Optimal share contracts with moral hazard on effort and in output reporting: Managing the double Laffer curve effect

By Alain de Janvry* and Elisabeth Sadoulet+

* Department of Agricultural and Resource Economics, University of California at Berkeley
e-mail: alain@are.berkeley.edu
* Department of Agricultural and Resource Economics, University of California at Berkeley
e-mail: Sadoulet@are.berkeley.edu

We explore in this paper the design of optimal share contracts when there is a double moral hazard, one on inputs exclusively provided by the agent (such as effort) and the other in reporting the level of output to be shared with the principal, and when there is a social efficiency cost to under-reporting. The optimal contract is second best in that it allows for residual moral hazard in both effort and output reporting. The model predicts that contract terms will vary with the value to the tenant of unreported output as well as with any capacity of the principal to directly supervise the agent. The model is written for a landlord-tenant share contract but applies as well for tax collection and franchising.

JEL classification: D02, D82, L14.

Address of corresponding author: Alain de Janvry, Giannini Hall 207, University of California, Berkeley, California 94720, USA. Email: alain@are.berkeley.edu.
1. Introduction: The contract fulfillment problem

Due to information asymmetries, opportunistic behavior is always a looming risk in transactions that involve an element of time as at least one party to the exchange can take advantage of this situation to cheat and deceive the other. Contracts, and their enforcement, attempt at overcoming this problem (North, 1990). Failure to fulfill contracts can prevent a transaction from happening altogether, limit the transaction to a spot exchange in a “flea market”-type economy (Fafchamps and Minten, 2001), or confine the transaction to segregated circles of confidence composed of family, friends, and clan members where partners share information about each others (Grief, 1994). Contract failures can thus result in huge inefficiencies, missed opportunities, and segmented markets. It is consequently not surprising that analyzing solutions to the contract fulfillment problem has been one of the most central themes to economics.

In this paper, we consider the situation where lack of third party verifiability of tenant behavior and one-time transactions require reliance on self-enforcing incentive contracts to solve the contract fulfillment problem. We consider the case where a share contract must be chosen because the tenant’s aversion to risk prevents use of a fixed rent contract. There are two dimensions of tenant behavior that cannot be contracted. One is the tenant’s level of effort, as in classical Marshallian theory. The other is output reporting before sharing is done. As Allen and Lueck (1992) observe in share contracting in Nebraska and South Dakota, “the farmer has an incentive to under-report the harvest to the landowner. Under-reporting may take the form of crop quality as well as quantity”. While effort and output reporting cannot be contracted, there is near full information in village economies on individual farming skills (Lanjouw, 1999), labor productivity (Foster and Rosenzweig, 1993), and honesty in fulfilling contracts (Hayami and Kawagoe, 1993). A landlord can thus be expected to know the tenant’s response functions for both effort and output reporting to the terms of the contact, even if they are not contractible because they are not third party verifiable. We consider the case where under-reporting has a social efficiency cost due to existence of transactions costs in disposing of unreported goods. The terms of the contract are thus set by the landlord to maximize his expected income subject to these two response functions. This corresponds to a double Laffer curve effect where the tax rate affects both the tax base (output) and the incentive to under-report on output. We then discuss how the contract is modified when some elements of coercion and cooperation are possible in seeking contract fulfillment.
In section 2, we review the literature on contract fulfillment in order to classify solutions to the problem and place our approach relative to these. In section 3, we provide factual evidence on the building blocks needed to construct an optimal self-enforcing incentive share contract when there is moral hazard on both effort and output reporting. The model is built in section 4 for the case of farming, but it has immediate analogies in other fields such as tax collection and franchising. We then consider in section 5 possible efficiency gains from non-linear contracts and in section 6 modifications of the incentive contract when some level of coercion can be exercised on the tenant and when cooperative response can be expected in seeking a more efficient solution to the contract fulfillment problem. Section 7 summarizes and concludes.

2. Solutions to the contract fulfillment problem

Contract fulfillment requires putting into place mechanisms that create incentives or obligations for the agent to provide the maximum possible gain to the principal. For the latter, there are basically three categories of solutions to this problem: design self-enforcing incentive contracts, rely on coercive measures to enforce contracts, or combine incentives and coercion when contracts are repeated over time.

The first solution to the contract fulfillment problem consists of defining a self-enforcing incentive contract. Relying on this solution for contract fulfillment is necessary when there is no third party verifiability or when verification is too expensive, and hence when a legal system or another punishment mechanism cannot be invoked for contract enforcement. When the contract is a one-time exchange, and there are aspects of behavior that are not contractible, then the terms of the contract seek to induce second best behavior. The second best contract chosen is the most efficient for the principal among those acceptable to the agent. A share contract in farming is typically of this type. The landlord cannot enforce effort, truthful reporting on the level of output reached, and possibly other aspects of behavior such as respect of the quality of the asset lent to the tenant. The contract is thus incomplete. Its use is motivated by the fact that alternative contracts would lead to even lower efficiency outcomes, perhaps because they do not allow risk reduction for the tenant (Newbery and Stiglitz, 1979), or access to non-marketed inputs (Eswaran and Kotwal, 1985), or relaxation of a liquidity constraint for the tenant (Laffont and Matoussi, 1995).

