Fit Risk: Secondhand Market versus Money-back Guarantee

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Abstract

Consumers face a great fit risk when they purchase products, especially when they purchase experience goods whose quality is unknown and is revealed after consuming. The fit uncertainty can be caused by possible discrepancies among idiosyncratic needs, expectations, and product attributes. This paper analyzes two post-purchase mechanisms on reducing the fit risk: money-back guarantee (MBG) and secondhand market (SHM). We identify key factors affecting the desirability of these two mechanisms from the perspective of retailers and consumers: (a) the perceived value difference between new and used products; (b) the transaction cost in a secondhand market; (c) return costs of an MBG option; (d) the marginal production cost relative to the maximum product benefit; and (e) consumers’ heterogeneity. The lower the value difference, the higher the transaction cost, and/or the lower the relative marginal production cost, the lower the value of a SHM to retailers and consumers. Particularly, we find the following results: (a) A SHM improve the welfare of both retailers and consumers if it has a sufficiently low transaction cost and/or retailers have a relatively high marginal production cost; and (b) providing an MBG is more desirable from the perspective of retailers and consumers if a SHM has a sufficiently high transaction cost and the sum of marginal return costs to consumers and retailers is sufficiently low, or if the sum of marginal return costs equals the transaction cost.

Key words: fit risk, product fit, money-back guarantee, secondhand market, secondhand product

1. Introduction

Consumers face a great risk arise from uncertainty about product quality, durability, performances, sales conditions (Roselius 1971; Heiman, McWilliams, and Zilberman 2001a), especially when they purchase
experience goods whose quality is unknown and is revealed after consuming (Tirole 1988). Fit risk is a loss resulting from the misfit between products and individual idiosyncratic needs, lifestyle, or social feedbacks even though a product is priced reasonably and performs properly and effectively (Heiman, McWilliams, and Zilberman 2001a; Heiman, McWilliams, Zhao, and Zilberman 2002; Jin, Zilberman, and Heiman 2005). Fit risk is an important components of purchase uncertainty. For example, when purchasing clothes, buyers are not completely sure whether the new clothes fit into their daily life and the rest of their wardrobe; children may not like the musical instrument that their parents bought for them; consumers may face uncertainty when they buy a treadmill for their “back to shape” campaign. Justin (1996) revealed that the majority of multimedia sales were returned resulting in only 16% of the sales profitable during the 1995 Christmas season, and there was a significant portion of returns resulting from misfit and complicated installation.

To reduce the fit risk, consumers engage in activities including learning and information acquisition (Nelson 1970; Nelson 1974; Stigler 1963; Cremer and Khalil 1992; Lewis and Sappington 1997; Cremer, Khalil, and Rochet 1998a; Cremer, Khalil, and Rochet 1998b). However, these studies mainly focus on quality risk rather than fit risk. Retailers utilize various mechanisms to allow consumers to gain consumption experience to reduce fit risk ex ante like product demonstrations (Smith and Swinyard 1983; Dipak, Mahajan, and Muller 1995; Heiman, McWilliams, and Zilberman 2001a) or ex post like money-back guarantees (MBGs) (Davis, Gerstner, and Hagerty 1995; Che 1996; Heiman, McWilliams, and Zilberman 2001a; Heiman, McWilliams, Zhao, and Zilberman 2002; Matthews and Persico 2005; Jin, Zilberman, and Heiman 2005). Product demonstrations allow buyers to test products directly before purchase without any obligation. An MBG allows consumers to try the product in their own environment at a convenient time. If they find a misfit for whatever the reason, buyers can return the product and receive either a full or partial refund of the purchase price.1

We argue that consumers can use secondhand markets (SHMs) to dispose their unwanted products due to misfit and, thus reduce the fit risk. It is common for high-end consumers to sell low-end consumers products, or for consumers to trade excessive stock. A rich literature of SHMs, concentrating on the roles of SHMs in improving market efficiency, focus on products whose used ones preserve much of their original price, including automobiles (Purohit 1992; Huang, Yang, and Anderson 2001) and textbooks (Miller 1974; Chevalier and Goolsbee 2005; Ghose, Smith, and Telang 2005). There are at least two evolving features of SHMs since electronic marketplace was introduced: (a) it is more convenient to have transactions. For example, Amazon.com lists new and used products side-by-side on their product pages (See Figure 5(a)); and (b) People use SHMs against fit risk, i.e., they may sell their unwanted products because of misfit with their idiosyncratic needs. For example, one may want to sell a concert ticket in a SHM because of schedule conflicts.2 It is not uncommon to find secondhand product sales listed in a “new” status in Amazon.com

1See Heiman, McWilliams, and Zilberman (2001a) for the discussion of tradeoffs between demonstration and MBGs.
(See Figure 5(b)). As a result, a significant transaction volume are through electronic marketplace (Siegel and Siegel (2004) shows that the proportion of used books sold through the Internet channel increases from 11.3% in 2001 to 54.4%.), and more varieties of products are involved in SHMs including copyrighted products. In the United States, the First Sale Doctrine (17 U.S.C. §109) allows for the resale of copyrighted products such as books and CDs. A significant portion of these transactions are due to misfit and, hence, we argue that SHMs allow consumers against the fit risk. These unwanted products are not necessarily wore out, greatly depreciated, or obsolete. Instead, they could be completely new or only have a gentle use or a few trials. Therefore, the phrase “used products” we use in the rest of this paper refers unwanted products.

Both MBGs and SHMs can serve as a post-purchase mechanism against the fit risk. It is of interest of both retailers and consumers to compare the welfare impact of these two mechanisms. We investigate the following research questions: (a) does and under what conditions will a SHM or an MBG increase the welfare of retailers and/or consumers; and (b) does and under what conditions is it more profitable for retailers to provide an MBG than welcome a SHM? We identify key factors affecting the desirability of these two mechanisms: (a) the perceived value difference between new and used products; (b) transaction costs; (c) the marginal production cost; and (d) consumers’ heterogeneity. Particularly, we find the following results: (a) A SHM improve the welfare of both retailers and consumers if it has a sufficiently low transaction cost and/or retailers have a relative high marginal production cost; and (b) providing an MBG is more desirable from the perspective of retailers if a SHM has a sufficiently high transaction cost while redemptions of MBGs have a sufficiently low transaction cost; (c) when SHMs and MBGs have an equal transaction cost, it is more profitable for retailers to provide an MBG than welcome the SHM.

The rest of this paper is organized as follows. We present the model in section 2 and investigate the welfare impacts of SHMs and MBGs in section 3. We show that a SHM does not necessarily improve consumer welfare, and retailers may benefit from a well-functioning SHM with a low transaction cost. Under certain conditions, an MBG option is more desirable to retailers and consumers than a SHM. The last section presents concluding remarks and market implications.

2. The Model

We assume that retailers are monopolistic, and they have a zero fixed cost and a constant marginal production cost $c$ including the payment to manufacturers or upstream distributors, marginal sale cost, etc. Retailers maximize their profits by choosing the optimal product price and the MBG arrangement given the heterogeneity among consumers.

We assume that there are $N$ consumers, and each individual consumes at most one unit of the product to maximize his/her expected benefit. Before purchasing, an individual consumer knows that his/her product valuation is $x$ if the new product fits needs and zero otherwise. $x$ varies among consumers and follows a

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distribution with the density function \( f(x) \) such that \( \int_{x_{\bar{}}}^{x_{\bar{}}} f(x) dx = 1 \), where \( x_{\bar{}} \) and \( x_{\bar{}} \) are the lower and upper bounds of \( x \). There is a likelihood that the product does not fit the need. We assume the probability of fit is the same for all the consumers that is denoted by \( q \in \{0, 1\} \). Our assumption about consumer heterogeneity differs from Davis, Gerstner, and Hagerty (1995), Che (1996) and Matthews and Persico (2005) in a sense that consumers have different distribution of product valuation in our model.\(^3\) To examine and compare the impacts of MBGs and SHMs, we investigate the following three cases:\(^4\)

- **Case 1, without MBGs and SHMs**: When both SHMs and MBGs are not available, consumers buy either new products or nothing, and buyers can only discard or scrap their unwanted products.

- **Case 2, with MBGs only**: Consumers buy either new products bundled with a MBG option or nothing. If they find a misfit, buyers return the product to retailers and get a refund of the purchase price.

- **Case 3, with SHMs only**: When a SHM exists but MBGs are not available, consumers buy either the new product from retailers, or the used product in the SHM, or nothing. Regardless whether consumers buy the new or used products, they can sell their unwanted product in the SHM.

