Plan

A) Effect of taxes with equilibrium bw supply and demand?
   ◯ Equivalence result
   ◯ Tax incidence
   ◯ Deadweight loss (DWL)
   ◯ Ramsey rule
   ◯ Import tariff
   ◯ Leakage from carbon tax

B) Illustrations of the theory of the second best
Chapter on tax policy in equilibrium

Taxes with production side, and more than one markets:

- We have already examined the effect of taxes on consumers, taking prices and supply side as given

⇒ Now: accounting for adjustments on supply side (production and trade)

- Optimal taxes? Effects of taxes across agents?

⇒ Tax incidence, Ramsey rule, terms of trade effects

Taxing supply or demand?

- Should we tax brands or consumers?
- Should we tax employees or employers?

Two results you should know:

- In equilibrium, it is equivalent to tax supply and demand
- Who is most affected by the tax does not depend on who directly pays for it but depend on relative elasticity of demand and supply.
A simple equivalence result

Setting:

- Downward-slopping Demand \( D(p) \)
  Upward-slopping supply \( S(p) \)

- Equilibrium with a tax \( t \) on consumers: \( D(p + t) = S(p) \)
  market price is \( p \) but consumers have to pay \( p + t \).

- Equilibrium with a tax \( t \) on suppliers: \( D(p') = S(p' - t) \)
  market price is \( p' \) but suppliers only perceive \( p' - t \).

This yields the same equilibrium conditions with \( p' = p + t \).
Then, who is most affected? who sees the largest price change?

Equilibrium with and without tax
Tax incidence

- Equilibrium without tax: \( D(p^*) = S(p^*) \)
- Equilibrium with tax: \( D(p + t) = S(p) \)
- Elasticity of demand: \( \eta = -\frac{p}{D(p)} \frac{dD(p)}{dp} \)
- Elasticity of supply: \( \sigma = \frac{p}{S(p)} \frac{dS(p)}{dp} \)
- **Tax incidence**: around \( p^* \), suppliers see a price decrease:
  \[
  \frac{dp}{dt} = -\frac{\eta}{\eta + \sigma}
  \]
  while consumers see a price increase:
  \[
  \frac{d(p + t)}{dt} = \frac{\sigma}{\eta + \sigma}
  \]
  ⇒ Inelastic parties endure the largest effect

Deadweight loss (DWL)

DWL when suppliers adjust their prices?
(Consumers: let’s ignore income effects and focus on price effects)

- Taxes generate revenues \( T = t \cdot D(p + t) = t \cdot S(p) \)
- Loss for consumers: change in consumer surplus
  \[
  \Delta CS = \int_{\nu=p^*}^{p+t} D(\nu)d\nu
  \]
- For suppliers: change in “producer surplus”, i.e. profits (again this comes from Shephard’s Lemma applied to profits)
  \[
  \Delta \Pi = \int_{\nu=p}^{p^*} S(\nu)d\nu
  \]
- Dead-weight loss \( DWL = \Delta CS + \Delta \Pi - T \)
Second-order approximation

- At first order, DWL is negligible: for small taxes, DWL depend on $t^2$
- See blackboard:

$$DWL \approx \frac{1}{2} \frac{\sigma \eta}{\sigma + \eta} \frac{q}{p} t^2$$
Optimal tax: which sectors to tax?

Suppose that one needs to tax amount $G$ in taxes

- Problem: minimize sum of DWL's, with $\sum_i t_i q_i = G$
  (or maximize $G$ such that $\sum_i DWL_i = \text{cst}$)

- Assumptions:
  - No lump-sum tax (would be optimal)
  - At least one sector (e.g. sector $i = 0$) is not tax
    Hence we cannot tax all sectors uniformly (would also be optimal)
  - Zero cross-price elasticities (in Hicksian demand) to simplify the problem
    and ignore interactions between sectors

- Lagrangian for minimization problem, including second-order terms:
  \[
  L = \sum_i DWL_i - \lambda \left[ \sum_i q_i t_i \left(1 + \frac{dq_i}{dp_i} \frac{t_i}{q_i} \right) - G \right]
  \]