The second solution to the contract fulfillment problem is coercive enforcement using the legal system. This requires that contract violations be third party verifiable. Following Becker’s
(1968) work on crime and punishment, the design of optimal policies to enforce social contracts (i.e., to combat illegal behavior in Becker’s case) depends on the cost of catching and convicting offenders, the nature of punishment including fines and imprisonment, and the behavioral response of offenders to changes in enforcement. Because the enforcement effort is costly, optimum implementation of the law should imply imposing maximally large fines to save on enforcement costs (Polinsky and Shavell, 2000). If the level of fines is constrained by considerations of fairness, low sanctions must be met by higher enforcement expenditures to raise the probability of detection. For individuals with low assets who are unable to pay high fines, imprisonment becomes a costly alternative punishment to fines. Comparison of theory to practice shows, however, two interesting regularities. One is that penalties are lower than they could be, leading to socially wasteful high enforcement costs or to less public enforcement of the law than is socially optimum (Fehr and Gächter, 2000; Feld, 2003). The other is that the coercive enforcement of contracts, for instance of tax payments by individuals, is insufficient to explain observed behavior, suggesting that something else than formal coercion must be at play in explaining contract fulfillment.

In certain cases, incentives and coercion can be optimally combined, providing a third solution to the contract fulfillment problem. If contracts are repeated over time, “punishment” can derive from interruption of the contract, with the result that the exchange will be sustained for as long as the short run gain from cheating is inferior to the long run gain from contract fulfillment. In the efficiency wage theory (Akerlof, 1984; Shapiro and Stiglitz, 1984), the probability of being caught cheating leads to the threat of retaliation under the form of termination of the exchange. In this case, there is a trade-off between a higher wage reward above the market opportunity cost and the intensity of monitoring to detect cheating. A firm will consequently choose the combination of wage premium and monitoring cost that maximizes effective labor per unit of cost, the efficiency wage hypothesis. A landlord can similarly sustain an efficient long-term share contract with reciprocal gift exchange, even when the contract is not enforceable (Radner, 1981; Sadoulet, Fukui, and de Janvry, 1994).

Repeated transactions also allow the fulfillment of contracts when non-compliance leads to the threat of loss of reputation in a segregated social group that shares information on members’ behavior. If every agent in the group acts honestly and punishes any agent who has cheated a member of the group in the past, a multilateral reputation mechanism becomes established that leads to an “honesty equilibrium” (Platteau, 2000). Grief (1994) shows how
contracts can be enforced informally in collectivist segregated societies where personalized reputation and trust govern exchanges within a religious, ethnic, or kinship group that imposes collective punishment if there is disclosure of opportunistic behavior. Generalized morality is achieved when trust extends to the whole society, allowing to sustain anonymous market exchange with little risk of non-fulfillment of contracts (Platteau, 2000). Trust in government and feelings of fairness and reciprocity can thus be invoked in explaining high rates of compliance in tax payment in spite of low threats of punishment (Fehr and Gächter, 2000). In this case, greater coercion in enforcing payments can “crowd out” tax morale, and lead to lower aggregate tax collection (Frey, 2003; Feld 2003). We explore in this paper how considerations of fairness and reciprocity can affect the specification of share contracts.

3. Building blocks for a model of share contract with under-reporting

There are seven building blocks that we use in constructing a model of optimal share contracting with moral hazard in effort and output reporting. For each, we refer to empirical evidence in support of the assumptions made. Because the tax payment problem is similar to the share contract problem, some of our empirical evidence in support of the building blocks is derived from that literature.

Observation 1. There are multiple sources of moral hazard in share contracts that are not fully contractible

Share contracts are motivated by the fact that incentives need to be provided when some of the aspects of tenant behavior cannot be contracted. In all cases, the tenant’s reaction functions to the terms of the contract for these non-contractible aspects of behavior are known to the landlord. Sources of moral hazard include:

1) Shirking on labor effort: this is the most fundamental source of moral hazard, leading to the well-known Marshallian inefficiency in sharecropping (Marshall, 1956).

2) Shirking in the provision of other non-shared inputs, such as management and supervision (Eswaran and Kotwal, 1985). In this case, when each of these inputs is exclusively provided by one party to the contract, the optimal share contract results in a double moral hazard model.
3) Under-reporting on output or gross revenue before sharing. This is discussed further below, and has been introduced in contract choice by Allen and Lueck (1992) and by Datta, O’Hara, and Nugent (1986).

4) Over-invoicing on the cost of shared purchased inputs. Input cost sharing allows to reduce moral hazard in provision, but leaves open the possibility of “creative accounting” on costs before net revenue sharing is done. This is a widely denounced practice in revenue sharing with actors by movie companies (Prendergast, 1999).

5) Deviation of use of shared purchased inputs toward one’s own plot when a tenant cultivates both his own land and a share rented plot. This practice has been exposed by Arcand and Rambonilaza (2001) in their work on Tunisia.

6) Land mismanagement with depletion of the quality of the asset provided by the landlord. Concern with this source of abuse affects the choice of contract (Allen and Lueck, 1992) as well as the design of the optimum share contract (Dubois, 2002).

In this paper, we consider two sources of moral hazard: one is on effort, which creates a deadweight loss for the two partners in the classical Mashallian fashion; the other is on output reporting. If under-reporting leads to a loss of value for the two partners because of the way the tenant disposes of the unreported product, it is also a source of inefficiency.

These sources of moral hazard can affect the contract choice between fixed rent, share rent, and a wage contract; the contract design in sharecropping, affecting both the tenant’s share \( \alpha \) and the fixed rent \( \beta \) he pays; and efforts to reduce moral hazard directly through coercion (monitoring and sanctions) or calls on cooperation.

