ARE 202, T. Fally - Problem set 5 – Krugman model with CES

We assume that all consumers have the same preferences:

$$U = \left[\int_i x_i^\rho \, di \right]^{1/\rho}$$

with $\rho < 1$ and where i denotes a product variety. We can also define $\sigma = \frac{1}{1-\rho} > 1$. Each consumer earns an income w. On the supply side, we assume that each single variety i is produced by a single firm (hence we also index firms by i). To produce a quantity y_i, firm i has to incur total costs:

$$C_i = cw\, y_i + fw$$

where w denotes the wage. All firms have the same marginal cost c and fixed cost $f > 0$ in terms of labor requirements. We consider that the number of firms N is very large (approximated as a continuous variable).

We start by examining a closed economy with L consumers such that total GDP equals L (with wage $w = 1$ normalized unity) and such that $y_i = Lx_i$. Questions:

1. Show that the price p_i for each variety i has a constant markup over the marginal cost.
2. What is the number of firms N in equilibrium with free entry?

   How does firm scale vary with market size L?
3. How does welfare depend on population L, in equilibrium with free entry?

Now, suppose that the country opens to trade with another country with population L^* where consumers have similar tastes and local firms have access to the same technology, but foreign varieties are **distinct** from domestic varieties. Goods can be traded freely and prices are identical across countries.

4. Why are nominal wages equal in both countries? In other words, why can we normalize wages to unity in both countries?
5. Explain why firm scale is the same as in Autarky.
6. In equilibrium with free trade, show that the welfare gains from trade in each country (relative to Autarky) are given by:

$$\frac{U_{Trade}}{U_{Aut}} = \lambda^{-\frac{1}{\sigma-1}}$$

   where λ denotes the share of domestic goods in consumption (one minus the share of imports).

Notes: See Krugman (1979) and chapter 5 in Feenstra's book (2003) for a variation of this model where firm scale and markups do vary with trade. See Arkolakis, Costinot and Rodriguez-Clare (2012) for a generalization of the formula (above) for welfare gains from trade.