

## ARE 202 T. Fally – Problem set 2 – Choke price

This problem set examines again directly-additive utility functions of the form:

$$U = \sum_i u_i(x_i)$$

where  $x_i$  denotes the consumption of good  $i$ . Let's denote by  $w$  the consumer's income. We will also denote by  $\lambda$  the Lagrange multiplier associated with the budget constraint. We assume that:

$$u'_i(x) > 0 \quad \text{and} \quad u''_i(x) < 0$$

And here we also suppose that marginal utility is bounded by  $\alpha_i$ :

$$u'_i(0) = \alpha_i < +\infty$$

Questions:

1. Show that the demand for good  $i$  is zero if its price  $p_i$  is larger than the *choke price*  $p_i^*$  defined by:

$$p_i^* \equiv \frac{\alpha_i}{\lambda}$$

2. Show that  $\lambda$  decreases with income.
3. What does the previous question imply for the number of product varieties purchased by rich vs. poor households?
4. For this question, suppose that utility is parameterized by:  $u_i(x) = (x + a_i)^{b_i}$ . Comparing a rich and a poor household with the same preferences, which one has a higher price elasticity of demand for a given good  $i$ ? (assuming that both the rich and poor households consume positive amounts of this good)<sup>1</sup>
5. Now, suppose that we have a large number of goods approximated by a continuum and indexed by  $\omega \in [0, +\infty)$ . Suppose that all goods have the same price  $p$ :

$$p_\omega = p$$

Suppose that preferences take the form:

$$U = \int_{\omega=0}^{+\infty} \log(x_\omega + \omega^\kappa) d\omega$$

with  $\kappa > 0$ . What is the range of goods consumed by a consumer with income  $w$ ? What is the demand for each good  $\omega$  as a function of  $p$  and  $w$ ?

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<sup>1</sup>Here for simplicity you can assume that the expenditure share of good  $i$  is small hence  $\frac{\partial \lambda}{\partial p_i}$  is negligible.