**Case 1: Without SHMs and MBGs**

We first consider the benchmark case where both SHMs and MBGs are not available. Assuming that retailers sell the new product at \( p_{0} \), an individual buyer will obtain a benefit of \( x - p_{0} \) if it fits his/her idiosyncratic needs; and lose \( p_{0} \) otherwise. Thus, an individual consumer with fit probability \( q \) gains an expected net benefit \( B_{0}(p_{0}) \):

\[
B_{0}(p_{0}) = q(x - p_{0}) + (1 - q)(-p_{0}) = qx - p_{0}.
\]

Consumers will buy the new product if and only if their expected product valuation is greater than the purchase price \( (qx > p_{0}) \). The critical value of product valuation above which consumers will buy the product is a function of \( p_{0} \):

\[
x_{0}(p_{0}) = \frac{p_{0}}{q}.
\]

There is a rich literature on aggregation (see the survey paper by Felipe and Fisher (2003) on production economics and Blundell and Stoker (2005) on macroeconomics). Like Mussa and Rosen (1978) we adopt

\(^3\)Davis, Gerstner, and Hagerty (1995), Che (1996) and Matthews and Persico (2005) assume that even though consumers do not know the product valuation before the purchase is made, their product valuation follows the same distribution that is known to both consumers and producers.

\(^4\)There may exist a case where both MBGs and SHMs are available. We do not discuss it due to the complexity of analytical discussion. However, the comparison among the proposed three cases will provide sufficient insights to our research questions.
the aggregation technique to derive the initial aggregate demand under the one-dimension heterogeneity:

\[ D_0(p_0) = N \int_{x_0(p_0)}^{\pi} f(x)dx. \] (3)

Retailers choose the optimal product price to maximize profits. Assuming a zero fixed cost and a constant marginal production cost \( c \), retailers’ maximum profit is

\[ \pi_0(p^*_0) = \max_{p_0} \{ (p_0 - c) D_0(p_0) \}, \] (4)

where \( p^*_0 \) is the optimal price. \( p^*_0 \) is achieved when the marginal revenue \( MR_0 \) equals the marginal cost \( c \),

\[ MR_0 = p_0 + \frac{D_0(p_0)}{dD_0(p_0)/dp_0} = p_0 - \frac{q \int_{x_0(p_0)}^{\pi} f(x)dx}{f(x_0(p_0))} = c. \] (5)

The expected consumer surplus is written as

\[ CS_0 = \int_{x_0(p^*_0)}^{\pi} B_0(p^*_0)dx = N \int_{x_0(p^*_0)}^{\pi} (qx - p^*_0) f(x)dx = N q \int_{x_0(p^*_0)}^{\pi} x f(x)dx - \int_{x_0(p^*_0)}^{\pi} p^*_0 D_0(p^*_0), \] (6)

where \( D_0(p^*_0) \) is the equilibrium quantity. Equation (7) shows that the aggregate expected consumer surplus is the total expected benefits net of the purchase cost.

**Case 2: With MBGs only**

MBG contracts have two main components, a refund amount and a grace period. These two components vary across products and retailers. The grace period ranges from two weeks to three months. We assume that the grace period is long enough for consumers to identify all possible misfit before the return option expires. The amount of refund can be full or partial (charging “restocking fee or imposing penalty) refund of the purchase price. Since the majority of the current MBG practice offers a full refund of the purchase price in the United States (Heiman, McWilliams, and Zilberman 2001b), we assume a full refund of an MBG option.5 In particular, we assume that retailers sell a product bundled with an MBG option at price \( p_1 \). Consumers keep the product and obtain a net benefit of \( x - p_1 \) if the product fits their needs. Otherwise, they redeem the MBG option and obtain a full refund of the purchase price. The redemptions of MBG options impose costs on consumers which is denoted by \( RC \). Consumers’ return cost \( RC \) depends on factors such as distance traveled to return the product, time spent waiting in line to process the return, finding a convenient time to return the products, etc. For simplicity, we assume a constant and identical \( RC \) among consumers. To exclude the case such that buyers do not return their unwanted products because the refund cannot cover

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5 Davis, Hagerty, and Gerstner (1998) investigate the optimal hassle cost when offering an MBG; and Matthews and Persico (2005) discuss whether an MBG with a full refund is excessive.
their return cost, we assume $RC < p_1$. An individual consumer with fit probability $q$ obtains an expected net benefit of buying the product $B_1(p_1)$:

$$B_1(p_1) = q(x - p_1) + (1 - q)(-RC). \quad (7)$$

Consumers will buy the product if and only if $B_1(p_1) \geq 0$. The critical value of product valuation above which consumers will buy the product is a function of $p_1$,

$$x_1(p_1) = p_1 + \frac{1 - q}{q} RC. \quad (8)$$

The initial demand hence is

$$D_1(p_1) = N \int_{x_1(p_1)}^{x} f(x) \, dx. \quad (9)$$

In this case, both consumers and retailers can recycle unwanted products. That is, consumers can return their unwanted products to the retailers, and their expected amount of returns is $(1 - q)D_1(p_1)$. The majority of current practice in the supply chain is that retailers can return unsold products to manufacturers or upstream distributors under the so-called buyback policy (Choi, Li, and Yan 2004). Unlike Matthews and Persico (2005) we assume that retailers can recycle all the returned products, i.e., retailers can return all the returned product to the manufacturer or upstream distributor and obtain a full refund. Thus, the nominal salvage value of returned product to retailers equals the marginal production cost, and the net salvage value is the difference between the marginal production cost and the return cost. The assumption of recycling returned products does not affect the main results since we can always adjust retailers’ return cost to account for the salvage value of the partial refund case. For each unit of initially sold products, retailers obtain a profit of $p_1 - c$ for a fit and bear a constant return cost $R$ if it does not fits needs. Therefore, the expected profit per unit of initial sold product is $q(p_1 - c) - (1 - q)R$, and retailers’ maximum profit is:

$$\pi_1(p_1^*) = \max_{p_1} \{ [q(p_1 - c) - (1 - q)R] D_1(p_1) \}, \quad (10)$$

where $p_1^*$ is the optimal product price. $p_1^*$ is achieved when the marginal revenue $MR_1$ equals the marginal cost $MC_1$, where $MC_1$ consists of the marginal production cost and the adjusted return cost:

$$MR_1 = p_1 + \frac{D_1(p_1)}{dD_1(p_1)/dp_1} = p_1 - \frac{\int_{x_1(p_1)}^{x} f(x) \, dx}{f(x_1(p_1))} = c + \frac{1 - q}{q} R = MC_1. \quad (11)$$

6There are studies in the supply chain literature investigating responsibility of the unsold inventory between manufacturers or upstream distributors and retailers and obtain a full refund (Spendler 1950; Pasternack 1985; Padmanabhan and Png 1995; Kandel 1996; Marvel and Peck 1995; Emmons and Gilbert 1998; Lau and Lau 1999; Lau, Lau, and Willett 2000; Donohue 2000; Webster and Weng 2000; Choi, Li, and Yan 2004). The common practice is that retailers can return any unsold product the the manufacturer or upstream distributor under the so-called buyback policy (return policy).
The profitability of an MBG increases as the return cost declines \( \left( \frac{d\pi}{dq}(p_1) < 0 \right) \) or the perceived fit probability goes up \( \left( \frac{d\pi}{dR}(p_1) < 0 \right) \). The expected consumer surplus is

\[
CS_1 = \int_{x_1(p_1^*)}^{\pi} B_1(p_1^*)dx = Nq \int_{x_1(p_1^*)}^{\pi} x f(x)dx - \text{Expected purchase costs} - \text{Expected return costs}.
\]

Equation (13) shows that the expected consumer surplus is the total expected product valuation minus the sum of the expected purchase cost and the expected return costs.

**Case 3: With SHMs only**

The relationship between new and used markets was first discussed by Benjamin and Koramendi (1974) who argue that monopolists can maintain their marketing power by restricting used markets. Miller (1974) show that publishers and authors are not necessarily better off by killing off markets for used textbooks. Bulow (1982) analyze the difference between the optimal level of quality chosen in monopolistic and competitive markets for new and used products. Waldman (1996a) investigate the degree to which the price monopolists can charge for new goods are constrained by the prices in the secondhand market. However, in these studies consumers sell products mainly because these products are wore out, greatly depreciated, or obsolete. Focusing on fit risk, we model SHMs as a mechanism to dispose unwanted products because of misfit. These unwanted products can be completely new or only have gentle use or a few trials, therefore comparable with new products. Let \( p_n^s \) and \( p_s^2 \) denote the prices of new and used products, respectively. Figure 1 summarizes the corresponding payoffs of purchase alternatives (buy new products, used ones, or nothing).

