Chapter 4

STATIC POLITICAL-ECONOMIC ANALYSIS

4.1 Introduction

Models of economic systems involving government intervention by definition must incorporate policy instruments. Frequently, economists have tended to view these variables as exogenously determined. While this specification is appropriate for some analytical problems, it certainly abstracts from the realities of political-economic life. For most applications, economic policy is dependent on the economic structure, with policy variables codetermined with endogenous economic variables.

Where government intervention has continued for a sufficiently long duration, it often exhibits certain regularities which may be captured by incorporating causal governmental behavior. As a result, for explanation and prediction purposes, it is often desirable to establish hypotheses concerning the formation of the observed political regularities.

Political behavior may be viewed as a process of accommodation among conflicting interests. We develop a conceptual formulation of the political economy which explicitly recognizes the various policymaking centers and interest groups in the system. Our theory views the political economy as a bargaining game among organized groups with conflicting interests. A political equilibrium is identified with the solution of the corresponding bargaining game. In this game, some groups are too poorly organized for bargaining, but even
so their uncoordinated reaction to policy choices affects the policy process formation.

4.2 Organization of the Political System

A political system, or a *polity*, arises whenever market coordinated individual actions are superseded by some form of nonmarket collective action. This is true even when the sole motivation for collective activity is to change a market relationship. To be effective, every organization for collective action must satisfy certain *organizational imperatives*; in particular, it must feature a policymaking and coordination *center* and *peripheral participants* whose actions are controlled by the center. In a minimal political economy, the center consists of policymakers in government while all other economic agents (households, producers, etc.) are peripheral participants. Decisions taken by the center determine resource allocation and income distribution in the political economy and thereby the peripheral participants’ levels of well-being.

Certain groups of individual peripheral participants share common interests in the center’s policy choices and thus have incentives to form interest groups. As joint group action may enhance the group’s power over the center’s policy decisions, some interest groups organize for collective lobbying activity in order to pursue their common interest. Such pursuits can be hampered, however, by organizational set-up costs and by individual proclivities to “free ride” (Olson, 1965). Hence, some potential interest groups are unable to achieve an organizational form that actually influences decisions designed and implemented by the center.

In this setting and our analytical framework, four types of groups may be distinguished in a political economy:

1. A *center consisting of policymakers*. We shall usually assume that only one center exists, but often a *polycentric structure*, involving several centers, may be constitutionally established. Each center is presumed to have a well-defined objective function
representing central decision agents’ preferences over the policy space. Each center also has the capacity to negotiate and enter binding agreements (tacit or explicit) with other organized groups, including other centers.

2. **Organized interest groups.** Such groups characteristically evolve a group choice mechanism, including a particular governance structure and effective leadership capable of rallying group members and of negotiating and entering into binding agreements with other organized groups, including policy centers.

3. **Unorganized but responsive interest groups.** Such groups fail to evolve any machinery for collective choice, but individual group members actively respond to the center’s policy choice.¹

4. **Politically inert interest groups.** Members of such groups have a common political-economic interest. Yet, not only do such groups lack any mechanism for coordinated joint action, their members are also unresponsive to the center’s policy choices. Inert interest groups are rather rare; generally we do not expect individual members of any interest group to be totally oblivious to policy decisions affecting their well-being, and in a democratic society with highly developed communication media and active political parties, the affected individuals will respond, at least in the voting booth.

We shall use the terms *power groups* in referring to the first three group types and *organized groups* in referring to policymaking centers and organized interest groups. As indicated, the latter groups are characterized by group choice and resource mobilization mechanisms along with the capacity to negotiate and enter binding agreements.

This classification of groups determines the nature of the political process. As an organized interest group is capable of negotiating and entering a binding agreement with a center,

¹The reaction of the unorganized but responsive groups is not necessarily the optimal response strategy to the center’s policy. In fact, group members’ responses may be emotional and not rational (e.g., urban dwellers’ riot in reaction to increased food prices). The only theoretical requirement in the present context is that group responses are characterized by predictable behavioral patterns.
the relationship between these two organized groups defines a reciprocal power situation in which each party employs its power in a bargaining process. When there are \( n \)-organized interest groups and a single center, a political-economic equilibrium is a solution to a \((n + 1)\)-person bargaining game. In a polycentric group configuration, every center with an interest in the relevant political outcome takes an active part in the corresponding \((g + n)\)-person bargaining game, where \( g \) is the number of interested policy centers and \( n \) is the number of organized interest groups.

Consider a political economy consisting merely of a single policymaking center and a single unorganized but responsive interest group. The relationship between the two defines a unilateral power situation. That is, the policymaking center is aware of the unorganized group’s reaction function and unilaterally selects a policy that would maximize the center’s policy objective function given the unorganized interest group reaction pattern.

But how is the political-economic equilibrium determined under a group configuration involving all forms of power groups, that is, \( g \) interested policymaking centers, \( n \) organized interest groups, and \( m \) unorganized but responsive interest groups? Under these circumstances, bargaining is conducted between \( n + g \) ”Stackelberg leaders” who take into account the reaction functions of the \( m \) unorganized but responsive interest groups, who should actually be regarded as Stackelberg followers. Politically inert interest groups play no political role – affecting the political-economic outcome solely through their economic responses which are embedded in the economic structural relations. Finally, it is worth noting that the bases of any group’s power, especially the economic and political bases of power, depend on the ability of the group’s leaderships to overcome members’ propensity to ”free ride” and to mobilize members’ resources for the joint interest serving activities. The means of achieving members’ mobilization are discussed elsewhere (Olson, 1965; Hardin, 1982) and will be examined in Chapter 7.
4.3 The Political-Economic Structure

Our formulation focuses on the formation of quantitative policy (Tinbergen, 1956). We shall restrict the analysis in this chapter to static systems, with the dynamic generalization presented in Chapter 5. Specifically, let

\[ F(y, x_0; z) = 0 \]

represent the economic structure consisting of \( G \) independent structural relations; \( y \) is a \( G \) vector of endogenous variables, \( z \) is a \( K \)-vector of exogenous variables, and \( x_0 \) is an \( m \) vector of policy instruments. The value of \( x_0 \) is determined by policymakers in the center assigned constitutional responsibility. The \( G \)-vector, \( y \), of endogenous variables as function of \( x_0 \) is determined by the economic structure, (4.1).

Initially, the political economy is presumed to be comprised of a single policymaking center and \( n \) organized interest groups. Hence, only reciprocal power relations prevail. Moreover, let \( X_0 \subset \mathbb{R}^m \) be the set of politically feasible values of \( x_0 \). \( X_0 \) is restricted in several ways: some variables must be nonnegative (prices, outputs, etc.); others are constrained by administrative and technical considerations; and, finally, there are modes of intervention which are unanimously regarded as illegitimate, and therefore, are ruled out as politically unacceptable (e.g., lump sum transfers may be constitutionally prohibited). Thus, the main political resources at the disposal of policymakers are: legitimate power, the coercive power of the state, and the technical and administrative ability to carry out the various policies.³

Thus, bases of power also determine \( X_0 \).

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²In a dynamic formulation, current values of the instrumental as well as other variables are functions of historical and current values of the endogenous, exogenous, and policy variables. The analysis, which in the static version is performed in terms of the policy variables alone, may have to be carried out in terms of policy parameters in the dynamic formulation. Alternatively, optimal control theory could be employed in making the political-economic structure dynamic. (Rausser and Hochman, 1979)

³Policymakers ordinarily control other political resources as well. These have been ignored in the present analysis. However, their effects will, presumably, show up in the groups’ strength of power and the cost of power.
Depending on members’ ability to organize for a concerted political action, their wealth, socioeconomic status, historical status, political representation, etc., each interest group controls certain economic and political resources which constitute its base of power. Let \( x_i \) denote the actions (means of power) taken by the \( i \)th group. These may consist of actions such as: going on strike, supporting a particular individual in an internal party struggle, supporting the political opposition, blocking legislative measures favored by group policymakers, contributions to election funds, setting prices under the group’s control at particular levels, etc. Accordingly, let \( X_i \) be the set of feasible actions open to the \( i \)th interest group. Clearly, \( X_i \) also depends on the group’s base of power.