**Observation 2. There is extensive under-reporting of product or revenue in share contracts**

In any share contract where the level of output or revenue to be shared is difficult to monitor, under-reporting can be expected. This has been extensively described in sharecropping, tax payment, and franchising. Allen and Lueck (1992) thus observe that “a crop share contract is more likely to occur when the costs of dividing the crop are relatively low”. As a consequence, there is more share contracting for cash crops that are delivered to elevators than for hay crops that are sold directly by the tenant. Bouquet and Colin (1999) describe how, in Mexican ejidos where access to new potato varieties is obtained through sharecropping with individuals who have access to the new seeds and knowledge of the marketing channels for the new crop, under-reporting by the tenant of the value of the sale is widely practiced. In this contract, no
enforcement mechanism exists as use of the venal court system is not even considered and reputation mechanisms with one-time tenants who are not community members do not apply. In spite of this, sharecropping is widely practiced as it provides landlords access to the new technology, seeds, and markets, at a high cost due to opportunistic behavior by the tenant. Finally, Salih (1993) reports how farmers in the Sudan from the 1950s through the 1980s had to deliver to the government a set share of their reported wheat production at a price below the free market price. This created an incentive to under-report production. Aggregate under-reporting was estimated to have reached up to 12% of harvest according to years.

The tax evasion literature deals extensively with the problem of under-reporting. In the US, the tax gap (the difference between the federal income taxes households actually owe and what they report and pay voluntarily on a timely basis) was 17% in 1992 (Andreoni et al., 1998) and about 40% of households under-reported their taxable income. In the Philippines, Krugman et al. (1992) estimated that 50% of income taxes due went unpaid. Cheating on farm income earned tends to be higher than on non-farm income as it is exempt from reporting requirements during the fiscal year. However, taxpayers are more honest than might be expected given the low levels of audits and penalties. Hence, while tax compliance is higher than predicted by existing levels of coercion, under-reporting on taxable income is nevertheless extensive.

Finally, the franchising literature has also been concerned with under-reporting (Pénard et al., 2003). Franchisees pay royalty as a fraction of gross revenues. As a consequence, like sharecroppers and taxpayers, they have an incentive to under-declare actual revenues to minimize royalties due. Devising optimal franchise contracts and monitoring systems for contract fulfillment are thus important aspects of that literature (Mathewson and Winter, 1985).

**Observation 3. There is near full information on the behavior of others in village communities**

While effort and output reporting are not contractible by the landlord, there is ample evidence that local landlords have near full information about tenant behavior. The tenant’s reaction functions for effort and output reporting to the terms of the contract can thus be expected to be known to the landlord. In a test of the role of information in land leasing in India, Lanjouw (1999) found that knowledge of the relative farming skills of households in the community is widely shared: “villagers are aware of those who are better farmers and lease in and out in accordance with this information”. As a consequence, costly screening of tenants by level of
entrepreneurial ability through the use of share contracts (Allen, 1985) is not necessary (see also Bardhan, 1984). Similarly, Foster and Rosenzweig (1996) do a test of information about worker productivity by comparing piece rate wages that reflect true productivity and time rate wages that reflect the employers’ perceptions of productivity. They find that 88% of the variation in worker productivity is known to employers in the village. Finally, Hayami and Kowagoe (1993) find that there is full information about misconduct in contracting in Indonesian village communities. As they say, “in the village community, everyone is watching everyone. Gossip about one’s misconduct is circulated by word and mouth faster than any modern means of communication.” This does not mean, of course, that there is no private information in village communities. Ligon (1998) shows that a private information model of risk sharing better explains observed consumption than full information models in two of three Indian villages, and Conley and Udry (2005) find that social learning about new crops in Ghanean villages is confined to specific information networks. However, the assumption of sufficiently perfect information within village communities to expect landlords to know their tenants reactions functions for effort and output reporting to the terms of the contract seems tenable for the majority of situations.

Observation 4. The benefit of under-reporting to the tenant is less than the cost of under-reporting to the landlord due to costly actions to dispose of unreported product

Theft has been widely recognized to be socially inefficient due to four indirect costs associated with the action: (1) the cost in time and money spent in protection by the owner in an effort to prevent theft, (2) the cost in time and resources of engaging in theft by the criminal, (3) the potential destruction of product in the process of stealing, the loss of value of the product between owner and thief, and the transaction cost in the resale of stolen goods or the laundering of money, and (4) the redirection of production toward goods less prone to be stolen, creating a deadweight loss in production (Usher, 1987; Hasen and McAdams, 1997). We are concerned here with the third type of indirect cost. Deriving benefit from stolen goods or money often requires the resale of goods or the laundering of illegally gained money. These activities are necessary to transform the illegally gained goods or revenues into effective purchasing power (Masciandaro, 1999). Income for the thief is the value of the stolen good less the transaction cost in selling and laundering, with a unit cost that can be expected to increase with the amount stolen. Hence, there would be decreasing returns in disposing of stolen goods and in laundering.
Note that while we consider here the general situation where under-reporting (theft) is socially inefficient, this is not always the case. Under-reporting can create a net social gain if the marginal utility of the unreported goods or their effective prices are higher for the (poor) tenant than for the (rich) owner. No loss of goods is involved if the poor tenant directly consumes at home the unreported food crop. Thus, while there are exceptions, the situation where under-reporting is a source of social inefficiencies can be considered the dominant one, especially when sale of unreported goods is involved.

