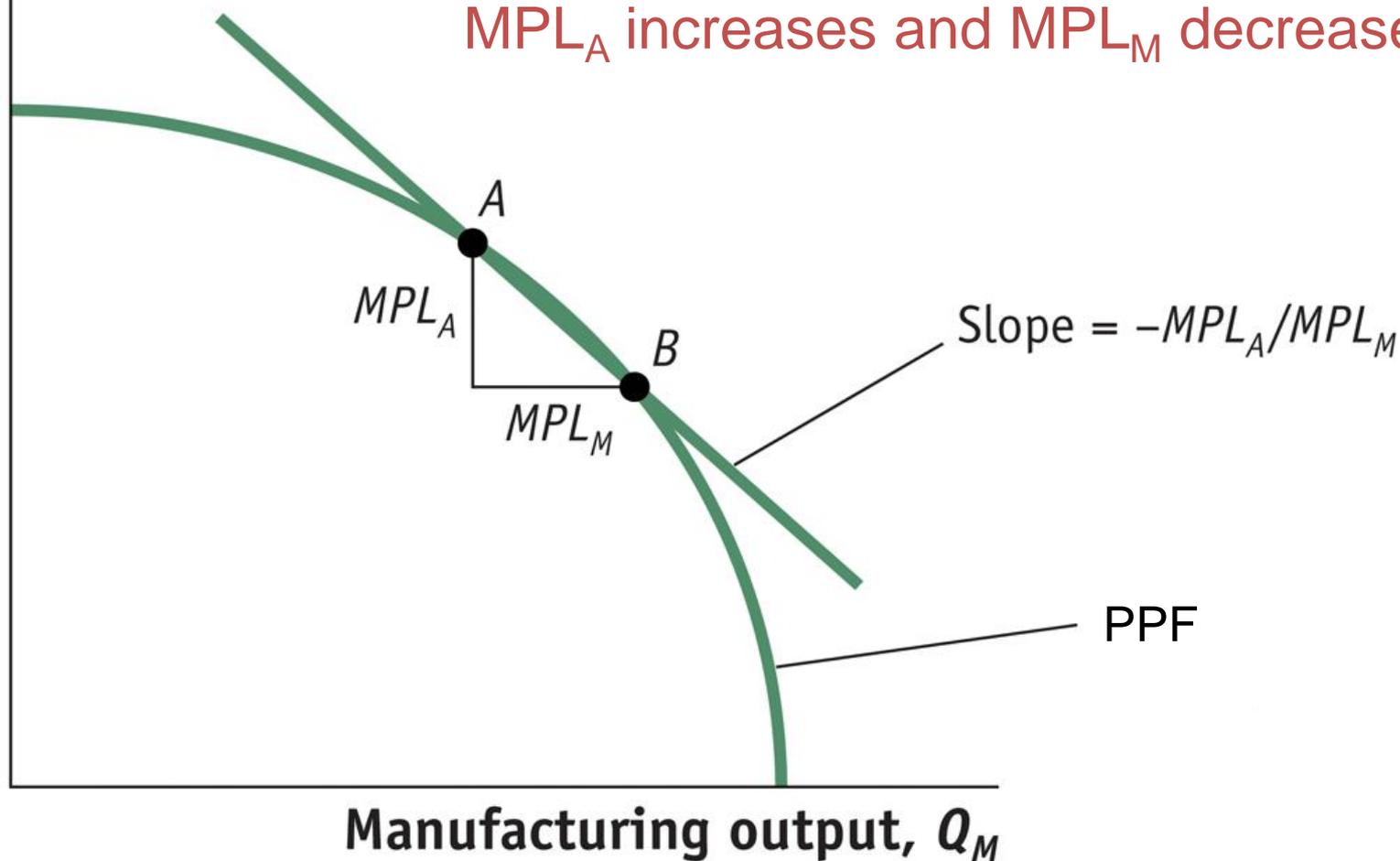
Why does the slope increase?



Slope of PPF reflects the opportunity cost of manuf. output:

Agriculture
output, Q_A

If one worker moves from A to B:
Why does the slope increase?
 MPL_A increases and MPL_M decreases



Manufacturing output, Q_M

1 Setup of Factor-Specific Model

Slope of PPF

Why does the slope increase from point A to B?

- Slope equals MPL_A/MPL_M
- As L_A decreases, MPL_A increases
- As L_M increases, MPL_M decreases

→ Hence the ratio increases!