To simplify the presentation, we shall, henceforth, ignore the exogenous variables \( z \), so that \( y = y(x_0) \). Since the endogenous variables, \( y \), are determined by the policy instruments, \( x_0 \), the state of the political-economic system is fully described by the vector,

\[
x = (x_0, x_1, ..., x_n) \\
x_i \in X_i \quad i = 0, 1, ..., n.
\]

It is assumed that the preference ordering of the states of the political-economic system as evaluated by each interest group may be represented by the group’s policy objective function as defined over political and economic outcome space.

Since the actions of each interest group are aimed at the policymakers, and in order to simplify the analysis, we shall assume additive objective functions. For the policymaking center,

\[
U_0 = U_0(x) \\
= u_0(x_0) + \sum_{i=1}^{n} s_i(x_i, \delta_i)
\]

where \( u_0(x_0) = \bar{u}(y(x_0), x_0) \), \( \bar{u}_0 : \mathcal{R} \times X_0 \rightarrow \mathcal{R}, y(x_0) \), is the \( G \)-vector of endogenous variables whose values are determined by \( x_0 \), \( \delta_i \) is a strategy indicator variable indicating whether a ”reward” or ”penalty” strategy has been adopted in the strategic interaction by
the corresponding organized group, and $s_i(\cdot)$ represents the strength or influence of interest
group $i$. The index $i = 0$ is reserved for the policymaking center, and $i = 1, 2, ..., n$ for the
$n$ organized interest groups. That is,

\begin{equation}
U_i = U_i(x)
= u_i(x_0) - w_i(x_i, \delta_i) = u_i(x_0) - c_i \quad i = 1, 2, ..., n
\end{equation}

where $u_i(x_0) = \bar{u}_i(y(x_0), x_0)$; $\bar{u}_i : \mathbb{R}^G \times X_0 \to \mathbb{R}$ and $c_i$ represents the cost to interest group
$i$ of exercising strength or influence. The subjective cost of power to the $i^{th}$ interest group
of attempting to influence the policymakers is given by the function $w_i$ in equation (4.2b)
which is positive whenever an active influence attempt is made by the group.

The $i^{th}$ group’s extended political objective function is measured by $U_i$. The $U_i$’s are
scalar functions expressed in terms of a common numeraire, say, dollars or pounds.\footnote{No inter-group utility comparability is implied by our specification of the objective functions. The definition of $U_i$’s in terms of a common numeraire implicitly assumes that members of each group are able to evaluate changes in the state of the political-economic system in terms of dollars and cents. This is a subjective value reflecting the group’s own preferences; i.e., $u_i(x_0)$ is a ”money metric” utility function (see Varian, 1992: 109).} Hence, each $u_i(x_0)$ is group $i$’s evaluation of the state of the economic system given the policy
choice, $x_0$. These performance measures may be equated with economic quantities such as
disposable income, consumer’s surplus, government expenditure, and the like. The strength
function of the $i^{th}$ group over the policymakers, $s_i$, in equation (4.2a) is positive when the $i^{th}$
interest group pursues a ”reward” policy, and negative when the $i^{th}$ interest group penalizes
policymakers.

Each interest group will seek to minimize the cost of power, $c_i$, for given levels of pressure
(strength and $\delta_i$) it exerts on the policymakers. This is achieved by a proper selection
of actions, $x_i (i = 1, 2, ..., n)$. Let $x_i^0$ be the ”cost of power” minimizing combination of
actions by interest group $i$. Recalling that $x_i^0$ may represent either a ”reward strategy” or a
”penalizing strategy,” the strength function, $s_i$, depends on the embedded cost of power, $c_i$,
and the nature of the strategy $x_i^0$, signified by $\delta_i$. The foundation for these observations is

\begin{equation}
\begin{aligned}
c_i &= c_i(\delta_i, s_i) = \min_{x_i \in X_i} w_i(x_i, \delta_i)
\end{aligned}
\end{equation}

where $s_i$ is the level of group $i$’s strength of power over the central policymakers and $\delta_i$ is the indicator of the nature of the ”strength of power” function; that is, $\delta_i = \alpha_i$ when group $i$ is employing a ”reward” policy and $\delta_i = \beta_i$ when group $i$ employs a ”penalty policy.” That is, $c_i$ refers to group $i$’s least-cost combination of the means of power necessary for the $i^{th}$ group to achieve the ”strength”, $s_i$, over the central policymakers while $\delta_i$ indicates the nature of the policy adopted by group $i$. Hence, the strength of power function, $s_i(\ldots)$, may be represented as follows:

\begin{equation}
\begin{aligned}
s_i &= \left\{ \begin{array}{ll}
\alpha_i [c_i(\delta_i, \alpha_i)] , & \text{when } \delta_i \text{ is ”reward”, (i.e., } \delta_i = \alpha_i) \\
-\beta_i(c_i) = -\beta_i [c_i(\beta_i, \delta_i)] , & \text{when } \delta_i \text{ is ”penalty” (i.e., } \delta_i = \beta_i) 
\end{array} \right.
\end{aligned}
\end{equation}

We shall employ the notation $U_i(x_0)$ and $u_i(x_0)$ to denote the $n+1$ vector-valued functions $[U_i(x_0)]$ and $[u_i(x_0)]$, respectively. It is assumed that the $u_i$’s ($i = 0, 1, ..., n$) are such that the set of feasible $u_i(x_0)$ (i.e., $\{u_i(x_0) \in \mathbb{R}^{n+1} : x_0 \in X_0\}$) is compact and convex; the $s_i$’s are concave in $c_i$, and all functions are twice differentiable.

Each groups’ policy objective functions are faithfully presumed to reflect the policy preferences of the group members, and the concept of ”interest groups” is not construed as a denial of the ”methodological individualism” doctrine, which insists that only individuals, but no social aggregate, possess preferences. Rather, the policy objective functions in fact reflect the political preferences of each group’s leadership which is strongly influenced by individual members’ almost identical preferences.

The objective function, $U_i$, referred to as ”group $i$’s extended objective function,” may consist of some or all of the following three components: (i) the organized group policy
objective function, \( u_i(x_0) \); (ii) the pressure function, \( \sum_{i=1}^{n} s_i(x_i, \delta_i) \); and (iii) the cost of power, \( c_i \), to the \( i \)th group. Note that in the structural specification, the political pressure function and the explicit cost of power function never appear jointly in any single extended objective function.\(^5\)

Two concepts of the efficiency frontier may be distinguished: (a) the economic efficiency frontier\(^6\) – the set of efficient points, \( \{ u(x_0) \in \mathbb{R}^{n+1} : x_0 \in X_0 \text{ and } u(x_0) \text{ is not dominated by any } u(x^*), x^* \in X_0 \} \), and (b) the political efficiency frontier – the set of efficient points, i.e., the following set \( \{ U(x_0, \delta_1, \ldots, \delta_n, c_1, \ldots, c_n) \in \mathbb{R}^{n+1} : x_0 \in X_0, \delta_i = \alpha_i, c_i \geq 0 (i = 1, 2, \ldots, n) \} \) is not dominated by \( \{ U(x^*_0, \delta^*_1, \ldots, \delta^*_n, c^*_1, \ldots, c^*_n) \in \mathbb{R}^{n+1} : x^*_0 \in X_0, \delta^*_i = \alpha^*_i, c^*_i \geq 0 (i = 1, 2, \ldots, n) \} \).

The first set consists of efficient combinations of the policy objective functions attainable under the constraints imposed by the economic structure and technical and political feasibility, i.e., here political rewards, or penalties, are not considered at all. The second efficiency concept is obtained from the first by allowing interest groups to offer central policymakers rewards or to impose penalties. The economic feasibility set \( \{ u(x_0) \in \mathbb{R}^2 : x_0 \in X_0 \} \) is depicted by the shaded area \( F \) in Figure 4.1. It represents the feasible combinations of the policymakers and the single interest group policy objective functions when neither rewards nor penalties are allowed. In Figure 4.1 the policymaking center objective (utility) function, \( u_0 \), is plotted on the horizontal axis while the policy objective (utility) function of the single interest group, \( u_1 \), is represented on the vertical axis. The northeastern, outer border of the economic feasibility set is the economic efficiency frontier. It consists of the feasible policy objective functions combinations \( (u_0, u_1) \) that cannot be dominated by any other feasible vector \( [u_0(x^*), u_1(x^*)] \) such that \( x^* \in X_0 \), too. That is, \( (u_0, u_1) \in E \) cannot be dominated by any feasible vector \( (u^*_0, u^*_1) \in F \).

