ARE 202, Spring 2018
Welfare: Tools and Applications

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Lecture notes 02 – Price and Income Effects

Plan

1. Preferences and utility
   - Preferences and utility, Debreu's theorem
   - Marshallian demand
   - Examples of utility functions and demand
2. About aggregation and RUM
3. Duality
   - Hicksian Demand
   - Shephard's Lemma and Roy's Identity
   - Giffen goods: example from Jensen and Miller (2008)

1) Preferences, Utility and Demand

- Preferences and utility
- Marshallian demand
- Demand and price elasticities
- Illustrating income effects
- Examples of utility functions

Some definitions

Rational preferences: Preferences $\succsim$ on X are rational if:

- Completeness: For all $x, y \in X$, we have $x \succsim y$ and/or $y \succsim x$
- Transitivity: For all $x, y, z \in X$, $x \succsim y$ and $y \succsim z$ implies $x \succsim z$

Other definitions:

- Preferences $\succsim$ on X are **monotone** if $x \gg y$ implies $x \succ y$, and strictly monotone if $x \geq y$ and $x \neq y$ implies $x \succ y$
- Preferences $\succsim$ on X are **continuous** if for all $\{x_n, y_n\}$ such that $x_n \succsim y_n$, $x_n \to x$ and $y_n \to y$, then $x \succsim y$.
- Preferences $\succsim$ on X are locally **non-satiated** if for every $x \in X$ and $\varepsilon > 0$, there is a $y \in X$ such that $\|x - y\| < \varepsilon$ and $x \succ y$
- Preferences $\succsim$ on X are **convex** if for every $\alpha \in (0, 1)$, $y \succsim x$ and $z \succsim x$ then $\alpha y + (1 - \alpha)z \succsim x$ ($\succ$ if **strictly** convex)
- Preferences are **homothetic** if for any $\alpha > 0$, $x \sim y$ implies $\alpha x \sim \alpha y$

Utility representation:

Utility function such that: $x \succsim y \Leftrightarrow U(x) \geq U(y)$

Debreu's theorem:
Let $X \subset R^n$. Preferences $\succsim$ on X have a continuous utility representation if and only if these preferences are (check needed conditions):

- ☐ rational?
- ☐ monotone?
- ☐ strictly monotone?
- ☐ continuous?
- ☐ locally non-satiated?
- ☐ convex?
- ☐ strictly convex?
- ☐ homothetic?

Counter-example: preferences that don't have a continuous rep'?

Homothetic preferences:

Preferences such that, for any $\alpha > 0$, $x \sim y$ implies $\alpha x \sim \alpha y$

Proposition:
Any homothetic, continuous and monotonique preference relation can be represented by a utility function that is homogeneous of degree one.

Utility maximization problem:

$$\max u(x)$$
$$\text{such that: } p.x \leq w$$

Proposition:
It has a unique solution $x(p, w)$ (Marshallian or Walrasian demand) if (check what is needed):

- ☐ $u(x)$ is continuous?
- ☐ $u(x)$ is strictly quasi-concave?
- ☐ preferences are homothetic?
- ☐ corresponding preferences are locally non-satiated?

Notes:

- Example of preferences that we will use but does not satisfy all these conditions: Leontief preferences
- $p.x \leq w$ is the budget constraint (a.k.a. Walrasian set)

Properties of Marshallian demand

We have:

- $x(w, p)$ is homogeneous of degree zero

Moreover, if preferences are locally non-satiated:

- $p.x(w, p) = w$

In general, we will also assume that $u(x)$ is differentiable as many times as needed.

Indirect utility and marginal utility of wealth

- **Indirect utility:** Utility associated with the chosen bundle $x(p, w)$:

$$V(p, w) = U(x(p, w)) = \max U(x) \quad \text{such that } p.x \leq w$$

- Marginal utility of wealth:

$$\frac{\partial V}{\partial w} = \sum_i \frac{\partial U}{\partial x_i} \cdot \frac{\partial x_i}{\partial w} =$$

More definitions

Def 1: Price elasticity: $\varepsilon_i^P = \frac{\partial \log x_i}{\partial \log p_i}$

- $\varepsilon_i^P < 0$ Giffen good
- $\varepsilon_i^P > 1$ Price-elastic good

Def 3: Income elasticity: $\varepsilon_i^I = \frac{\partial \log x_i}{\partial \log w}$

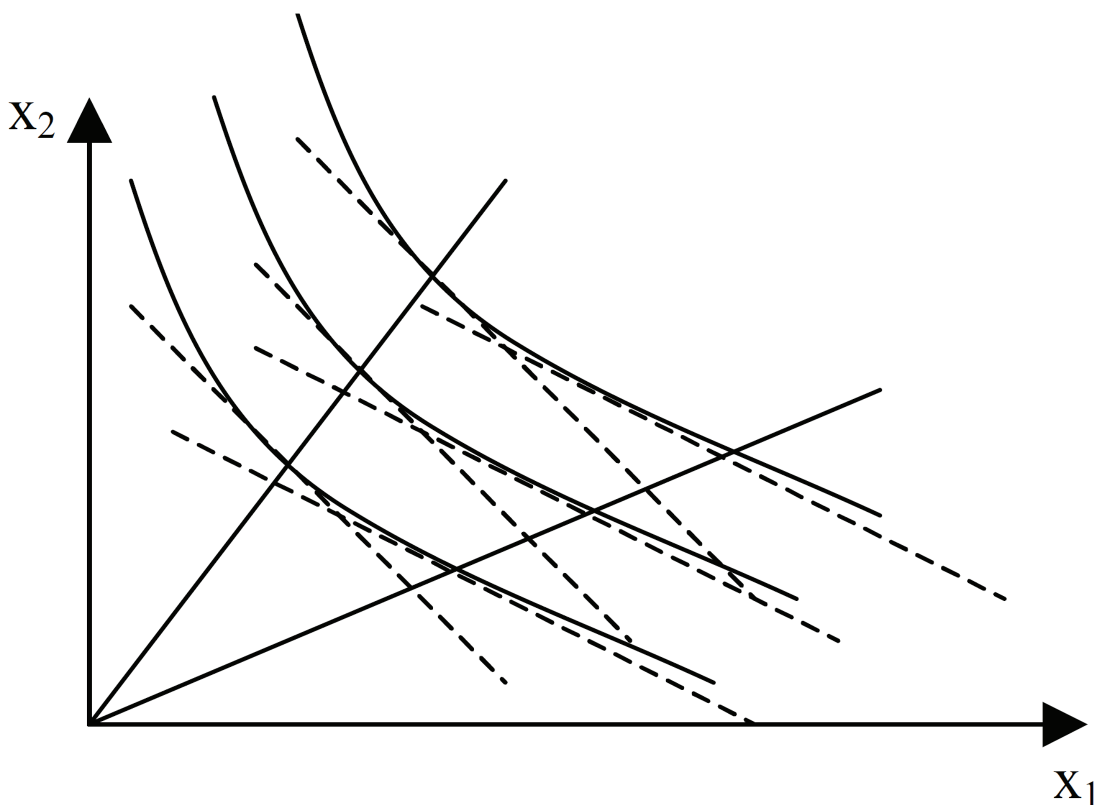
- $\varepsilon_i^I < 0$ Inferior good
- $\varepsilon_i^I \in [0, 1]$ Normal good
- $\varepsilon_i^I > 1$ Luxury good

Def 3: Elasticity of substitution: $\sigma_{ji} = \frac{d \log(x_j/x_i)}{d \log(U_i/U_j)} = \frac{d \log(x_j/x_i)}{d \log(p_i/p_j)}$
(curvature of indifference curve)

Illustrating wealth effects

- **Engel curves**
  Consumption against income
- **Wealth expansion paths**
  Optimal consumption baskets as income varies
  (holding prices constant)