<table>
<thead>
<tr>
<th>Consumers</th>
<th>Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not buy</td>
<td>0</td>
</tr>
<tr>
<td>Buy new</td>
<td>( x - p_n^2 )</td>
</tr>
<tr>
<td>Fit</td>
<td>( x - p_n^2 )</td>
</tr>
<tr>
<td>No fit</td>
<td>( -p_n^2 + p_s^2 - T_s )</td>
</tr>
<tr>
<td>Buy used</td>
<td>( \beta x - p_s - T_b )</td>
</tr>
<tr>
<td>Fit</td>
<td>( \beta x - p_s - T_b )</td>
</tr>
<tr>
<td>No fit</td>
<td>( -T_s - T_b )</td>
</tr>
</tbody>
</table>

Figure 1: Consumers’ purchase alternatives and payoffs

Under the first purchase choice, consumers buy nothing and obtain a zero benefit. Under the second purchase choice, consumers buy the new product from retailers at \( p_n^2 \). If the product fits their personal needs, they obtain a benefit of \( x - p_n^2 \); otherwise, they discover a misfit and sell the unwanted product in the SHM. Sellers receive the used product price \( p_s^2 \) at a cost of \( T_s \). \( T_s \) represents time and efforts that consumers...
spend when they sell the used products, and it also embodies the value that consumers could get if they had perfect information about the SHM. Because buyers of the new product will not sell their unwanted products if the sale price cannot cover the transaction cost, we assume $T_s < p_s^s$. Hence, buyers of new products obtain a net benefit of $-p_n^n + p_s^s - T_s$ if the product does not fit their needs. The expected net benefit of buying the new product is

$$B_n(p_n^n, p_s^s) = q(x - p_n^n) + (1 - q)(-p_n^n + p_s^s - T_s).$$

(13)

Solving $B_n(p_n^n, p_s^s) \geq 0$ yields the critical value of product valuation above which consumers have a higher expected net benefit from buying the new product than not buying at all,

$$x_n^*(p_n^n, p_s^s) = \frac{p_n^n - (1 - q)(p_s^s - T_s)}{q}.$$

(14)

Finally, under the third purchase alternative, consumers purchase the used product at $p_s^s$. Consumers perceive a value difference between the new and used products. They place a relative low value on used products because of the following possible reasons: (a) A product may lose value to consumers simply because it is not new any more (a brand new car loses its value at the moment it is sold); and (b) a used product may not be comparable with a new one even with a gentle use or a few trials. Because we focus on fit risk, the newness effect, i.e., consumers value the new products more simply because they are new, is an essential factor for the value difference. We assume that consumers perceive a value of $\beta x$ if the used product fits their needs, where $\beta \in [0, 1]$ is a valuation ratio of used products relative to new products and $(1 - \beta)$ is the depreciation ratio.\(^7\) When consumers buy the used product, they bear a search cost of obtaining information of product price and characteristics. Because consumers will not buy the used product if the sum of the used product price and the search costs is higher than the new product price, we assume $T_b < p_n^n - p_s^s$. If used product buyers discover a misfit, they sell the product in the SHM and receive the purchase price $p_s^s$, and their loss is the transaction cost $TT$ that is the sum of the search cost $T_b$ and the resale cost $T_s$. The expected net benefit of buying a used product is

$$B_s(p_n^n, p_s^s) = q(\beta x - p_s^s - T_b) + (1 - q)(-TT).$$

(15)

Solving $B_s(p_n^n, p_s^s) \geq 0$ yields the critical value of product valuation above which consumers are better off buying the used product than not buying at all,

$$x_s^*(p_n^n, p_s^s) = \frac{qp_s^s + (1 - q)T_s + T_b}{\beta q}.$$

(16)

\(^7\)The value of depreciation ratio of secondhand products is affected by product durability, the rate at which new products are introduced ($\beta$ decreases with a higher product turnover), related technological progress rate (personal computers have a low value of $\beta$, despite its relatively high reliability), and seasonality (used costumes have a high $\beta$ as events such as Halloween are approaching). Focusing on fit risk, we argue that consumers sell their unwanted products in the SHM because the product does not fit their needs, and these products are likely comparable with the new ones. Hence, the physical depreciation of the product is not a critical issue, and used products lose their value relative to the new product mainly because they are not new any more.

8
All prospective consumers with $B_n(p^n_2, p^n_2) \geq B_s(p^n_2, p^n_2)$ would prefer buying the new product over the used one, which results in the critical value of product valuation above which consumers receive a higher expected net benefit from buying the new product than the used one:

$$x^{ns}_2(p^n_2, p^n_2) = \frac{p^n_2 - p^n_s - T_b}{(1 - \beta)q}. \quad (17)$$

Figure 2 illustrates the range of product valuations of market segments. The horizontal intercepts determine the critical values of product valuation above which consumers are better off purchasing the product than nothing: $x^{ns}_2(p^n_2, p^n_2)$ for the new product and $x^{ns}_2(p^n_2, p^n_2)$ for the used product. The market segments are characterized below: (a) Those with high product valuations $x \geq x^{ns}_2(p^n_2, p^n_2)$ will buy the new product since $B^n_2(p^n_2, p^n_2) \geq \max \{B^n_2(p^n_2, p^n_2), 0\}$; (b) those with intermediate product valuations $x^s_2(p^n_2, p^n_2) < x < x^{ns}_2(p^n_2, p^n_2)$ will buy the used product since $B^n_2(p^n_2, p^n_2) > \max \{B^n_2(p^n_2, p^n_2), 0\}$; and (c) those with low product valuations $x < x^s_2(p^n_2, p^n_2)$ will buy nothing. No one will buy the used product when $x^s_2(p^n_2, p^n_2) > x^{ns}_2(p^n_2, p^n_2)$, which is not an interesting case. Thus, to ensure that $x^s_2(p^n_2, p^n_2) < x^{ns}_2(p^n_2, p^n_2)$ holds, the valuation ratio of used products $\beta$ has to be greater than $\beta_{min}$:

$$\beta_{min} = \frac{p^n_2 - (1 - q)(p^n_s - T_s) + T_b}{p^n_2 - (1 - q)(p^n_s - T_s)}. \quad (18)$$

Since a low $\beta$ suggests a great value depreciation if consumers buy the used products, no one want to buy the used product if $\beta$ is sufficiently low. To ensure the demand of the used product, $\beta > \beta_{min}$ must hold.

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**Figure 2:** Market segments of new and used products when a SHM exists

A SHM creates a market segmentation, and consumers self-select into new and used product buyers according to their product valuations. The initial demand of new and used products, $D^n_2(p^n_2, p^n_2)$ and $D^s_2(p^n_2, p^n_2)$, are given below:

$$D^n_2(p^n_2, p^n_2) = N \int_{x^{ns}_2(p^n_2, p^n_2)}^{\pi} f(x)dx, \quad (19a)$$

$$D^s_2(p^n_2, p^n_2) = N \int_{x^s_2(p^n_2, p^n_2)}^{x^{ns}_2(p^n_2, p^n_2)} f(x)dx. \quad (19b)$$
Although a product can be resold in the SHM many times, we assume it loses value only once and the loss is the perceived difference of product valuation between the new and used product. Since buyers will sell the products in the SHM if they find a misfit, the supply of the used product $S_2(p_{n2}^*, p_{s2}^*)$ resulting from misfit equals the expected amount of resale among new and used product buyers:

$$S_2(p_{n2}^*, p_{s2}^*) = (1 - q) D_{n2}^N(p_{n2}^*, p_{s2}^*) + (1 - q) D_{s2}^U(p_{n2}^*, p_{s2}^*) = N (1 - q) \int_{x_{s2}^2(p_{n2}^*, p_{s2}^*)}^{x_{n2}^2(p_{n2}^*, p_{s2}^*)} f(x) dx.$$  \hspace{1cm} (20)

Consumers’ fit probability is $q$ regardless of whether they buy new or used products. The actual amounts of resale of both the new and used products are random; hence, quantity available in the SHM is a random variable. Thereafter, the market-clearing price varies instantaneously. We assume an expected equilibrium, i.e., the used product price is achieved when the demand equals the expected supply:

$$S_2(p_{n2}^*, p_{s2}^*) = D^N_{n2}(p_{n2}^*, p_{s2}^*) \Rightarrow (1 - q) \int_{x_{n2}^*(p_{n2}^*, p_{s2}^*)}^{x_{n2}^2(p_{n2}^*, p_{s2}^*)} f(x) dx = q \int_{x_{s2}^*(p_{n2}^*, p_{s2}^*)}^{x_{s2}^2(p_{n2}^*, p_{s2}^*)} f(x) dx.$$  \hspace{1cm} (21)

$p_{n2}^*$ and $p_{s2}^*$ are consistent with the expected market equilibrium. The actual prices may deviate from $p_{n2}^*$ and $p_{s2}^*$. Retailers will respond to the use product price since rational consumers will consider the salvage value (which is the used product price net of the resale cost) when they purchase the product. Differentiating equation (22) with respect to $p_{n2}^*$ and $p_{s2}^*$ yields a price response function between the new and used products,

$$\frac{dp_{n2}^*}{dp_{s2}^*} = 1 + \frac{(1 - \beta)q^2}{\beta} \frac{f(x_{n2}^*)}{f(x_{s2}^*)}.$$  \hspace{1cm} (22)

Equation (23) shows that the level of the price responsiveness depends on the valuation ratio $\beta$, the fit probability $q$, and the probability of consumers at the margin, $f(x_{n2}^*)$ and $f(x_{s2}^*)$. Any change in the used product price results in at least the same amount of change in the new product price since $\left(\frac{dp_{n2}^*}{dp_{s2}^*} > 1\right)$. If product valuation is uniformly distributed, equation (23) becomes

$$\frac{dp_{n2}^*}{dp_{s2}^*} = 1 + \frac{(1 - \beta)q^2}{\beta}.$$  \hspace{1cm} (23)

Equation (24) suggests the price difference becomes wider by $\left(\frac{1 - \beta}{\beta}q^2\right)$ units if there is an one-unit increase in the used product price. Furthermore, the price different between the new and used products becomes larger when consumers have a high fit probability $\left(\frac{d(dp_{n2}^*/dp_{s2}^*)}{dq} > 0\right)$ or a higher depreciation ratio of the used products $\left(\frac{d(dp_{s2}^*/dp_{n2}^*)}{d\beta} < 0\right)$.