Figure 4.1 represents the feasible combinations of the policymakers’ and one organized

\(^5\)Nevertheless, in a polycentric political configurations, reciprocal power relations between different centers will give rise to extended objective functions comprising all three components (see the Appendix to this chapter).

\(^6\)The term, ”economic efficiency,” used here is not necessarily synonymous with overall social Pareto optimality in the traditional economic sense. Rather, it applies only to the organized groups participating in our particular political economy.
Figure 4.1: The economic feasibility set in the \((u_0, u_1)\) space
interest group’s policy objective function, before taking into account the ability of the organized interest group to reward and penalize the policymakers in the center according to the functions $\alpha_i(c_i)$ or $-\beta_i(c_i)$ respectively.

If there is only one interest group (i.e., $n = 1$), and its extended objective function, $U_1$, is plotted on the vertical axis, while the policymakers’ extended objective function, $U_0$, is plotted on the horizontal axis, then the reward function appears in Figure 4.2 and the penalty function in Figure 4.3. Note that in both figures $c_1$ is measured downward along the vertical axis.

The essence of the political problem is the resolution of the conflict arising between the various groups, each attempting to influence policymakers to adopt a policy, $x_0 \in X_0$, maximizing the group’s political objective function. Thus, $X_0$ is the scope of all organized interest groups. The domain of each interest group consists of a single actor – the policymaking center. The groups may employ their power bases to exert political pressure by promising central decision agents ”rewards” for a policy favored by each group and threatening ”penalties” in response to a policy considered harmful to each group’s cause.

How is the conflict resolved? Since cooperation, rather than confrontation, is the governing phenomenon in political-economic systems, one must search for a cooperative solution. Following Harsanyi (1962a), we shall, therefore, adopt the Nash solution to the two-person bargaining game (Nash, 1953) and Harsanyi’s generalizations to $n$-person ($n > 1$) games (Harsanyi 1963). Formally, the political-economic equilibrium is defined as the joint solution to the cooperative game and the structural economic representations.

\footnote{Of course, noncooperative behavior is quite common also. However, it is believed by us to be far less frequent than cooperation, ordinarily arising only when the parties in conflict have divergent perceptions of their mutual power positions. Furthermore, a noncooperative solution should never be viewed as an equilibrium point, since at the moment the parties decide to cooperate, their behavior, in principle, changes and the point before that could not be a sustainable equilibrium.}

\footnote{See also Chapter 3.}
Figure 4.2: The “reward” function when $n = 1$, plotted in the $(u_0, u_1)$ space
Figure 4.3: The "penalty" function when $n = 1$, plotted in the $(u_0, u_1)$ space
4.4 Conflict Resolution and the Equilibrium Relations

In this section we explore the crucial properties of the equilibrium solutions and their implications for the analysis of power relations. The case of two players – the policy center and one interest group – is investigated first, to be followed by an analysis of the \((n + 1)\) players’ general case.

4.4.1 The policy center and one organized interest group \((n = 1)\)

According to Nash (1953), the cooperative game is preceded by a noncooperative game, where the disagreement payoffs \([t_0(\tilde{x}), t_1(\tilde{x})]\) are determined by the players’ threat strategies where \(\tilde{x} \in X\). Given the disagreement payoffs, the solution to the cooperative game is the joint strategy \(\bar{x}_i \in X\) which maximizes the product \([U_0(\bar{x}) - t_0][U_1(\bar{x}) - t_1]\) such that \(U_i(\bar{x}) - t_i \geq 0\), \((i = 0, 1)\). As shown (in Chapter 3 (Harsanyi, 1977)), the equilibrium threat strategies \(\tilde{x} = (\tilde{x}_0, \tilde{x}_1)\) are such that

\[
(4.5) \quad t_0(\tilde{x}) - H_1 t_1(\tilde{x}) = \max_{x_0 \in X_0} \min_{x_1 \in X_1} [U_0(x) - H_1 U_1(x)],
\]

where \(x = (x_0, x_1)\) and \(H_1\) is a constant such that

\[
(4.6) \quad H_1 \geq 0,
\]

and

\[
(4.7) \quad U_0(\bar{x}) + H_1 U_1(\bar{x}) \max_{x \in X} [U_0(x) + H_1 U_1(x)]
\]
where \( \bar{x} \) is the vector of cooperative solution strategies of both parties. At the optimal solution point, \( H_1 \) is the slope of the political efficiency frontier, i.e.,

\[
H_1 = -\frac{\partial U_0}{\partial U_1} \bigg|_{x=\bar{x}}
\]

Expressing the objective functions in terms of (4.2a) and (4.2b), Condition (4.5) becomes

\[
(4.5') \quad t_0(\bar{x}_0, \bar{c}_1, \bar{\delta}_1) - H_1 t_1(\bar{x}_0, \bar{c}_1) = \max_{x_0 \in X_0} \min_{c_1 \geq 0, \delta_1 \in \{\alpha, \beta\}} \{u_0(x_0) + s_1(c_1, \delta_1) - H_1[u_1(x_0) - c_1]\}
\]

The Kuhn-Tucker conditions imply that a necessary condition for (4.5') is

\[
(4.8a) \quad -\frac{\partial \beta_1(\bar{c}_1)}{\partial c_1} + H_1 \geq 0,
\]

and the complementarity condition is

\[
(4.8b) \quad (-\frac{\partial \beta(\bar{c}_1)}{\partial c_1} + H_1)\bar{c}_1 = 0
\]

where the strict equality of (4.8a) holds whenever \( \bar{c}_1 > 0 \). Notice that, due to the additivity of the objective functions, the interest group will always adopt a penalizing threat strategy under disagreement, namely, \( \bar{\delta}_1 = \beta \).

Stating (4.7) in terms of (4.2a) and (4.2b), it turns out that, due to additivity, the maximization of \( U_0 + H_1 U_1 \) in the cooperative game consists of:

(i) the interest group adopts a reward policy, i.e., \( \bar{\delta}_1 = \beta \);

(ii) \( \bar{c}_i \) is selected so as to maximize \( \alpha_1(\bar{c}_1) - H_1 \bar{c}_1 \);

(iii) \( x_0 \) is selected so as to maximize \( u_0(x_0) + H_1 u_1(x_0) \).
Consequently, the following condition holds:

\[
\frac{\partial \alpha_1(\bar{c}_1)}{\partial c_1} - H_1 \leq 0,
\]

and the complementarity condition is

\[
(\frac{\partial \alpha_1(\bar{c}_1)}{\partial c_1} - H_1)\bar{c}_1 = 0
\]

where the strict equality of (4.9) holds whenever \( \bar{c}_1 > 0 \). The model describing the Nash solution of the game for the case \( \bar{c}_1 > 0, \bar{\tilde{c}}_1 > 0 \) is presented graphically in Figures 4.4 through 4.11.

When plotted in the \((U_0, U_1)\) space, the concave reward function might look something like Figure 4.2 and the penalty function, like Figure 4.3. Note that in both figures \( c_1 \) is measured downward along the vertical axis representing \( U_1(x) \).

In order to put these three figures together, it must be kept in mind that the interest group can dole out rewards or penalties given any vector, \( x_0 \), which is chosen by the policymakers. The one political feasibility set, \( \Phi \), is therefore constructed by finding all the extended objective functions combinations that can be attained through rewards or penalties starting from any utility combination in the economic feasibility set \( F \). That is, \( U_0 = u_0(x_0) + \alpha_1(\bar{c}_1) - \beta_1(\bar{c}_1) \) is feasible if \( x_0 \) is feasible, and \( U_1 \) is feasible if \( u_1(x_0) - c_1 \) is feasible. Graphically, this is done by sliding the origin of Figures 4.4 and 4.5 along the outer boundary of the economic feasibility set, and marking the outer envelope of points thus reached. This is done consecutively in Figures 4.6 and 4.7.

As drawn here, the resulting political feasibility set has a nonconvexity. If one also assumes free disposal of utility, however, this nonconvexity disappears, as shown in Figure 4.8.