1 Setup of Factor-Specific Model

Labor market and relative prices

- Labor is mobile across sectors
- Hence **wages** are equalized:

$$W = P_M \cdot MPL_M$$

$$W = P_A \cdot MPL_A$$

- And should be the same across sectors. Hence:

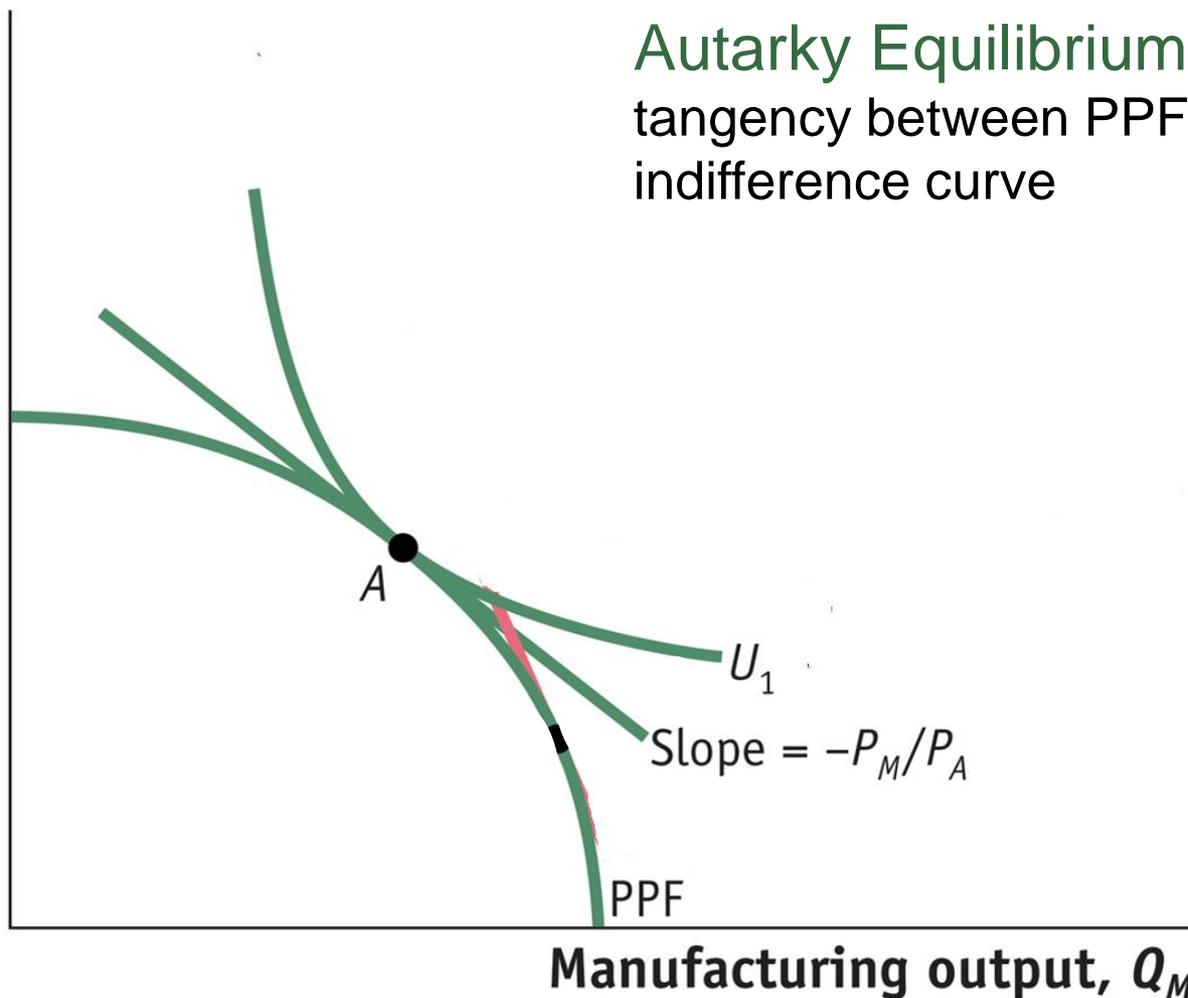
$$\frac{P_M}{P_A} = \frac{MPL_A}{MPL_M}$$

= Slope of the PPF

1 Setup of Factor-Specific Model

Equilibrium in Autarky:

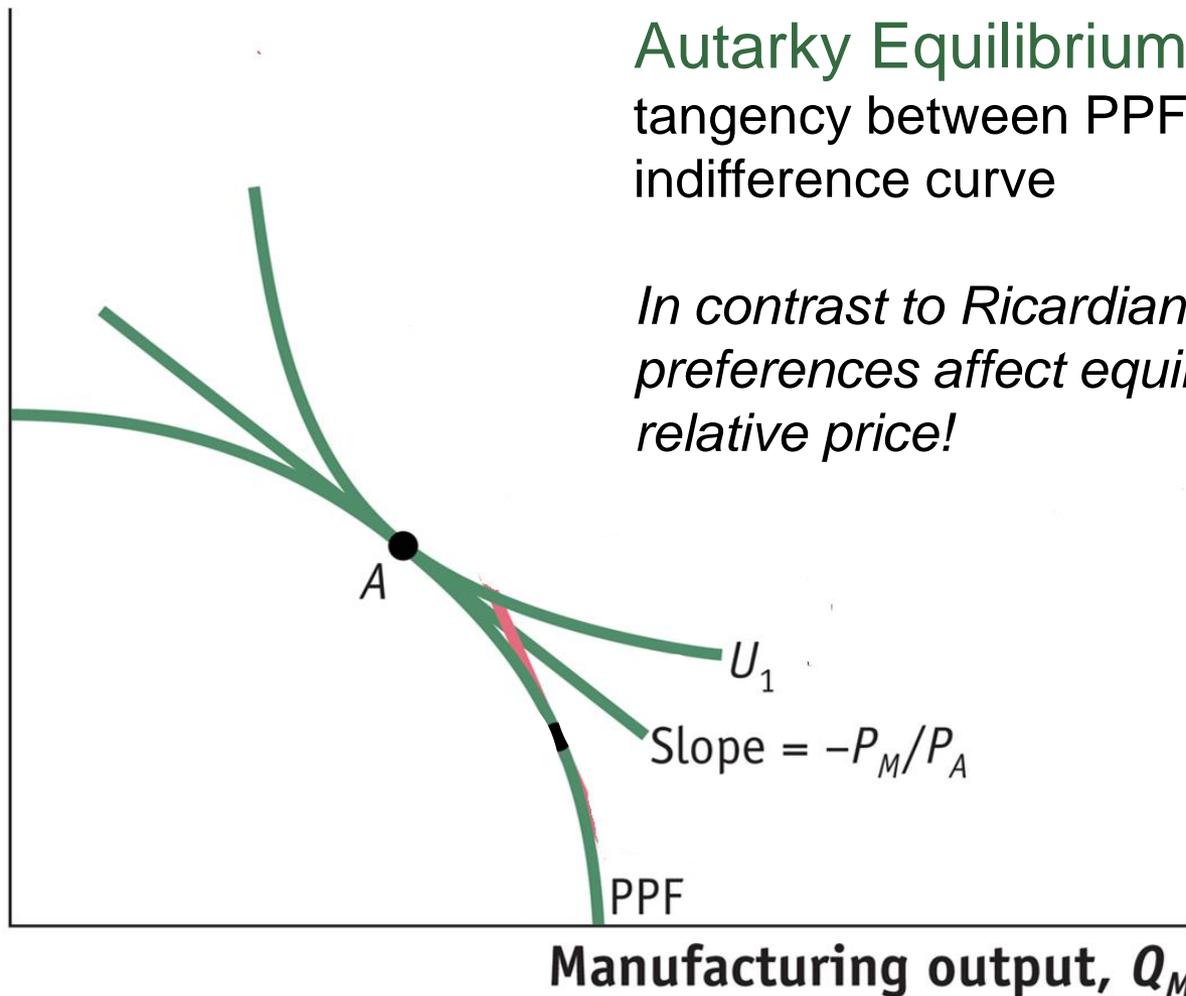
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1 Setup of Factor-Specific Model

Equilibrium in Autarky:

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- **CHAPTER 3: Road map:**

- Setting up the specific factor model

→ Change in production and employment

- Aggregate gains from trade
- Effect on labor wages
- Effect on returns to K and Land

2 Effect of Trade on production

The Foreign Country

- Let us assume that Home has a comparative advantage in manufacturing

⇔ *Equivalent to assuming that the Home no-trade relative price of manufacturing is lower than Foreign rel. price:*

$$(P_M / P_A) < (P^*_M / P^*_A).$$

New world price?

