ARE 202: Welfare: Tools and Applications
Spring 2018

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Lecture Notes 05 – Production and Industry Equilibrium

Production side

Topics for a brief intro:

- **Returns to scale**: crucial in applied micro, growth, development, trade
- **Producer surplus** and profits as measure of welfare in partial eq.
- Industry equilibrium with **free entry**: yields zero profits in long term, and other implications for aggregate production function, etc.
- Beyond firms: I want to talk about equilibrium on **factor markets** and return to factors

  Putting firms and factors together: "Production Possibility Frontier"
- Imperfect competition: monopoly pricing and monopolistic competition

Plan

A- Tools

1) Returns to scale
2) Producer surplus and duality in supply side
3) Industry equilibrium with free entry
4) Equilibrium on factor markets
5) Monopolistic competition

B- Illustration

- Costinot and Donaldson (2016):

  Illustrate how to recover PPF and equilibrium prices in Agriculture from output and sales data across US counties, and how to infer gains from economic integration and productivity improvements (1880-1997)

Returns to scale

Consider production function $Y = f(L_1, L_2, ...)$ with inputs L_f

- Constant returns to scale (CRS):

$$f(\lambda L_1, \lambda L_2, ...) = \lambda F(L_1, L_2, ...) \quad \text{for any} \quad \lambda > 0$$

- CRS, increasing or decreasing returns to scale?
  - ☐ $f(L) = aL - b$?
  - ☐ $f(L, K) = [a_L L^\rho + a_K K^\rho]^{\frac{1}{\rho}}$?
  - ☐ $f(L, K) = aL^\alpha K^\beta$?
  - ☐ $\log f(L, K) = a_L \log L + a_K \log K + a_{KL}(\log K - \log L)^2$? (see also translog production functions)

- IRS often modeled with a fixed cost (see IRS example above)
- In these examples: elasticity of substitution between K and L?

Returns to scale

- CRS often assumed in macro or trade (see later slides with free entry)
- Note: Any DRS industry can be re-interpreted as CRS:
  Just add a specific factor (hypothetical or not).

  Suppose that an industry has production function $F(X)$ decreasing returns in vector of inputs X. Define z as the new (specific) input and define a new production function such that:

  $$\widetilde{F}(X, z) = z/z_0.F(Xz_0/z)$$

  and assume that the supply of this input is fixed at z_0.

Duality on supply side

- Inputs and outputs chosen to maximize profits
- Dual: For a given output level, inputs also minimize total cost
  - Solve for cost function $C(Y, w)$ for the examples provided earlier
- Like for consumption side, envelop theorem can be applied to link supply (output), demand for inputs with the **changes** in profits and costs

$\Rightarrow$ Can be useful for welfare analysis on supply side
and analysis of equilibrium among factors of production

Shephard's Lemma (again)

- Linking input requirements and cost function: **Shephard's lemma** (notation: w_f unit price of factor of production L_f)

$$L_f(Y,w) = \frac{\partial C(Y,w)}{\partial w_f}$$

- With CRS production function:
  - Denote $c(w)$ the *unit cost* function (homogeneous of degree 1)

$$c(w) = \min \left\{ \sum_f w_f L_f | f(L) \geq 1 \right\}$$

  - Shephard's lemma provides *per unit* input requirements:

$$a_f(w) = \frac{\partial c(w)}{\partial w_f}$$

  homogeneous of degree 0, with $c(w) = \sum_f w_f a_f(w)$

Linking profits and output

In competitive equilibrium, taking output prices and factor prices as given:

- You already know well that the price is equal to marginal cost

$$MC_i(Y_i,w) \equiv \frac{\partial C_i(Y_i,w)}{\partial Y_i} = p_i$$

- We also get **Hotelling's Lemma**: $\frac{\partial \pi}{\partial p_i} = Y_i(p)$

  Proof: apply envelop theorem knowing that firms maximize profits $\pi = \max \sum_i [p_i Y_i - \sum_f w_f L_{if}]$

$\Rightarrow$ Since supply increases in prices, profits are convex

$\Rightarrow$ "Producer surplus" is the area to the left of the supply curve (holding constant factor prices). Note: it omits fixed costs (hence $\neq$ profits).