**Observation 5. Landlords have to provide a contract that protects the tenant against risk**

If there were no market failure else than in effort and output reporting, a fixed rent contract would be chosen, avoiding the problems of labor shirking and diversion of product. If, however, there are additional market failures, a share contract may be chosen. A classical case in the developing country context is absence of an insurance market available to risk-averse poor tenants, and the necessity for landlords to offer contracts that protect them against the worst outcomes (Scott, 1976). One of the ways in which tenants are insured against such outcomes is through a limited liability clause, which leaves the tenant no less than a minimum whatever the realization of output or prices (Basu, 1992).

**Observation 6. Landlords adjust contract design to protect themselves from shirking on effort and from under-reporting on product**

Sharecropping models endogenize contract design as a second best option in the face of market failures. In the contract, the optimal share and fixed fee are thus affected by the relative risk aversions of landlord and tenant (Newbery and Stiglitz, 1979), Marshallian responses to exclusively provided non-traded inputs (Eswaran and Kotwal, 1985), liquidity constraints (Laffont and Matoussi, 1995), and tendency to asset abuse by the tenant (Dubois, 2002). While these arguments have been extensively used to analyze contract choice empirically, there has been little work on their impact on contract design. This is because we observe considerable contract uniformity, with in particular a concentration of shares around 50% over a wide range of

---

1 In terms of model, it is well established that the optimal contract when the tenant is risk averse and effort is not contractible is a sharecropping contract. Using the limited liability clause also leads to the choice of a sharecropping contract, even with a utility function linear in income. This is because it reflects an extreme risk aversion for any outcome lower than the threshold minimum income. We will choose this modeling, as it is analytically easier to use for deriving results.
circumstances (Bell, 1977; Sharma and Drèze, 1996). As suggested by Young and Burke (2001), this may be due to customs that codify what is considered to be fair in a particular region. Contracts only show discontinuity across regions and under sharply different levels of input provision by the landlord. Hence, variation in the terms of the contract would only be visible at the boundaries of large domains of contractual uniformity.

The tax literature, by contrast, is replete with attempts at linking tax collection to an optimum tax rate. Existence of a quadratic relation between tax rate and tax base has been proposed by Laffer (Wanniski, 1978). More recently, a second Laffer-type curve on the relation between tax rate and income disclosure by taxpayers has been evidenced (Sanyal et al., 2000): a higher tax rate induces lower income disclosure by taxpayers and lower net revenues for the tax collecting agency. We use this double Laffer effect to specify the corresponding optimal share contract. For both taxpayers and sharecroppers, a higher extraction rate creates a disincentive on effort, reducing output, and a disincentive on disclosure, increasing under-reporting.

**Observation 7. Monitoring of effort and of output reporting by the landlord is limited by cost and by the crowding-out effect of coercion on reciprocity/cooperation**

There is extensive evidence that landlords attempt to monitor their tenants, for both effort and truthful output reporting. This is pushed to the extreme in Cheung’s (1969) sharecropping model, where the landlord can observe the tenant’s effort, and effort is contractible (third party verifiable and enforceable). In this case, sharecropping only serves for optimum risk sharing between landlord and tenant. Otsuka and Hayami (1988) observe that this holds when farming systems and technologies are simple, with little impact of the level of effort on output, and when the landlord resides in the community and has comparability benchmarks through the outcome of his own cultivation or the output of other tenants. Landlords also typically come to witness the harvest, and sometimes sleep in the field at harvest time to prevent removal of part of the crop by the tenant.

However, there is also concern that excessive supervision will hurt the tenant’s feeling of fairness and thus discourage reciprocity. Hayami (personal communication) reports that, in the village in the Philippines that he monitored for the last 30 years (see Hayami and Kikuchi, 1982), many landlords display “proper” behavior in showing trust in their tenants by not coming to the farm when the harvest is gathered and shared. This display of trust in workers’ honesty is also the hallmark of supervisor-worker relations in Japanese business firms. Excessive supervision
would be considered harassment, reducing workers’ incentives to work hard. The landlord, however, knows that some cheating will result. He protects himself from excessive cheating by two mechanisms. One is coming to visit the farm under some convenient pretext before the harvest to have some visual assessment of the expected yield. The other is by inducing gossiping by others about the level of cheating which they would readily report if it were to exceed reasonable norms.

In the tax literature, Allingham and Sandmo (1972)’s classical model predicts that higher audit probabilities and higher penalties decrease cheating. Verification of this proposition has been obtained in experimental studies (Andreoni et al., 1998). While empirical studies typically suffer from endogeneity of the probability of audit, they also tend to confirm that cheating decreases with both audit probability and level of penalties (Andreoni et al., 1998; Pommerehne and Weck-Hannemann, 1996). However, it has been widely confirmed that observed tax compliance cannot be explained by current levels of coercion (Fehr and Gätcher, 2000). Hence, explaining the low rates of tax evasion requires calling on psychological determinants: the pervasiveness of “tax morale” (voluntary disclosure) which is induced by greater political participation rights for citizens (Feld and Frey, 2002), more respectful treatment of taxpayers by tax authorities (ibid), greater confidence in fairness in the use of tax revenues by government (Uslader, 2003), and expectations of greater cooperation and reciprocity by other taxpayers. In this context, coercion can crowd out tax morale: a higher tax collection effort can lead to smaller tax revenues (Frey, 1997; Sanyal et al., 2000; Feld and Frey, 2002).

These seven observations enable us to develop, in section 4, a model where the terms of the share contract are optimally set in order to simultaneously induce effort, reduce the cost of insurance, and reduce the cost of under-reporting on the level of output achieved. We consider in section 6 extensions of the model to the case where the landlord can combine an optimal incentive contract with some coercive enforcement to reduce under-reporting.