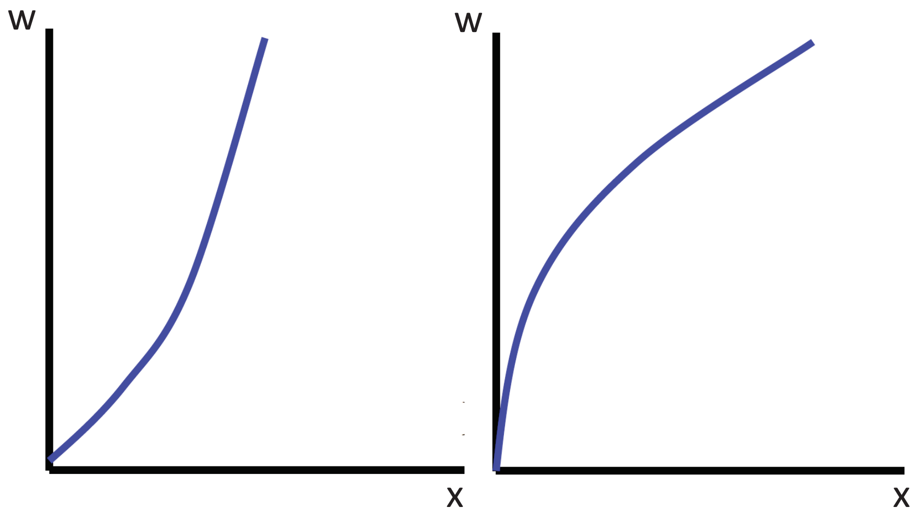
Wealth expansion paths with homothetic preferences



Engel curves

- Definition: Consumption against income

Left: income-inelastic good; Right: luxurious good



Engel aggregation

- The weighted average of income elasticities has to equal unity:

$$\frac{\sum_i p_i x_i \varepsilon_i^{Inc}}{\sum_i p_i x_i} = 1$$

There can't be only inferior goods or only luxury goods

Complementarity

- Goods i and j are "gross substitutes" if $\frac{\partial x_i}{\partial p_j} > 0$ (e.g. gas and cars), and "gross complements" otherwise (e.g. different brands of a good).

  Note: better definition ("net substitutes"): using Hicksian demand

Examples to know well (1/5):

- Cobb-Douglas: $u(x) = \sum_i \alpha_i \log x_i$ with $\sum_i \alpha_i = 1$

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- Cobb-Douglas: $u(x) = \sum_i \alpha_i \log x_i$ with $\sum_i \alpha_i = 1$
- We get: $x_i(p, w) = \frac{\alpha_i w}{p_i}$
- Elasticities:
  - Income elasticity $= 1$ for all goods
  - Own price elasticity $= 1$ for all goods, cross price elasticity $= 0$
  - Elasticity of substitution $= 1$

Examples to know well (2/5):

- Stone-Geary: $u(x) = \sum_i \alpha_i \log(x_i - \phi_i)$ with $\phi_i > 0$
- with choke price if $\phi_i < 0$

Examples to know well (2/5):

- $u(x) = \sum_i \alpha_i \log(x_i - \phi_i)$
- We get: $x_i(p, w) = \frac{\alpha_i(w - \sum_j p_j \phi_j)}{p_i} + \phi_i$ if $\frac{\alpha_i w}{p_i} > -\phi_i$
- Elasticities:
  - $w - \sum_j p_j \phi_j =$ "disposable income"
  - Income and price elasticities depend on relative size of ϕ_i's
  - Example: with $\phi_i = -\phi < 0$, only low-price commodities are consumed. Consumption of higher-price commodities positive only above a certain threshold of wealth (see problem set 2).

Examples to know well (3/5):

- Leontief: $u(x) = \min_i \{x_i/\alpha_i\}$
- Linear: $u(x) = \sum_i \alpha_i x_i$

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- Leontief: $u(x) = \min_i \{x_i/\alpha_i\}$

  $\Rightarrow \; x_i = \frac{\alpha_i w}{\sum_j \alpha_j p_j}$, and therefore $\frac{x_i}{x_j} = \frac{\alpha_i}{\alpha_j}$

- Linear: $u(x) = \sum_i \alpha_i x_i$

  $\Rightarrow \; x_i = \frac{w}{p_i}$ if $\frac{p_i}{\alpha_i} = \min_j \left\{ \frac{p_j}{\alpha_j} \right\}$, $x_i = 0$ otherwise

Examples to know well (4/5):

- CES: $u(x) = \left[\sum_i x_i^\rho\right]^{\frac{1}{\rho}}$
  Limit cases when $\rho = 0$? $\rho = 1$?

Examples to know well (4/5):

- CES: $u(x) = \left[\sum_i x_i^\rho\right]^{\frac{1}{\rho}}$
  - we get: $x_i = \left(\frac{p_i}{P}\right)^{-\sigma} \frac{w}{P}$,
    with $\sigma = \frac{1}{1-\rho}$ and price index $P = \left[\sum_i p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$
  - Income elasticity $= 1$
  - Own price elasticity $= \sigma - (\sigma - 1)s_i$ (s_i share of i in expenditures)
    Cross price elasticity $= -(\sigma - 1)s_j$ (w.r.t. price of good j)
    Own (resp. cross) elasticity equals $\approx \sigma$ (resp. 0) if market share is small
  - Elasticity of substitution $= \sigma$

Examples to know well (5/5):

- Separable and quasi-linear: $u(x) = x_0 + \sum_i u_i(x_i)$

Examples to know well (5/5):

- Separable and quasi-linear: $u(x) = x_0 + \sum_i u_i(x_i)$

  - "Numeraire" x_0: we get: $\lambda = p_0$ for the numeraire

    Practical to normalize $p_0 = 1 \Rightarrow$ Lagrange multiplier λ equals one

  - $u_i'(x_i) = p_i$ yields demand: $x_i = D_i(p_i)$ that only depends on price p_i

    In turn, consumptino of numeraire is $x_0 = w - \sum_j p_j D_j(p_j)$

  - No income effect (income elast. $= 0$)

    except for numeraire (income elast. > 1)

Continuous versions and combinations

Integrating over goods i, often indexed by $i \in [0,1]$:

- Cobb-Douglas: $u(x) = \int_i \alpha_i \log x_i \, di$ with $\int_i \alpha_i \, di = 1$
- Stone-Geary: $u(x) = \int_i \log(x_i - \phi_i) \, di$
- Linear: $u(x) = \int_i \alpha_i x_i \, di$
- CES: $u(x) = \left[\int_i x_i^\rho \, di\right]^{\frac{1}{\rho}}$
- Separable and quasi-linear: $u(x) = x_0 + \int_i u_i(x_i) \, di$

and combinations, e.g.:

- $u(x) = \sum_k \alpha_k \log \left[\int_i x_{ik}^\rho \, di\right]^{\frac{1}{\rho}}$

Other examples (1/4):

- CRIE: $u(x) = \sum_i x_i^{\frac{\sigma_i - 1}{\sigma_i}}$ (see Caron, Fally and Markusen 2014)
  Advantage: constant ratio of income elasticity $\frac{\sigma_i}{\sigma_j}$ across two goods

Pigou's law

- With separable utility $u(x) = \int_i u_i(x_i)\, di$, the price elasticity is proportional to the income elasticity:

$$\frac{\partial \log x_i}{\partial \log w} = \frac{\partial \log x_i}{\partial \log p_i} \frac{\partial \log \lambda}{\partial \log w}$$

when good i has a negligible market share. Hence:

$$\frac{\frac{\partial \log x_i}{\partial \log w}}{\frac{\partial \log x_j}{\partial \log w}} = \frac{\frac{\partial \log x_i}{\partial \log p_i}}{\frac{\partial \log x_j}{\partial \log p_j}}$$

- This is a strong restriction
  This feature is often rejected in the data (Deaton 1974)
- Implicit utility functions as above can address this issue
  see Comin, Lashkari and Mestieri (2017)

Other examples (2/4):

- Gorman implicit utility: $1 = \sum_i \left(\frac{x_i}{g_i(u)}\right)^\rho$ (Comin et al. 2017)
  Advantage: Non-separable, no link bw income and price elasticities
  (Note: one can actually have ρ depend on u, see Fally 2017)

Other examples (3/4):

- Quality w outside good: $u(x) = U(u_G(x,z), z)$ (Faber and Fally 2017) with $u_G(x,z) = \left[\int_i (\varphi_i(z) x_i)^{\rho(z)} \, di\right]^{\frac{1}{\rho(z)}}$
  Advantage: Price elasticity $\sigma(z)$ and Quality valuation $\varphi_i(z)$ of good i vary with consumption of outside good z and therefore income.

Other examples: AIDS (Deaton Mullbauer)

"Almost Ideal Demand System" (acronym chosen in the 70's)

- Assume: $\log e(p,u) = a(p) + ub(p)$
  We get market shares: $\frac{x_i}{w} = A_i(p) + B_i(p) \log w$

- Further imposing:

$$a(p) = \alpha_0 + \sum_j \alpha_j \log p_j + \frac{1}{2} \sum_j \sum_k \gamma_{jk} \log p_j \log p_k$$

$$b(p) = \beta_0 \prod_k p_k^{\beta_k}$$

$$\text{with} \quad 0 = 1 - \sum_j \alpha_j = \sum_j \beta_j = \sum_j \gamma_{jk} \;\; \textit{and} \;\; \gamma_{kj} = \gamma_{jk}$$

we get: $\frac{x_i}{w} = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(w/P)$
with $\log P \approx \sum_i \frac{x_i}{w} \log p_j$.