Ramsey Rule

- First simple case: with perfectly-elastic supply ($\sigma_i = +\infty$)
  \[
  L = \sum_i \frac{\eta_i}{2} q_i \frac{t_i^2}{p_i} - \lambda \sum_i q_i t_i \left(1 - \frac{\eta_i t_i}{p_i} \right)
  \]
  FOC:
  \[
  \frac{t_i}{p_i} = \frac{\lambda}{1 + 2\lambda} \frac{1}{\eta_i} \propto \frac{1}{\eta_i}
  \]
  \[
  \Rightarrow \text{Optimal tax rate must be inversely proportional to price elasticity}
  \]

- Similarly, with imperfectly-elastic supply ($\sigma_i < +\infty$):
  \[
  L = \sum_i \frac{\sigma_i \eta_i}{2(\sigma_i + \eta_i)} q_i \frac{t_i^2}{p_i} - \lambda \sum_i q_i t_i \left(1 - \frac{\sigma_i \eta_i t_i}{\sigma_i + \eta_i \frac{t_i}{p_i}} \right)
  \]
  Optimal tax proportional to $\frac{1}{\sigma_i} + \frac{1}{\eta_i}$
Example of tax policy for an open economy: Optimal tariffs

- **Vocabulary: quick reminder**
  - **Tariff**: basically a tax on trade. It can be a tax on exports or, more often, on imports. Often defined at the 6-digit level of HS classification
  - **Export subsidies**: negative tax on exports
  - **Terms of trade effect** (TOT): change in the price of exports relative to imports

- **Quizz**: Are import tariffs equivalent to:
  - \( A \) export subsidies?
  - \( B \) tax on exports?

Welfare analysis

- **Quasi-linear preferences with numeraire good** \( q_0 \) (easy to generalize to multiple industries):
  \[
  U = q_0 + u(q) \quad \Rightarrow \quad q = D(p) ; q_0 = I - D(p)
  \]
  (we’ll keep the notation \( x \) for exports)

- **Consumer welfare** \( U = I + CS(p) \)
  with \( CS(p) \equiv u(D(p)) - p.D(p) \)

- **Free trade and constant unit cost of production for** \( q_0 \)
  Domestic price \( p \) equals international price \( p^* \) plus tariff \( t \)

- **Production with convex cost function** \( C(Y) \)
Sources of revenues

- **Labor** $L$, wage normalized to one

- **Profits**: $\Pi = p.y - C(y)$
  Under perfect competition $y(p)$ verifies: $C'(y(p)) = p$

- **Tariffs**: $t.m$
  where $m$ represents net imports: $m = q - y$

- Envelop theorem (Shephard’s Lemma) provides:
  
  \[
  \frac{\partial CS(p)}{\partial p} = -D(p) \\
  \frac{\partial \Pi(p)}{\partial p} = y(p)
  \]
  (the latter identity is only valid under perfect competition)

Impact of tariffs on welfare

- **Total welfare**: $W = S + L + t.m + \Pi$

- Under **perfect competition**, we obtain:
  
  \[
  \frac{dW}{dt} = -q \cdot \frac{dp}{dt} + m + t \cdot \frac{dm}{dt} + y \cdot \frac{dp}{dt}
  \]
  
  Using $1 - \frac{dp}{dt} = -\frac{dp^*}{dt}$ and $q - y = m$, we have:
  
  \[
  \frac{dW}{dt} = t \cdot \frac{dm}{dt} - m \cdot \frac{dp^*}{dt}
  \]
  (the first term is negative (imports decrease with tariffs)
  
  The second term can be positive only for a large country ($\frac{dp^*}{dt} = 0$ for a small country). It’s a **term-of-trade** effect
Optimal tariff

- FOC: Tariff is optimal when $\frac{dW}{dt} = 0$
- Using the above expression, we obtain:
  $$t^* / m = \frac{dp^*}{dt} / \frac{dm}{dt}$$
- When foreign firms are price-taker, we can consider the supply curve $x(p^*)$. Since $m = x(p^*)$, we obtain:
  $$\frac{dm}{dt} = \frac{\partial x}{\partial p^*} \frac{dp^*}{dt}$$
  (Note the distinction between total and partial derivatives!)
- Optimal tariffs can be expressed as:
  $$\frac{t}{p^*} = \left(\frac{p^*}{x} \frac{\partial x}{\partial p^*}\right)^{-1}$$

Small open economy (taking $p^*$ as given)

- (a) Home Market
- (b) Import Market

  ![Diagram](image)