Plan

A- Tools

1) Returns to scale
2) Producer surplus and duality in supply side
3) **Industry equilibrium with free entry**
4) Equilibrium on factor markets
5) Monopolistic competition

B- Illustration

- Costinot and Donaldson (2016): recovering PPF in Agriculture

Competitive equilibrium with free entry

What if new producers can freely enter/exit an industry?

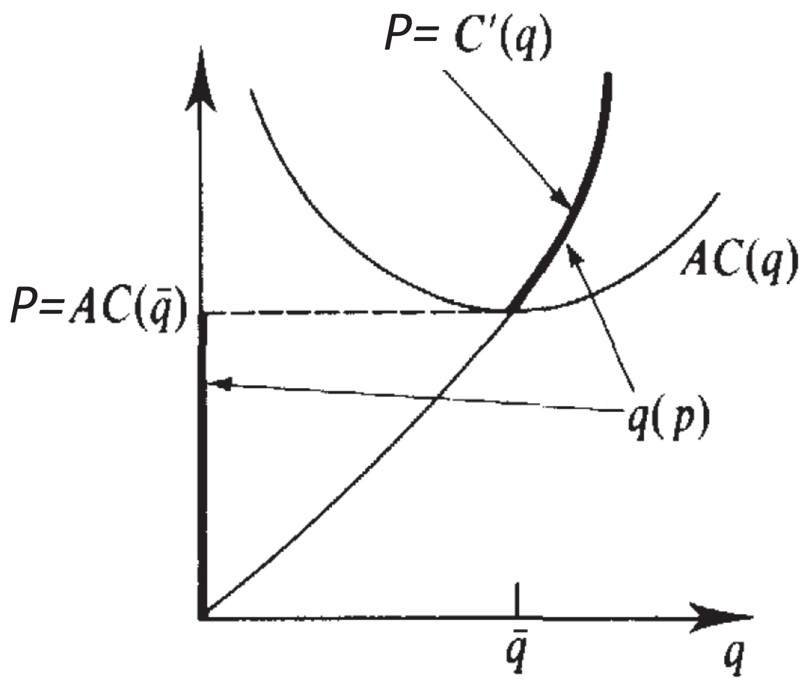
- For well-defined equilibrium, suppose that costs are convex and suppose that there are fixed costs: $C(0, w) > 0$.
- When profits are positive: new producers enter
  When profits are negative: new producers exit

$\Rightarrow$ Profits must be zero at equilibrium

- We get then:
  - Marginal cost equals price p: $MC(Y, w) \equiv \frac{\partial C(Y,w)}{\partial Y} = p$
  - With free entry, we also get: $AC_i(Y_i, w) \equiv \frac{C(Y_i,w)}{Y_i} = p_i$
    (hence AC$=$ MC $=$ p)
  - Noteworthy: AC meets MC at its minimium

Competitive equilibrium with free entry

P = MC = AC



Aggregation of many firms with free entry

What is the aggregate production function when firms can freely enter?

- Previous equalities yield a (cheap) way to obtain optimal firm scale, characterized by $AC = MC$
- "Replication argument":
  If industry size is much larger than this optimal firm scale, more firms enter. The total cost on aggregate (for the industry) is then the sum of costs across these firms and is proportional to total industry output

$\Rightarrow$ Justifies assumption of CRS technologies in Macro

To see this, consider the aggregate cost function (ignore that N is a discrete number, assuming N is sufficiently large): $\widetilde{C}(Y, w) = \min_N\{N \,.\, C(Y/N, w)\}$
We can rewrite it as cost function that is linear in Y (hence CRS):

$$\widetilde{C}(Y, w) = Y/y^*(w).C(y^*(w), w)$$

where y^* is the that $MC(y^*) = AC(y^*)$, and $N = Y/y^*(w)$

Factor markets

Equilibrium on input markets:

- Factor market clearing (sum across industries i), with endowment $\bar{L}_f$ for each factor of production f (e.g. K vs. L):

$$\sum_i L_{if} = \bar{L}_f$$

- Equilibrium (with mobility across sectors) implies:

$$p_i \frac{\partial Y_i}{\partial L_i} = w_f$$

$\Rightarrow$ "Marginal product" of each factor of production is equalized across industries (equal to factor price w_f).