4. Optimal share rental contract under moral hazard on effort and in output reporting

How will a landlord exposed to moral hazard on effort and output reporting set up an optimal contract? To answer this question, we develop a model in which the tenant chooses both the level of effort and the quantity of output he reports to the landlord. Production is increasing and concave in effort. Return to under-reporting is increasing and concave in the amount unreported.
The landlord does not engage in direct supervision (at this stage of the model), and controls effort and reporting through the terms of a linear contract. The contract terms influence both effort and reporting decisions, the share parameter representing the effective price received for the output reported to the landlord and the opportunity cost of under-reporting.

The tenant produces an output $q$ with effort $e$. Output is a stochastic variable, with probability density $f(q|e)$ conditional on effort $e$ and defined over the interval $[0, \bar{q}]$. We assume that higher effort decreases the probability of having a return lower than any given value, and this with decreasing return to scale. The production function is thus represented by:

$$f(q|e), \quad \text{with } F_e(q|e) < 0 \text{ and } F_{ee}(q|e) > 0, \forall q \in [0, \bar{q}],$$

where $F$ is the cumulative distribution function of $q$, and $F_e$ and $F_{ee}$ are first and second derivatives of $F$ with respect to $e$.

The tenant has the possibility of reporting to the landlord a production of only $\tilde{q}$ out of effective output $q$ and of keeping the remaining $(q - \tilde{q})$ either for own consumption or for sale. The value to the tenant of unreported output is a function $k(.)$ such that:

$$k(q - \tilde{q}), \quad \text{with } 0 \leq k' < 1 \text{ and } k'' < 0.$$

Both landlord and tenant are risk neutral. There is a limited liability clause in the contract so that the landlord must leave to the tenant no less than a minimum amount $L$ for any realization of output:

$$\alpha \tilde{q} + \beta + k(q - \tilde{q}) \geq L, \forall q,$$

where $\alpha$ and $\beta$ are the terms of the contract.

4.1. Tenant's optimal choice of effort and reporting

The tenant chooses a level of effort prior to knowing the random shock that will affect output, but reports to the landlord a level of output $\tilde{q}$ only after realized output $q$ is known. The tenant’s problem is thus solved by backward induction, first establishing the optimal reporting strategy and then the optimal effort. The optimal reporting is determined by:

---

2 The limited liability clause is an indirect way of representing the tenant’s risk aversion despite a utility function linear in income. It is chosen here for the ease of analytical derivation that it offers.
\[
\max_q \alpha \tilde{q} + \beta + k(q - \tilde{q}), \text{ at given } q.
\]

The tenant allocates his realized output between reported \( \tilde{q} \) and unreported \( s = q - \tilde{q} \) to equalize marginal returns given by the share \( \alpha \) that the contract leaves him, provided both lie in the feasible range \([0, q]\). The corresponding first order condition is \( k'(s) = \alpha \), and the optimal levels of unreported \( s \) and reported \( \tilde{q} \), are:

\[
\begin{cases}
  s = s^*(\alpha) \equiv k^{-1}(\alpha), \quad \tilde{q} = q - s^*(\alpha) & \text{for } k'(q) < \alpha < k'(0), \text{ some non-reporting} \\
  s = 0, \quad \tilde{q} = q & \text{for } \alpha \geq k'(0), \text{ full reporting} \\
  s = q, \quad \tilde{q} = 0 & \text{for } \alpha \leq k'(q), \text{ full non-reporting.}
\end{cases}
\]

The non-reporting function \( s^* \) is akin to an input demand derived from the production function \( k(\cdot) \), with \( \alpha \) being the opportunity cost of the input \( q - \tilde{q} \). It is decreasing and convex in \( \alpha \). The realized unreported \( s \) and reported \( \tilde{q} \) are function of both the contract share \( \alpha \) and the realized output \( q \), namely \( s = s(\alpha, q) \) and \( \tilde{q} = \tilde{q}(\alpha, q) \).

For a given realization of output, the optimal non-reporting and reporting are represented in Fig. 1 as a function of the contract share. At low share, \( \alpha \leq k'(q) \), the tenant's optimum is to report no output. The higher the share he gets from reported output, the less incentive he has to under-report, and for a share above the return to the first unit of unreported good, \( \alpha \geq k'(0) \), the tenant will fully report all output.

For a given contract share \( \alpha \), the tenant's reporting behavior varies with the realized level of output, as illustrated in Fig. 2. At low output realization \( q \ (q \leq k^{-1}(\alpha)) \), it is optimal to report zero output and declare complete loss. At higher realized output, the optimal amount of under-reporting is such that the marginal return to under-reporting is equal to the share the tenant would receive from the landlord.

Prior to knowing the realization of output, the expected values of unreported output, return to under-reporting, and reported output are thus:

\[
Es = E(\tilde{q}|q \leq k^{-1}(\alpha)) + s^*(\alpha)(1 - F(k^{-1}(\alpha))),
\]

\[
Ek(s) = E(k(q)|q \leq k^{-1}(\alpha)) + k(s^*(\alpha))(1 - F(k^{-1}(\alpha))),
\]

and \( E\tilde{q} = E(q|q \geq k^{-1}(\alpha)) - s^*(\alpha)(1 - F(k^{-1}(\alpha))) \).
The tenant’s optimal effort is dictated by maximization of expected income, anticipating output not reported:
\[
\max_e E[\alpha(q - s) + \beta + k(s) - w]e, 
\]
where \(w\) is the opportunity cost of effort and \(s = s(\alpha, q)\) is determined in (1) above.