Demand with single aggregator

Gorman (1972), Fally (2017)

- If demand depends on own income w, own price p_i and a common price aggregator Λ, then it must take one of these two forms:

$$q_i(w, p_i, \Lambda) = D_i(F(\Lambda)p_i/w)/H(\Lambda)$$

or:

$$q_i(w, p_i, \Lambda) = G_i(\Lambda)(p_i/w)^{-\sigma(\Lambda)}$$

- Conversely, these demand systems are integrable if:
  - $\varepsilon_{Di}\varepsilon_F < \varepsilon_H$ for all p_i/w and Λ
  - $G_i(\Lambda)$ increases sufficiently fast with Λ (see Fally 2017 for conditions)

Notes on the choice of preferences mentioned earlier

- **Cobb-Douglas**: Used across broad categories of goods when we want to hold constant expenditures shares for simplicity and tractability.
- **CES**: workhorse in Macro, Trade, etc. as it is homothetic and works very well with monopolistic competition.
- **Stone-Geary**: Most simple way to get non-homotheticity, but income effects converge very quickly for higher levels of income, substitution effects are too restrictive.
- **AIDS**: Very-widely used even today. Flexible income effects and price effects, but not very tractable. An advantage to Stone Geary is that expenditure shares depend on log income. Problems with bounds (corner solutions for expenditure shares)
- **Fieler (2011)**, Ligon (2016), Caron et al (2014), etc.: Behave almost like AIDS w.r.t income (log expenditures vary with log income), but subject to Pigou's law
- **Comin et al (2016)**: Behave almost like AIDS w.r.t income (log expenditures vary with log income), simple price effects as in CES, yet avoids Pigou's curse.
- **Faber and Fally (2017):** very amenable to empirical estimation, do not impose how income affect substitution and price effects, flexible income effect through quality.
- **Kimball (1995):** Homothetic with very flexible own-price elasticities (PS1 part B).

Plan

1. Preferences and utility
2. **Aggregation**
3. Duality

2) Aggregation

- When can we express aggregate demand as a function of prices and aggregate wealth?
- Discrete-choice models

Gorman's Aggregation Theorem

- When can we express aggregate demand as a function of prices and aggregate wealth, irrespective of the distribution of wealth?

  Answer:
  When one can express indirect utility with a Gorman form:

$$v_h(p, w_h) = a_h(p) + b(p)w_h$$

- Note: Weaker restrictions can be imposed if we specify the distribution of wealth.
- Examples: quasi-linear preferences, identical Stone-Geary preferences, identical homothetic pref. (where $v_i(p, w_i) = b(p)w_i$)

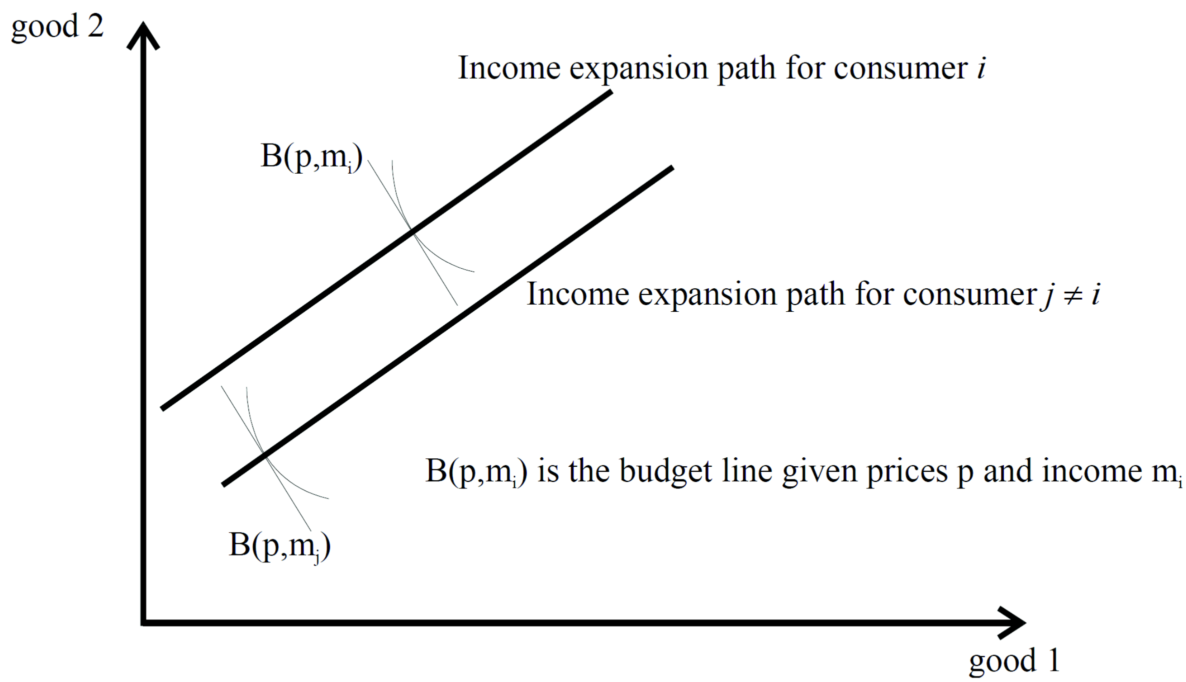
Implication for wealth expansion paths

- With Gorman form, Marshallian demand is linear in wealth
  This can be shown easily using Roy's identity:

$$x_i^h = -\frac{\frac{\partial v^h(p,w_h)}{\partial p_i}}{\frac{\partial v^h(p,w_h)}{\partial w_h}} = -\frac{1}{b(p)} \cdot \frac{\partial v^h(p, w_h)}{\partial p_i}$$

- This implies linear wealth expansion paths with Gorman
- linear Engel curves
- and expenditure functions linear in u
- Gorman form: sometimes called "quasi-homothetic"

Wealth expansion paths with Gorman preferences:



Discrete-choice models

or "Random Utility Models", pioneered by McFadden

- Each consumer only buys one good (within a category)
  Focus on a specific industry and often assume quasi-linear pref.
- Individuals may differ in their taste for attributes of goods
  and have idiosyncratic taste shock for each good
- Typically, indirect utility of consumer z with choice i is:

$$U_{zi} = \alpha_z(w_z - p_i) + \phi_z(Z_i) + \epsilon_{zi}$$

  hence income effects drop out (quasi-linear preferences)
- See Berry, Levinsohn and Pakes (1995), aka BLP, for estimation
  Core topic in IO and Environmental Econ courses

Discrete-choice models

- We can also mix discrete choice (each consumer buys one brand) with continuous quantities (how much it buys from that brand).
- Discrete but continuous quantity choice with type-II extreme value distribution for ε_{zi} leads exactly to CES on aggregate

$$U_z = \max_{i \in \Omega,\, q_{zi}} [\log q_{zi} + \log \varphi_i + \mu \epsilon_{zi}]$$

... equivalent to:

$$U = \log \left[\sum_{i \in \Omega} (q_i \log \varphi_i)^{\frac{\sigma-1}{\sigma}} \right]$$

after aggregating across individuals (Anderson, de Palma, Thisse 1987) with elasticity of substitution $\sigma = 1 + \frac{1}{\mu}$

Plan

1. Preferences and utility
2. Aggregation
3. **Duality**

3) Price effects and duality:

- Why do we need other tools?
  Problems with indirect utility and Marshallian demand
- Definitions:
  - Dual problem
  - Hicksian Demand
  - Expenditure function
- Properties:
  - Shephard's lemma
  - Slutsky equation
- Application: Are there Giffen Goods?
  Jensen and Miller (2008)

Indirect utility

Indirect utility: Utility associated with the chosen bundle $x(p, w)$:

$$V(p, w) = U(x(p, w)) = \max U(x) \quad \text{such that } p.x \leq w$$

Issues with V?

- We can redefine utility up to any increasing function $f(U(x))$ which would yield $f(V(p, w))$ for indirect utility.

$\Rightarrow$ Then how to interpret V if any other $f(V(x))$ would also work?

- How to compare individuals?
- How to put a dollar value on V?