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8 However, since SHMs play various roles, there are multiple supply sources in SHMs, including those that are worn out, greatly depreciated, or obsolete and those that are relatively comparable to new products but sold because of misfit.
Retailers choose the new product price $p_2^*$ to maximize their profit, and their profit is written as

$$\pi_2^*(p_2^*, p_2^{**}) = \max_{p_2^*} \left\{ (p_2^* - c) D_2^*(p_2^*, p_2^{**}) \right\}, \quad (24)$$

where $p_2^*$ and $p_2^{**}$ are the optimal prices of the new and used products. $p_2^{**}$ is achieved when the marginal revenue $MR_2$ equals the marginal cost $c$:

$$MR_2 = p_2^{**} + \frac{D_2^*(p_2^*, p_2^{**})}{dp_2^*} = p_2^{**} - (1 - \beta) \left( \frac{dp_2^*}{dp_2^*} - 1 \right) \frac{q \int_{x_2^{**}}^{x_2^{*}} f(x)dx}{f(x_2^{**})} = c. \quad (25)$$

Substituting equation (23) into equation (26) yields

$$p_2^{**} - \int_{x_2^{**}}^{x_2^{*}} f(x)dx \left[ (1 - \beta)q f(x_2^{**}) + \beta \frac{q f(x_2^{**})}{f(x_2^{**})} \right] = c. \quad (26)$$

Solving equations (22) and (27) simultaneously yields the optimal prices of new and used products. Thereafter, we calculate the expected welfare of consumers who buy either new or used products,

$$CS_2 = \int_{x_2^{**}}^{x_2^{*}} B_n(p_2^*, p_2^{**}) f(x)dx + \int_{x_2^{**}}^{x_2^{*}} B_s(p_2^*, p_2^{**}) f(x)dx,$$

$$= Nq \int_{x_2^{**}}^{x_2^{*}} f(x)dx + N\beta q \int_{x_2^{**}}^{x_2^{*}} f(x)dx + (1 - q)(p_2^{**} - T_s)(D_2^{**} + D_2^{**}) - [p_2^{**}D_2^{**} + (p_2^{**} + T_s)D_2^{**}],$$

where $D_2^{**} = D_2^*(p_2^*, p_2^{**})$ and $D_2^{**} = D_2^*(p_2^*, p_2^{**})$ are the initial equilibrium quantity of new and used products, respectively. The total expected consumer surplus consists of the expected product valuation and the net expected resale benefits subtracting the total purchase costs.

Both MBGs and SHMs provide an insurance against the fit risk. Redemptions of MBGs impose return costs on retailers and consumers. If the sum of the marginal return costs is higher than the marginal production cost, it is less profitable for retailers to provide MBGs. Sellers of used products bear a resale cost, and buyers incur a search cost of finding price and characteristic information of used products. A high resale cost decreases consumers’ willingness to sell; and a high search cost discourages consumers from buying used products. A sufficiently high transaction cost results in a decrease in retail profits, or even eliminates the role of SHMs on reducing fit risk.

Define the profit difference between with MBGs and with SHMs: $\Delta \pi = \pi_1(p_2^*) - \pi_2^*(p_2^{**}, p_2^{**})$.

**Proposition 1:** The profit of an MBG relative to a SHM increases as the marginal return cost of MBGs decreases ($\frac{d\Delta \pi}{dR} < 0$ and $\frac{d\pi_2}{dR} < 0$); the transaction cost increases ($\frac{d\Delta \pi}{dT} > 0$ and $\frac{d\pi_2}{dT} > 0$); and/or the perceived value difference increases ($\frac{d\Delta \pi}{d\beta} > 0$ when $\beta > \frac{qa_2^{**} + (1-q)T_s + T_b}{a_2^{**} + (1-q)T_s}$).
Proof: See Appendix A.

Our intuition for Proposition 1 are given below: (a) a higher return cost truncates consumers with lower product valuations from the buyer segment. Hence, the initial demand decreases, which may lead to a lower retail profits;\(^9\) (b) A higher transaction cost reduces the value of a SHM to retailers; and (c) When the product valuation ratio \(\beta\) is higher, some new product buyers may switch to the used products, which may lower retail profits. However, a high \(\beta\) may increase the used product price, which will likely result in an increase in the new product price and, hence, retail profits increase. If \(\beta > \frac{q p_2^*+(1-q)T_s+T_b}{p_2^*+(1-q)T_s}\) is satisfied, an increase in \(\beta\) will make MBGs more attractive to retailers. Proposition 1 implies that retailers are more likely to provide an MBG to restraint the competition from a SHM if the marginal return cost is sufficiently low, the transaction cost is sufficiently high, and/or depreciation rate of used products is sufficiently low.

3. Impacts of SHMs and MBGs when Product Valuation is Uniformly Distributed

To obtain further insights about the impacts of MBGs and SHMs, we make one more assumption that consumers’ product valuation \(x\) is uniformly distributed from \(x_0\) to \(x_3\). Define \(M_i\) for \(i = 0, 1, 2\) as the maximum expected social benefit for each unit of product sold under each case \(i\). When retailers provide MBGs, \(M_1\) equals the maximum expected benefit net of the production cost and the expected return costs. When a SHM exists, the expected maximum benefit consists of \(q(x - c)\) if the new product fits needs, and \(\beta(1-q)(x - c)\) if it does not fit needs and is sold in the SHM, subtracting the adjusted transaction cost and the marginal production cost. Therefore, we obtain \(M_i\) below:

\[
M_i = \begin{cases} 
M_0 = q(x - c) & \text{without MBGs and SHMs}, \\
M_1 = q(x - c) - (1-q)(R + RC) & \text{with MBGs only, and} \\
M_2 = [q + \beta(1-q)]x - \frac{1-q}{q}TT - c & \text{with SHMs only.}
\end{cases}
\]  

(28)

Table 1 presents the equilibria under three cases and shows that the difference of welfare distribution largely depends on the maximum social benefit per unit of product produced \(M_i\).

3.1. Outcomes when an MBG is provided and a SHM does not exist

Jin, Zilberman, and Heiman (2005) investigate the welfare impacts of an MBG in great detail. One of their findings when consumers share an identical fit probability but differ by their uniformly distributed product valuation is summarized in the following proposition.

\(^9\)We assume that consumers will return the product if they find a misfit and the return probability is \(1-q\) as long as consumers’ return cost is smaller than the product price. Thus, consumers’ return cost will not directly affect the return rate.
Table 1: Summary of equilibria under different cases

<table>
<thead>
<tr>
<th>Cases</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3 (^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SHM</td>
<td>absent</td>
<td>absent</td>
<td>exists</td>
</tr>
<tr>
<td>MBG</td>
<td>absent</td>
<td>provided</td>
<td>absent</td>
</tr>
<tr>
<td>New product price</td>
<td>(p^*_0 = \frac{1}{2}(M_0 + 2c))</td>
<td>(p^*_1 = \frac{1}{2q}M_1 + c + \frac{1-q}{q}R)</td>
<td>(p^*_2 = \frac{1}{2}(M_2 + 2c))</td>
</tr>
<tr>
<td>Used product price</td>
<td>/</td>
<td>/</td>
<td>(p^**_2 = \frac{\beta}{2q}[c + (2\gamma - \beta - q + \beta q)x])</td>
</tr>
<tr>
<td>New product quantity</td>
<td>(D^*_0 = \frac{1}{2\Delta q}M_0)</td>
<td>(D^*_1 = \frac{1}{2\Delta q}M_1)</td>
<td>(D^*_2 = \frac{q}{2\Delta q}M_2)</td>
</tr>
<tr>
<td>Used product quantity</td>
<td>/</td>
<td>/</td>
<td>(D^**_2 = \frac{1-q}{2\Delta q}M_2)</td>
</tr>
<tr>
<td>Expected retail profit</td>
<td>(\pi_0 = \frac{1}{4\Delta q}M^2_0)</td>
<td>(\pi_1 = \frac{1}{4\Delta q}M^2_1)</td>
<td>(\pi_n = \frac{q}{4\Delta q}M^2_2)</td>
</tr>
<tr>
<td>Expected consumer surplus</td>
<td>(CS_0 = \frac{1}{4\Delta q}M^2_0)</td>
<td>(CS_1 = \frac{1}{4\Delta q}M^2_1)</td>
<td>(CS_2 = \frac{q}{8\Delta q}M^2_2)</td>
</tr>
<tr>
<td>Expected social welfare</td>
<td>(SW_0 = \frac{3}{8\Delta q}M^2_0)</td>
<td>(SW_1 = \frac{3}{8\Delta q}M^2_1)</td>
<td>(SW_2 = \frac{3q}{8\Delta q}M^2_2)</td>
</tr>
</tbody>
</table>

\(^a\) \(\lambda = \beta + (1 - \beta)q^2\) and \(\Delta = \pi - \bar{\pi}\).