Using the expression for \(Es\) and \(Ek(s)\) derived above, the corresponding first order condition is written:
\[
(\text{IC})^e \quad \frac{d}{de} Eq - w + \frac{d}{de} E[k(q)] - \alpha q \leq k'(1)(\alpha) = 0. 
\]

The third term is positive. Hence, for any given \(\alpha\), the optimal effort and output are higher than in a sharecropping contract without under-reporting. Product under-reporting thus gives the tenant a greater incentive to effort since he receives a larger share of the product of his efforts. Note, however, that this additional incentive only comes from the case where the realization of output is so low that the tenant declares complete loss and keeps all the output for himself. For all other outcome realizations, the marginal return to output is \(\alpha\) and thus incentives are not affected by under-reporting. This is because the amount of unreported output is defined by \(k'(s) = a\), and hence all extra production is reported to the landlord.

Fig. 3 represents the tenant’s decisions. Equation (2) solves for the optimal effort \(e\) and the expected supply function \(Eq(\alpha)\), which is increasing and concave in \(\alpha\). Realized output \(q\) is randomly distributed around the expected value \(Eq(\alpha)\). The locus of realized output is represented by a vertical segment corresponding to each expected value of supply, and one such value is marked on the graph. Given realized output, equation (1) gives the optimal level of unreported output \(s'(\alpha)\), which is decreasing and convex in \(\alpha\). This function, described in Fig. 1, is reproduced in Fig. 3. For very low \(\alpha\), under-reporting will reach the maximum value \(q\), while for values of \(\alpha\) beyond the marginal return to under-reporting \(k'(0)\), there will be no theft. The reported quantity is then the difference between the realized output and the unreported quantity.

Hence, there can be no contract if the share parameter is set below a threshold level \((\alpha \leq \alpha_{\min}\) on the graph), as all output would be non-reported. Beyond this value, and for
\( \alpha \leq k'(0) \), the sharecropping contract entails an optimal level of under-reporting. And for \( \alpha \geq k'(0) \), there will be full reporting.

4.2. Landlord’s choice of contract terms

The landlord’s program is to choose the contract terms \((\alpha, \beta)\) to maximize his expected income:

\[
\max_{\alpha, \beta} W = (1 - \alpha) \int \tilde{q} f(q) dq - \beta,
\]

subject to the limited liability constraint (LL), and the effort (IC)⁰ and reported output (IC)⁰ response functions decided by the tenant and known to him:

(LL) \[ \alpha(q - s) + \beta + k(s) \geq L \]

(IC)⁰ \[ \alpha E_q q + E_e k(q) - \alpha q \tilde{q} \leq k^{-1}(\alpha) \]

Substituting this constraint in the landlord’s welfare gives:

\[ W = (1 - \alpha) E(q - s | q \geq s) - L. \] (3)

Let us first look at the optimal value \( \alpha^{nc} \) of \( \alpha \) when there is no under-reporting. The interior solution is solution to:

\[
\frac{dW^{nc}}{d\alpha} = -Eq + (1 - \alpha) E_e \frac{de}{d\alpha} = -Eq + \frac{(1 - \alpha)}{\alpha} \frac{E_e^2}{-E_{ee}}, \] (4)

where \( E_e \) and \( E_{ee} \) are the first and second derivatives of \( Eq \) with respect to effort \( e \), and the subscript \( nc \) to the welfare function indicates that under-reporting is not considered. The optimal share is implicitly given by:

\[ \alpha^{nc} = \frac{E_{e}^2}{E_{e}^2 - E_{ee} Eq}. \]
This expression shows that, whenever there is some risk in production \( (E q > q_{\text{min}} = 0) \), the optimal contract will be a sharecropping contract with \( \alpha^{nc} < 1 \). We represent two cases on Fig. 4, labeled \( W^{nc1} \) (Fig. 4.1) and \( W^{nc2} \) (Fig. 4.2), depending on how the optimal share \( \alpha^{nc} \) compares to \( k'(0) \). The shapes of these curves display the standard Laffer curve effect. If \( \alpha^{nc} \geq k'(0) \) (\( W^{nc1} \) on Fig. 4.1), this is the optimal contract. No under-reporting is incentive compatible for the tenant, as under-reporting could not improve on the welfare he can obtain with full reporting.

If, however, \( \alpha^{nc} < k'(0) \) (\( W^{nc2} \) on Fig. 4.2), then the tenant will have an incentive to under-report. The best contract that can be offered to completely suppress under-reporting would be to increase his share to \( \alpha = k'(0) \). This solution, however, needs to be compared to the optimal contract when under-reporting occurs.

With under-reporting, the landlord’s FOC is:

\[
\frac{dW^c}{d\alpha} = -E\bar{q} + (1 - \alpha) \frac{d}{d\alpha} E\bar{q} = 0. \tag{5}
\]

How do the landlord’s welfare functions with no under-reporting and under-reporting compare? These functions are:

\[
W^{nc} = (1 - \alpha)E q^{nc} - L,
\]

\[
W^c = (1 - \alpha)E (q^c - s) - L,
\]

where \( q^{nc} \) and \( q^c \) are the optimal output level with no under-reporting and under-reporting, respectively. Although under-reporting induces some increase in the optimum level of effort, we assume that for any given \( \alpha \), the expected reported quantity \( E (q^c - s) \) with under-reporting is lower than \( E (q^{nc}) \) with no under-reporting.\(^3\) In that case, for any level of \( \alpha \), the welfare of the landlord is lower with under-reporting than with no under-reporting, and the landlord’s welfare curve with under-reporting \( (W^c) \) is everywhere below the landlord’s welfare curve with no

\[^3\text{Recall that the increase in output is minimal, as it is solely due to cases where output is very low and completely kept by the tenant who declares total loss. Hence, this assumption is not restrictive.}\]
under-reporting (the $W^{nc^2}$ curve), as represented on Fig. 4.2. The shape of this curve represents the double Laffer curve effect on landlord’s welfare: the effect of output sharing on effort (original Laffer effect) and on reporting (extended Laffer effect suggested by Sanyal et al. (2000)).