Price effects with Marshallian demand

Price effect: Why is the sign of $\frac{\partial x_i}{\partial p_i}$ not always positive?
We were told that a demand curve is downward slopping...

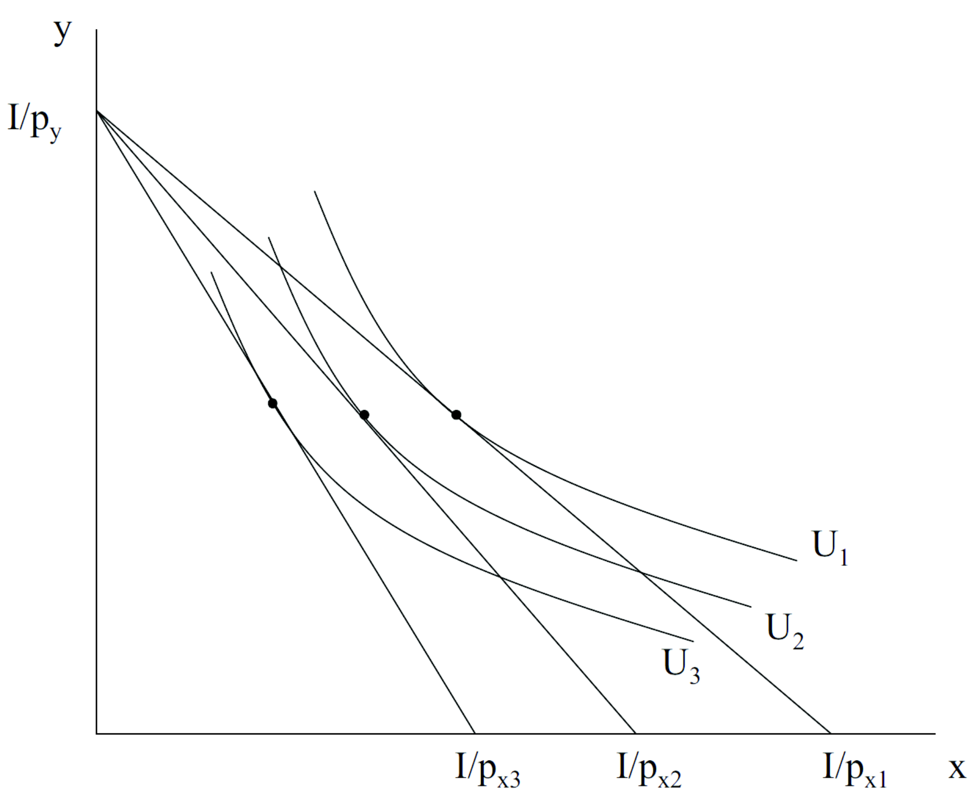
This price effect depends on various things:

- Curvature of indifference curve
- Difference between indifference curves at different utility levels

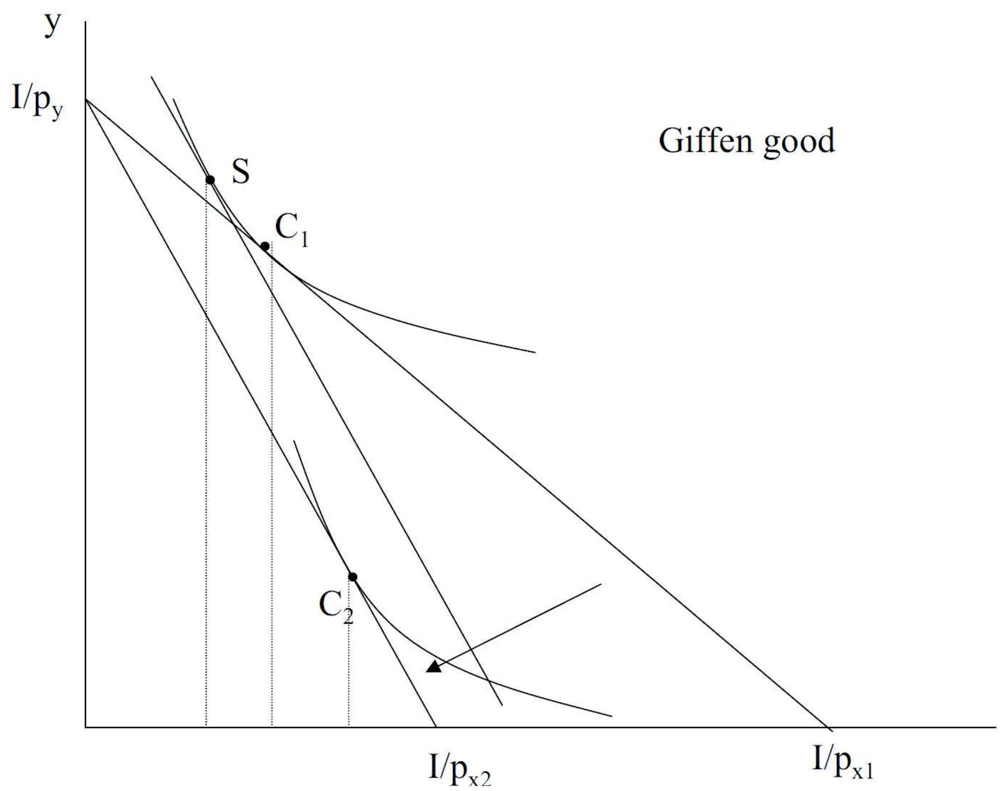
$\Rightarrow$ Not so easy to illustrate / understand

Note: In comparison, wealth effects $\frac{\partial x_i}{\partial w}$ are easy to understand with Marshallian demand: budget set shifts in or out by preserving relative prices and marginal rate of substitution $\frac{\partial U}{\partial x_1} / \frac{\partial U}{\partial x_2}$.

Price effect with Marshallian demand:



Example of positive price effect:



New tools needed

It is easier to capture movements along indifference curves
i.e. holding Utility fixed

This leads to a set of new tools:

- **Expenditure function** $e(p, u)$: wealth required to get utility u with prices p (Note: e is concave in p)
- **Hicksian demand** $h_i(p, u)$: demand for good i as a function of utility u and prices p

$$h_i(p, u) = x_i(p, e(p, u))$$

  also called "compensated demand function"

For the story, Marshall was the first one to draw demand demand and supply curves. Hicks was the first one to carefully examine price effects.

Dual problems

- Utility maximization problem

$$\max U(x)$$
$$\text{such that: } p.x = w$$

leads to Marshallian demand $x(p,w)$ and indirect utility $v(p,w)$

- Expenditures minimization problem

$$\min p.x$$
$$\text{such that: } U(x) = u$$

leads to Hicksian demand $h(p,u)$ and expenditure fctn $e(p,u)$

Understanding welfare and price effects

- Using expenditure function to examine welfare: "Lecture notes 04" (compensating variations and equivalent variations)

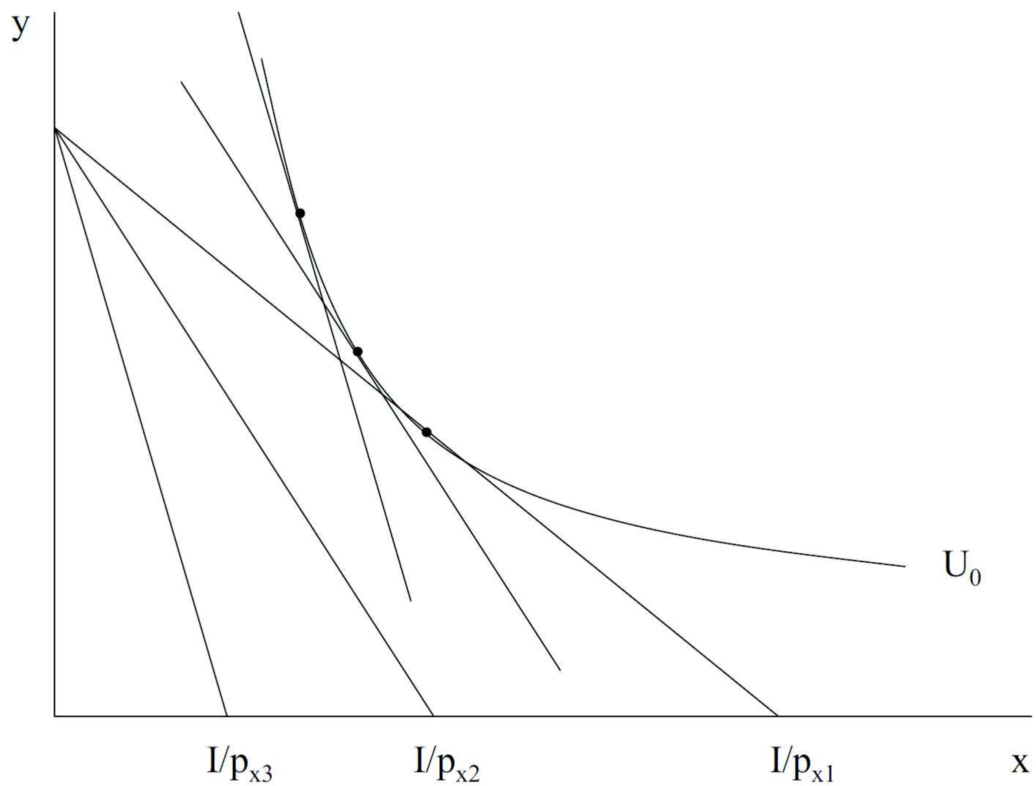
- Today: focus on the price effect

First property:
The price effect with the Hicksian demand is always negative:

$$\frac{\partial h_i(p,u)}{\partial p_i} < 0$$

as long as preferences are convex (i.e. utility quasi-concave)