2 Effect of Trade on production

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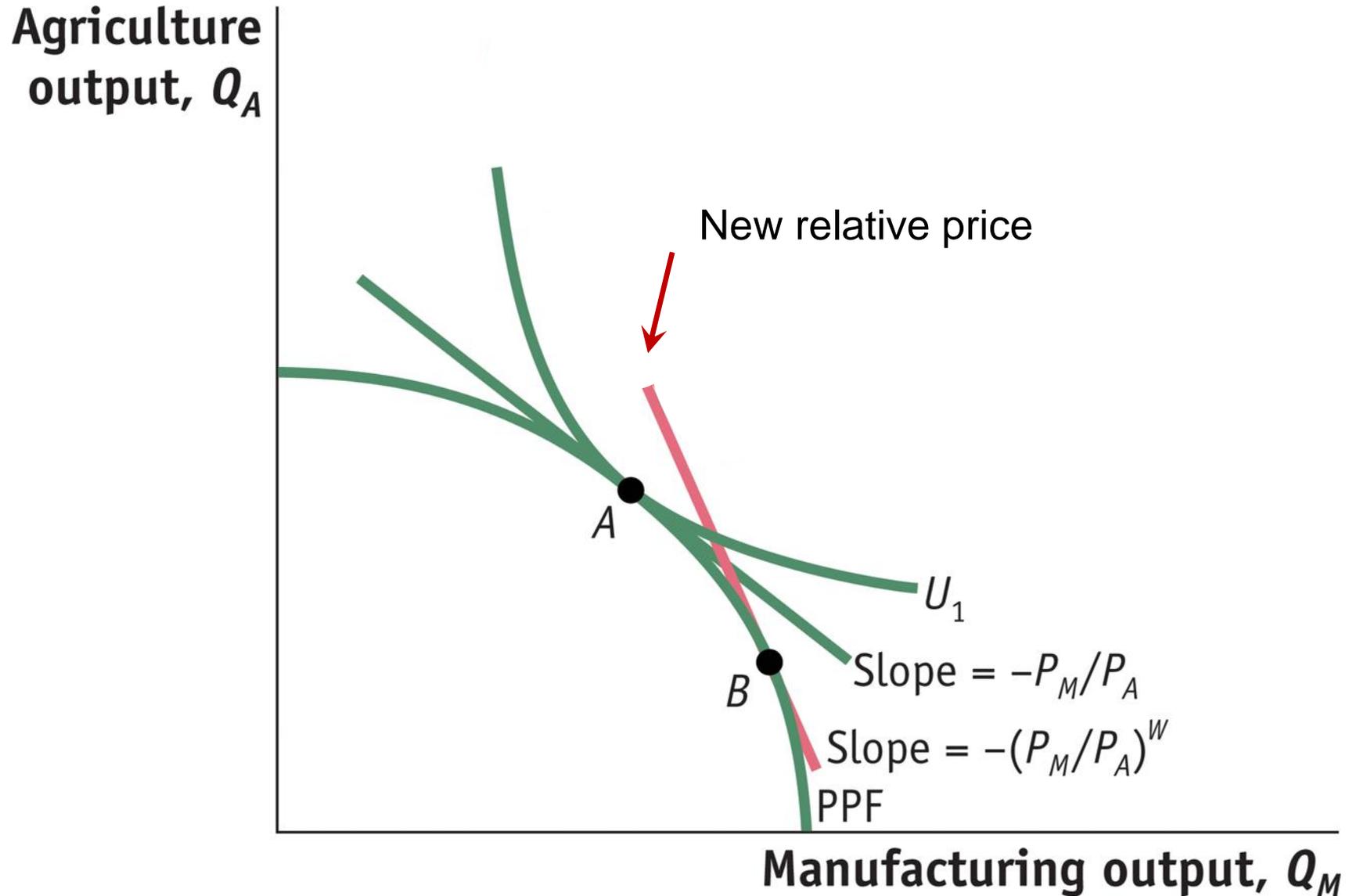
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New world price:

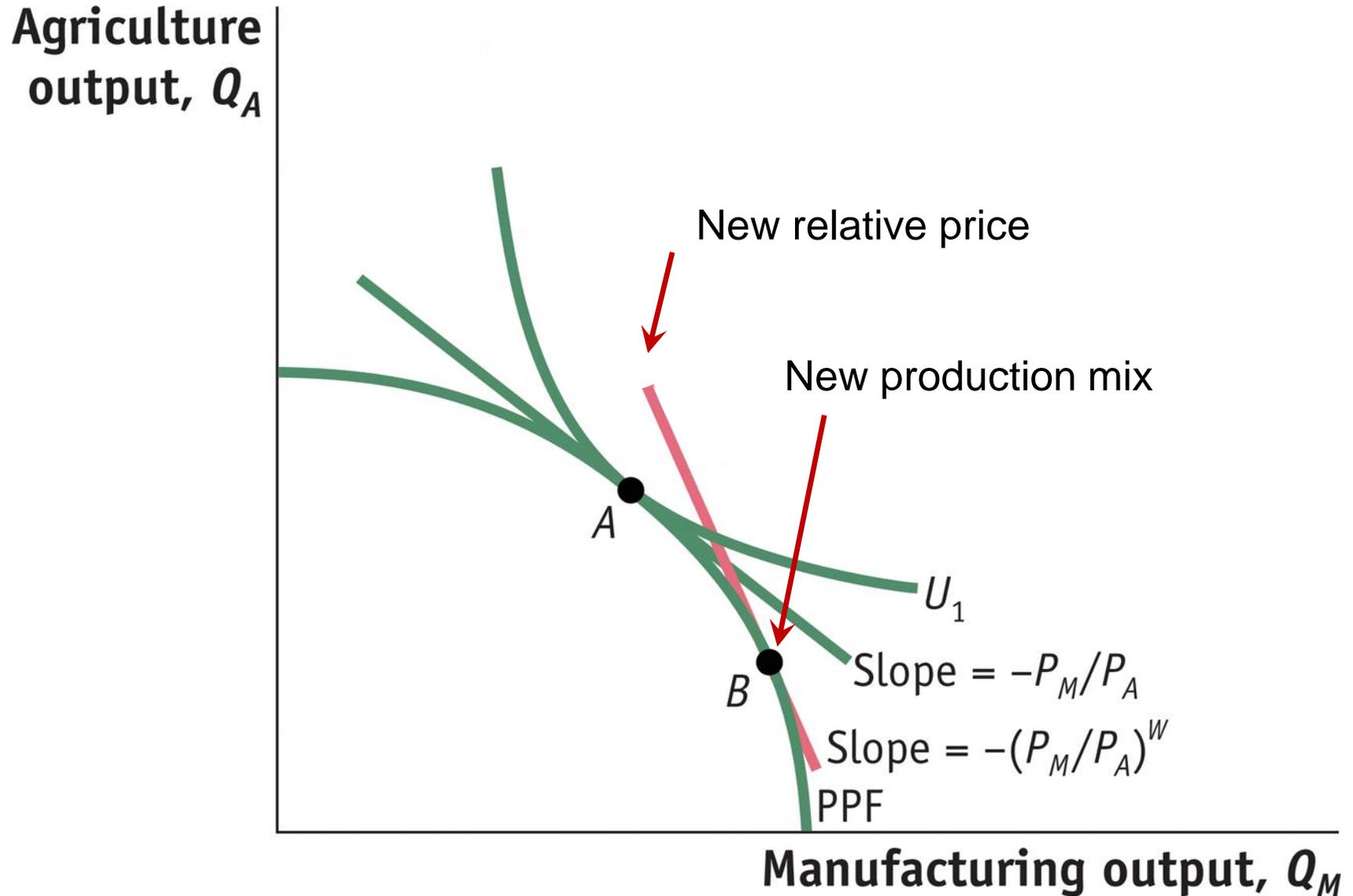
$$(P_M/P_A) < (P_M/P_A)^W < (P^*_M/P^*_A).$$

Effect on production?

2 Effect of Trade on production



2 Effect of Trade on production



2 Effect of Trade on production

Quantitative example:

In the next example with Cobb-Douglas production, I would like to show you:

- How to link ratio of MPL to employment
 - How to link ratio of MPL to prices
- ➔ How to link employment to prices

2 Effect of Trade on production

Quantitative example:

- Manufactures: $Y_M = a_M K^{1/3} L_M^{2/3}$
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Marginal product of Labor:

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→ Slope of PPF:
$$Slope = \frac{MPL_A}{MPL_M} = \frac{a_A T^{1/3}}{a_M K^{1/3}} \left(\frac{L_M}{L_A} \right)^{1/3}$$

2 Effect of Trade on production

Quantitative example:

- Slope of PPF:
$$\text{Slope} = \frac{MPL_A}{MPL_M} = \frac{a_A T^{1/3}}{a_M K^{1/3}} \left(\frac{L_M}{L_A} \right)^{1/3}$$

Constant term x Employment ratio

2 Effect of Trade on production

Quantitative example:

- Slope of PPF: $Slope = \frac{MPL_A}{MPL_M} = \frac{a_A T^{1/3}}{a_M K^{1/3}} \left(\frac{L_M}{L_A} \right)^{1/3}$
- At equilibrium: $Slope = \frac{P_M}{P_A}$
- How does a change in prices affects $\frac{L_A}{L_M}$?