We saw above that, for $\alpha \geq k'(0)$, it is optimal for the tenant to fully report the realized output, even when under-reporting is possible. Hence, the two curves are identical above the threshold $\alpha = k'(0)$. We will now establish that the maximum of the $W^c$ curve is to the right of the maximum of the $W^{nc}$ curve, as represented on Fig. 4.2. Consider the derivatives of expected reported output under the two regimes:

$$\frac{d}{d\alpha} E(q^{nc}) \text{ and } \frac{d}{d\alpha} E(q^c - s).$$

Both output $q^{nc}$ and $q^c$ increase with $\alpha$, although $q^c$ less than $q^{nc}$ as the additional incentive given by under-reporting decreases. Unreported output $s$ decreases with $\alpha$. We make the reasonable assumption that the change in incentive given by under-reporting does not compensate for the decline in under-reporting itself, and hence that:

$$\frac{d}{d\alpha} E(q^{nc}) < \frac{d}{d\alpha} E(q^c - s).$$

Under these two assumptions, the derivative of the landlord’s welfare with respect to the share $\alpha$ is higher with under-reporting (equation 5) than with no under-reporting (equation 4) for all values of $\alpha$. Hence, the maximum of the welfare curve $W^c$ is to the right of the maximum of $W^{nc^2}$. The effect of under-reporting is to increase the contract share from $\alpha^{nc^2}$ to $\alpha^c$.

The intuition behind this result is the following. Suppose that in absence of under-reporting the optimal contract would be a share $\alpha^{nc^2}$. Introduce now the possibility of under-reporting by the tenant. Under-reporting takes place because the marginal value of the unreported good to him is higher than his share. The landlord should then reconsider the terms of the contract. Reducing the tenant’s share would both reduce supply and increase under-reporting. Therefore, any adjustment to the contract necessarily entails increasing the tenant’s share. As in

\[\text{This assumption is again not really restrictive since the difference in incentive with and without under-reporting is solely due to cases where the production outcome falls below the threshold under which the tenant declares complete loss.}\]
the standard tax problem, where the tax rate affects both effort (the tax base) and truthful reporting, the maximum to the extended Laffer curve is to the right of the original Laffer curve. Landlords (the tax collection agency) are better off raising the tenant’s share (reducing the tax rate) to reduce the level of under-reporting and increase their welfare (tax collection).

4.3. Comparative static on contract terms

Let $z$ be a shifter in the $k(\cdot)$ function. We can think of it as an ability to under-report that would shift the diversion curve and increase the marginal return to under-reporting. Total differentiation of the first order condition (5) gives the impact of an increase in $z$ on the optimal contract:

$$\frac{d^2W^c}{d\alpha dz} d_z + \frac{d^2W^c}{d\alpha^2} d_\alpha = 0 .$$

This shows that $d_\alpha$ has the sign of the cross derivative $\frac{d^2W^c}{d\alpha dz}$.

Derivation of (5) gives:

$$\frac{d^2W^c}{d\alpha dz} = \left[ -\frac{dE(q)}{dz} + (1-\alpha) \frac{d^2E(q)}{d\alpha dz} \right] + \left[ \frac{ds(\alpha)}{dz} - (1-\alpha) \frac{d^2s(\alpha)}{d\alpha dz} \right].$$

The terms in the first bracket represent the influence of the tenant’s increasing under-reporting ability on production level and the terms in the second bracket its influence on under-reporting. Standard assumptions on input demand functions (input demand increases with fixed factors, and the second derivative with respect to price and fixed factor is negative) establishes that the second bracket is positive. The first bracket, however, could be of any sign, but recall that this effect is only due to the marginal cases of very low output. We can thus safely assume that it is negligible and that the overall expression is positive. Therefore, tenants with higher ability to or return from under-reporting have to be offered a higher share $\alpha$, i.e. a better contract to deter them from under-reporting.

5. Beyond linear contracts?

Could a more general non-linear contract $t(\hat{q})$ prevent under-reporting? To address this issue, we simplify the analysis to the case where there is no moral hazard on effort. Output takes a
stochastic value not under the tenant’s control, and the tenant only chooses the level of reporting. Consider a contract $t(.)$ that induces truthful reporting of the realized output $q$ by the tenant. The contract can thus be written as a function of realized $q$. For the tenant not to under-report, it must be the case that at any value of realized output the marginal payment by the landlord is higher than the return to under-reporting any small amount of output:

$$t'(q) \geq k'(0).$$

The contract is thus:

$$t(q) = \int_0^q t'(x) dx + t_0 \geq \int_0^q k'(0) dx + t_0 = k'(0)q + t_0.$$

Adding the limited liability constraint, the lowest value for $t_0$ is $L$.

Hence, the optimal contract that would induce full reporting is the linear contract:

$$t(q) = k'(0)q + L.$$

Yet, as we have seen above, the linear contract with full reporting is not always better than a contract with some under-reporting. The intuition for this result is that preventing under-reporting forces the landlord to give at least $k'(0)$ at the margin, for all values of output $q$, while limited liability prevents him from extracting the surplus with a lump sum tax.