Price effect with Hicksian demand:



Other properties

- Lagrange multiplier in EMP inversely related to Lagrange multiplier in UMP:

$$\lambda^{EMP} = \frac{p_i}{\frac{\partial U}{\partial x_i}} = \frac{1}{\lambda^{UMP}}$$

- Note also that:

$$\lambda^{EMP} = \frac{\partial e(p,u)}{\partial u}$$

- Moreover, the envelop theorem then gives:

$$\frac{\partial e(p,u)}{\partial p_i} = h_i(p,u)$$

  This is called **Shephard's Lemma**

Roy's Identity

Equivalent of Shephard's Lemma for Marshallian demand?

- Exercise: Show that:

$$-\frac{\frac{\partial V(p,w)}{\partial p_i}}{\frac{\partial V(p,w)}{\partial w}} = -\frac{-\lambda^{UMP} . x_i(p,w)}{\lambda^{UMP}} = x_i(p,w)$$

- Applications: sometimes it is practical to specify indirect utility rather than demand and preferences. Roy's identity then yields demand.
- Example: Addilog:

$$V(p,w) = \int_i v(p_i/w)di$$

CES is a special case with iso-elastic $v(.)$, other cases are non-homothetic

Price effect

Linking Hicksian and Marshallian demand

- Recall that $h_i(p,u) = x_i(p, e(p,u))$. Differentiating, we get:

$$\frac{\partial h_i(p,u)}{\partial p_j} = \frac{\partial x_i(p,w)}{\partial p_j} + \frac{\partial e(p,u)}{\partial p_j} . \frac{\partial x_i(p,w)}{\partial w}$$

Rearranging: $\frac{\partial x_i(p,w)}{\partial p_j} = \frac{\partial h_i(p,u)}{\partial p_j} - \frac{\partial e(p,u)}{\partial p_j} . \frac{\partial x_i(p,w)}{\partial w}$

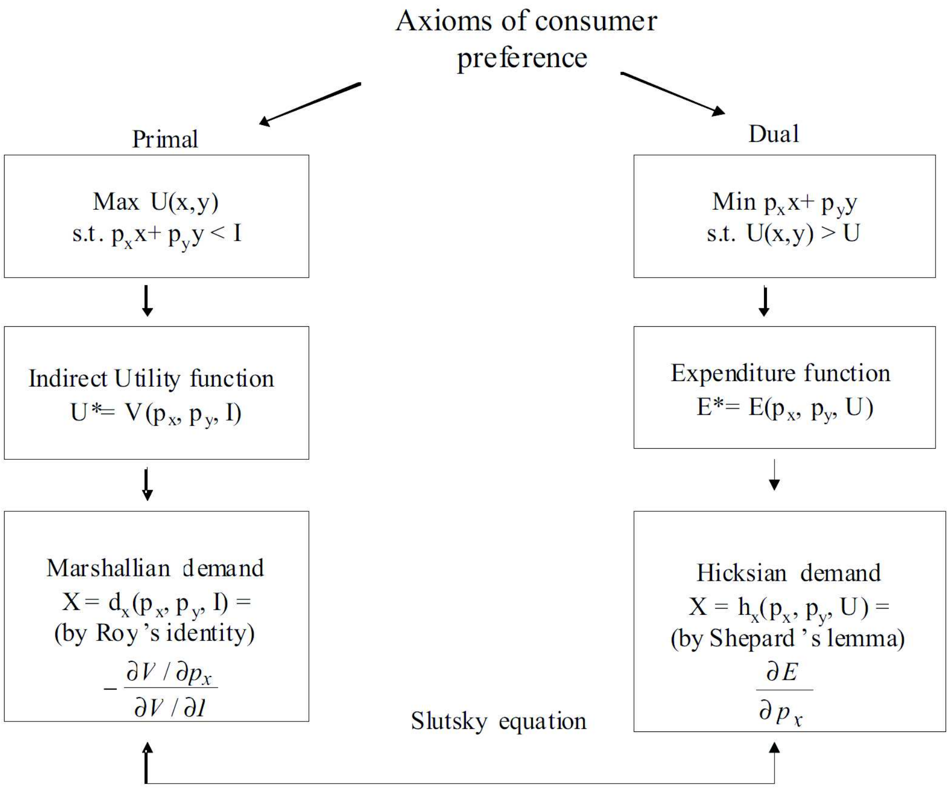
- Using Shephard's Lemma, we obtain **Slutsky Equation**:

$$\frac{\partial x_i(p,w)}{\partial p_j} = \frac{\partial h_i(p,u)}{\partial p_j} - h_j . \frac{\partial x_i(p,w)}{\partial w}$$

Price effect = Substitution - Income effect

- In elasticities: $\varepsilon_{ij}^{Marshall} = \varepsilon_{ij}^{Hicks} - s_j . \varepsilon_{iw}^{Marshall}$

Summing up:



Price effects: three cases

Let's come back to different types of goods depending on their wealth effects:

- Normal goods: Income effect reinforces the substitution effect

$$\frac{\partial x_i(p,w)}{\partial p_i} = \frac{\partial h_i(p,u)}{\partial p_i} - h_i \,.\, \frac{\partial x_i(p,w)}{\partial w} < \frac{\partial h_i(p,u)}{\partial p_i} < 0$$

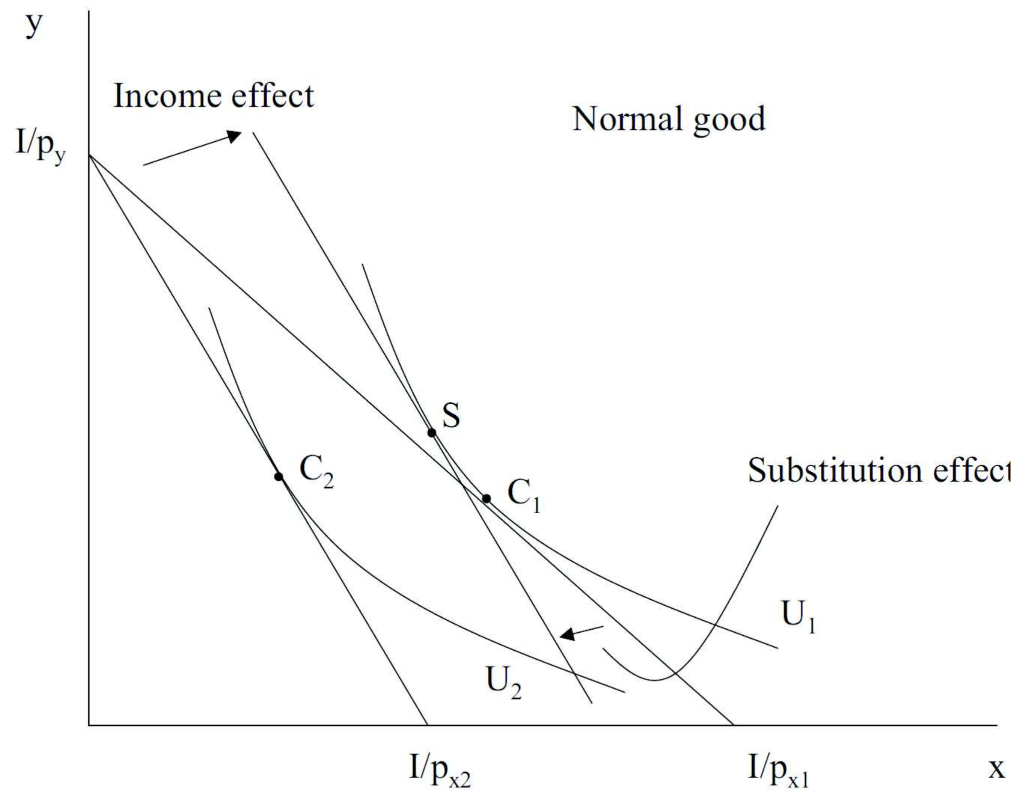
- Inferior goods: Income effect mitigate substitution effect

$$\frac{\partial x_i(p,w)}{\partial p_i} = \frac{\partial h_i(p,u)}{\partial p_i} - h_i \,.\, \frac{\partial x_i(p,w)}{\partial w} > \frac{\partial h_i(p,u)}{\partial p_i} \quad (\text{but still} < 0)$$