2 Effect of Trade on production

Quantitative example:

- Slope of PPF: $Slope = \frac{MPL_A}{MPL_M} = \frac{a_A T^{1/3}}{a_M K^{1/3}} \left(\frac{L_M}{L_A} \right)^{1/3}$
- At equilibrium: $Slope = \frac{P_M}{P_A}$
- How does a change in prices affects $\frac{L_A}{L_M}$?

$$\frac{P_M}{P_A} = \frac{a_A T^{1/3}}{a_M K^{1/3}} \left(\frac{L_M}{L_A} \right)^{1/3} \Rightarrow \frac{L_A}{L_M} = \frac{a_A^3 T}{a_M^3 K} \left(\frac{P_A}{P_M} \right)^3$$

Clicker question:

If the relative price of manufacturing goods increases by 1%, relative employment in manufacturing L_M / L_A increases by:

- a) A negative percentage, i.e. decreases!!
- b) Increases by 1%
- c) Increases by 0.33%
- d) Increases by 3%

Answer:

Answer:

If the relative price of manufacturing goods increases by 1%, relative employment in manufacturing L_M / L_A increases by:

Some useful algebra...

Quantifying changes with exponents, etc.:

- Suppose $Z = a X^\beta$
- If X increases by 1% then Z increases by β %.
- If Z increases by 1% then X increases by $1/\beta$ %.

- Suppose $Z = X \cdot Y$

- If X increases by x %

- If Y increases by y %

→ Then Z increases by: $x+y$ %.

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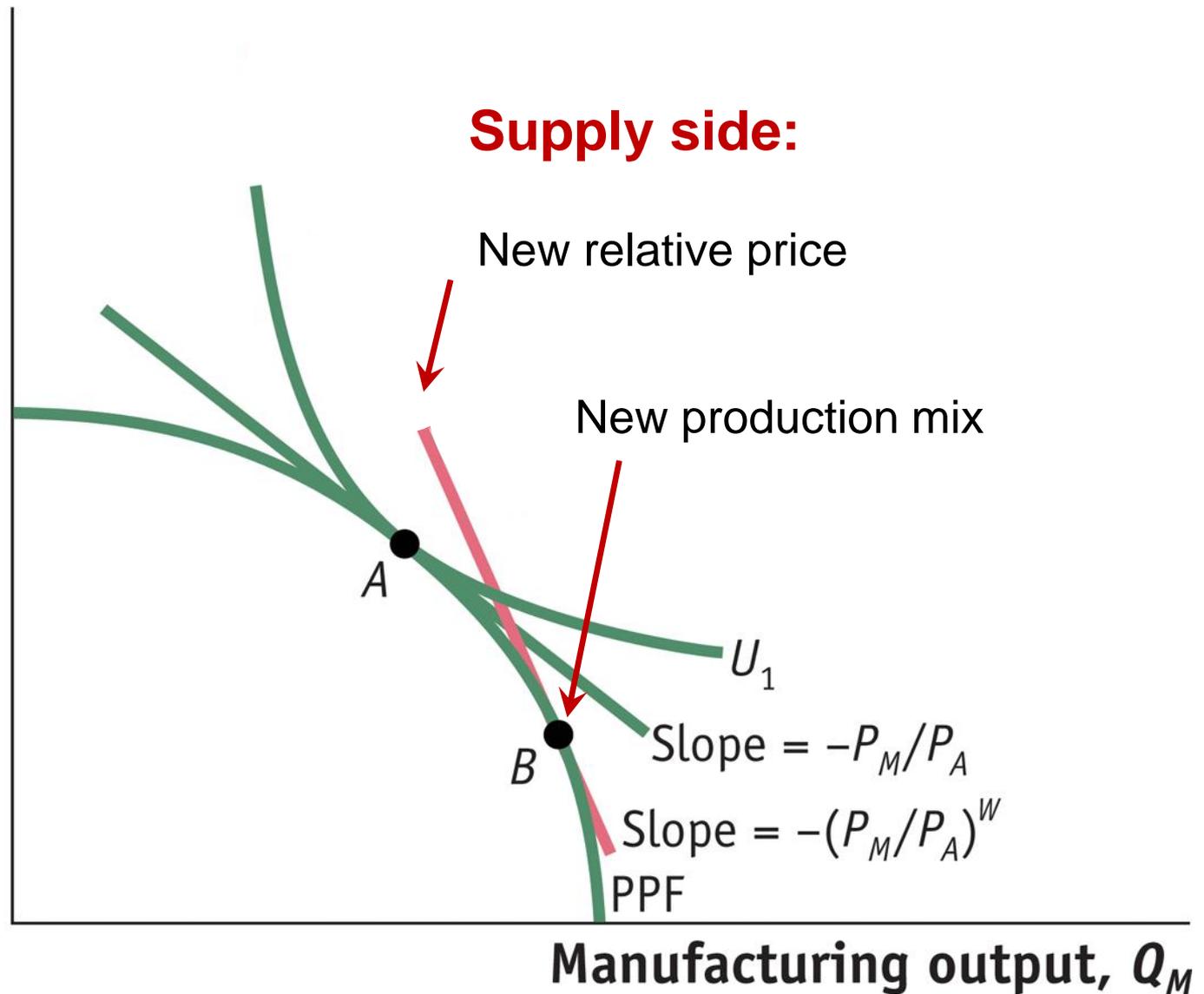
3 Gains from Trade

Overall Gains from Trade?