**Proposition 2:** An MBG increases both retail profits and consumer welfare if the sum of marginal return cost to consumers and retailers is lower than the marginal production cost \((R + RC < c)\). Otherwise, an MBG reduces the welfare of both retailers and consumers if \(R + RC > c\).

**Proof:** Solving \(\pi_1 > \pi_0\) and \(CS_1 > CS_0\) shown in Table 1 results in the above results.

Proposition 2 shows that the privately optimal MBG arrangement is also socially optimal. An increase in the return cost and/or a decrease in the marginal product cost reduces the value of an MBG to retailers and society as well.

### 3.2. Outcomes when a SHM exists and an MBG option is not available

A SHM will not exist if it lacks the demand or supply. On the demand side, consumers are less likely to buy the used product if they perceive a high value loss, and/or it is very costly to find used products. Therefore, \(\beta > \beta_{\text{min}}\) has to be satisfied. On the supply side, buyers will not sell the product if the used product price cannot cover the resale cost. Therefore, the amount of transactions in the SHM may decline or even drop to zero with a sufficiently low valuation ratio and/or a sufficiently high transaction cost. To have both \(\beta > \beta_{\text{min}}\) and \(p^*_2 > T_s\) hold, the transaction cost has to be lower than \(TT_0\) such that

\[
TT_0 = \beta q\bar{\pi} + \frac{\beta q(q\bar{\pi} - c)}{\beta + \beta q + 2(1 - \beta)q^2}.
\]  

(29)
Define \( c_x \) as a ratio of the marginal production cost and the maximum product valuation. Namely, \( c_x \) is the relative marginal production cost.

**Proposition 3:** A SHM exists only if (a) the transaction cost is sufficiently low \((0 < TT < TT_0)\); and (b) the relative marginal production cost is sufficiently high \((c_x > (1 - \beta)(1 - q)(1 + 2q) - 1)\).

**Proof:** See Appendix B.

If the relative marginal production cost is sufficiently low, it may not be worthwhile to recycle the products, and the value of a SHM declines. A sufficiently low \( c_x \) may eliminate the SHM on reducing fit risk. On the other hand, a sufficiently high transaction cost will kill off the SHM on its role of reducing fit risk since it is not worthwhile to buy or sell the used products. Proposition 3 suggests that the ratio of the marginal production cost and the maximum product valuation has to be sufficiently high, and the transaction cost has to be sufficiently low to have a SHM play its role on reducing fit risk.

We discuss the impacts of a SHM on the price, equilibrium quantity, and welfare distribution in Propositions 4, 5, and 6, respectively.

☐ **The effects on the new product price**

**Proposition 4 (price impacts):** A SHM increases the new product price. The price gain increases as the transaction cost decreases \( \left( \frac{d(p_n^* - p_0^*)}{dT} < 0 \right) \), and/or the perceived depreciation ratio decreases \( \left( \frac{d(p_s^* - p_0^*)}{d\beta} > 0 \right) \).

**Proof:** See Appendix C.

Since rational consumers anticipate the resale value of their products if a SHM exists, retailers can charge a higher price (Miller 1974; Swan 1980). However, the substitution between new and used products may drive the new product price lower (Waldman 1997; Hendel and Lizzeri 1999). According to equations (3) and (15), the new product price with and without a SHM are given

\[
p_0 = qx_0 \quad \text{without a SHM,} \tag{30a}
\]

\[
p_2^n = qx_2^n + (1 - q)(p_s^n - T_s) \quad \text{with a SHM,} \tag{30b}
\]

where \( x_0 \) and \( x_2^n \) are the production valuation of the marginal buyers who are indifferent to buying the new product or nothing at all. The price difference is \( p_2^n - p_0 = q(x_2^n - x_0) + (1 - q)(p_s^n - T_s) \). \((p_s^n - T_s)\) is positive since consumers will not sell their unwanted products if the product price cannot recover their resale cost. If indeed a SHM decreases the initial equilibrium quantity of the new product, i.e. \( x_2^n > x_0 \), new product price increases. If \( x_2^n < x_0 \), the tradeoff between \( q(x_2^n - x_0) \) and \( p_s^n - T_s \) determines whether

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the new product price increases. Proposition 4 suggests that regardless whether a SHM increase or decrease the initial equilibrium quantity of the new product, the price gain from a SHM dominates, and retailers always charge a higher price when a SHM exists. Furthermore, the price gain increases when there is an decrease in the transaction cost or the depreciation ratio \(\left(\frac{d(p^*_n - p^*_0)}{dT} < 0\right.\) and \(\left.\frac{d(p^*_n - p^*_0)}{d\beta} > 0\right)\). The reason that \(\frac{d(p^*_n - p^*_0)}{d\beta} > 0\) holds is given below: Consumers demand more used products when they perceive a smaller valuation depreciation (a high \(\beta\)) and, thus, the used product price may go up. According to the price response function written in equation (24), if the used product price increases by one unit, the new product price will increase by \(1 + \frac{1-\beta}{\beta}q^2\) units. Therefore, the price difference goes up by \(\frac{1-\beta}{\beta}q^2\) units, and retailers charge a much higher price.

□ The effects on equilibrium quantity

Proposition 5 (quantity impacts): A SHM increases the initial total equilibrium quantity of new and used products. It decreases the initial equilibrium quantity of the new product \(D_2^n\) when the relative marginal production cost is sufficiently small \(\left(\frac{c}{x} < \frac{q}{1+q}\right)\); and increases \(D_2^u\) when \(\frac{c}{x} > \frac{q}{1+q}\) and \(0 < TT < TT_q\), where

\[
TT_q = \beta \left[ \frac{1 + q}{q} c - x \right].
\]

Proof: See Appendix D.

The introduction of a SHM will affect consumer segments: Consumers who cannot afford the new product may purchase the used one at a lower price; those with relative low product valuations who buy the new product may switch to the used product; and those who do not buy if there is no option to sell in the SHM may purchase the new product. Figure 3 identifies the range of product valuations for market segments. The two upper plots show that only consumers whose product valuation is higher than \(x_0(p_0^n) = \frac{p^*_0}{q}\) will buy the product when a SHM does not exist. The two bottom plots show that a SHM creates a market segmentation: Consumers whose product valuation is greater than \(x_{ns}(p^*_n, p^*_s)\) will purchase the new product; those with intermediate product valuations \(x^*_2(p^*_n, p^*_s) < x < x^*_2(p^*_n, p^*_s)\) will buy the used product; and the remaining consumers will not buy at all. One scenario in the right panel of Figure 3 shows that a SHM decreases the equilibrium quantity of the new product when the relative marginal production cost is sufficiently small \(\left(\frac{c}{x} < \frac{q}{1+q}\right)\) since consumers whose product valuation is between \(x^*_2(p^*_n, p^*_s)\) and \(x_0(p_0^n)\) switch to the used product. On the other hand, when the relative marginal production cost is sufficiently high \(\left(\frac{c}{x} > \frac{q}{1+q}\right)\), retailers may gain consumers with product valuations \(x^*_2(p^*_n, p^*_s) < x < x_0(p_0^n)\) if the transaction cost is sufficiently low \(0 < TT < TT_q\) (the left pair in Figure 3), or loss buyers if the transaction cost increases above \(TT_q\) (\(TT_0 > TT > TT_q\)) as shown in the right pair of Figure 3. Proposition 5 suggests
that a SHM likely decreases the initial equilibrium quantity of the new product with a low relative marginal production cost and/or a high transaction cost.

□ The effects on retail profits and consumer welfare

**Proposition 6 (welfare impacts):** A SHM decreases retail profits and consumer surplus regardless of the transaction cost when the relative marginal production cost is sufficiently low \( \frac{c}{p} < q \left( 1 - \frac{\beta(1-q)}{\sqrt{\beta + (1-\beta)q^2-q}} \right) \).