- The same result would be obtained by optimizing the use of factors of production to max joint profits (shadow cost $\equiv w_f$): first best

Notation in macro: MPL for labor, MPK for capital

Production possibility frontier

Examples of PPF: see blackboard

- 2 CRS industries, one input
- 2 decreasing returns to scale industries, one input
- 2 IRS industries, 1 input
- 2 CRS industries, 2 specific inputs, 1 shared input
- 2 CRS industries, 2 inputs

Interpretation of slope of PPF? Competitive equilibrium?

Production possibility frontier

In all case, the competitive equilibrium is defined by tangency with PPF

- 2 CRS industries, one input: linear
  Slope = ratio of prices in Autarky
  If prices differ: solution is in one of the two corners
- 2 decreasing returns to scale industries, one input: concave
- 2 IRS industries, 1 input: convex (hence two corner equilibria)
- 2 CRS industries, 2 specific inputs, 1 shared input: concave
  Slope = - ratio of marginal product of the shared (mobile) input
- 2 CRS industries, 2 inputs: concave
  Slope = - ratio of MP for each mobile input

Example: two factors and two goods

Practical tips: Equilibrium in multiple factor markets & goods market with CRS

- If all production functions are CRS, use ratios and homogeneity properties: K/L, Y_2/Y_1, r/w, etc. E.g. ratio $c_2(r,w)/c_1(r,w)$ only depends on r/w
- How does $c_2(r,w)/c_1(r,w)$ depend on r/w?

$$\frac{\partial(c_2/c_1)}{\partial(r/w)} > 0 \qquad \Longleftrightarrow \qquad a_{2K}(r,w) > a_{1K}(r,w)$$

  Increases in r/w if industry 2 is more intensive in K.
- How does the relative demand for K/L depend on production y_2/y_1?

$$\frac{K}{L} = \frac{y_1 a_{1K}(r,w) + y_2 a_{2K}(r,w)}{y_1 a_{1L}(r,w) + y_2 a_{2L}(r,w)}$$

  Increases in y_2/y_1 if industry 2 is more intensive in K.
- Will be useful for Problem Set 4!

Factor market

Additional comments:

- Partial equilibrium analysis: usual trick is to assume outside sector with p=1 and constant marginal product for that factor $\Rightarrow$ fixed wages
  - Often combined with quasi-linear preferences to also neutralize wealth effects in consumption

- Adjustments affecting factor markets:
  - Occupational choice as in Lucas (1978): workers vs. entrepreneurs
  - Endogenizing technology, labor demand can end up being upward sloping, e.g. Acemoglu (2002) with $F(H,L) = [A_L L^\rho + A_H H^\rho]^{\frac{1}{\rho}}$ and factor biased technology:
  $$\frac{A_H}{A_L} = \left(\frac{H}{L}\right)^\beta$$
  Examples: Innovations in Ag often in response to shortage of a factor

Pricing and free entry under imperfect competition

A quick intro to pricing decisions:

- Monopoly pricing
- Monopolistic competition with CES
- Free entry under monopolistic competition

= Minimum that you need to know for Problem Set 5:

- Product varieties matter for consumer welfare in GE
- Useful for future courses in Macro, Development and Trade

More will be done next semester with Jeff Perloff:

- Less trivial / more realistic models of imperfect competition and pricing

Monopoly - quick review

- Firms maximize profits: first order condition leads to marginal costs (MC) being equal to marginal revenues (MR).
- When firms are price takers (perfect competition), marginal revenues from additional unit are equal to the price. Equilibrium:

$$MC(q) \;=\; MR(q) \;=\; p$$

- However, when demand is downward slopping, producing additional units leads to lower prices.

