

Estimation of the mean of a population and construction of a confidence interval (cont.)

4. Compute a confidence interval for the true value μ

4.1. Assume that we know the true value of for the standard deviation of x : $\sigma = 10771$

$$\text{var}(\bar{x}) = \frac{1}{n} \text{var}(x) \text{ and thus } \text{se}(\bar{x}) = \frac{\text{sd}(x)}{\sqrt{n}} = \frac{10771}{\sqrt{2234}} = 228$$

Then, we can construct intervals that have 95% chance to contain μ

Here are some values for $\bar{x}$:

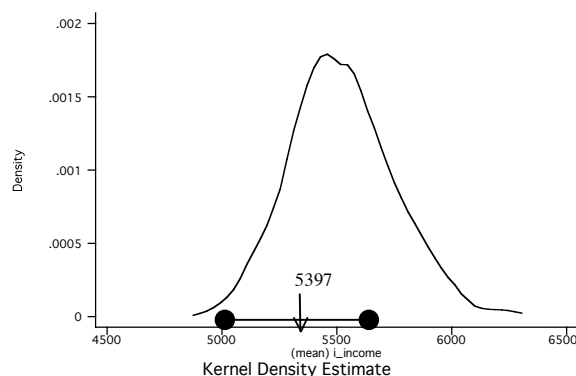
	minc	$\bar{x} - 1.96 * 228$	$\bar{x} + 1.96 * 228$	Does it contains μ ?
1	5397	4950	5843	Yes
2	5283	4837	5730	Yes
3	5434	4987	5881	
4	5403	4956	5850	
5	5510	5063	5957	
6	5697	5250	6144	
7	5331	4885	5778	
8	6027	5580	6474	No
9	5386	4939	5832	
10	5123	4676	5570	
11	5688	5241	6134	
12	5597	5151	6044	
13	5440	4993	5887	
14	5471	5024	5918	
15	5537	5091	5984	

4.2. If you don't know the true value of σ , then use s , and the Normal distribution when n very large

Then, construct intervals that have 95% chance to contain μ

	minc	s	$\bar{x} - 1.96 * s / \sqrt{n}$	$\bar{x} + 1.96 * s / \sqrt{n}$	Does it contains μ ?
1	5397	8011	5064	5729	Yes
2	5283	8817	4918	5649	Yes
3	5434	7596	5119	5749	
4	5403	7406	5096	5710	
5	5510	14259	4919	6101	
6	5697	9979	5283	6111	
7	5331	8408	4983	5680	
8	6027	14905	5409	6645	
9	5386	9455	4993	5778	
10	5123	7188	4825	5421	No
11	5688	14232	5097	6278	
12	5597	15076	4972	6223	
13	5440	9030	5066	5814	
14	5471	9091	5094	5848	
15	5537	14116	4952	6123	

What does the confidence interval means:



5. Special case of a binary variable

x only takes values 0 or 1.

Let p be the *true but unknown* proportion of 1 in the population.

For one random observation: $E(x) = p$ and $\text{var}(x) = p(1-p)$

If one collects a sample of n observations, and compute the average $\bar{x}$ of these observations, then

$$E(\bar{x}) = p \quad \text{and} \quad \text{var}(\bar{x}) = \frac{p(1-p)}{n}$$