In order to solve the model for the case with a single-interest group, we adopt Nash’s
Figure 4.4: Constructing the combined "reward" function and the successive efficiency frontier $\varepsilon_1, \varepsilon_2, \varepsilon_3$ in the $(u_0, u_k)$ space ($k = 1, 2, 3$)
Figure 4.5: Rewarding cost and the value of "rewards" to the policy making center
Figure 4.6: The political feasibility set if only "rewards" are possible, when \( n = 1 \)
Figure 4.7: The political feasibility set if both "rewards" and "penalties" are possible, when \( n = 1 \)
Figure 4.8: The political feasibility set if both "rewards" and "penalties" are possible, and utility vectors are freely disposable, when \( n = 1 \)
solution for a two-person, two-stage bargaining game (Chapter 3). In the first stage, the policymakers and the interest group decide noncooperatively what threats to invoke if no agreement is reached in the second stage. In other words, they independently chose a vector of threat strategies, \( \tilde{x} \in X \), which determines the vector of disagreement payoffs, \([t_0(\tilde{x}), t_1(\tilde{x})]\).

In the second stage, this ”disagreement point” is taken as given, and Nash’s solution for a one-stage bargaining game with an exogenously given disagreement point is applied. This solution consists of the joint strategy, \( \tilde{x} \in X \), that maximizes the product, \([u_0(x) - t_0][u_1(x) - t_1]\), such that \( u_i(\tilde{x}) - t_i(\tilde{x}) \geq 0, i = 1, 2 \).

Algebraically, the solution to the second-stage game with the above specifications can be obtained in the following way. First, let \( \epsilon_i \) denote the political efficiency frontier, defined as all points in the political feasibility set not dominated (in the weak sense) by any other point in the same set. Write the equation for \( \epsilon_i \) as

\[
H(u_0, u_1) = 0
\]

where we have assumed that the constraints, \( u_i - t_i \geq 0, i = 0, 1 \), are not binding. The associated first-order conditions for \( u_0 \) and \( u_1 \) can be combined to yield the equation,

\[
\frac{u_0 - t_0}{u_1 - t_1} = \frac{H_1}{H_0}
\]

For the case with more interest groups, not discussed here, one may adopt Harsanyi’s (1963) generalization of this solution to \( n \)-person (i.e., \( n > 1 \)) games.
where
\[ H_i \equiv \frac{\partial H(u_0, u_1)}{\partial u_i} \quad (i = 0, 1) \]

Normalizing the ratio, \( \frac{H_1}{H_0} \) in Equation (4.13) by setting \( H_0 = 1 \) and recalling that
\[ -\frac{du_1}{du_0} = \frac{H_0}{H_1} = \frac{1}{H_1}, \]
one gets from Equation (4.13)
\[ \frac{u_1 - t_1}{u_0 - t_0} = -\frac{du_1}{du_0} \bigg|_{U=\bar{U}} = \frac{1}{H_1}. \]  

There is a useful graphical interpretation of this equality. It states that the line segment joining the Nash bargaining solution to the second-stage game with the disagreement point \((t_0, t_1)\) must have a positive slope equal to the absolute slope of the political efficiency frontier at the solution point, \(\bar{U}\). By implication, the second-stage solution would have been the same had the disagreement point been any other point on that same line segment or its extension into the interior of the political feasibility set (this line has also been called the isolation fiber). This in turn suggests a way to solve the entire game graphically, by working backward from the second stage.

First, construct for any arbitrary point \(z\) on the political efficiency frontier the line tangent to the frontier at \(z\). Next, find the line through \(z\) that has the same positive slope that is equal to the absolute slope of the tangent line at \(z\) (that is, the normal through \(z\)). The resulting line in Figure 4.9 is then in the set \(C(\bar{u})\) of Chapter 3.

Repeating this process for many points, \(z_i\), yields a set of line segments on the political feasibility set that ”fan out” if the political feasibility set is strictly convex, as in Figure 4.8.

The crucial point to note is that in the first stage of the game, given the property of the second-stage solution expressed in Equation (4.13), the policymakers and the interest group will only care which of these ”disagreement lines,” \(c(z_i)\), their threat strategies will take them to their best bargain. Given that the disagreement point lies anywhere on a particular one of
Figure 4.9: Method of constructing the disagreement line set, \( c(z) \) ("isolation fiber"), through the disagreement point \((t_0, t_1)\) and in the political space and bargaining solution point \((\bar{U}_0, \bar{U}_1) = z\)
Figure 4.10: Method of constructing the disagreement line sets, $c(z_i)$
Figure 4.11: Graphically derived solution to the game as a whole when $n = 1$
these lines, \( c(z_i) \), the solution to the game as a whole will then be given by the point at which that particular line crosses the political efficiency frontier. It is obvious from Figure 4.11 that the policymakers will then prefer the disagreement point to lie on a line \( c(z_i) \) that is as far as possible to the southeast, whereas the organized interest group prefers a disagreement line as far as possible to the northwest. This is the content of our Equation (4.5) and (4.5').

In order to minimize the expression in braces on the right-hand side (RHS) of condition (4.5'), the interest group will always adopt a penalizing conflict strategy, i.e., \( \delta_1 = \beta \), and it will also choose \( \tilde{c}_1 \) as in Equations (4.8a) and (4.8b).

In order to maximize the same expression, the governmental center will choose \( \tilde{x}_0 \) such that \( u(\tilde{x}_0) \) lies in the far, bottom-right corner of the economic feasibility set. More precisely, \( u(\tilde{x}_0) \) will be the point where the economic feasibility set touches the southeastern disagreement line, contrary to the construction in Zusman (1976, Figure 1), where the origin of the interest group’s penalty function, \(-\beta(c)\), lies considerably above the bottom-right corner of the economic feasibility set. The correct diagram is Figure 4.11.\(^{10}\)

As shown in Figure 4.11, the origin of the interest group’s penalty function coincides with the bottom-right corner of the economic efficiency frontier. Starting from that corner, the interest group will choose the penalty that reaches the most northwestern disagreement line, thereby satisfying condition (4.8a). The disagreement point then is point \((t_0, t_1)\) in the diagram, where the interest group’s penalty function is tangent to the highest disagreement line.

The conflict strategies \( \tilde{x}_0 \) and \( \tilde{c}_1 \) are virtual strategies that are never actually carried out.\(^{11}\) Instead, the policymakers and the interest group cooperate in stage two of the game to attain the point, \( U(\tilde{x}) \), where the disagreement line ("isolation fiber") on which \((t_0, t_1)\) lies, crosses the political efficiency frontier. In order to attain this point, the policymakers implement the vector of policy instruments, \( \tilde{x}_0 \), thereby inducing the vector of utilities \( u(\tilde{x}_0) \).

\(^{10}\)Apparently, Zusman’s construction was due to an unexplained binding constraint in the general restriction, \( x_0 \in X_0 \).

\(^{11}\)It should be emphasized that a threat need not be carried through. In fact, it need not be at all explicit. All that is required is the perception of a potential threat by the participants (see Nagel, 1968).
on the economic efficiency frontier. The interest groups, in turn, reward the policymakers with additional utility, $\alpha_1(\bar{c}_1)$, at a subjective cost, $\bar{c}_1$, to themselves.

Figure 4.11 also illustrates our main result, that can be summed up as follows: to find the solution to the bargaining game find the line tangent to the political efficiency frontier at $U(\bar{x}_0)$ and the line tangent to the economic efficiency frontier at $u(\bar{x}_0)$. These lines are obtained by finding the solution $\bar{c}_1, \bar{c}_1$ and $\bar{x}_0$ in

$$
\max_{\bar{x}_0 \in X_0 \atop \bar{c}_1 > 0} \bar{c}_1 > 0 \ W = u_0(x_0) - \frac{\partial \beta_1(\bar{c}_1)}{\partial \bar{c}_1} u_1(x_0)
$$

$$
= u_0(x_0) + \frac{\partial \alpha(\bar{c}_1)}{\partial \bar{c}_1} u_1(x_0)
$$

Under the assumptions of our model, the bargaining process can occur at the economic feasibility set alone. All relevant results can be expressed in terms of the economic space and not in terms of the political space.