$$MC(q) \;=\; MR(q) \;=\; p + q \, . \, \frac{dp}{dq} \;<\; p$$

- Maximizing profits leads to markups equal to the inverse of price elasticity of demand $\eta(q) \equiv \frac{\partial \log q}{\partial \log p}$:

$$\frac{p - MC}{p} = \frac{1}{\eta(q)}$$

Monopolistic competition with CES

- Assumptions:
  - CES demand across product varieties (one variety = one firm)

$$x_i = \left(\frac{p_i}{P}\right)^{-\sigma} \frac{E}{P}$$

  - Other firms affect demand curve only through price index P
  - Firms take price index as given
    $\Rightarrow$ each firm has a tiny market share $s_i \approx 0$
- In this case, the price elasticity is:

$$\frac{\partial \log x_i}{\partial \log p_i} \;=\; -\sigma$$

$$\text{Hence:} \qquad p_i \;=\; c_i \, . \, \frac{\sigma - 1}{\sigma} \;=\; \frac{c_i}{\rho}$$

- Yields a very tractable framework with constant markups, prices not affected by other firms' decisions (horrible in the eyes of IO economists)

Free entry and firm scale

- We have seen how to have free entry in the perfect competition framework. With perfect competition, we need:
  - increasing marginal costs
  - fixed costs

  With imperfect competition, increasing marginal costs are not required: we can get the required convexity from the demand.

- Suppose that firms have the same constant marginal costs c but each firm has to pay a fixed cost f to enter the industry:

$$C(q) = f + cq$$

- Free entry implies zero profits in equilibrium

$\Rightarrow$ Optimal number of firms N? Firm scale in equilibrium?

Free entry with CES

- With CES, the price is given by $p = \frac{\sigma}{\sigma-1}\, c$
- Average cost is given by: $AC = c + \frac{f}{q}$
- Free entry imposes $AC = p$, which yields:

$$\frac{\sigma}{\sigma - 1}\, c = c + \frac{f}{q} \quad \Rightarrow \quad cq = (\sigma - 1)f$$

$\Rightarrow$ Firm scale is fixed, independent of market size

$\Rightarrow$ Number of firms is proportional to market size

See PS5 for similar statements in GE with trade and several countries

Plan

A- Tools

B- **Illustration**

- Costinot and Donaldson (2016): recovering PPF in Agriculture

Costinot and Donaldson (2016)

- Lecture on WARP: Gains from trade depend on price changes relative to Autarky and rely on concavity of PPF.
- Here: estimate shape of PPF in US Agriculture to compute gains from market integration across US counties 2,600 from 1880 to 1997
- Challenge: recovering missing info using theory and assuming competitive equilibrium in goods and factor markets