However, a SHM benefits both retailers and consumers if \( \frac{c}{p} > q \left( 1 - \frac{\beta(1-q)}{\sqrt{\beta + (1-\beta)q^2-q}} \right) \) and \( 0 \leq TT < TT_n \), where

\[
TT_n = \beta q \bar{x} - \frac{\sqrt{\beta + (1-\beta)q^2-q}}{1-q} (q\bar{x} - c) .
\]

**Proof:** See Appendix E.

On the retailers’ side, a SHM may induce a greater demand of the new product since it provides an insurance, or cause a shrink in the demand because of the competition from the used products. On the consumers’ side, three groups of consumers may benefit from a SHM: those who afford used products but not the new ones, those who switch to used products, and those who find a misfit and sell their unwanted products. However, buyers of new products may lose surplus since they may pay a higher price; buyers of the used product bear a search cost of price and product information; and new product buyers who find a misfit bear a cost to sell their unwanted products. Whether a SHM improves retail profits and consumer welfare depends on the relative marginal production cost and the transaction cost. Proposition 6 suggests the following results: (a) A SHM with a high transaction cost will hurt retailers; and (b) A SHM increases both the welfare of both retailers and consumers if the relative marginal product cost is sufficiently high, \( \frac{c}{p} > q \left( 1 - \frac{\beta(1-q)}{\sqrt{\beta + (1-\beta)q^2-q}} \right) \), and the transaction cost is sufficiently low, \( 0 < TT < TT_n \).
Figure 4 provides a graphic summary about the impacts of a SHM on the equilibrium outcome presented in Propositions 4, 5, and 6. A SHM increases the new product price, and it also has the following impacts on the equilibrium outcome that are summarized according to the critical value of the relative marginal production cost:

- **Low relative marginal production cost**, $\frac{c}{\bar{c}} < q \left(1 - \frac{\beta(1-q)}{\sqrt{\beta(1-\beta)q^2}}\right)$: A SHM increases the initial equilibrium quantity of the new product and decreases the welfare of both retailers and consumers.

- **Intermediate relative marginal production cost**, $\frac{q}{1+q} > \frac{c}{\bar{c}} > q \left(1 - \frac{\beta(1-q)}{\sqrt{\beta(1-\beta)q^2}}\right)$: A SHM decreases the initial equilibrium quantity of the new product, and it improves (decreases) the welfare of retailers and consumers if $0 < TT < TT_0$ ($TT_0 > TT > TT_\pi$).

- **High relative marginal production cost**, $\frac{c}{\bar{c}} > \frac{q}{1+q}$: SHM increases the initial equilibrium quantity of the new product if $0 \leq TT < TT_\pi$, and increases both retailer profits and consumer surplus if $0 \leq TT < TT_\pi$.

![Figure 4: Impacts of a SHM on market equilibrium and welfare](image)

Our results show that whether a SHM will hurt retailers or not depends on the value of the transaction cost, the depreciation ratio, the fit probability, and the distribution of product valuation. Retailers may have an incentive to restrain the competition from the SHM if it decreases retail profits, or to lower the transaction cost to facilitate the transaction in the SHM otherwise. The detailed discussion about retailers’ strategies to facilitate transactions or interfere with the SHM is presented in section 4.
3.3. SHMs versus MBGs

There are similarities and differences of SHMs and MBGs in their role on reducing the fit risk:

- Both of them recycle unwanted products. That is, consumers return their unwanted products to retailers when an MBG is available, or consumers sell their unwanted products in the SHM. Therefore, both MBGs and SHMs provide an insurance against the fit risk to consumers. However, there is a key difference in terms of unwanted products due to misfit: retailers cannot directly control the competition between the new and used products if a SHM exists. However, they are in control of returned products if they provide an MBG option.

- Both redemption of an MBG option and transactions in the SHM are costly, i.e. return costs to both consumers and retailers in the case of an MBG, and a search cost to buyers and a resale cost to sellers of the used product. However, there is a key difference of these cost components: Retailers share redemption costs of an MBG, but do not explicitly share the transaction cost of the SHM.

Figure 4 shows that a SHM with a sufficiently high transaction cost ($TT > \max\{0, TT\pi\}$) hurts retailers and, thus, retailers may have an incentive to restrain the competition from the SHM by providing an MBG option. On the other hand, even if a SHM increases retail profits when $\frac{x}{\bar{x}} > q \left( 1 - \frac{\beta(1-q)}{\sqrt{\beta + (1-\beta)q^2 - q}} \right)$ and $0 < TT < TT\pi$ (see Proposition 6), it is of retailers’ interest to find out whether an MBG is more profitable.

To make an MBG an effective tool to reduce the product supply in the SHM, we make the following two assumptions: (a) An MBG option has a sufficiently long grace period for consumers to discover all the possible misfit between the product and their idiosyncratic needs; and (b) $p_1 - RC > p_s^x - T_s$ is satisfied, which implies that consumers have a higher gain if they return the product to retailers rather than sell the product in a SHM. Comparing retail profits and consumer surplus when either MBGs or SHMs are available yields the critical value of $TT$ above which an MBG option is more desirable:

$$TT_{MBG} = \sqrt{\beta + (1-\beta)q^2(R + RC)} - (1 - \beta)q\bar{x} - \frac{q}{1-q}(\bar{x} - c) \left( \sqrt{\beta + (1-\beta)q^2} - 1 \right).$$ (33)

**Proposition 7 (MBGs versus SHMs):**

- An MBG is more desirable to both retailers and consumers when the transaction cost is sufficiently high ($\max\{0, TT_{MBG}\} < TT < TT_0$) and the sum of marginal return costs is lower than the marginal production cost ($c > R + RC$), or the sum of marginal return costs equals the transaction cost.

- If the sum of marginal return costs and the transaction cost are both sufficiently high, $c < R + RC$ and $\max\{0, TT_{\pi}\} < TT < TT_0$, it is socially and privately optimal to have no MBGs or SHMs.

**Proof:** See Appendix F.
Proposition 7 suggests that retailers have incentives decrease the supply of SHMs and, thus restrain the competition from the SHM in the following two scenarios:

- **Scenario 1 when a SHM has a high transaction cost**, \( \max\{0, TT_{MBG}\} < TT < TT_0 \): Consumers are less willing to buy used products with a high search cost, and buyer are less willing to sell unwanted products with a high resale cost. Thus, a SHM with a high transaction cost may hurt retailers, and retailers may provide an MBG to interfere with SHMs.

- **Scenario 2 when the total marginal return costs of MBGs equals the transaction cost in the SHM**, \( TT = R + RC \): Let’s interpret the sum of return costs to both retailers and consumers as a transaction of redeeming the return option. When the transaction cost of both mechanisms (MBGs and SHMs) are equal, an MBG increases the new product price more since it provides a full insurance and also induces a greater demand since there is no competition from the used products. Therefore, it is more profitable for retailer to provide MBGs than welcome the SHM.

Furthermore, retailers and society gain the marginal production cost for each returned product but incur return costs. If the sum of the marginal return costs is higher than the production cost, the gain in the welfare from providing an MBG is lower than redemption costs, thereby an MBG reduces the welfare of retailers and society. On the other hand, if a SHM has a high transaction cost, \( \max\{0, TT_N\} < TT < TT_0 \), both retailers and consumers lose their welfare. Therefore, the absence of both SHMs and MBGs is socially and privately desirable when the sum of marginal return costs and the transaction cost are both sufficiently high, \( c < R + RC \) and \( \max\{0, TT_{MBG}\} < TT < TT_0 \) as suggested by Proposition 7.

### 4. Conclusions and Market Implications

Both SHMs and MBGs are mechanisms to reduce the loss of misfit. When prospective consumers have an identical fit probability and different product valuations, we obtain the following results:

- Both retailers and consumers prefer the market without MBGs and SHMs if SHMs cause a substantial cost and redemptions of MBGs result in a sufficiently high return cost.

- A SHM sorts prospective consumers into three groups: Those with high product valuations will buy the new product; those with intermediate product valuations will buy the used product; and those with low product valuations will not buy at all. However, retailers cannot segment prospective consumers by providing an MBG bundled with the product. Compared with the base case where there is no MBG nor SHM, (a) an MBG increases the product price and improves retail profits if the marginal production cost is lower than the sum of the marginal return costs to consumers and retailers; and (b) a well-functioning SHM with a sufficiently low transaction cost increases retail profits.
• When an MBG is offered, a SHM is likely of no relevance in terms of fit risk since consumers tend to return their unwanted products to retailers to get the purchase price refunded rather than sell in the SHM at a lower price. If an MBG is not available but a SHM exists, it is of retailers’ interest to assess whether the SHM decreases their profitability: (a) Retailers can obtain a higher profit by providing an MBG if the SHM has a high transaction cost, retailers have a low relative marginal production cost, and the marginal cost is greater than the sum of the marginal return costs to consumers and retailers; (b) An MBG is more profitable if the transaction cost equals the sum of the marginal return costs to consumers and retailers; and (c) a well-functioning SHM with a low transaction cost may increase retail profit in the primary market and, thus, retailers have incentives to facilitate transactions in SHMs.

These results shed light on several situations:

• The existence of an MBG for clothing but not car. In the case of clothing, consumers face a significant fit risk even after trying clothes on at a store because they may desire to get feedback from others and gain time to evaluate the product fit. Goodwill stores serve as a SHM for clothing. However, the existence of SHMs for clothing may hurt retailers because consumers perceive a great value loss to the used clothing in goodwill stores and the transaction cost is not trivial. On the other hand, when providing an MBG retailers bear a relative low return cost since they can sell returned clothing to manufacturers or upstream distributors. Therefore, retailers provide an MBG option against the fit risk to interfere with the SHM of clothing.