We have the following relationship among the $(n + 1)$ vectors,

$$
U(x_0) = u(x_0) + V_n
$$

and

$$
TU(x_0) = T[u(x_0) + V_n] \quad T : \epsilon_1 \rightarrow E;
$$

where

$$
u(x_0) \in E \subseteq F
$$

and $T$ is the order preserving linear transformation $[U(x_0) - V_n] = T$. $T$ is an order preserving linear transformation of the vector $U(x_0)$, since it merely involves the subtraction of a $(n+1)$ vector of constants, $V_n$, from the $(n+1)$ vector $U(x_0)$. By axiom LINV in Chapter 2, we then have that the solution of the new bargaining game, $G^*$ (in $F$), is the image of the solution of the old bargaining game, $G, \bar{U}$, under $T$, i.e., $\bar{u}_1(x_0) = T\bar{U}(x_0)$. Thus, one can obtain $\bar{u}_1(x_0)$
in the economic space or \( U x_0 \) in the political space (i.e., \( U ⇔ \bar{u} \)). But since concentrating on \( \bar{u} \) is simpler and more direct than concentrating on \( U \), the former solution is preferred.

The relevant geometrical relations for the case of \( n = 1 \) are presented in Figure 4.11, where one observation is crucial; namely, that the slope of the political efficiency frontier, \( \epsilon_1 \), at \( \bar{U}(x_0) \) is, by construction, equal to the slope of the economic efficiency frontier, \( E \), at \( \bar{u}_1(x_0) \). This "slope preserving" property is actually derived from the equivalence of the representation in the economic and political space. This equivalence provides much simplification at the minimal cost of some information regarding the political moves in the cooperative solution of the political bargaining game. Note, also, that the governance function, \( W \), in Equation (4.15) is also expressed in terms of the economic space.

Graphically, because of the way the political efficiency frontier is constructed in the \( (U_0, U_1) \) space by sliding the origin of the reward function and the penalty function along the economic efficiency frontier, the slopes of the two efficiency frontiers are always equal at points such as \( u(\bar{x}_0) \) and \( U(\bar{x}_0) \) that always differ from each other by the vector \( [\alpha_1(\bar{c}_1), -\bar{c}_1] \).

From Equation (4.14), the absolute slope at the solution point, \( U(\bar{x}) \), and hence also at the point, \( u(\bar{x}_0) \), equals the reciprocal of \( H_1 \). The lines tangent to either point are therefore given by the function \( u_0(x_0) + H_1 u_1(x_1) \). From Equation (4.14), \( H_1 \) in this function can be substituted for by \( \frac{\partial \beta_1(\bar{c}_1)}{\partial c_1} \), or by \( \frac{\partial \alpha_1(\bar{c}_1)}{\partial c_1} \), and the result follows. Note that this

\[
\frac{\partial \beta_1(\bar{c}_1)}{\partial c_1} - \frac{\partial \alpha_1(\bar{c}_1)}{\partial c_1} = b_1
\]

(4.16)

is the power coefficient \( b_1 \). Hence, \( \bar{c}_1 \) determines \( -\frac{\partial \beta_1(\bar{c}_1)}{\partial c_1} \) and \( \frac{\partial \alpha_1(\bar{c}_1)}{\partial c_1} \) is determined by \( \bar{c}_1 \) and consequently the political-economic solution is given by \( \bar{x} \in X_0 \) such that

\[
W(\bar{x}_0) = \max_{x_0 \in X_0} [u_0(x_0) + b_1 u_1(x_0)]
\]

Note, however, that the weight on the interest group’s utility, \( u_1 \), in this "political governance function” depends very much on the shape of the economic feasibility set, the reward
function, and the penalty function. In other words, the equilibrium of a political-economic system consisting of the policy center and one organized interest group is associated with the maximization of the sum of the policymakers’ objective function and the organized interest group’s "political objective function," each group’s objective function weighted by the marginal strength of its power over the policymakers, \(-\frac{\partial \beta_1(\tilde{c}_1)}{\partial c_1} = \frac{\partial \alpha_1(\bar{c}_1)}{\partial c_1} = b_1\). The equilibrium weight is regarded constant. It is worth noting that the equilibrium value of \(\bar{x}_0\) is invariant under order-preserving linear transformations of the objective functions. We shall refer to \(W(x_0)\) as the political governance function. Note that the present theory assumes that the political power of the policymaking center which serves as a number and its value is normalized to 1, i.e., \(b_0 \equiv 1\).

### 4.4.2 One policy center and \(n\)-organized interest groups

The case of \(n\)-organized interest groups is analyzed with the aid of the solution concept proposed by Harsanyi (1963). We shall refrain from repeating the full development of the concept and will limit the presentation to the final set of conditions defining the solution.

According to Harsanyi (1963) (see Chapter 3), the solution to the overall game depends on the solutions to subgames among all possible coalitions. The subgames determine a cooperative payoff to each member of the coalition, which, in turn, affects the disagreement payoffs as a member in higher order coalitions. Let \(N\) denote the set of all players, i.e., \(N = \{0, 1, 2, \ldots, n\}\), \(S\) a subset of \(N\) and \(\bar{S} = N - S\), the complement of \(S\) in \(N\). Let \(U_i^S\) denote player \(i\)'s cooperative payoff when he is a member of \(S\). Likewise, let \(t_i^S\) denote player \(i\)'s disagreement payoff as a member of \(S\), when \(S\) and \(\bar{S}\) fail to agree, each coalition employing its threat strategies. Let also \(\tilde{x}^S\) be the threat strategy of coalition \(S\), where \(x^S\) denotes the variable \(x\) under the control of coalition \(S\) and \(X^S\) the feasible strategy space of
\(S\), i.e., \(\bar{x}^S \in X^S\). The solution is then given by the following conditions:\(^{12}\)

\[(4.17a) \quad H_i \geq 0 \quad i \in N, \quad \text{where} \quad H_i \equiv \frac{\partial(U)}{\partial U_i} \]

\[(4.17b) \quad \sum_{i \in N} H_i U_i^N = \max_{x \in X} \sum_{i \in N} H_i U_i(x) \]

such that:

\[(4.17c) \quad H_i(U_i^N - t_i^N) = H_j(U_j^N - t_j^N), \quad i, j \in N \]

\[(4.17d) \quad U_i^S = U_i(\bar{x}^S, \bar{x}) \quad i \in S, \quad S \subseteq N \]

\[(4.17e) \quad t_i^S = \sum_{\substack{i \in R, \ R \subseteq S}} (-1)^{s-r+1} U_i^R \quad s = |S| > 1, \quad i \in S, \quad S \subset N, \quad r = |R| \]

where \(t_i^S(\bar{x}^S)\) and \(t_j^S(\bar{x})\) are determined by the following relation,

\[(4.17f) \quad \sum_{i \in S} H_i U_i(\bar{x}^S, \bar{x}) - \sum_{j \in \bar{S}} H_j U_j(\bar{x}^S, \bar{x}) = \max \min \left[ \sum_{i \in S} H_i U_i(\bar{x}^S, \bar{x}) - \sum_{j \in \bar{S}} H_j U_j(\bar{x}^S, \bar{x}) \right] \]

and subject to:

\[(4.17g) \quad \begin{cases} 
H_i(U_i^S - t_i^S) = H_k(U_k^S - t_k^S) & i, k \in S \\
H_j(U_j^S - t_j^S) = H_m(U_m^S - t_m^S) & j, m \in \bar{S} 
\end{cases} \]

where, for the purpose of the \(\max\) and \(\min\) operation, the quantities \(H_i, H_k, H_j, H_m\) and \(t_i^S, t_k^S, t_j^S, t_m^S\) are regarded as constants. Let the symbols \(s\) and \(r\) refer to the number of members in coalitions \(S\) and \(R\), respectively. Notice, also, that \(U_i = U_i(\bar{x})\) is the \(i^{th}\) participant’s cooperative payoff in the overall game. The constants, \(H_i\), are normalized by dividing through by \(H_0\).