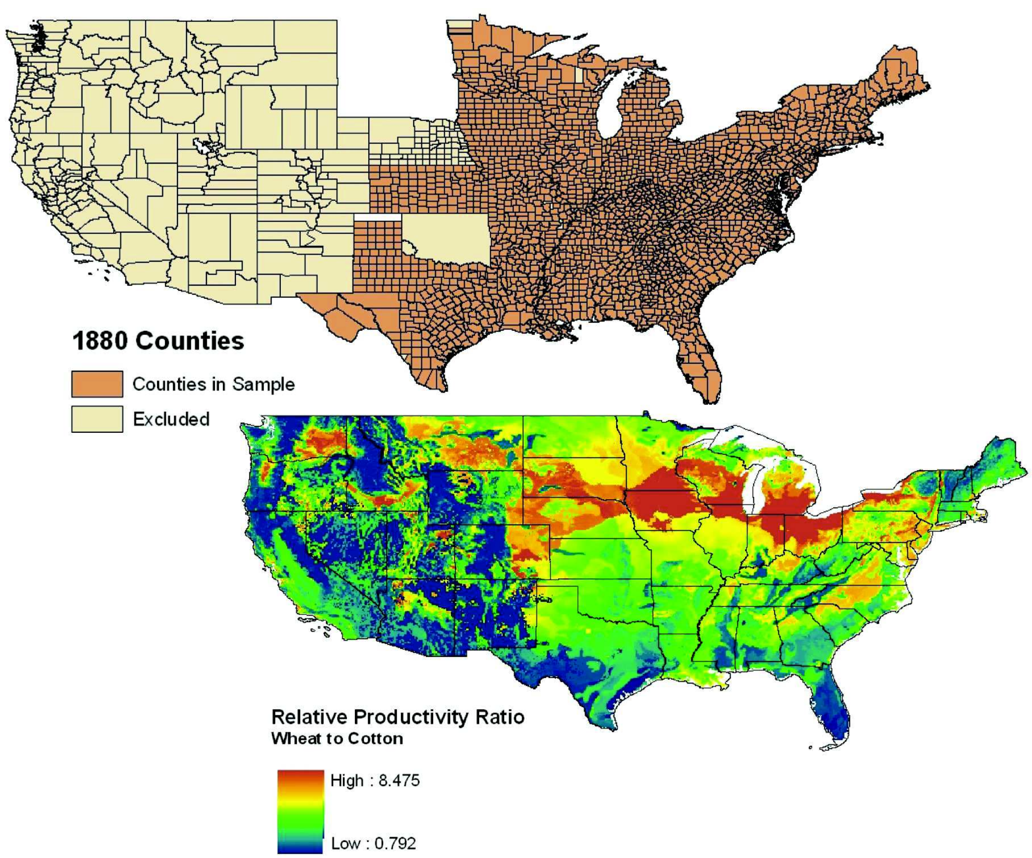
Data

- Historical Data:
  - Quantities $\hat{Q}_{it}^k$ produced by crop k and county i at time t
  - Land use $\hat{L}_{if}^k$ by county i and crop k
  - Sales $\hat{S}_{it}$ by location, but not by crop
  - but no price data (some incomplete data at state level but not by county)
- GAEZ data:
  - Productivity estimate $\hat{A}_{i,2011}^{fk}$ of land by crop at each field in 2011
    Each county i is the collection of many fields $f \in \mathcal{F}_i$
  - Still need to recover true productivity, assuming

$$A_{it}^{fk} = \alpha_{it}^k \hat{A}_{i,2011}^{kf}$$

Q: How to infer price p_{it}^k and productivity shifter α_{it}^k assuming equilibrium?

Each US county = location i



Characterization of equilibrium

Characterization of equilibrium on goods and factor markets

- Setting
  - Available land for each field f normalized to 1: $\sum_k L_{it}^{fk} = 1$
  - Production function: $Q_{it}^k = \sum_f A_{it}^{fk} L_{it}^{fk}$
  - Profits: $\Pi_{it}^k = p_{it}^k \left(\sum_f A_{it}^{fk} L_{it}^{fk}\right) - \sum_f r_{it}^f L_{it}^{fk}$
  - Prices p_{it}^k are taken as given and exogenous
- Competitive equilibrium implies:
  - Non-positive profits: $p_{it}^k A_{it}^{fk} \leq r_{it}^f$
  - Zero profits when $L_{it}^{fk} > 0$: $p_{it}^k A_{it}^{fk} = r_{it}^f$
  - Full employment of resources: $\sum_k L_{it}^{fk} = 1$

Characterization of equilibrium

- Usual questions facing theorists are
  - How to solve for equilibrium (+ prove uniqueness/existence)
  - Find tractable cases
  - Comparative statics in the model parameters
- But applied theorists/empirists would rather ask:
  - Assume equilibrium, etc.
  - What underlying parameters are consistent with observed equilibrium outcomes?

  You need to be able to "reverse-engineer" the model.

Recovering prices and productivity shifters

Find α_{it}^k and p_{it}^k such that:

- Equilibrium:
  (1) $p_{it}^k \alpha_{it}^k \hat{A}_{i,2011}^{kf} \leq r_{it}^f$
  (2) $p_{it}^k \alpha_{it}^k \hat{A}_{i,2011}^{kf} = r_{it}^f$ when $L_{it}^{fk} > 0$
  (3) $\sum_k L_{it}^{fk} = 1$
- Observed moments:
  (4) $\sum_f \alpha_{it}^k \hat{A}_{i,2011}^{kf} L_{it}^{fk} = \hat{Q}_{it}^k$
  (5) $\sum_f L_{if}^{fk} = \hat{L}_{if}^k$
  (6) $\sum_k p_{it}^k \hat{Q}_{it}^k = \hat{S}_{it}$