Considering linear contracts, we have established that, if the optimal contract turns out to be one in which the contract share is sufficiently high for the tenant ($\alpha^{nc} \geq k'(0)$), he will have no incentive to under-report. If, by contrast, this optimal contract would provide a low share to the tenant, then he will have an incentive to under-report and, in order to reduce somewhat this under-reporting, the landlord will optimally raise the contract share. The outcome is a share contract with an optimal level of under-reporting, implying a double inefficiency, one in effort and the other in reporting. We have also shown that suppressing all under-reporting through incentives would not be optimum for the landlord, even under a more general non-linear contract. This raises the question of whether some level of costly supervision would help the principal.

6. Coercive contract enforcement to reduce under-reporting

In the previous section, we established the optimum self-enforcing incentive contract that takes into consideration the non-contractibility of effort and of output reporting, using only the terms of the share contract (fix and share parameters) to seek contract fulfillment without any direct
enforcement mechanism. As discussed in the literature review (section 2), contract fulfillment is, however, generally ensured by an optimal mix of incentive and coercive enforcement mechanisms. The specific instruments of supervision (random audit vs. supervision at harvest or just prior to harvest time to preserve goodwill) and punishment (cancellation of a repeated contract or of an interlinked contract, imposition of a fine, ostracization in a segregated social group, etc.) that constitute the contract enforcement mechanism of choice vary with the specificity of the context and the cost of alternative supervision instruments. It is beyond the scope of this paper to specify the choice of a mechanism and to establish the corresponding optimal contract. Using instead an ad-hoc formulation, we can qualitatively explore how the availability of an enforcement mechanism would modify both the optimal share contract parameters and the resulting optimal level of effort and output reporting. All enforcement mechanisms entail a cost for the landlord, noted by \( t \). Supervision increases the tenant’s cost of under-reporting, as he has to use more expensive means to dissimulate his action and faces a residual expected cost equal to the punishment multiplied by the risk of being caught. In the framework of the model constructed in section 4, this cost can simply be included in the return to under-reporting which is now written:

\[
k(s,t), \text{ with } k_t(s,t) < 0 \text{ and } k_{st}(s,t) < 0.
\]

As supervision decreases the return to under-reporting, the equilibrium with supervision will imply less under-reporting, and a higher reported share of output than the optimum without supervision. How much supervision is optimal for the landlord to exercise needs to be established by comparing its costs and benefits.

Most landlords will admit that they know their tenants are under-reporting. Similarly, white-collar crime, provided it remains of “reasonable” magnitude, is known and accepted in the workplace (Clark and Hollinger, 1983). We have seen that tax collectors know that there is extensive under-reporting of taxable income. This could simply be the optimal equilibrium level, given the cost of supervision and enforcement. Another argument to accept this level of under-reporting is that pounding too hard on the cheater to reduce cheating further may create bad relationships that would be detrimental to the quality of work (Dickens et al., 1989) and to tax morale (Fehr and Gächter, 2000). This feeling of “revenge” against a harsh landlord or tax collector can be modeled as an increasing desire to under-report to the principal when supervision becomes more pervasive. The return to under-reporting can then be written as:
\[ k(s, z^p, t), \text{ with } k(s, z^p, t) \geq 0 \text{ and } k(s, z^p, t) \geq 0. \]

The optimum solution implies a lower level of supervision despite a higher level of under-reporting. This explains why supervision is restrained, as observed by Hayami and in the tax literature.

7. Conclusion

Contract fulfillment is one of the fundamental problems in sustaining efficient exchange. We have seen that it can be addressed through the design of incentive compatible contracts, monitoring and punishment, and various combinations of the two particularly when exchanges are repeated. We explored here the fulfillment of incentive share contracts when there is a double moral hazard, on inputs exclusively provided by one partner (such as effort) and in reporting the level of output to be shared with a principal (a landlord or a tax collector), and when there is a social efficiency cost to under-reporting. This paper advances the literature on share contracts by specifying how the landlord’s optimal contract will be determined to achieve contract fulfillment while restraining these two sources of moral hazard. The optimal contract is second best in that it allows for residual moral hazard in both effort and output reporting. The model predicts that contract terms will vary with the tenant’s desire and ability to under-report. This implies designing the contract to manage a double Laffer-curve effect: on effort (tax-base) and on reported output (income disclosure). The terms of the contract are altered toward a reduction of the tenant’s share when the landlord can engage in costly supervision and punishment. Use of coercion may be limited by crowding out on morale, implying greater reliance on contract incentives. However, use of incentives can also be limited by perceptions of fairness, as indicated by rigidity in the 50% share in sharecropping contracts, implying that adjustment may then well have to come from increased surveillance.
Unreported and reported quantities

Fig. 1. Optimal reporting as a function of contract term for given $q$

Unreported and reported quantities

Fig. 2. Optimal reporting as a function of output realization for given $\alpha$
Fig. 3. Tenant’s choice of supply and reporting

Fig. 4.1. Optimal contract with no under-reporting

Fig. 4.2. Optimal contract with under-reporting

Fig. 4. The optimal contract
Acknowledgements

This paper was prepared for the symposium honoring the distinguished career of Eithan Hochman, a graduate from the University of California at Berkeley and professor at the Hebrew University of Jerusalem in Israel. We are indebted to Cécile Aubert and Ethan Ligon for useful suggestions.

References