Now, stating the objective function in terms of (4.2a) and (4.2b), we find that, due to ad-
ditivity, the maximization of $\sum_{i \in N} H_i U_i$ in Equation (4.17b) implies that in the (cooperative) bargaining solution:

(i) Interest groups adopt a reward policy, i.e.,

$$\bar{\delta}_i = \alpha \quad i = 1, 2, \ldots, n$$

(ii) $\bar{c}_j$ is selected so as to maximize

$$\alpha_j(\bar{c}_j) - H_j \bar{c}_j \quad j = 1, 2, \ldots, n$$

(iii) $\bar{x}_0$ is selected so as to maximize the following equation

$$W(\bar{x}_0) = \max_{x_0 \in X_0} \left[ u_0(x_0) + \sum_{i=1}^n H_i u_i(x_0) \right]$$

That is, $u(\bar{x}_0)$ is on the economic efficiency frontier. Consequently, the following conditions hold:

(4.18a) $$\frac{\partial \alpha_j(\bar{c}_j)}{\partial c_j} - H_j \leq 0 \quad j = 1, 2, \ldots, n$$

and the complementary relation

(4.18b) $$\left( \frac{\partial \alpha_j(\bar{c}_j)}{\partial c_j} - H_j \right) \bar{c}_j = 0$$

Hence, by complementarity, the strict equality of (4.18a) holds whenever $\bar{c}_j > 0$.

Consider, now, the subgame between $S = \{i\}$ and $\bar{S} = N - \{i\}$ and then, the additivity assumption condition (4.17f) implies a penalizing threat policy on the part of $i$, namely, $\delta_i^{(i)}$.
and

\[(4.19a) \quad -\frac{\partial \beta_i(\tilde{c}_i)}{\partial c_i} - H_i \geq 0 \quad i = 1, 2, ..., n\]

and the complementary relation

\[(4.19b) \quad \left(-\frac{\partial \beta_i(\tilde{c}_i)}{\partial c_i} - H_i\right) \tilde{c}_i = 0\]

Hence, by complementarity, the strict equality of (4.19a) holds whenever \(\tilde{c}_i > 0\). The policy applies to all interest groups when facing a coalition of all other groups.

For reasons which have already been discussed, one expects the subgame to involve a positive threat, that is, \(\tilde{c}_i > 0\), and the equality in (4.19a) holds. Combining Equations (4.19a) and (iii) above, it is found that the overall solution is associated with a maximization of

\[W = u_0(x_0) + \sum_{i=1}^{n} -\frac{\partial \beta_i(\tilde{c}_i)}{\partial c_i} u_i(x_0) = u_0(x_0) + \sum_{i=1}^{n} b_i u_i(x_0),\]

where

\[b_i = -\frac{\partial \beta_i(\tilde{c}_i)}{\partial c_i} = \frac{\partial \alpha_i(\tilde{c}_i)}{\partial c_i} \geq 0\]

with respect to \(x_0 \in X_0\).

In other words, the equilibrium of a political-economic system is associated with the maximization of the sum of the policymakers’ policy objective functions (the interest groups’ policy objective functions) weighted by their marginal strength of power, \(\{b_i : i = 1, 2, ..., n\}\), over the policymakers. The equilibrium non-negative weights \((b_0, b_1, ..., b_n)\), where \(b_0 \equiv 1\), are regarded as constants, although their values are endogenously determined.

The theory thus predicts a maximizing behavior of the political-economic system. However, the quantity being maximized (i.e., the political governance function of the single
center-n interest groups’ political economy) is not necessarily the policymakers’ objective function. It is, rather, a reflection of the social power structure and the interests of the various organized groups.

### 4.4.3 A polycentric configuration

Consider a group configuration comprising $g$ interested policymaking centers and $n$ organized interest groups. Let $j = 1, 2, ..., g$ index the policymaking centers and $i = 1, 2, ..., n$ index the organized interest groups.

Also, let $x_0 = (x_0^1, x_0^2, ..., x_0^n)$ be the vector of policy instruments controlled by the various policymaking centers. That is, we assume that each policymaking center is constitutionally vested with the authority to determine the value of some specific policy instruments. Furthermore, it is presupposed that a reciprocal power relationship prevails among the various centers so that each center has some power over all other centers. Hence, the extended objective functions of the policymaking centers are

\[(4.20) \quad U_j = u_j(x_0) + \sum_{i=1}^{n} s_{ij}(c^j_i, \delta^j_i) + \sum_{k \neq j} S_{kj}(c^j_k, \delta^j_k) - \sum_{k \neq j} c^k_j, \quad j, k = 1, 2, ..., g\]

where $u_j(x_0)$ is the policy objective function of center $j$ reflecting the center’s decision agents’ preferences over the entire policy space, $X_0$, $s_{ij}(c^j_i, \delta^j_i)$ is the strength of power of the $i^{th}$ interest group over the $j^{th}$ center, $S_{kj}(c^j_k, \delta^j_k)$ is center $k$’s strength of power over center $j$; $c^j_i$, $c^j_k$ and $c^j_j$ are, respectively, the costs of power of the $i^{th}$ interest group over the $j^{th}$ center, and the $k^{th}$ center over the $j^{th}$ center, and the $j^{th}$ center over the $k^{th}$ center. $\delta^j_i$ and $\delta^j_j$ are strategy indicator variables indicating whether a “reward” or “penalty” strategy has been adopted in the strategic interaction between the corresponding organized groups. The extended objective functions of the organized interest groups are
$$U_i = u_i(x_0) - \sum_{j=1}^{g} c_j^i \quad i = 1, 2, ..., n$$

where $u_i(x_0)$ is organized interest group $i$’s policy objective function defined over $X_0$. Notice that $x_0$ consists of an array of $g$ vectors, each of which is decided by a separate policymaking center, each belonging to a polycentric system of $g$ centers, each of which is interested in accommodating $n$ interest groups.

Since reciprocal power relationships prevail among all organized groups, the equilibrium solution of the political economy is a solution to the corresponding $(g + n)$-person bargaining game. In the Appendix to the present chapter, the solution to this $(g + n)$-person simple bargaining game (where all disagreement payoffs $t_0^i, t_0^j$ are treated as given) is explored.

### 4.4.4 One policymaking center, $n$-organized interest groups, and $K$ unorganized but responsive interest groups

Under the group configurations considered hitherto, inter-group power relationships were all reciprocal. By introducing unorganized but responsive interest groups, $K$ unilateral power relationships are added to the power structure. We shall assume first that the reaction of the $k^{th}$ unorganized but responsive interest group affects the well-being of the policymakers alone, the $n$ organized interest groups are assumed to be indifferent to the unorganized group reaction. This assumption, and the assumption of a single policymaking center, are made in the interest of simplicity and brevity; they could be easily relaxed. As will become evident subsequently, the required change in the analysis warranted by the relation is minimal and obvious.

Let $r_k(x_0)$ denote the reaction function of the unorganized but responsive interest group $k$, and let $r(x_0) = [r_1(x_0), ..., r_K(x_0)]$. Under the present assumptions, the reaction function affects the policymaking center alone. We may then express the center’s (index $i = 0$) policy
objective function as follows

\[(4.22) \quad u_0 = u_0(x_0, \pi(x_0)) = \hat{u}_0(x_0)\]

Hence, substituting \(\hat{u}_0(x_0)\) for \(u_0(x_0)\) everywhere in Sections 4.3 and 4.4, one obtains the relevant analytic results. Extending the analysis to other group configurations is similarly implemented. Note, for instance, that when all interest groups are unorganized but responsive, we have the special case,

\[(4.22') \quad u_0 = u_0(x_0, r(x_0)) = \hat{u}_0(x_0),\]

where \(r(x_0) \in \mathbb{R}^K\) is the responses of the \(K\) unorganized but responsive interest group, to the policy \(x_0\), and the policymaking center is a single Stackelberg leader. More generally, if the well-being of members of all organized groups is influenced by every reaction of the unorganized but responsive groups, i.e.,

\[r_i(x_0) = [r_{i1}(x_0), ..., r_{iK}(x_0)]\]

\(i\) is not a subscript of any other index. There are all together \(K\) unorganized but responsive groups and \(r_i(x_0)\) is the vector of responses of the unorganized but responsive groups on the \(i^{th}\) organized group.

The analytically crucial conclusion of the preceding analysis is that, for a fairly broad spectrum of group configurations, the political equilibrium is associated with the maximization of the political governance function appropriately modified to allow the unorganized but responsive interest groups’ reaction.

Accepting the basic tenets of the methodological individualism doctrine, any teleological
interpretation of this result should be avoided – the political economy, as a social aggregate, has no objective of its own. The maximization result is strictly an "as if" outcome originating from strategic interactions among individual rational actors.