Recovering prices and productivity shifters

Three-step procedure:

1. 1st step: construct unadjusted PPF using $\hat{A}_{i,2011}^{kf}$
   - Suppose that we have two crops k and k'. Crop k is produced in fields with highest $\frac{A_{it}^{k'f}}{A_{it}^{kf}}$ which are given by $\frac{\hat{A}_{i,2011}^{k'f}}{\hat{A}_{i,2011}^{kf}}$ (the α's cancel out).

   $\Rightarrow$ Determines the shape of the PPF

2. 2nd step: adjust productivity shifters α_{it}^k to fit production and land use
   - From land use across crops L_{it}^k, we can deduce crop choice by field f
   - Conditional on land use, decreasing α_{it}^k for a crop k and county i leads to shrinking the PPF along the dimension of crop k (equation 4)

   $\Rightarrow$ The α's adjust the scale on each dimension (each Q_{it}^k)

Note: productivity shifters are not identified for crops that are not produced.

Recovering prices and productivity shifters

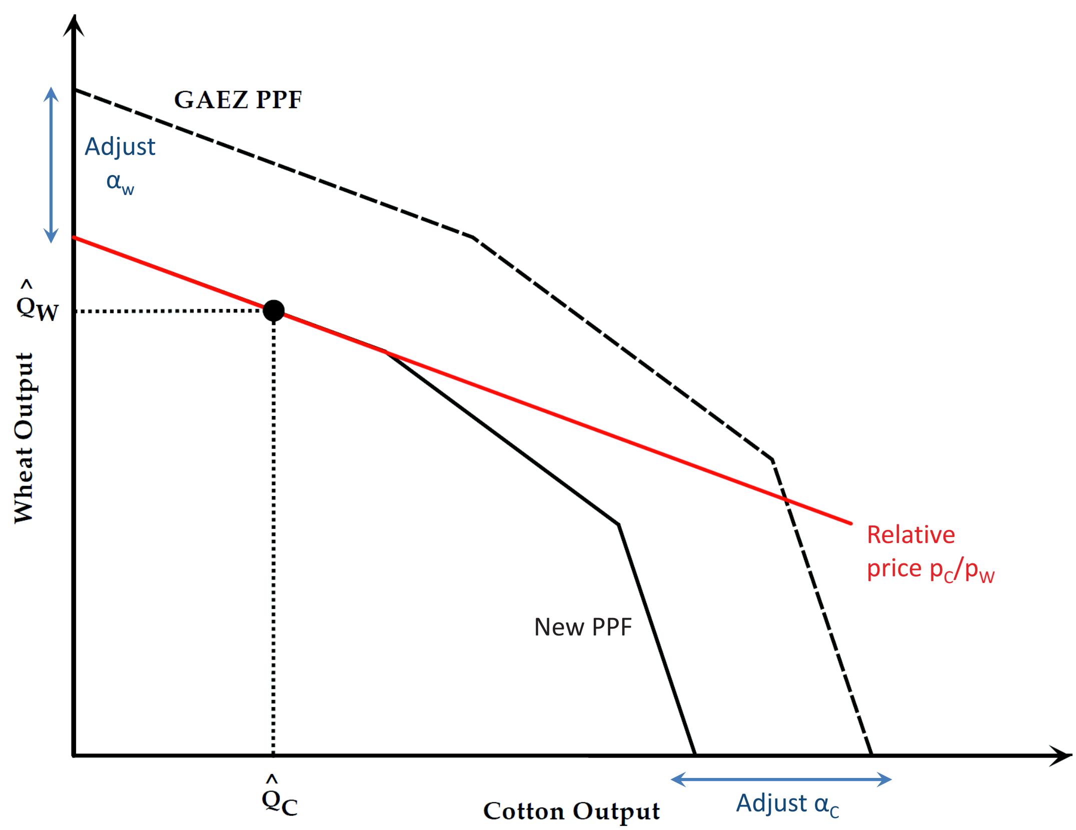
By the second welfare theorem, for each efficient allocation there is exists a price vector such that it corresponds to a competitive equilibrium.
Here, this implies that we can solve for prices in a separate step:

3. 3rd step: find prices
   - Average price across crops given by total sales $\hat{S}_{it}$ (equation 6)
   - Assuming that there exists is a field f that produces two crops k and k', we get relative prices across crops using equation (2):

$$p^k_{it}\alpha^k_{it}\hat{A}^{kf}_{i,2011} = p^{k'}_{it}\alpha^{k'}_{it}\hat{A}^{k'f}_{i,2011} = r^f_{it} \implies \frac{p^k_{it}}{p^{k'}_{it}} = \frac{\alpha^{k'}_{it}\hat{A}^{k'f}_{i,2011}}{\alpha^k_{it}\hat{A}^{kf}_{i,2011}}$$

There are potentially some uniqueness issues when we end up on a vertex (0.01% of the cases in practice), where we can only get a *range* of prices

Example with three fields f, and positive output for two crops k



Estimated Prices

Table 1: Estimated farm-gate prices (relative to observed New York prices)

	1880		1920		1954		1997	
Crops	U.S. average (1)	Std. dev. (2)	U.S. average (3)	Std. dev. (4)	U.S. average (5)	Std. dev. (6)	U.S. average (7)	Std. dev. (8)
Barley	0.55	0.26	0.76	0.29	–	–	0.88	0.20
Buckwheat	0.54	0.32	0.69	0.22	0.87	0.18	0.95	0.15
Corn	0.60	0.37	0.86	0.27	0.81	0.24	0.83	0.19
Cotton	0.72	0.29	0.75	0.17	0.79	0.23	0.87	0.25
Groundnuts	–	–	0.66	0.29	0.94	0.31	1.10	0.28
Oats	0.43	0.40	0.81	0.30	0.91	0.25	1.05	0.21
Potatoes	0.71	0.41	0.85	0.34	0.84	0.15	0.98	0.17
Rice	0.70	0.58	0.64	0.29	0.71	0.10	0.79	0.08
Rye	0.59	0.24	0.75	0.31	0.80	0.22	0.84	0.24
Sorghum	–	–	0.84	0.42	0.90	0.25	0.91	0.20
Soybeans	–	–	–	–	0.66	0.36	0.77	0.34
Sugar beets	–	–	0.74	0.28	0.94	0.33	0.86	0.28
Sugar cane	0.57	0.39	0.70	0.36	0.79	0.34	0.91	0.31
Sunflower	–	–	–	–	–	–	0.94	0.29
Sweet potatoes	0.69	0.42	0.74	0.31	0.70	0.24	0.85	0.19
Wheat	0.71	0.37	0.81	0.30	0.94	0.18	1.12	0.15

Notes: Values of $p_{it}^k / \bar{p}_t^k$ for the indicated years t, crops k, averaged (and standard deviation) across all sample U.S. counties i. Entries indicated by "–" are those for which the crop was not reported as produced in any county.

Model fit

Correlation between inferred and actual prices at the state level

Table 3: Correlation between estimated farm-gate prices and observed state-level prices

	Estimated (state-average) farm-gate price from model							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Observed (state-level) farm-gate price	0.810***	0.713***	0.680***	0.692***	1.049***	0.842***	0.804***	0.835***
	(0.019)	(0.022)	(0.034)	(0.040)	(0.006)	(0.022)	(0.026)	(0.051)
Logs or levels	Levels	Levels	Levels	Levels	Logs	Logs	Logs	Logs
Constant	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Crop fixed effect	No	No	Yes	Yes	No	No	Yes	Yes
Year fixed effect	No	No	No	Yes	No	No	No	Yes
R-squared	0.408	0.285	0.370	0.431	0.929	0.352	0.614	0.714
Observations	2,766	2,766	2,766	2,766	2,766	2,766	2,766	2,766

Notes: Columns (1) and (5) report uncentered R-squared values. Columns (1)-(4) report regressions of our estimates of farm-gate prices, p_{it}^k, (averaged across all counties i in a state) on state-level farm-gate price data. Columns (5)-(8) do the same but with both variables in logs. Robust standard errors, clustered at the state-level, are reported in parentheses. *** indicates statistically significant at 0.1% level.