- Giffen goods: Income effect dominates the substitution effect

$$\frac{\partial x_i(p,w)}{\partial p_i} = \frac{\partial h_i(p,u)}{\partial p_i} - h_i \,.\, \frac{\partial x_i(p,w)}{\partial w} > 0$$

Overall price effect:



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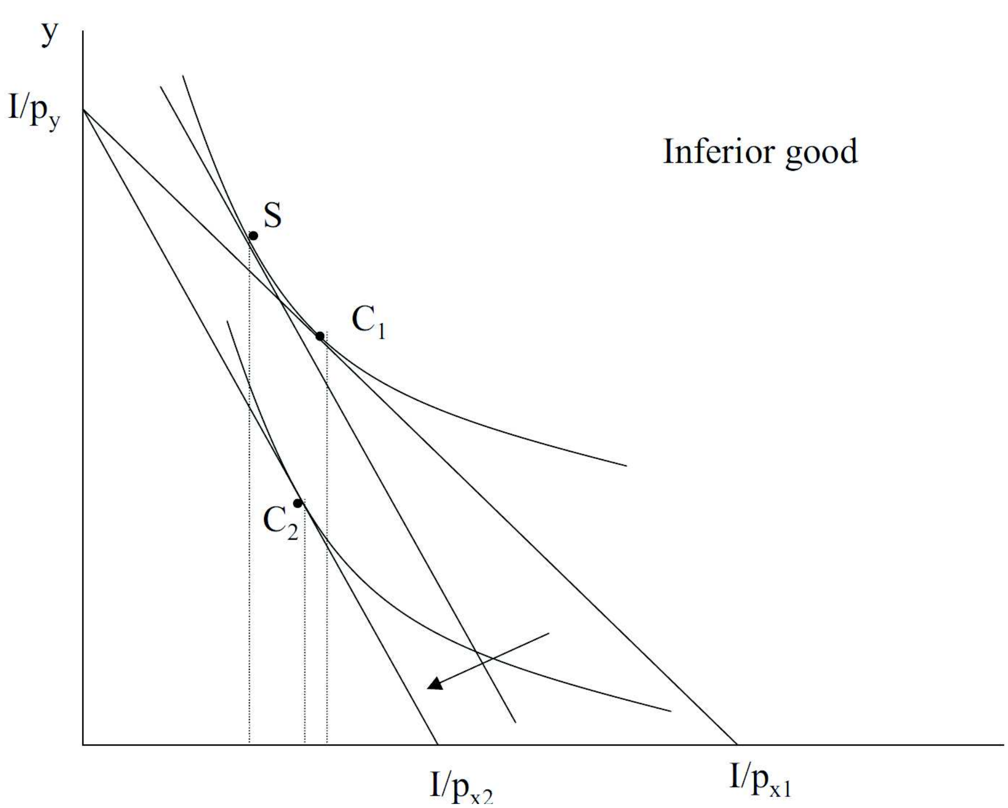

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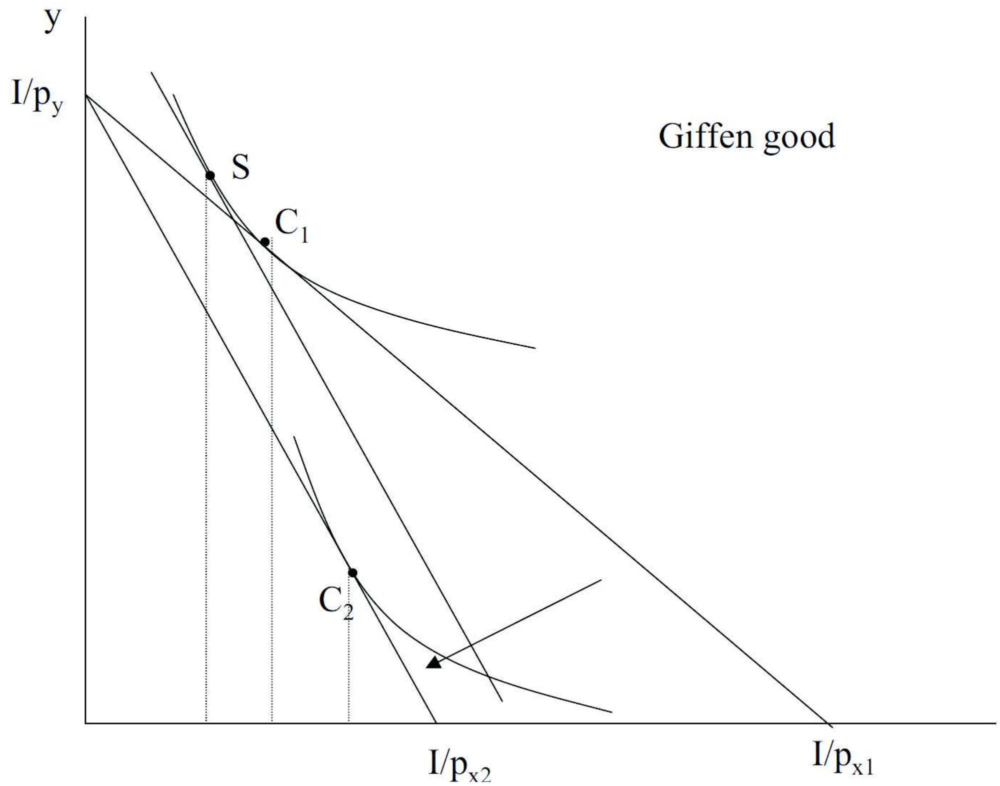
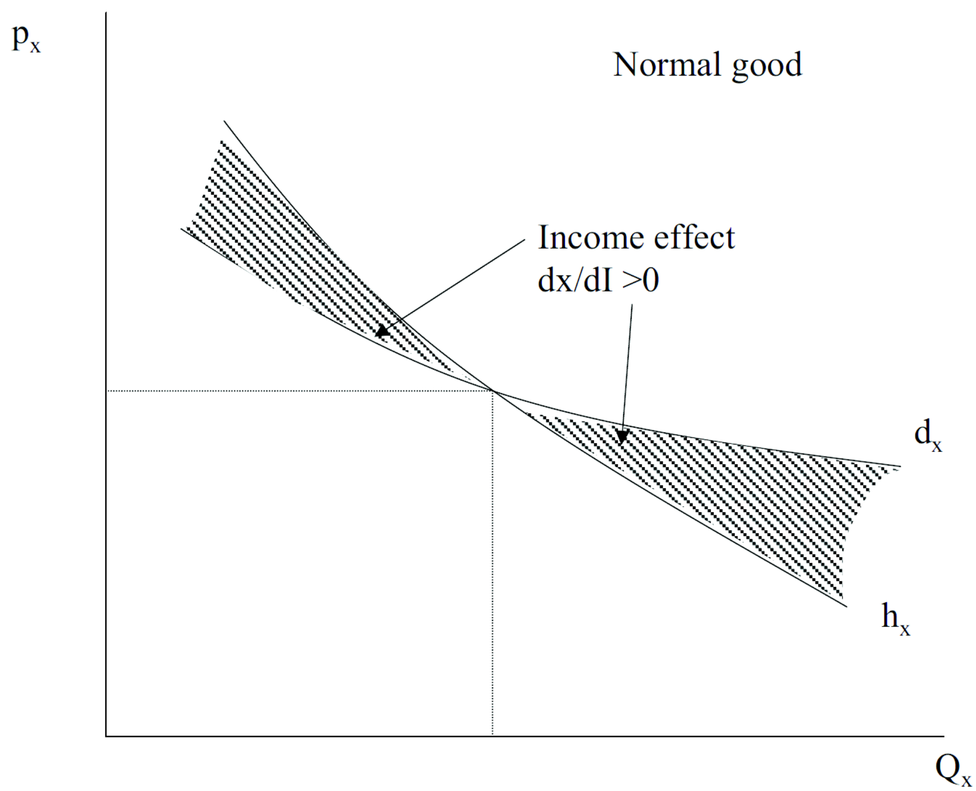
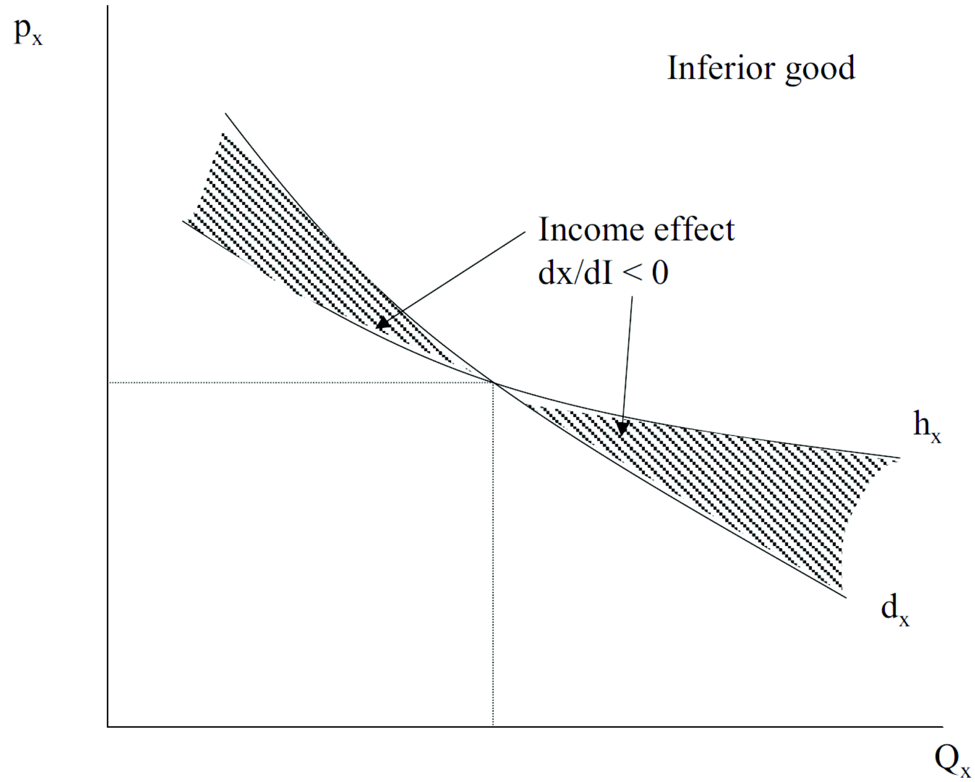