We have thus uncovered a special type of political power externality whereby each organized interest group benefits from an increase in the number, \( n \), of organized interest groups, without any group’s contribution in this respect. Furthermore, only a jointly agreed upon strategy could fully internalize this type of political power externality. However, our theory is in this respect inadequate, as it ignores the potentialities of inter-group strategy coordination (by forming coalitions of groups with similar interests, say).

4.5 Conclusion

The theory expounded in this chapter seeks to endogenize economic policy formation. To this end, a political economy is conceived as a system of interacting interest power groups with conflicting objectives – the evolving policy representing a resolution of the underlying conflict.

According to this conception, the constitutional order, whether formal or informal, establishes a center, or centers, with policy selection and implementation authority. These centers are usually identified with the government. As the chosen policies affect the well-being of many individual economic actors in various ways, interest groups consisting of individuals sharing common interests are formed. Some of the groups may be organized for joint political action by ”political entrepreneurs” seeking to advance their own private interests (see Chapter 6).

Depending on their level of organization and responsiveness to the center(s)’ policy choices, interest groups participate in the policy formation process. The ensuing political-economic equilibrium is a solution to a bargaining game among organized groups, including all relevant policymaking center(s). The responses of unorganized interest groups are taken
into account by the participants in the bargaining game. The resulting policy choice depends on the interests and political power of the organized groups. The model, therefore, applies to a broad spectrum of possible interactions, including the case of political competition among political parties and individuals.

As group rationality is presupposed, the cooperative solution of the bargaining game corresponds to the maximization of a certain "policy governance function" – a weighted sum of the various groups’ policy objective functions, where the weights, whose values are endogenously determined, may be interpreted as the marginal strength of the interest groups’ power over the policymaking center. The policy governance function should not necessarily be interpreted as the policymakers’ objective function, nor should it be identified with the public interest. Hence, the political-economic equilibrium is not necessarily efficient.\(^\text{13}\)

It should be noted that the "group rationality" postulate presently adopted, essentially implies that the bargaining game is a cooperative one. We have avoided a noncooperative solution to the political conflict since in the long-run, noncooperative solutions, which are often Pareto inefficient for the participating parties, cannot represent long-term social equilibria. This is because the opposing parties can always agree on a Pareto-superior solution through negotiation. The viability of such agreements in the absence of third-party enforcement then follows from the long-term repetitive nature of the underlying game (see also footnote 2).

Appendix 4.A The Solution to the Polycentric \((n + g)\)-person, Simple Bargaining Problem

Proposition 4.1 (a) Let \(F\) and \(\Phi\), respectively, be the economical and political feasibility sets. Both the economic efficiency frontier, \(E\), and the political efficiency frontier, \(E_1\) are the upper-right boundaries of the corresponding feasibility sets, \(F\) and \(\Phi\).

(b) Given that \(U^*\) is feasible but is dominated by some other feasible vector \(U \in \Phi\), say,

\(^{13}\)The normative political-economic aspects of the present theory are discussed in Chapter 5.
then the corresponding vector \( u^* \in F \) is not efficient.

**Proof.** To prove part (a) of the proposition, note that the economic feasibility set, \( F \), is ex hypothesis, compact and convex. We shall employ the notation \( U(x_0) \) and \( u(x_0) \) to denote the \( n+1 \) vector-valued functions \([U_i(x_0)]\), and \([u_i(x_0)]\), respectively. It is assumed that the \( u_i \)'s \((i = 0, 1, \ldots, n)\) are such that the set of feasible \( y(x_0) \) (i.e., \( \{u(x_0) \in \mathbb{R}^{n+1} : x_0 \in X_0\} \)) is compact and convex, the \( s_i \)'s are concave in \( c_i \), and all functions are twice differentiable.

Also, by definition, the economically efficient set, \( E \subseteq F \), consists of the \((n + 1)\) vectors, \( u(x_0) \in F \), which are not dominated by any other vectors, \( u^* \in F \). \( E \) must, therefore, consist of all vectors in the outer upper-right boundary of \( F \). To see this suppose, to the contrary, that \( u^*(x_0) \) is in the interior of \( F \); then there exists a vector \( u(x_0) \) in the outer upper-right boundary of \( F \) such that \( u(x_0) \) is feasible and, at least for one element, \( u_i \), of \( u(x_0) \), \( u_i > u_i^* \) and for all \( j \neq i \) \( u_j^* \leq u_j \). Hence, all vectors \( u^*(x_0) \) in the interior of \( F \) are dominated by vectors \( u(x_0) \), in the upper-right outer boundary of \( F \). On the other hand, the upper-right boundary of \( F \) is down-sloping to the right. For any pair of vectors, \( u \) and \( u^* \) in \( E \); such that \( u_i > u_i^* \) for some \( i \), implies that for some \( j \neq i \) \( u_j < u_j^* \) so that \( u \) and \( u^* \) do not dominate each other. Consequently, \( E \) consists of all vectors \( u(x_0) \) on the outer upper-right boundary of \( F \). The economic feasibility set, \( F \), is compact and convex. Hence, by construction, so is the political feasibility set, \( \Phi \) (See the construction of \( \Phi \) from \( F \) in Figures 4.3 to 4.8.) Then, by applying, essentially, the same arguments to the feasible political set, \( \Phi \) and the political efficiency frontiers, \( \epsilon_1 \), one proves part (a) of Proposition 4.1.

To prove part (b) of the proposition, let the \((n + 1)\) vector, \( V_n \), consist of the following components:

\[
v_0 = u_0(x_0) + \sum_{j=1}^{n} a_j(\bar{c}_j) \quad \text{and} \quad v_i = u_i(x_0) - \bar{c}_i \quad \text{for} \quad i = 1, 2, \ldots, n, \quad u(x_0) \in E
\]

Note that since in the cooperative bargaining solution \( \delta_i = \alpha \) for all \( i \), Equations (4.18a) and (4.18b) uniquely determine the values of \( \bar{c}_i \) for all \( i \)'s. Hence, the value of all \( v_i \)'s and thus,
$V_n$, are also unique. We may, therefore, write:

$$U(x_0) = u(x_0) + V_n$$

Consequently, $U(x_0)$ is feasible, too.

Suppose next that the political vector $U(x_0)$ dominates the political vector, $U^*(x_0) \in \Phi$, so that we may write:

$$U(x_0) = u(x_0) + V_n \geq u^*(x_0) + V_n = U^*(x_0)$$

where the inequality is strict for at least one element, $i$, of all the $(n+1)$ vectors in the inequality above. That is, for some $i$’s, $u_i(x_0) > u^*_i(x_0)$ and for all $j \neq i$,

$$u_j(x_0) \geq u^*_j(x_0)$$

which also implies that $u(x_0)$ dominates $u^*(x_0)$.

Hence, if the political vector, $U^*(x_0)$, is dominated by another politically feasible vector, $U(x_0)$, so do the corresponding economic vectors (i.e., $u(x_0)$ dominates $u^*(x_0)$). Hence, if the political vector, $U^*(x_0)$, is inefficient, so is the corresponding economic vector, $u^*(x_0)$.

Notice that the condition, $\frac{\partial \alpha_i(\bar{c}_i)}{\partial c_i} = H_i$, derived from Equations (4.18a) and (4.18b) implies that for all $\bar{c}_i > 0$, the slope of $\alpha_i(\bar{c}_i)$ equals the slope of the political efficiency frontier, $\epsilon_k$, where $k$ indexes the interest group ($k = 1, 2, ..., n$) in the space $(U_0, U_k)$ of Figure 4.8. Then, assuming the economic feasibility set, $F$, to be identical for all pairs of actors represented by $U_0$ and $U_k$ one can draw the following set of political efficiency frontiers in the two-dimensional space, $(U_0, U_k)$, of Figure 4.4, where $k$ increases as the analysis progresses to successive efficiency frontiers marked by the index, $k$.