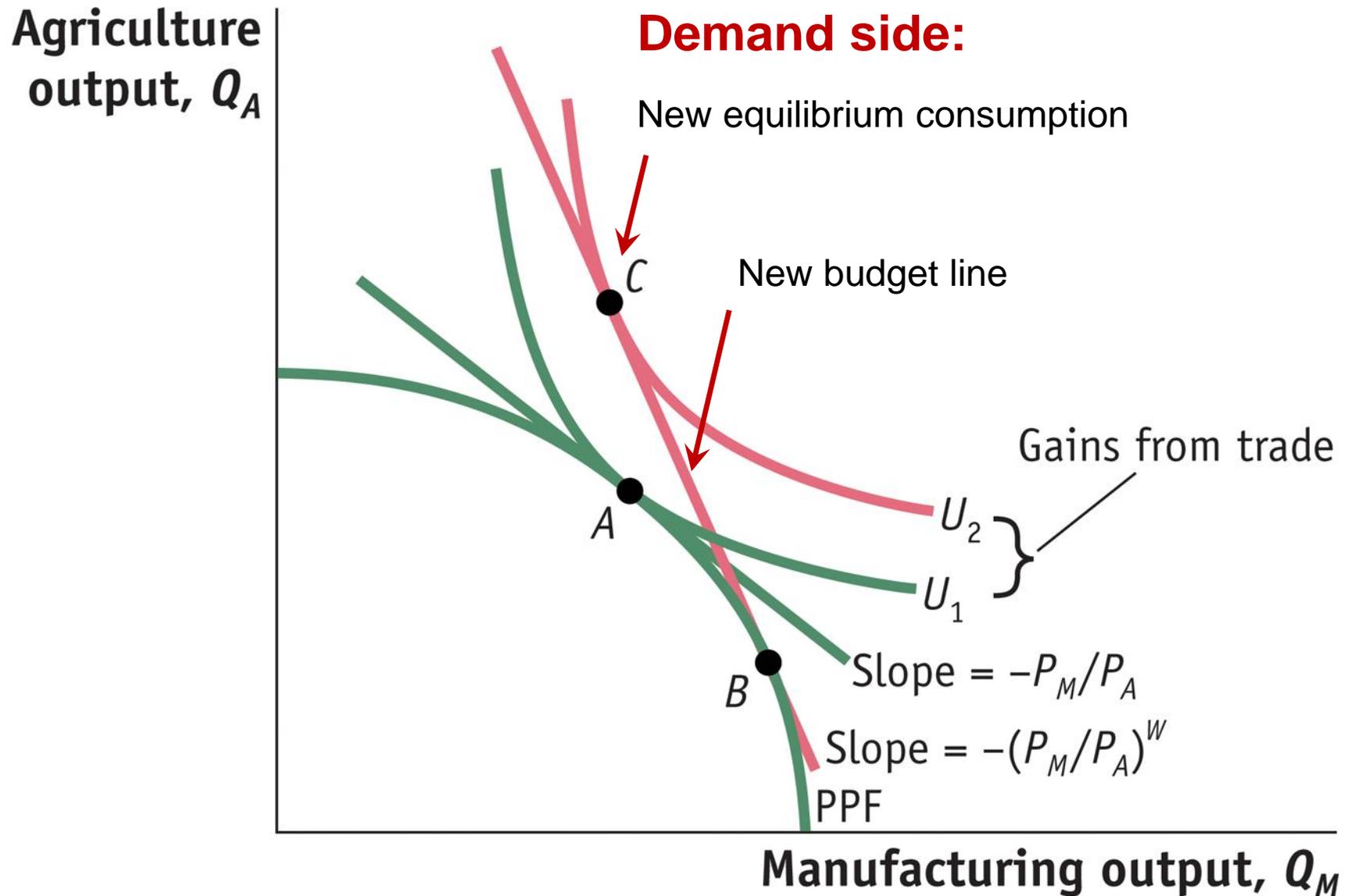
- We start by looking at the average consumer