Cars lose a significant amount of value at the moment it is bought, and a car with a few miles is still considered the used one. Therefore, if car dealers provide a 30-day MBG option, they bear an extremely high return cost because of the value loss. Hence, they will not provide an MBG option. Instead, a SHM is used to resolve fit risk problems and car dealers take actions to reduce the transaction costs of SHMs. For example, car dealers or manufacturers take trade-ins (van Ackere and Reyniers 1995) or engage in selling both new and used cars. By doing so, retailers gain more profits.

• CDs, books, and software: In cases of CDs, books and software, MBGs are unlikely to be available partly because of moral hazard problem. SHMs are the main channel to deal with the fit risk. Some sellers, including Amazon.com, sell both new and used CDs, books, and software. For example, Amazon.com offered new and used Microsoft Office Professional Edition 2003 packages with different prices (March 15, 2005). SimCity 3000 Unlimited sells 10 new (original package, factory-sealed) and used games. Retailers’ active involvement in SHMs reduce transaction cost and, thus, a SHM increases retail profits in the primary market.

When a SHM is beneficial to retailers in the primary market, retailers have an incentive to reduce the transaction cost. One option is to create an electronic SHM to allow geographically dispersed buyers and
sellers to trade used products. Electronic SHMs can also be used against fit risk including Ebay and Amazon. It is not uncommon to find sales of virtual new products in Ebay or Amazon. On the other hand, if a SHM hurts retailers in the primary market, retailers can use various strategies to reduce the magnitude of SHMs. One option is to provide an MBG to dramatically decrease the supply of used products because of misfit. Retailers can also affect the SHM through other strategies, including (a) endogenously choose the built-in durability of the new product (Hendel and Lizzier 1999; Purohit 1992); (b) introduce “planned obsolescence” by selling new products to make old units obsolete (Waldman 1993; Waldman 1996b; Choi 1994; Miller 1974; Levinthal and Purohit 1989); and (c) design the optimal contracts of leasing and control the availability of used goods (Desai and Purohit 1999). Although the last three strategies do not aim to reduce fit risk, they affect the performance of a SHM and, thus, indirectly affect the profitability of a SHM versus an MBG in managing fit risk.

We envision the following two extensions: (a) The length of the grace period of an MBG option play an important role in determining the relative performance of MBGs and SHMs as a mechanism against the fit risk. If an MBG has such a short grace period that the majority of consumers cannot tell whether the product fits their needs within this time period, a SHM is more suitable to reduce the cost of fit risk. Secondly, if retailers impose a penalty on returns (partial refund of the purchase price or restocking fee charge), consumers may be better off by selling their unwanted products in a SHM. Thus, it is important for retailers to endogenously design an appropriate grace period and conditions of an MBG; and (b) It is common for retailers to be engaged in multiple channels: they can sell new products in the primary market and sell returned products in a SHM such as Dell, or retailers can even be engaged in rental and leasing. Thus, retailers have several revenue streams. Hence, changes of the model specifications are needed.

References


Choi, J. (1994). Network externality, compatibility choice and planned obsolescence. *Journal of Indus-


(a) New and used products side-by-side at Amazon.com

(b) Used products in a good (new) status

Figure 5: Used and new product sales in Amazon.com
Appendices

A. Proof of Proposition 1

Differentiating $\pi_1(p_1^*)$ in equation (11) with respect to $R$ and $RC$ and evaluating at the optimal price yields the following inequalities:

$$
\frac{d\pi_1(p_1^*)}{dR} = \frac{\partial \pi_1}{\partial p_1} \frac{dp_1}{dR} + \frac{\partial \pi_1}{\partial R} = - (1-q)R D_1(p_1^*) < 0, \tag{A.1a}
$$

$$
\frac{d\pi_1(p_1^*)}{dRC} = \frac{\partial \pi_1}{\partial p_1} \frac{dp_1}{dRC} + \frac{\partial \pi_1}{\partial RC} = - (1-q) \left( p_1^* - c - \frac{1-q}{q} R \right) f(x') < 0, \tag{A.1b}
$$

where $\frac{\partial \pi_1}{\partial p_1} = 0$ is the first order condition of equation (11); $\left( p_1^* - c - \frac{1-q}{q} R \right) > 0$ is the profit per unit of the final sales; and $f(x') = f \left( p_1^* + \frac{1-q}{q} RC \right) > 0$ is the proportion of consumers who are indifferent to buying and not buying the product. Therefore, we obtain two inequalities above.

Differentiating the retail profit $\pi_2(p_n^{n*}, p_2^{n*})$ in equation (25) with respect to $p_2^{n*}$, $T_s$, $T_b$, and $\beta$ yields the following equations:

$$
\frac{\partial \pi_2(p_n^{n*}, p_2^{n*})}{\partial p_2^{n*}} = \frac{1}{(1-\beta)q} f(x_2^{n*}) (p_2^{n*} - c), \tag{A.2a}
$$

$$
\frac{\partial \pi_2(p_n^{n*}, p_2^{n*})}{\partial T_s} = 0, \tag{A.2b}
$$

$$
\frac{\partial \pi_2(p_n^{n*}, p_2^{n*})}{\partial T_b} = \frac{f(x_2^{n*})}{(1-\beta)q} (p_2^{n*} - c), \tag{A.2c}
$$

$$
\frac{\partial \pi_2(p_n^{n*}, p_2^{n*})}{\partial \beta} = -\frac{p_2^{n*} - p_2^{b*} - T_b}{(1-\beta)^2 q} f(x_2^{n*}) (p_2^{n*} - c) = -\frac{x_2^{n*}}{1-\beta} f(x_2^{n*}) (p_2^{n*} - c). \tag{A.2d}
$$

Differentiating equation (22) with respect to $p_2^{n*}$ and $T_s$, $p_2^{n*}$ and $T_b$, $p_2^{n*}$ and $\beta$, and $p_2^{b*}$, yields the following:

$$
\frac{dp_2^{n*}}{dT_s} = -\frac{(1-\beta)(1-q)f(x_2^{n*})}{(1-\beta)q^2 f(x_2^{n*}) + \beta f(x_2^{n*})}, \tag{A.3a}
$$

$$
\frac{dp_2^{n*}}{dT_b} = -\frac{(1-\beta)q f(x_2^{n*}) + \beta f(x_2^{n*})}{(1-\beta)^2 q^2 f(x_2^{n*}) + \beta f(x_2^{n*})}, \tag{A.3b}
$$

$$
\frac{dp_2^{n*}}{d\beta} = -\frac{(1-\beta)q^2 x_2^{n*} f(x_2^{n*}) + \beta q x_2^{n*} f(x_2^{n*})}{(1-\beta)^2 q^2 f(x_2^{n*}) + \beta f(x_2^{n*})}. \tag{A.3c}
$$

Totally differentiating the retail profit $\pi_2(p_n^{n*}, p_2^{n*})$ with respect to $T_s$, $T_b$, and $\beta$, and evaluating at the optimal prices yields the following equations:

$$
\frac{d\pi_2(p_n^{n*}, p_2^{n*})}{dT_s} = \begin{bmatrix} \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial T_s} \\ \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial T_s} \\ \frac{\partial \pi_2}{\partial T_s} & \frac{\partial \pi_2}{\partial T_s} & \frac{\partial \pi_2}{\partial T_s} \end{bmatrix} = \begin{bmatrix} \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial T_s} \end{bmatrix}, \tag{A.4a}
$$

$$
\frac{d\pi_2(p_n^{n*}, p_2^{n*})}{dT_b} = \begin{bmatrix} \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial T_b} \\ \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial T_b} \\ \frac{\partial \pi_2}{\partial T_b} & \frac{\partial \pi_2}{\partial T_b} & \frac{\partial \pi_2}{\partial T_b} \end{bmatrix} = \begin{bmatrix} \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial T_b} \end{bmatrix}, \tag{A.4b}
$$

$$
\frac{d\pi_2(p_n^{n*}, p_2^{n*})}{d\beta} = \begin{bmatrix} \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial \beta} \\ \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial \beta} \\ \frac{\partial \pi_2}{\partial \beta} & \frac{\partial \pi_2}{\partial \beta} & \frac{\partial \pi_2}{\partial \beta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \pi_2}{\partial p_n^{n*}} & \frac{\partial \pi_2}{\partial p_2^{n*}} & \frac{\partial \pi_2}{\partial \beta} \end{bmatrix}. \tag{A.4c}
$$
where $\frac{\partial \pi^1}{\partial p^1_s} + \frac{\partial \pi^2}{\partial p^2_s} \frac{dp^2_s}{dp^2_s}$ is the first-order condition of equation (25), and it equals zero at the optimal price.