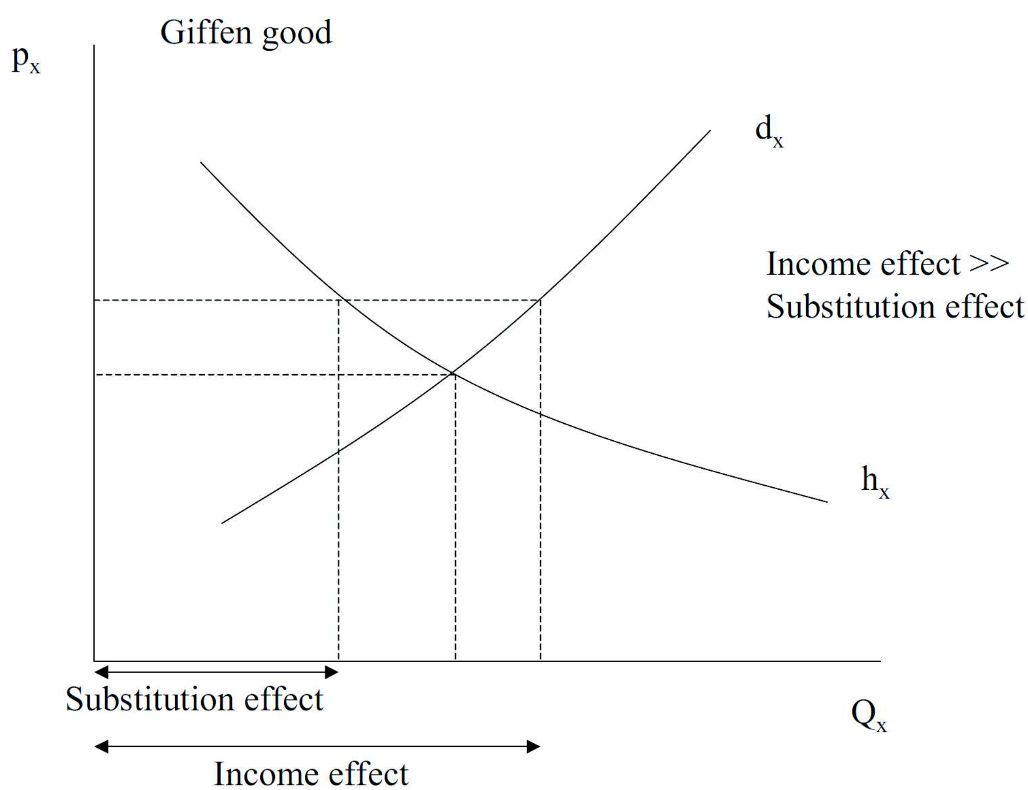
Slope of demand



Slope of demand



Slope of demand



Examples

[see blackboard: expenditure function, Hicksian demand]

Using the utility functions examined previously:

- Leontief and linear: $u(x) = \min_i \alpha_i x_i$ and $u(x) = \sum_i \alpha_i x_i$
- Cobb-Douglas: $u(x) = \sum_i \alpha_i \log x_i$ with $\sum_i \alpha_i = 1$
- Stone-Geary: $u(x) = \sum_i \log(x_i - \phi_i)$
- CES: $u(x) = \left[\sum_i x_i^\rho\right]^{\frac{1}{\rho}}$

Examples

Examples

⇒ Expenditure functions:

- Leontief: $e(u,p) = u.\sum_i \alpha_i p_i$
- Linear: $e(u,p) = u.\min_i\{p_i/\alpha_i\}$
- Cobb-Douglas: $e(u,p) = u.\prod_i \left(\frac{p_i}{\alpha_i}\right)^{\alpha_i}$
- Stone-Geary: $e(u,p) = \sum_i p_i\phi_i \; + \; u.\sum_i \prod_i \left(\frac{p_i}{\alpha_i}\right)^{\alpha_i}$
- CES: $e(u,p) = u.\left[\sum_i p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$

Giffen good: theoretical artifact?

Giffen behavior would require:

1. Very negative income elasticity:
   Staple for the Poor, substituted by other products by the Rich
2. Large consumption by the poor: the income effect in Slutsky equation is larger for larger consumption shares.
3. Low substitution $\frac{\partial h_i(p,u)}{\partial p_i}$ with other staples

Potatoes during the Great Irish Famine? (1845-52)

Jensen and Miller (AER 2008)

First study to carefully show evidence of Giffen behavior

Population:

- about 1,300 households living with less than \$1/day in Hunan and Gansu provinces in 2005

Food items:

- Hunan: rice; Gansu: wheat

Identification issues:

- Demand shocks usually lead to positive correlations between prices and quantities when prices respond to changes in demand – this is not the proof that there are Giffen goods

Experimental setting:

- Randomly give lower prices (rice or noodles) to some households during 5 months (discounts worth about 10-25%)

Jensen and Miller (AER 2008)

They argue that they need the following conditions:

C1 Households are poor enough to face subsistence nutrition concerns

C2 Simple diet, including a basic and a fancy food

C3 a) This basic food constitutes a large part of the diet (e.g. rice)
b) Basic food is cheapest source of calories and has no ready substitute