Counterfactual analysis

Two counterfactual exercises using the inferred prices and PPFs:

- Gains from market integration:
  Let us interpret the ratio $\frac{p_{it}^k}{\bar{p}_t^k}$ relative to New York prices as trade costs. What would be output and sales if those trade costs are reduced to those of a later period t'? Solve for new competitive equilibrium and quantities $(Q_{it}^k)^T$ at new prices and compute:

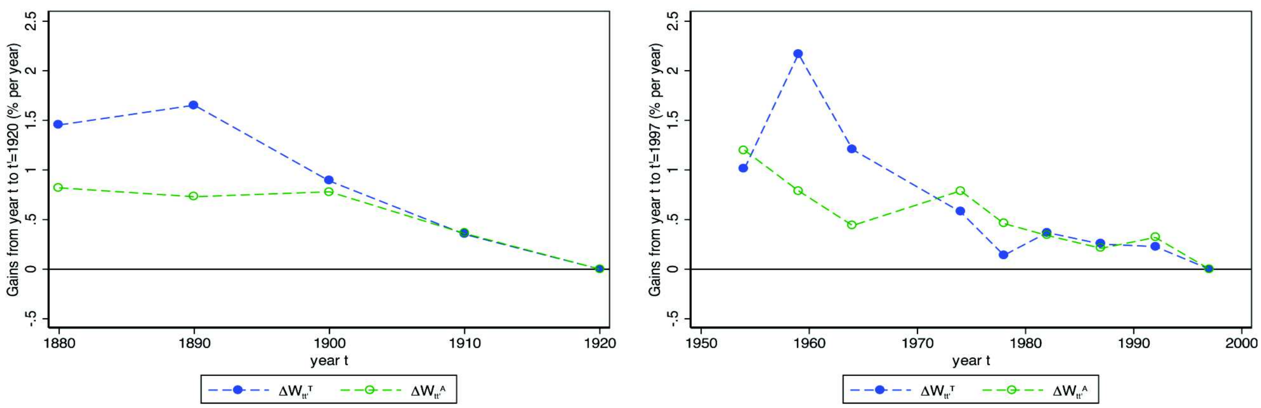
$$\Delta W^T = \frac{\sum_i \sum_k (Q_{it}^k)^T (p_{it}^k)^T}{\sum_i \sum_k \hat{Q}_{it}^k p_{it}^k} - 1$$

- Gains from productivity improvements:
  What if productivity shifters α_{it}^k were equal to those of a later period t'? Similarly, solve for counterfactual $(Q_{it}^k)^A$ and compute:

$$\Delta W^A = \frac{\sum_i \sum_k (Q_{it}^k)^A (p_{it}^k)^A}{\sum_i \sum_k \hat{Q}_{it}^k p_{it}^k} - 1$$

Counterfactual analysis

Gains from market integration and productivity improvements
Each equivalent to 1-2% annual sales growth



(a) Early Period, 1880-1920

(b) Late Period, 1954-1997

Remarks:

- Great use of theory and data! (FYI, it's R&R at Econometrica)
  - Similar approach used in Costinot, Donaldson and Smith (JPE 2016)
- In the paper, the authors also account for:
  - outside good (to account for unused land),
  - alternative interpretation of price gaps (e.g. taxes and quotas),
  - additional factor of production (labor), etc.
- but they do not:
  - describe much the fitted productivity shifters relative to existing data on relative productivity across crops (crop-specific technological change)
  - check that prices were correlated with proximity to major markets (they did it in an older version of the paper)
  - check that changes in prices correspond to changes in trade costs

Next chapters:

Welfare analysis in general equilibrium, putting consumption side and production side together in GE:

Ch6 Trade: Comparative advantage, Gains from trade

Ch7 Public policy, taxation in GE
Theory of second best: inefficiency in one market can be welfare improving when another market is also inefficient