Substituting equations (A.2b) and (A.3a) into equation (A.4a); equations (A.2a), (A.2c), and (A.3b) into equation (A.4b); and equations (A.2a), (A.2d), and (A.3c) into equation (A.4c) yields the following inequalities:

\[
\begin{align*}
\frac{d\pi^2_0}{dT_s} & = \frac{\partial \pi^2}{\partial p^2_s} \frac{dp^2_s}{dT_s} = \frac{\partial \pi^2}{\partial p^2_s} \frac{\partial p^2_s}{\partial T_s} = -\frac{1 - q}{\partial f(x^2_{n^*})} (1 - \beta) f(x^2_{n^*}) (p^2_{n^*} \circ c) < 0, \\
\frac{d\pi^2_0}{dT_b} & = \frac{\partial \pi^2}{\partial p^2_s} \frac{dp^2_s}{dT_b} + \frac{\partial \pi^2}{\partial T_b} = -\frac{1 - q}{\partial f(x^2_{n^*})} (1 - \beta) f(x^2_{n^*}) (p^2_{n^*} \circ c) < 0, \\
\frac{d\pi^2_0}{d\beta} & = \frac{\partial \pi^2}{\partial p^2_s} \frac{dp^2_s}{d\beta} + \frac{\partial \pi^2}{\partial T_b} = \frac{q f(x^2_{n^*}) (p^2_{n^*} \circ c)}{1 - \beta} f(x^2_{n^*}) (p^2_{n^*} \circ c) (x^2_{n^*} - q x^2_{n^*}) \\
& \left\{ \begin{array}{l} > 0 \text{ if } \beta < \frac{q f(x^2_{n^*}) (p^2_{n^*} \circ c)}{1 - \beta} f(x^2_{n^*}) (p^2_{n^*} \circ c) (x^2_{n^*} - q x^2_{n^*}) \\ < 0 \text{ otherwise} \end{array} \right. 
\end{align*}
\]

Define the profit difference as $\Delta \pi = \pi^1_0(p^1) - \pi^2_0(p^2, p^2)$. Based on equations (A.1a), (A.1b), (A.5a), (A.5b), and (A.5c), we obtain the following results:

\[
\begin{align*}
\frac{d\Delta \pi}{dR} & = -(1 - q)RD_1(p^1) < 0, \\
\frac{d\Delta \pi}{dRC} & = -(1 - q)\left(p^1_1 - c - \frac{1 - q}{q} R\right) f\left(p^1_1 + \frac{1 - q}{q} RC\right) < 0, \\
\frac{d\Delta \pi}{dT_s} & = (1 - q)q(p^2 - c) f(x^2_{n^*}) \frac{dp^2}{dp^2_s} > 0, \\
\frac{d\Delta \pi}{dT_b} & = (1 - q)(p^2 - c) f(x^2_{n^*}) \frac{dp^2}{dp^2_s} > 0, \\
\frac{d\Delta \pi}{d\beta} & = -q(p^2 - c) f(x^2_{n^*}) (x^2_{n^*} - q x^2_{n^*}) \frac{dp^2_s}{dp^2_s} \\
& \left\{ \begin{array}{l} > 0 \text{ if } \beta < \frac{q f(x^2_{n^*}) (p^2_{n^*} \circ c)}{1 - \beta} f(x^2_{n^*}) (p^2_{n^*} \circ c) (x^2_{n^*} - q x^2_{n^*}) \\ < 0 \text{ otherwise} \end{array} \right. 
\end{align*}
\]

\[\Box\]

**B. Proof of Proposition 3**

Consumers will not buy used products if $\beta < \beta_{min}$; and buyers will sell their unwanted products only if $p^2_s > T_s$. Substituting the optimal prices, $p^1_s$ and $p^2_s$, into these two conditions yields the following:

\[TT < TT_0 - \beta q \bar{x} + \frac{q}{1 - q} (\bar{x} q - c) \text{ and } TT < TT_0 = \beta q \bar{x} - \frac{\beta q (\bar{x} q - c)}{2 \lambda - \beta + \beta q}.\]

Thus, $TT_0 - TT_0 = \frac{2q\lambda(\bar{x} q - c)}{(2\lambda - \beta + \beta q)(1 - q)} > 0$ suggests that $TT_0 < TT_0'$. Solving $TT_0 > 0$ yields that $\frac{\bar{x}}{\bar{x}} > \beta + q - \beta q - 2\lambda$. Thus, a SHM exists only if $\frac{\bar{x}}{\bar{x}} > \beta + q - \beta q - 2\lambda$ and $TT < TT_0$.

\[\Box\]
C. Proof of Proposition 4

The comparison of the new product price with and without a SHM shows that a SHM increases the new product price if $TT < TT_p$ where $TT_p = \beta q \bar{\pi}$. We know $TT_p > TT_0$, and the proof is given below:

$$TT_p - TT_0 = \beta q \bar{\pi} - \left[\frac{\beta q ((2\lambda - \beta + \beta q - q) \bar{\pi} + \bar{c})}{2\lambda - \beta + \beta q}\right] = \frac{\beta q (q \bar{\pi} - \bar{c})}{2\lambda - \beta + \beta q} > 0.$$ 

When a secondary market exists ($TT < TT_0$), retailers charge a higher price because $TT < TT_0 < TT_p$.

Differentiating $p_2^{n*} - p_2^{s*}$ with respect to $\beta$ and $TT$ yields the following inequalities:

$$\frac{d (p_2^{n*} - p_2^{s*})}{d\beta} = \frac{1}{2} (1 - q) \bar{\pi} > 0,$$  \hspace{1cm} (C.7a)

$$\frac{d (p_2^{n*} - p_2^{s*})}{dT} = \frac{1 - q}{2q} < 0.$$ \hspace{1cm} (C.7b)

Equations (C.7a) and (C.7b) show that the price difference is much higher if consumers perceive a low value loss of used products, and/or a SHM has a low transaction cost.

D. Proof of Proposition 5

The comparison of the initial new product demand with and without a SHM shows that a SHM decreases the initial equilibrium quantity of the new product if $TT > \max\{0, TT_q\}$. First, we claim that $TT_q < TT_0$, and the proof is given below:

$$TT_q - TT_0 = -\beta (q \bar{\pi} - \bar{c}) \frac{\beta + 2(1 - \lambda^2)(1 - q) + (1 - \beta) q^2}{(2\lambda - \beta + \beta q) q} < 0.$$ 

Secondly, we obtain the following inequality:

$$TT_q > 0 \Rightarrow \frac{c}{\bar{\pi}} > \frac{q}{1 + q}.$$ 

If both $\frac{c}{\bar{\pi}} > \frac{q}{1 + q}$ and $0 < TT < TT_q$ are satisfied, a SHM increases the equilibrium quantity of the new product.

E. Proof of Proposition 6

The comparison of retail profits and consumer surplus with and without a SHM shows that a SHM decreases the welfare of both retailers and consumers if $TT > \max\{0, TT_\pi\}$. First, we can prove that $TT_\pi < TT_0$.

Secondly, we obtain the following inequality:

$$TT_\pi > 0 \Rightarrow \frac{c}{\bar{\pi}} > q \left(1 - \frac{\beta (1 - q)}{\sqrt{\lambda - q}}\right).$$ 

If both $\frac{c}{\bar{\pi}} > q \left(1 - \frac{\beta (1 - q)}{\sqrt{\lambda - q}}\right)$ and $0 < TT < TT_\pi$ are satisfied, a SHM increases retail profits and consumer surplus.
F. Proof of Proposition 7

Comparing retail profits and consumer surplus with and without an MBG yields the following result: An MBG is more profitable if $c > R + RC$. Similarly, the comparison of retail profits with SHMs and MBGs shows that an MBG increases retail profits and consumer surplus if $TT > \max\{0, TT_\pi\}$. Therefore, an MBG is more profitable if $c > R + RC$ and $TT > \max\{0, TT_\pi\}$.

Assuming $TT = R + RC = A$, we obtain the profit difference $\pi^n_2 - \pi_1$.

\[
\pi^n_2 - \pi_1 = \frac{q}{4\Delta x} \left[ \frac{1}{\sqrt{\beta+(1-\beta)q^2}} \left( (\beta+q-\beta q)\bar{x} - \frac{1-q}{q}A - c \right) \right]^2 - \frac{q}{4\Delta x} \left[ \bar{x} - \frac{1-q}{q}A \right]^2
\]

\[
= \frac{q}{4\Delta x} \left[ \left( \frac{\beta+q-\beta q}{\sqrt{\beta+(1-\beta)q^2}} - 1 \right) \bar{x} - \left( \frac{1}{\sqrt{\beta+(1-\beta)q^2}} + 1 \right) (A + c) \right]
\]

where $\frac{\beta+q-\beta q}{\sqrt{\beta+(1-\beta)q^2}} - 1 = -\frac{\beta(1-\beta)(1-q)^2}{\sqrt{\beta+1-\beta}q^2} < 0$ and $\frac{1}{\sqrt{\beta+(1-\beta)q^2}} + 1 > 0$. Therefore, $\pi^n_2 < \pi_1$, implies that an MBG is more profitable if the total return cost per unit equals the transaction cost.