  - change in consumer surplus: $- (a+b+c+d)$
  - change in producer surplus: $+ a$
  - Tariff revenues: $+ c$

  TOTAL: (DWL) $- (b+d)$
**Large economy (negative effect on $p^*$)**

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^*$</td>
<td>$S_1$</td>
</tr>
<tr>
<td>$p^*$ + $t$</td>
<td>$S_2$</td>
</tr>
<tr>
<td>$p^w$</td>
<td>$D_1$</td>
</tr>
</tbody>
</table>

(a) Home Market
- No-trade equilibrium
- $A$ as the equilibrium price
- $B^*$ as the foreign demand at $p^w$
- $C^*$ as the foreign supply at $p^w$
- $D_1$ as the domestic demand at $p^*$
- $S_1$ as the domestic supply at $p^*$
- $S_2$ as the domestic supply at $p^*$

(b) World Market
- $X^*$ as the world supply at $p^*$
- $B^*$ as the foreign demand at $p^*$
- $C^*$ as the foreign supply at $p^*$

**Intuition**

- For a large country, DWL is small (second order) compared to tariff revenues, hence a non-zero optimal tariff. For a small country, free trade is optimal.

- This equality is valid for both imports ($x$ is the foreign supply) and exports ($x$ is the foreign demand): it yields positive taxes on imports or exports.

- Formula similar to a **monopsony** charging a positive markup when it internalizes the effect on supply.

  By taxing exporters it raises the country's gains while reducing trade when the elasticity of the foreign supply is low.
Other remarks

- Note that such a positive tariff does not take into account the foreign country’s welfare. For total welfare, the usual DWL applies.

- This result on optimal tariff and market power is also valid in a general equilibrium framework without numeraire good.

- In a two-country version where each country sets its tariff unilaterally, we need to look at the Nash equilibrium (see Bagwell and Staiger, 1999).

- Under certain conditions, repeated interactions between countries may help enforce smaller tariffs.

Negative Leakage

- See Karp (2013), also a possibility in Fullerton et al (2011).

- In partial equilibrium, taxing the “dirty” sector leads to positive leakage (i.e. increase in pollution) in other sector or other country.
  
  - Increasing costs of a dirty sector leads to increases in pollution in other sectors (substitution if there is a price change).
  
  - and/or increases in imports of dirty goods in other countries (especially with exogenous prices, as in small open economies).

- Opposite results can be obtained in general equilibrium, by endogenizing income or demand for other factors of production.
### Negative Leakage

- Assume a simple setting:
  - Price exogenously fixed by international markets (small open economy)
  - Homothetic preferences, relative demand for clean good $\alpha$
  - At zero carbon tax, production production of clean good $\beta$
  - $\varepsilon^x_t$ the elasticity of production of each good $x$, dirty or clean, w.r.t tax $t$, accounting for returns to all factors (in GE).

$\Rightarrow$ Reduction in dirty goods imports if $\frac{\beta}{\alpha} \varepsilon^{clean}_t < \varepsilon^{dirty}_t$

- For instance, if $\varepsilon^x_t < 0$ for both goods:
  - A pollution tax generates a decrease in total income larger than the decrease in production of the dirty good, which leads to a decrease in dirty goods imports if $\alpha$ is high.

- How can we have $\varepsilon^{clean}_t < 0$? Can happen for instance if the switch to cleaner inputs drains the clean sector (Fullerton et al 2010).

### Plan

- Part A - Effect of taxes with equilibrium bw supply and demand?
- Part B - Some illustrations of the theory of the second best
  - Markups on N-1 markets
  - Carbon tax
  - Carbon tax + Tariffs
  - Tariffs under imperfect competition
Theory of the second best: illustrations

Key message:

- If we have a failure/distortion in one market, removing failures in other markets does not necessarily improve welfare.

In some cases, this can justify government intervention.

Typical examples rely on externalities, but there are examples that do not involve externalities.