The reward technology employed is presented in Figure 4.5. The following definitions are employed in Figure 4.5.
\[ c_i = \text{the cost of rewarding the center by interest group } i. \]
\[ \bar{c}_i = \text{the best reward level by group } i \text{ (point } \alpha(\bar{c}_i) \text{ is tangential with } \epsilon_i). \]
\[ C_n = \sum_{i=1}^{n-1} \bar{c}_i + c_n = \text{total cost used optimally (except for } c_n \text{ which is not restricted.)} \]
\[ \bar{C}_n = \sum_{i=1}^{n} \bar{c}_i = \text{total optimal reward cost for } n \text{ organized interest groups.} \]
\[ \alpha_i(c_i) \text{ the reward function as described in Figure 4.5.} \]
\[ S_k = \sum_{i=1}^{k-1} \alpha_i(\bar{c}_i) + \alpha_k(c_k) = \text{the combined reward function.} \]
\[ \equiv \text{the heavily drawn curve in Figure 4.5.} \]
\[ \equiv \text{the upper envelope curve over the optimally related} \]
\[ \text{rewarding efforts in each interval except the last one,} \]
\[ c_i = \bar{c}_i \text{ (} i = 1, 2, \ldots, k - 1), \text{ and in the } k^{th} \text{ interval } c_k \text{ is unrestricted.} \]
\[ S_n = \sum_{i=1}^{n} \alpha_i(\bar{c}_i) \equiv \text{total reward when the optimal reward program by all } n \]
\[ \text{organized interest groups is performed.} \]

As can be gathered from Figures 4.4 and 4.5, the optimal level of rewarding effort is
provided by the sequence, \( \bar{c}_1, \bar{c}_2, \ldots, \bar{c}_n; \) that is, given the total cost of rewarding the policy-
making vector, it is best performed when the cost to the \( i^{th} \) interest group is \( \bar{c}_i, \) in which
\[ \text{case, total reward value is } S_i \equiv \sum_{i=1}^{n} \alpha_i(\bar{c}_i). \]

The reward function should be tangent to the corresponding political efficiency frontier,
\( \epsilon_i, \) and the full optimal reward, program yield, is \( \sum_{i=1}^{n} \alpha_i(\bar{c}_i). \) Clearly, it is not optimal from
the standpoint of the \( i^{th} \) organized interest group and the policymaking center to reward
the latter at any other cost levels. Thus, the solution of the political bargaining game is on
\( \epsilon_n, \) which is derived graphically through a sequence of rewards which may be drawn up as
points of tangency with \( \epsilon_1, \epsilon_2, \ldots, \epsilon_n. \)
Hence, the following proposition:

**Proposition 4.2** The more organized interest groups participate in the political process (namely, the greater is \( n \)) and the more rewarding effort each of them is able and willing to provide, the more the political solution is likely to be beneficial to each organized interest group and the more rewarding effort each of them is able and willing to provide, the more the political solution is likely to be beneficial to each organized interest group.

**Proof.** The proof is immediate, since by observing the sequence \( \epsilon_i \)'s \((i = 1, 2, ..., n)\) in Figure 4.4, it is immediately obvious that as the conditions of this proposition are fulfilled, the political efficiency frontier, \( \epsilon_n \), shifts to the right and upward so that each organized interest group benefits from the highest set line \((c(\bar{u}))\), the equilibrium locus of all disagreement payoffs.

**Proposition 4.3** Consider the political-economic system whose group configuration consists of \( n \) organized interest groups and \( g \) interested policymaking centers, each of which is constitutionally vested with the authority of deciding the \( j^{th} \) policy element of the policy vector, \( x_j^0 \), and each organized group has the following policy objective function: \( u_i(x_0) \) for \( i = 1, 2, ..., n \) and \( u_j(x_0) \) for \( j = 1, 2, ..., g \). All policy objective functions are defined over the feasible policy space, \( X_0 \). Then the political-economic equilibrium of this system is obtained by maximizing the following policy governance function with respect to \( x_0 \in X_0 \):

\[
W(x_0) = \sum_{i=1}^{n} B_i u_i(x_0) + \sum_{j=1}^{g} B_j u_j(x_0)
\]

**Proof.** Let

\[
\Gamma (U; t^0) = \prod_{i=1}^{n} [U_i - t^0_i] \prod_{j=1}^{g} [U_j - t^0_j];
\]

then the solution of the polycentric \((n + g)\)-person, simple bargaining game is obtained by maximizing \( \Gamma (U; t^0) \) with respect to \( x_0 \in X_0, \{c_i^0\}, \{c_k^j\} \), and \( \{c_k^j\} \), given that universal reward strategies prevail because only the cooperative solution is considered. Since \( \log \Gamma \) is
monotone increasing in $\Gamma$, maximizing

$$\ln \Gamma (U; t^0) = \sum_{i=1}^{n} \ln \left( U_i - t^0_i \right) + \sum_{j=1}^{g} \ln \left( U_j - t^0_j \right)$$

also maximizes $\Gamma$. Let the outer boundary of $X_0$ be given by $H(x_0) = 0$ and set the Lagrangian expression

$$L = \ln \Gamma (U; t^0) + \lambda H(x_0),$$

where $\lambda$ is a Lagrange multiplier. Assuming an interior solution, the first order conditions for maximum $\ln \Gamma$ are

(4.A.2a) \[ \frac{\partial L}{\partial x_0} = \sum_{i=1}^{n} \frac{1}{(U_i - t^0_i)} \frac{\partial u_i}{\partial x_0} + \sum_{j=1}^{g} \frac{1}{(U_j - t^0_j)} \frac{\partial u_j}{\partial x_0} + \lambda \frac{\partial H(x_0)}{\partial x_0} = 0; \]

(4.A.2b) \[ \frac{\partial L}{\partial c_i^j} = \frac{-1}{(U_i - t^0_i)} + \frac{1}{(U_j - t^0_j)} \frac{\partial s_{ij}(\bar{c}_i^j, \alpha_i^j)}{\partial c_i^j} = 0 \quad i = 1, 2, \ldots, n \]

(4.A.2c) \[ \frac{\partial L}{\partial c_k^j} = \frac{-1}{(U_k - t^0_k)} + \frac{1}{(U_j - t^0_j)} \frac{\partial s_{kj}(\bar{c}_k^j, \alpha_k^j)}{\partial c_k^j} = 0 \quad j = 1, 2, \ldots, g; \quad k, j = 1, 2, \ldots, g \quad k \neq j \]

where $\bar{U}_i$, $\bar{U}_j$ and $\bar{U}_k$ are the values of the corresponding, extended objective functions at the bargaining solution. Treating the values of $\bar{U}_i$, $\bar{U}_j$ and $\bar{U}_k$, too, as given constants, it is easily shown that maximizing $\Gamma (U; t^0)$ (i.e., finding the solution to the $(n+g)$-person, simple bargaining game) is equivalent to maximizing the following political governance function,

(4.A.1) \[ W(x_0) = \sum_{i=1}^{n} B_i u_i(x_0) + \sum_{j=1}^{g} B_j u_j(x_0), \]

with respect to $x_0 \in X_0$ (i.e., $H(x_0) \geq 0$), where

(4.A.3) \[ B_i = \frac{1}{\bar{U}_i - t^0_i} > 0 \quad \text{and} \quad B_j = \frac{1}{\bar{U}_j - t^0_j} > 0 \quad \text{for all } i \text{ and } j. \]
From (4.A.2b) and (4.A.2c), we also have

\[
\frac{B_i}{B_j} = \frac{\partial s_{ij} (c_i^j, \alpha_i^j)}{\partial c_i^j} \quad \text{and} \quad \frac{B_k}{B_j} = \frac{\partial s_{kj} (c_k^j, \alpha_k^j)}{\partial c_k^j} \quad i = 1, 2, \ldots, n
\]

\[
j, k = 1, 2, \ldots, g.
\]

Equation (4.A.4) suggests a power theoretic interpretation of the weights in the political governance function; namely, the ratio, \(B_i/B_j\), is the marginal strength of power of interest group \(i\) over political center \(j\) relative to a marginal increment in interest group \(i\)'s cost of power over policy center \(j\). The ratio, \(B_k/B_j\), is likewise interpreted in terms of the reciprocal power relationship between policy centers. Alternatively, the weight ratios, \(B_i/B_j\) and \(B_k/B_j\), may be interpreted as the corresponding groups' relative gains in the cooperative political-economic equilibrium compared to a conflict situation; i.e., \(B_i/B_j = (\bar{U}_j - t_j^0) / (\bar{U}_i - t_i^0)\) and \(B_k/B_j = (\bar{U}_j - t_j^0) / (\bar{U}_k - t_k^0)\) \((i = 1, 2, \ldots, n; \ j, k = 1, 2, \ldots, g)\). □