3 Gains from Trade

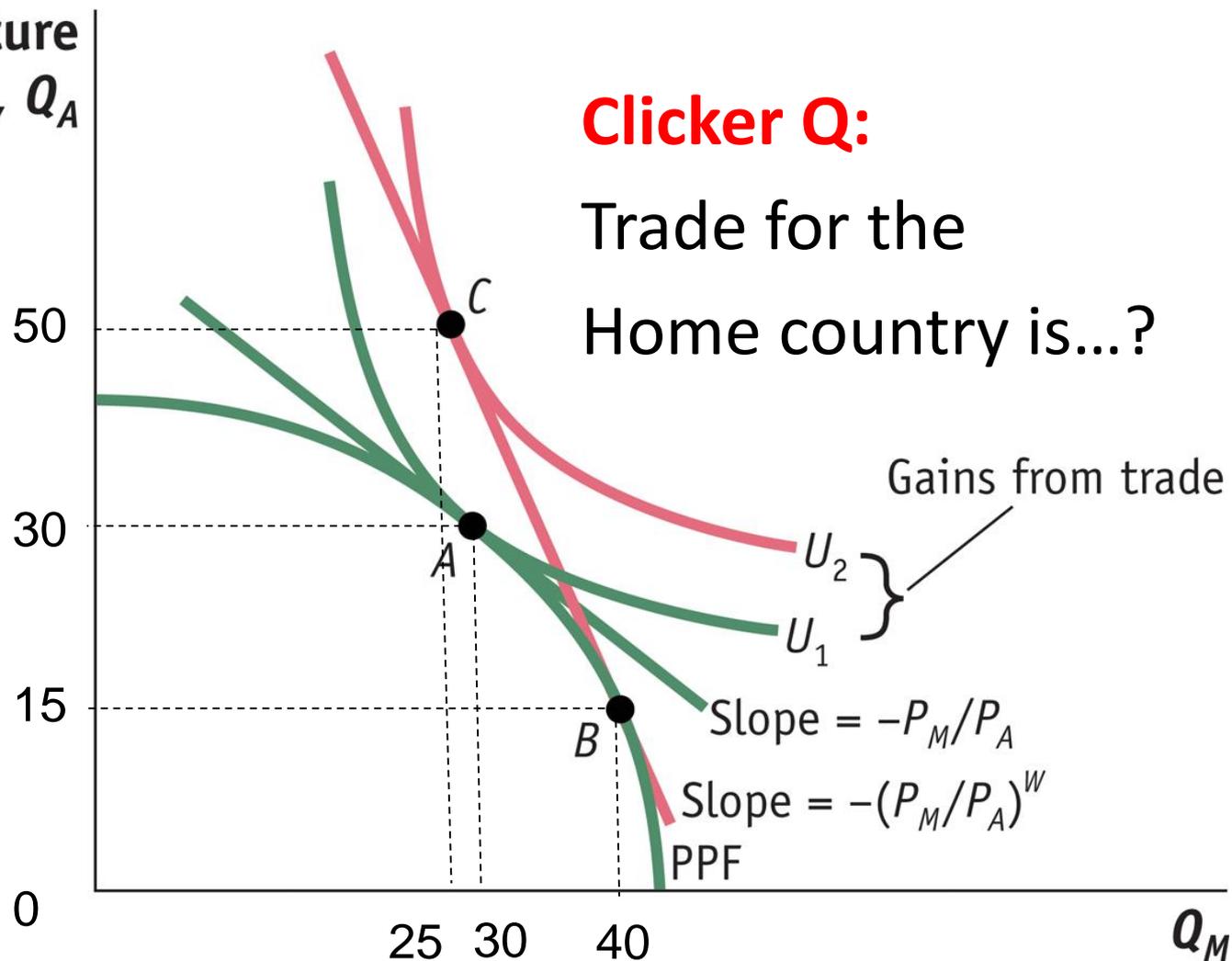
Agriculture output, Q_A



3 Gains from Trade



Agriculture
output, Q_A



- a) $X_M = 5, M_A = 20$; b) $X_M = 20, M_A = 20$;
c) $X_M = 15, M_A = 35$; d) $X_M = 20, M_A = 15$;

3 Gains from Trade

Overall Gains from Trade

So far, things are not very different from Ricardo:

New world price:

$$(P_M/P_A) < (P_M/P_A)^W < (P_M^*/P_A^*).$$

- Manufacturing goods are exported,
- Agricultural goods are imported
- For an average consumer, Home is better off with trade.

3 Gains from Trade

Gains for everyone?

- When there are gains from trade *on average*, it does not imply that everyone gains from trade
- The interesting part of the model is to examine what happens to the return to each factor:
 - 1) Labor wage
 - 2) Rental rate of Capital and Land

Do workers gain? Do land and capital owner gain?

- **CHAPTER 3 – Next lecture (part 2):**
 - Setting up the specific factor model
 - Change in production and employment
 - Aggregate gains from trade
- Effect on labor wages?
- Effect on returns to Capital and Land?