Application 1

- We have $L$ identical consumers/workers.
  Consumer revenues combine wages $w$ and profits
- Suppose that we have $N$ firms
- Preferences $U = U(q_1, q_2, ..., q_N)$ symmetrical in $q_i$’s
  For instance: $U = \left[ \sum_{i=1}^{N} q_i^\rho \right]^{1/\rho}$
- Producing one unit of a good requires $c$ workers
- Market failure:
  $N - 1$ firms charge the same markup $\frac{p_i}{cw} = \frac{p_j}{cw} = \mu > 1$

Q: Does it improve welfare to impose $p = cw$ for the $N^{th}$ firm?
Application 1: Solution

- At first best: all goods are consumed in the same quantities
- We are back to first best with $p_N = \mu cw = p_i$ for all $i$.
- We are strictly away from first best if $p_N = cw$ (if the $N$ goods are not perfect substitutes)
- Intuition:
  - At first best, production $q^* = \frac{L}{Nc}$.
  - This remains true as long as prices are identical (in GE when profits are shared with consumers)
  - A different price for good $N$ introduces a wedge similar to a tariff or a tax
  - Any price $p_N \in (cw, \mu cw]$ ("second best") improves welfare compared to $p_N = cw$

Application 2

- Household $h$ has preferences: $U^h = q_0^h + u(q^h) - E$ where $q_0$ is the numeraire and $E$ are total emissions
- Large number $L$ of households such that the impact of each household’s consumption on total emissions $E$ is negligible.
- A single firm produces $y = \sum h q_h$ and emits an amount $e$ of CO2 per unit of production
- The marginal cost of production is $c$ (constant)
- Market failure: No tax or constraint on CO2 emissions

Q: Does $p = c$ maximize consumer welfare? Gains from having higher markups and taxing revenues?
Application 2: Solution

- At first best: internalizing the cost for the environment:
  \[ u'(q) = c + Le \]

- We are back to first best if we impose taxes \( t = Le \) and if taxes are redistributed to consumers.

- Even if taxes are not redistributed to consumers, a positive tax may be welfare improving ("second best").

  **Intuition:**
  Here a positive tax improves welfare if demand elasticity is high, i.e. when a small tax has a large effect.

Application 3

Leakage risk:

- When a carbon tax is imposed, production partly reallocate to untaxed foreign markets.

- A import tariff would limit such "leakage risk"

- Reminder: tariffs decrease global welfare, + decrease a small economy’s welfare

  **Q:** Is zero tariff \( t = 0 \) still the optimal tariff under such externalities?

  (for simplicity: we’ll assume away consumption in foreign market)
Application 3: Solution

- Accounting for Foreign welfare + Externalities + Carbon tax:
  \[
  \frac{dW}{dt} = (t_C - e_d) \frac{dy_d}{dt} + (t - e_m) \frac{dm}{dt}
  \]
  \(t_C\): local carbon tax, \(e_d\) and \(e_m\): emissions externalities, \(L = 1\)

- Result: Yes, taxing imports is welfare improving: \(t^* = e_m\).

- Even more interesting:
  If it is not (politically) possible to tax imports, better to decrease domestic carbon tax (= domestic subsidy):
  Optimal subsidy: \(s^* = e_m \left( \frac{dy_d}{dt} / \frac{dm}{dt} \right)\)

Application 4

- Same setting as we used for optimal tariffs (quasi-linear preferences)

- Consider a small open economy:
  international price \(p^*\) is fixed for each good

- Suppose now that we are no longer under perfect competition among domestic suppliers, such that \(p > C'(y)\).

Q: Is zero tariff \(t = 0\) still the optimal tariff?
   If not, should we have a positive or negative tariffs?  
   (negative tariffs = import subsidies)
Application 4: Solution

- Under imperfect competition, we have an additional term:
  \[
  \frac{dW}{dt} = -q\frac{dp}{dt} + m + t\frac{dp}{dt} + y\frac{dp}{dt} + (p_d - C'(y))\frac{dy}{dt}
  \]

- After simplification, it yields:
  \[
  \frac{dW}{dt} = t\frac{dm}{dt} - m\frac{dp^*}{dt} + (p_d - C'(y))\frac{dy}{dt}
  \]

- When \( \frac{dp^*}{dt} = 0 \) (small economy) and \( p_d > C'(y) \), and given that \( \frac{dy}{dt} = \frac{dy}{dp} > 0 \), the optimal tariff is strictly positive:
  \[
  \frac{dW}{dt} > 0 \quad \text{when} \quad t = 0
  \]

- Intuition:
  Under imperfect competition, domestic firms do not produce enough. Higher prices push them to produce more.