C4 Households are not too poor either: they do not only consume the basic good

Indifference curves

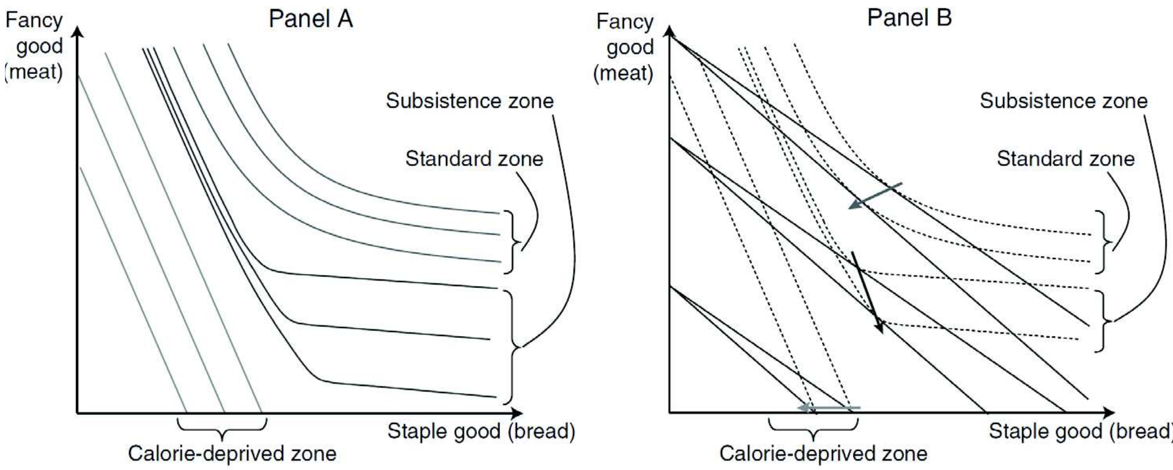


FIGURE 1. ZONES OF CONSUMER PREFERENCES

TABLE 2—DAILY CONSUMPTION PER CAPITA AND CALORIE SHARES FOR FOOD CATEGORIES

	Hunan		*Gansu*	
	Consumption (g)	Calorie share	Consumption (g)	Calorie share
Rice	330	0.64	35	0.07
	[125.4]	[0.17]	[69.5]	[0.13]
Wheat	42	0.08	344	0.69
	[60.2]	[0.12]	[134.3]	[0.17]
Other cereals	1.5	0.00	4.2	0.01
	[21.3]	[0.022]	[24.2]	[0.050]
Vegetables and fruit	341	0.05	232	0.07
	[194.6]	[0.044]	[141.6]	[0.045]
Meat (including eggs)	47	0.07	13	0.01
	[68.6]	[0.11]	[30.1]	[0.037]
Pulses	62	0.02	36	0.02
	[102.3]	[0.043]	[68.1]	[0.056]
Dairy	1	0.00	19	0.01
	[7.4]	[0.0031]	[56.6]	[0.039]
Fats	26	0.13	23	0.13
	[20.4]	[0.095]	[16.3]	[0.090]
Calories	1,805	—	1,710	—
	[591.7]		[517.4]	
Observations	644	644	649	649

Notes: Standard deviations in brackets. All consumption figures are in grams per capita. Calorie share is the percent of total calories attributable to the particular food category.

$$\%\Delta staple_{i,t} = \alpha + \beta\%\Delta p_{i,t} + \sum\gamma\%\Delta Z_{i,t} + \sum\delta County{*}Time_{i,t} + \Delta\varepsilon_{i,t}$$

TABLE 3—CONSUMPTION RESPONSE TO THE PRICE SUBSIDY: HUNAN

	Dependent variable: Rice							*Dependent variable: Meat*	
	Full sample (1)	Full sample (2)	ISCS ≤0.80 (3)	ISCS ≤0.80 (4)	ISCS >0.80 (5)	ISCS >0.80 (6)	ISCS 0.60–0.80 (7)	Full sample (8)	Initial intake >50g (9)
%ΔPrice(rice)	0.224	0.235*	0.451***	0.466***	−0.61**	−0.585**	0.640***	−0.325	−1.125*
	(0.149)	(0.140)	(0.170)	(0.159)	(0.296)	(0.262)	(0.192)	(0.472)	(0.625)
%Δ Earned		0.043***		0.047***		0.024	0.030	0.028	0.105
		(0.014)		(0.016)		(0.023)	(0.019)	(0.050)	(0.069)
%ΔUnearned		−0.044*		−0.038		−0.058	−0.053*	0.061	0.084
		(0.025)		(0.030)		(0.049)	(0.030)	(0.079)	(0.104)
%ΔPeople		0.89***		0.83***		1.16***	0.79***	−0.08	0.03
		(0.08)		(0.09)		(0.15)	(0.14)	(0.27)	(0.36)
Constant		4.1***		5.7***		−1.8	0.8	−12.3***	−49.0***
		(1.0)		(1.1)		(1.7)	(1.3)	(3.1)	(3.7)
Observations	1,258	1,258	997	997	261	261	513	997	452
R^2	0.08	0.19	0.09	0.20	0.15	0.33	0.24	0.09	0.28

Notes: Regressions include *County*Time* fixed effects. The dependent variable in columns 1–7 is the arc percent change in household rice consumption, and in columns 8–9 it is the arc percent change in household meat consumption. Standard errors clustered at the household level. %ΔPrice(rice) is the change in the subsidy, measured as a percentage of the average price of rice; %ΔEarned is the arc percent change in the household earnings from work; %ΔUnearned is the arc percent change in the household income from unearned sources (government payments, pensions, remittances, rent, and interest from assets); %ΔPeople is the arc percent change in the number of people living in the household. ISCS (Initial Staple Calorie Share) refers to the share of calories consumed as rice in the preintervention period. *Significant at 10 percent level. **Significant at 5 percent level. ***Significant at 1 percent level.

Staple price elasticities across households

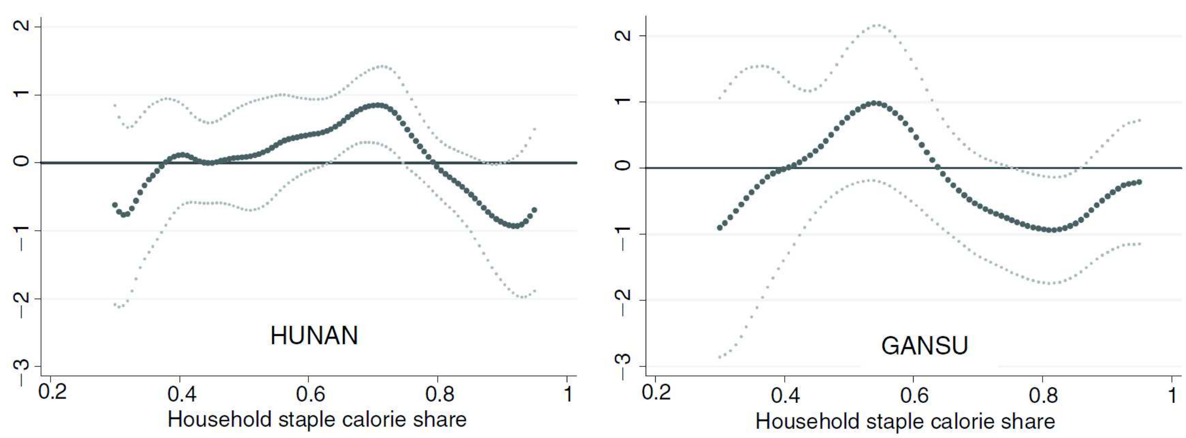


FIGURE 2. COEFFICIENT PLOTS

Jensen and Miller (AER 2008)

Conclusions

- Finally an example of Giffen good to provide in a micro class!!
  (other than potatoes during the great famine)

  Applies to most common goods in the most populated country

- Great identification strategy (not my role to discuss it here)

- Great use of micro-theory

$\Rightarrow$ Giffen behavior seems to happen where theory would predict:
Households that are poor but not starving, consuming a specific staple good as main source of calories

- I wish could more precisely disentangle price from wealth effects by combining price discount with random cash transfers.

Exercise

Build a simple example of utility with two goods (e.g. rice and meat) that generates Giffen goods as in Jensen and Miller (2008)?
[Get inspiration from criteria C1 to C4]

Hamilton Method

Can we retrieve real income from consumption patterns? (Hamilton 2001)

- Data: nominal income, approximation of relative prices
- Goal: estimate an inflation bias μ_t common to all goods k.
- Nakamura et al (2016) specify consumption shares as:

$$\omega_{i,t}^k = \psi_i^k + \beta_k \log(C_{i,t}/P_{i,t}) + \gamma_k \log(P_{i,t}^k/P_{i,t}) + \sum_x \Theta_x^k X_{i,t} + \epsilon_{i,t}$$

where $C_{i,t}$ denotes nominal expenditures at time t for individual(s) i.

- Issues with this approach?

Missing inflation?

Each obs = income group / year